A Mathematical Model of COVID-19 Using Fractional Derivative: Outbreak in India with Dynamics of Transmission and Control

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Abstract

Since the first case of 2019 novel coronavirus disease (COVID-19) detected on Jan 30, 2020, in India, the number of cases rapidly increased to 3819 cases including 106 deaths as of 5 April 2020. Taking this into account, in the present work, we are studying a Bats-Hosts-Reservoir-People transmission fractional-order COVID-19 model for simulating the potential transmission with the thought of individual social response and control measures by the government. The real data available about infectious cases from 14^{th} March to 26^{th} March 2020 is analysed and accordingly various parameters of the model are estimated or fitted. The Picard successive approximation technique and Banach's fixed point theory have been used for verification of the existence and stability criteria of the model. Numerical computations are done utilizing the iterative Laplace transform method. In the end, we illustrate the obtained results graphically. The purpose of this study is to estimate the effectiveness of preventive measures, predicting future outbreaks and potential control strategies using the mathematical model.

Keywords : mathematical models; coronavirus; caputo-fabrizio derivative; basic reproduction number; existence and stability; numerical simulations

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1 Introduction

Coronavirus disease is likely to emerge as a watershed moment in the history of planet. COVID-19, the abbreviation of coronavirus disease (2019) is caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) [1] hit the globe with a bang. In December 2019, the first outbreak was noticed in Hubei province, Wuhan, China [2]. On January 30, 2020, the World Health Organization (WHO) revealed the COVID-19 to be a public health emergency and identified it as a pandemic on 11 March 2020. The Symptoms of COVID-19 are not specific and many cases showed the infected

person may be asymptomatic. The majority of the cases have two common symptoms which includes dry cough (68%) and fever (88%). Some of the cases have symptoms that include fatigue, muscle and joint pain, respiratory sputum production (phlegm), sore throat, loss of the sense of smell, headache or chills and the shortness of breath. Moreover, the growth of the this infection can further proceed to acute respiratory distress syndrome, severe pneumonia, and death. The COVID-19 virus spreads at large extent between people in close contact with each other (within approximately 2m). The common incubation period ranges from 1 to 14 days [3]. In absence of a definite treatment modality like vaccine, physical distancing has been accepted globally as the most efficient strategy for reducing the severity of disease and gaining control over it.[4]. The concealment of physical contact in working environments, schools and other open circles is the objective of such preventive measures.

1.1 Timeline and Data analyses

On January 30, 2020, the first case of the COVID-19 in India was accounted. The nation revealed its initial three cases in the state of Kerala, every one of whom was students and had a travel history from Wuhan, China[5]. The transmission escalated within March when many reported cases throughout the country found to be connected to the people having travel history to the countries which are affected with COVID-19. On 11 March 2020, the Indian government starts taking strict actions by suspending all visas to India with the effect of 13 March 2020 till 15 April 2020. A 76-year-old man is the first victim of COVID-19 in the country who had returned from Saudi Arabia 0n March 12, 2020 [6]. On 15th March, the confirmed cases were 100 but it crossed 1,000 on 28 March and 2,000 on 2 April[7].

The Indian government has implemented high measures for a moderate outbreak. A day long countrywide public curfew was observed in India on March 22, 2020. Moreover, on March 24, the Prime Minister of India announced a countrywide lockdown for 21 days. In this study, we consider the reported cases of SARS-CoV-2, from March 14, 2020, till March 26, 2020, which are greater than compared to initial period [8] and shown in figures 1 and 2. We summarize the actual reported data from various resources of India in the following figures.

1.2 Mathematical Model

The application of computational mathematical methods to numerically simulate infections within the populations. Mathematical models of infectious disease dynamics have a deep history of more than 100 years. The most common mathematical formulations which represent the individual transition in a community between 'compartments' which attracts the situation of individual infection to surprisingly significant insight. These models of compartmental disease segregate a population into groups depend on each individual's infectious state and related population sizes with respect to time.

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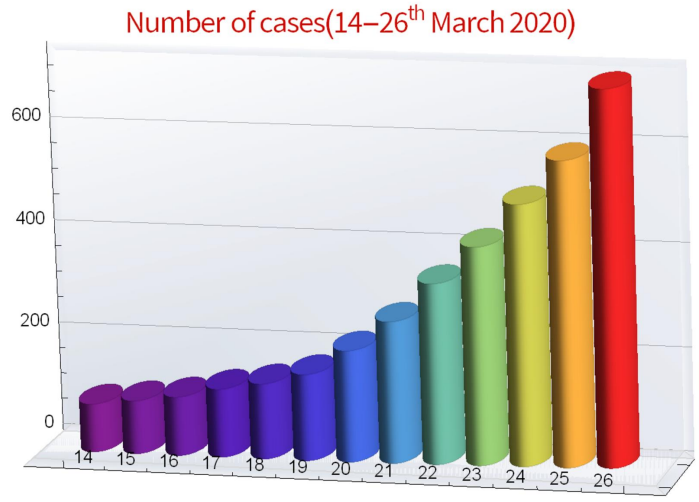

Figure 1: Confirmed cases of COVID-19 in India

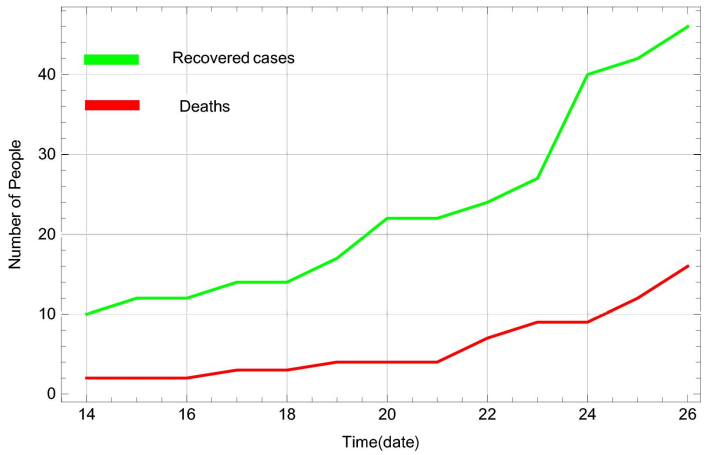

Figure 2: Number of recovered people and deaths during March 14 to March 26, 2020 in India

In [9], Lin et. al. suggested a conceptual model for the coronavirus disease 2019, which effectively catches the time line of the COVID-19 outbreak. A mathematical model for reproducing the stage-based transmissibility of a novel coronavirus is examined by Chen et.al in [10]. In [11] Khan et al. formulated Mathematical model of coronavirus versus people which is given as,

$$\begin{cases} D_tS(t) = \Delta - \lambda S - \dfrac{\alpha S(I+\beta A)}{N} - \gamma SQ, \\ D_tE(t) = \dfrac{\alpha S(I+\beta A)}{N} + \gamma SQ - (1-\phi)\delta E - \phi\mu E - \lambda E, \\ D_tI(t) = (1-\phi)\delta E - (\sigma+\lambda)I, \\ D_tA(t) = \phi\mu E - (\rho+\lambda)A, \\ D_tR(t) = \sigma I + \rho A - \lambda R, \\ D_tQ(t) = \kappa I + \nu A - \eta Q, \end{cases} \tag{1.1}$$

where N represents the total population of people. Further, N is segregated into five subclasses such as susceptible people $S(t)$, exposed people $E(t)$, infected (symptomatic), people $I(t)$, asymptotically infected $A(t)$ and and the removed or the recovered people $R(t)$. The people in the reservoir or market is denoted by $Q(t)$. The description of various parameters used in this model with there values and references is given in table 1

Mathematical models, utilizing ordinary differential equations with integer-order, have been demonstrated significantly for understanding the dynamics of biological systems [12, 13]. In any case, every single such model depends on classical derivatives that have some limitations related to the order of differential equations under consideration. To overcome these restrictions, many authors have looked for the help of a recently emerging area of mathematics known as fractional calculus. In fractional calculus the differential operators used are non-integer or fractional order which possesses memory impacts and are valuable to demonstrate many natural phenomena, nature-related truths and facts having non-local dynamics and anomalous behavior. The study of epidemiological dynamical processes involving memory effects is appropriate because such frameworks rely on strength of memory which is constrained by the order of fractional derivative operator.

In recent times, many researchers have developed and suggested efficient techniques to figure out real and approximate solutions of the differential equation involving fractional operator [14, 15, 16, 17, 18, 19, 20]. A lot of scholars are investigating epidemic models related to different infectious diseases involving fractional operator because it shows the reasonable biphasic decline of contamination of diseases. [21, 22, 23, 24, 25].

Recently, Caputo and Fabrizio [26] have recommended a unique fractional derivative operator having the exponential kernel and what's more Losada and Nieto [27] examined the properties of a recently proposed fractional derivative. The advantage of this operator is that it has nonlocal and nonsingular kernel and best suitable to describe and analyse dynamics of COVID-19. More literature about the Caputo-Fabrizio derivative operator is given in [28, 29, 30, 31, 32, 33].

Motivated by the above useful applications of the Caputo-Fabrizio (CF) operator in epidemic mathematical models, in this paper we are studying dynamics of novel coronavirus model suggested by Khan et.al.[11] in the form of the system of the nonlinear differential equations involving the CF fractional derivative operator of order τ such that $\tau \in (0, 1]$.

$$\begin{cases} {}^{CF}D_t^\tau S(t) = \Delta - \lambda S - \dfrac{\alpha S(I+\beta A)}{N} - \gamma SQ, \\ {}^{CF}D_t^\tau E(t) = \dfrac{\alpha S(I+\beta A)}{N} + \gamma SQ - (1-\phi)\delta E - \phi\mu E - \lambda E, \\ {}^{CF}D_t^\tau I(t) = (1-\phi)\delta E - (\sigma+\lambda)I, \\ {}^{CF}D_t^\tau A(t) = \phi\mu E - (\rho+\lambda)A, \\ {}^{CF}D_t^\tau R(t) = \sigma I + \rho A - \lambda R, \\ {}^{CF}D_t^\tau Q(t) = \kappa I + \nu A - \eta Q, \end{cases} \tag{1.2}$$

with initial conditions

$$S(0) = \theta_1,\ E(0) = \theta_2,\ I(0) = \theta_3,\ A(0) = \theta_4,\ R(0) = \theta_5\ Q(0) = \theta_6. \tag{1.3}$$

The approximate solution and graphical results are obtained by employing the iterative Laplace transform method (ILTM).

The structure of this paper is given as: In Sections 2, basic definitions and results of the fractional operator and the Laplace transforms are presented. In Section 3, the iterative scheme to find the solution of the above model using Laplace transforms and new iterative method (NIM) is given, moreover stability criteria by utilizing fixed point theory of Banach along with Picard successive approximation method is also discussed. In Section 4, Data fitting and estimation is done along with numerical simulations for various values of the fractional-order τ are displayed graphically. Section 5 is about discussion and finally we give our conclusions in section 6.

2 Preliminaries

Some fundamental definitions and results from fractional calculus are presented in this section.

Definition 2.1 *The Caputo fractional derivative operator of order* $\tau(\tau \geq 0)$ *and* $n \in \mathbb{N} \cup \{0\}$ *is defined as*

$$D_t^\tau(u(t)) = \frac{1}{\Gamma(n-\tau)} \int_0^t (t-\zeta)^{n-\tau-1} \frac{d^n}{dt^n} u(\zeta) d\zeta, \tag{2.1}$$

where $n-1 \leq \tau < n$.

Further, The fractional order derivative has been considered with exponential kernel by Caputo and Fabrizio in [26] and analysed by Losada and Nieto in [27] given by following definitions.

Definition 2.2 *Let* $u \in H^1(a,b), b > a, 0 < \tau < 1$, *the time fractional Caputo-Fabrizio fractional differential operator is defined by*

$${}^{CF}D_t^\tau u(t) = \frac{M(\tau)}{(1-\tau)} \int_a^t exp\Big[-\frac{\tau(t-\zeta)}{1-\tau}\Big] u'(\zeta) d\zeta, \quad t \geq 0, \qquad 0 < \tau < 1, \tag{2.2}$$

where $M(\tau)$ is a normalisation function which depends on τ and gives $M(0) = M(1) = 1$.

Definition 2.3 *The CF fractional integral operator of order $0 < \tau < 1$ is given by*

$$ {}^{CF}J_t^{\tau}u(t) = \frac{2(1-\tau)}{(2-\tau)M(\tau)}u(t) + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t u(\zeta)d\zeta, \qquad t \geq 0. \tag{2.3}$$

Where ${}^{CF}D_t^{\tau}u(t) = 0$, if u is a constant function.

Remark 2.1 *It has been observed from the previous definitions that fractional integral of a function with order $0 < \tau \leq 1$ is an average between respective functions and their integral of order one. It further gives*

$$\frac{2(1-\tau)}{(2-\tau)M(\tau)}u(t) + \frac{2\tau}{(2-\tau)M(\tau)} = 1.$$

The previous equation gives obvious formula for

$$M(\tau) = \frac{2}{(2-\tau)}, \qquad 0 \leq \tau \leq 1.$$

Definition 2.4 *The Laplace transform (LT) for the CF fractional operator of order $0 < \tau \leq 1$ for $m \in \mathbb{N}$ is given as*

$$\begin{aligned} L\Big({}^{CF}D_t^{m+\tau}u(t)\Big)(p) &= \frac{1}{1-\tau}L\big(u^{(m+1)}(t)\big)L\left(exp\left(-\frac{\tau}{1-\tau}t\right)\right) \\ &= \frac{p^{m+1}L(u(t)) - p^m u(0) - p^{m-1}u'(0) - \cdots - u^{(m)}(0)}{p + \tau(1-p)}. \end{aligned} \tag{2.4}$$

In particular, we have

$$\begin{aligned} L\Big({}^{CF}D_t^{\tau}u(t)\Big)(p) &= \frac{pL(u(t))}{p + \tau(1-p)}, \qquad m = 0. \\ L\big({}^{CF}D_t^{\tau+1}u(t)\big)(p) &= \frac{p^2L(u(t)) - pu(0) - u'(0)}{p + \tau(1-p)}, \qquad m = 1. \end{aligned}$$

3 Iterative scheme and Stability analysis

Consider the coronavirus model (1.2) along with initial conditions (1.3). The terms SI, SA and SQ in this model are nonlinear. Implementing the Laplace transform on both sides of system (1.2), we obtain,

$$\frac{pL(S(t)) - S(0)}{p + \tau(1-p)} = L\left(\Delta - \lambda S - \frac{\alpha S(I + \beta A)}{N} - \gamma SQ\right),$$

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$$\begin{aligned}
\frac{pL(E(t))-E(0)}{p+\tau(1-p)} &= L\bigg(\frac{\alpha S(I+\beta A)}{N}+\gamma SQ-(1-\phi)\delta E-\phi\mu E-\lambda E\bigg),\\
\frac{pL(I(t))-I(0)}{p+\tau(1-p)} &= L((1-\phi)\delta E-(\sigma+\lambda)I),\\
\frac{pL(A(t))-A(0)}{p+\tau(1-p)} &= L(\phi\mu E-(\rho+\lambda)A),\\
\frac{pL(R(t))-R(0)}{p+\tau(1-p)} &= L(\sigma I+\rho A-\lambda R),\\
\frac{pL(Q(t))-Q(0)}{p+\tau(1-p)} &= L(\kappa I+\nu A-\eta Q). \qquad (3.1)
\end{aligned}$$

Rearranging, we get

$$\begin{aligned}
L(S(t)) &= \frac{S(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L\bigg(\Delta-\lambda S-\frac{\alpha S(I+\beta A)}{N}-\gamma SQ\bigg),\\
L(E(t)) &= \frac{E(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L\bigg(\frac{\alpha S(I+\beta A)}{N}+\gamma SQ\\
&\quad -(1-\phi)\delta E-\phi\mu E-\lambda E\bigg),\\
L(I(t)) &= \frac{I(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L((1-\phi)\delta E-(\sigma+\lambda)I),\\
L(A(t)) &= \frac{A(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L(\phi\mu E-(\rho+\lambda)A),\\
L(R(t)) &= \frac{R(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L(\sigma I+\rho A-\lambda R),\\
L(Q(t)) &= \frac{Q(0)}{p}+\bigg(\frac{p+\tau(1-p)}{p}\bigg)L(\kappa I+\nu A-\eta Q). \qquad (3.2)
\end{aligned}$$

Further, the inverse Laplace transform on equations (3.2), yields

$$\begin{aligned}
S(t) &= S(0)+L^{-1}\bigg[\bigg(\frac{p+\tau(1-p)}{p}\bigg)L\bigg(\Delta-\lambda S-\frac{\alpha S(I+\beta A)}{N}-\gamma SQ\bigg)\bigg],\\
E(t) &= E(0)+L^{-1}\bigg[\bigg(\frac{p+\tau(1-p)}{p}\bigg)L\bigg(\frac{\alpha S(I+\beta A)}{N}+\gamma SQ\\
&\quad -(1-\phi)\delta E-\phi\mu E-\lambda E\bigg)\bigg],\\
I(t) &= I(0)+L^{-1}\bigg[\bigg(\frac{p+\tau(1-p)}{p}\bigg)L((1-\phi)\delta E-(\sigma+\lambda)I)\bigg],\\
A(t) &= A(0)+L^{-1}\bigg[\bigg(\frac{p+\tau(1-p)}{p}\bigg)L(\phi\mu E-(\rho+\lambda)A)\bigg],
\end{aligned}$$

$$R(t) = R(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\sigma I + \rho A - \lambda R)\right],$$
$$Q(t) = Q(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\kappa I + \nu A - \eta Q)\right]. \tag{3.3}$$

The series solutions achieved by the method are given by,

$$S = \sum_{n=0}^{\infty} S_n, \quad E = \sum_{n=0}^{\infty} E_n, \quad I = \sum_{n=0}^{\infty} I_n, \quad A = \sum_{n=0}^{\infty} A_n, \quad R = \sum_{n=0}^{\infty} R_n, \quad Q = \sum_{n=0}^{\infty} Q_n. \tag{3.4}$$

The nonlinearity SI, SA and SQ can be written as

$$SI = \sum_{n=0}^{\infty} G_n, \quad SA = \sum_{n=0}^{\infty} H_n, \quad SQ = \sum_{n=0}^{\infty} L_n,$$

whereas G_n, H_n and L_n are further decomposed as follows

$$G_n = \sum_{i=0}^{n} S_i \sum_{i=0}^{n} I_i - \sum_{i=0}^{n-1} S_i \sum_{i=0}^{n-1} I_i.$$
$$H_n = \sum_{i=0}^{n} S_i \sum_{i=0}^{n} A_i - \sum_{i=0}^{n-1} S_i \sum_{i=0}^{n-1} A_i.$$
$$L_n = \sum_{i=0}^{n} S_i \sum_{i=0}^{n} Q_i - \sum_{i=0}^{n-1} S_i \sum_{i=0}^{n-1} Q_i.$$

Using initial conditions, we get the recursive formula given by:

$$S_{n+1}(t) = S_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\Delta - \lambda S_n - \frac{\alpha S_n(I_n+\beta A_n)}{N} - \gamma S_n Q_n\right)\right],$$
$$E_{n+1}(t) = E_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\frac{\alpha S_n(I_n+\beta A_n)}{N} + \gamma S_n Q_n - (1-\phi)\delta E_n - \phi\mu E_n - \lambda E_n\right)\right],$$
$$I_{n+1}(t) = I_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L((1-\phi)\delta E_n - (\sigma+\lambda)I_n)\right],$$
$$A_{n+1}(t) = A_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\phi\mu E_n - (\rho+\lambda)A_n)\right],$$
$$R_{n+1}(t) = R_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\sigma I_n + \rho A_n - \lambda R_n)\right],$$
$$Q_{n+1}(t) = Q_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\kappa I_n + \nu A_n - \eta Q_n)\right]. \tag{3.5}$$

3.1 Stability analysis of proposed method

Consider $(\mathbb{B}, \|\cdot\|)$ be a Banach space with self-map T on $\mathbb{B}$. Also $\zeta_{n+1} = q(T, \zeta_n)$ represents exact recurring formula. Suppose fixed-point set on T is denoted by $U(T)$. Moreover, T has at least one element ζ_n, which converges to point $x \in U(T)$. Let $\{z_n \in \mathbb{B}\}$ and define $j_n = \|z_{n+1} - q(T, z_n)\|$. If $\lim_{n\to\infty} j^n = 0$ implies $\lim_{n\to\infty} z^n = x$, then given iteration method $\zeta_{n+1} = q(T, \zeta_n)$ is known as T-stable. In this manner, this sequence $\{z_n\}$ has an upper bound and the iteration is known to be Picard's iteration. Moreover, it is T-stable, if all of the above conditions are satisfied for $\zeta_{n+1} = T\zeta_n$.

Theorem 3.1 *Let $(\mathbb{B}, \|\cdot\|)$ as a Banach space with self-map T on $\mathbb{B}$ and satisfying*

$$\|T_a - T_b\| \leq \Gamma\|a - T_a\| + \varepsilon\|a - b\|$$

for all $a, b \in \mathbb{B}$ where $0 \leq \Gamma$, $0 \leq \varepsilon < 1$. Suppose T is Picard T-stable. Consider the equations (3.5) related to (1.2).

where $\dfrac{p + \tau(1-p)}{p}$ is a Lagrange's multiplier in fractional form.

Theorem 3.2 *Consider a self-map T defined as*

$$T(S_n(t)) = S_{n+1}(t) = S_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\Delta - \lambda S_n - \frac{\alpha S_n(I_n + \beta A_n)}{N} - \gamma S_n Q_n\right)\right],$$

$$T(E_n(t)) = E_{n+1}(t) = E_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\frac{\alpha S_n(I_n + \beta A_n)}{N} + \gamma S_n Q_n - (1-\phi)\delta E_n - \phi\mu E_n - \lambda E_n\right)\right],$$

$$T(I_n(t)) = I_{n+1}(t) = I_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L((1-\phi)\delta E_n - (\sigma + \lambda)I_n)\right],$$

$$T(A_n(t)) = A_{n+1}(t) = A_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\phi\mu E_n - (\rho + \lambda)A_n)\right],$$

$$T(R_n(t)) = R_{n+1}(t) = R_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\sigma I_n + \rho A_n - \lambda R_n)\right],$$

$$T(Q_n(t)) = Q_{n+1}(t) = Q_n(0) + L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\kappa I_n + \nu A_n - \eta Q_n)\right].$$

is T-stable in $L^1(a, b)$ if

$$\left(1-\lambda F(\tau)-\frac{\alpha}{N}(K_1+K_2)G(\tau)-\frac{\alpha\beta}{N}(K_1+K_3)H(\tau)-\gamma(K_1+K_4)J(\tau)\right)<1,$$
$$\left(1+\frac{\alpha}{N}(K_1+K_2)G(\tau)+\frac{\alpha\beta}{N}(K_1+K_3)H(\tau)-\big((1-\phi)\delta+\phi\mu+\lambda\big)J_1(\tau)\right)<1,$$
$$\left(1+(1-\phi)\delta G_1(\tau)-(\sigma+\lambda)H_1(\tau)\right)<1,$$
$$\left(1+\phi\mu G_2(\tau)-(\rho+\lambda)H_2(\tau)\right)<1,$$
$$\left(1+\sigma G_3(\tau)+\rho H_3(\tau)-\lambda J_3(\tau)\right)<1,$$
$$\left(1+\kappa G_4(\tau)+\nu H_4(\tau)-\eta J_4(\tau)\right)<1. \tag{3.6}$$

Proof. The proof begins by showing T has a fixed point. Therefore, for all $(m,n)\in N\times N$, we evaluate the followings.

$$\begin{aligned}
T(S_m(t))-T(S_n(t)) &= S_m(t)-S_n(t)+L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\Delta-\lambda S_m-\right.\right.\\
&\quad\left.\left.\frac{\alpha S_m(I_m+\beta A_m)}{N}-\gamma S_m Q_m\right)\right]-L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)\right.\\
&\quad\left.L\left(\Delta-\lambda S_n-\frac{\alpha S_n(I_n+\beta A_n)}{N}-\gamma S_n Q_n\right)\right],\\
T(E_m(t))-T(E_n(t)) &= E_m(t)-E_n(t)+L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\frac{\alpha S_m(I_m+\beta A_m)}{N}\right.\right.\\
&\quad\left.\left.+\gamma S_m Q_m-(1-\phi)\delta E_m-\phi\mu E_m-\lambda E_m\right)\right]-\\
&\quad L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L\left(\frac{\alpha S_n(I_n+\beta A_n)}{N}+\gamma S_n Q_n\right.\right.\\
&\quad\left.\left.-(1-\phi)\delta E_n-\phi\mu E_n-\lambda E_n\right)\right],\\
T(I_m(t))-T(I_n(t)) &= I_m(t)-I_n(t)+L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L((1-\phi)\delta E_m-\right.\\
&\quad\left.(\sigma+\lambda)I_m)\right]-L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L((1-\phi)\delta E_n-(\sigma+\lambda)I_n)\right],\\
T(A_m(t))-T(A_n(t)) &= A_m(t)-A_n(t)+L^{-1}\left[\left(\frac{p+\tau(1-p)}{p}\right)L(\phi\mu E_m-\right.
\end{aligned}$$

$$(\rho+\lambda)A_m)\Big] - L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L(\phi\mu E_n-(\rho+\lambda)A_n)\Big],$$

$$T(R_m(t))-T(R_n(t)) = R_m(t)-R_n(t)+L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L(\sigma I_m+\rho A_m - \lambda R_m)\Big] - L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L(\sigma I_n+\rho A_n-\lambda R_n)\Big],$$

$$T(Q_m(t))-T(Q_n(t)) = Q_m(t)-Q_n(t)+L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L(\kappa I_m+\nu A_m- \eta Q_m)\Big] - L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L(\kappa I_n+\nu A_n-\eta Q_n)\Big]. \quad (3.7)$$

Considering first equation of (3.7) and taking norm on both sides of it, without loss of generality, we get

$$\|T(S_m(t))-T(S_n(t))\| = \Big\|S_m(t)-S_n(t)+L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big)L\Big(\Delta-\lambda S_m - \frac{\alpha S_m(I_m+\beta A_m)}{N}-\gamma S_m Q_m\Big)\Big] - L^{-1}\Big[\Big(\frac{p+\tau(1-p)}{p}\Big) L\Big(\Delta-\lambda S_n-\frac{\alpha S_n(I_n+\beta A_n)}{N}-\gamma S_n Q_n\Big)\Big]\Big\|, \quad (3.8)$$

using triangular inequality and further simplifying (3.8) yields,

$$\|T(S_m(t))-T(S_n(t))\| \le \Big\|S_m(t)-S_n(t)\Big\| + L^{-1}\Big[\Big(\frac{s+\alpha(1-s)}{s}\Big)L\Big[\|-\lambda(S_m-S_n)\| + \|-\frac{\alpha}{N}S_n(I_m-I_n)\| + \|-\frac{\alpha}{N}I_m(S_m-S_n)\| + \|-\frac{\alpha\beta}{N}S_n(A_m-A_n)\| + \|-\frac{\alpha\beta}{N}A_m(S_m-S_n)\| + \|-\gamma S_n(Q_m-Q_n)\| + \|-\gamma Q_m(S_m-S_n)\|\Big]\Big]. \quad (3.9)$$

As both the solutions have the comparative influence, we assume that

$$\|S_m(t)-S_n(t)\| = \|E_m(t)-E_n(t)\|,$$
$$\|S_m(t)-S_n(t)\| = \|I_m(t)-I_n(t)\|,$$
$$\|S_m(t)-S_n(t)\| = \|A_m(t)-A_n(t)\|,$$
$$\|S_m(t)-S_n(t)\| = \|R_m(t)-R_n(t)\|,$$
$$\|S_m(t)-S_n(t)\| = \|Q_m(t)-Q_n(t)\|.$$

Replacing this in (3.9), we get the following relation

$$\begin{aligned}\|T(S_m(t)) - T(S_n(t))\| \leq \Big\|S_m(t) - S_n(t)\Big\| + L^{-1}\Bigg[\left(\frac{s+\alpha(1-s)}{s}\right)L\Big[\| - \lambda(S_m - S_n)\| \\ + \| - \frac{\alpha}{N}S_n(S_m - S_n)\| + \| - \frac{\alpha}{N}I_m(S_m - S_n)\| \\ + \| - \frac{\alpha\beta}{N}S_n(S_m - S_n)\| + \| - \frac{\alpha\beta}{N}A_m(S_m - S_n)\| \\ + \| - \gamma S_n(S_m - S_n)\| + \| - \gamma Q_m(S_m - S_n)\|\Big]\Bigg]. \qquad (3.10)\end{aligned}$$

Also the convergent sequences $S_n,\ I_m,\ A_m$ and Q_m are bounded. Next, we can obtain five different positive constants, $K_1,\ K_2,\ K_3,\ K_4$ and K_5 for all t such as,

$$\|S_n\| < K_1, \quad \|I_m\| < K_2, \quad \|A_m\| < K_3, \quad \|Q_m\| < K_4, \quad (m,n) \in \mathbb{N} \times \mathbb{N}. \qquad (3.11)$$

Further, consider equations (3.10) and (3.11), we get

$$\begin{aligned}\|T(S_m(t)) - T(S_n(t))\| \leq \Bigg(1 - \lambda F(\tau) - \frac{\alpha}{N}(K_1+K_2)G(\tau) - \frac{\alpha\beta}{N}(K_1+K_3)H(\tau) \\ - \gamma(K_1+K_4)J(\tau)\Bigg)\|(S_m - S_n)\|, \qquad (3.12)\end{aligned}$$

where $F,\ G,\ H$ and J are functions of $L^{-1}\left\{L\left(\frac{p+\alpha(1-p)}{p}\right)\right\}$. In the same manner, we can get

$$\begin{aligned}\|T(E_m(t)) - T(E_n(t))\| &\leq \Bigg(1 + \frac{\alpha}{N}(K_1+K_2)G(\tau) + \frac{\alpha\beta}{N}(K_1+K_3)H(\tau) \\ &\quad - \big((1-\phi)\delta + \phi\mu + \lambda\big)J_1(\tau)\Bigg)\|(E_m - E_n)\|, \\ \|T(I_m(t)) - T(I_n(t))\| &\leq \Big(1 + (1-\phi)\delta G_1(\tau) - (\sigma+\lambda)H_1(\tau)\Big)\|(I_m - I_n)\|, \\ \|T(A_m(t)) - T(A_n(t))\| &\leq \Big(1 + \phi\mu G_2(\tau) - (\rho+\lambda)H_2(\tau)\Big)\|(A_m - A_n)\|, \\ \|T(R_m(t)) - T(R_n(t))\| &\leq \Big(1 + \sigma G_3(\tau) + \rho H_3(\tau) - \lambda J_3(\tau)\Big)\|(R_m - R_n)\|, \\ \|T(Q_m(t)) - T(Q_n(t))\| &\leq \Big(1 + \kappa G_4(\tau) + \nu H_4(\tau) - \eta J_4(\tau)\Big)\|(Q_m - Q_n)\|. \qquad (3.13)\end{aligned}$$

Where,

$$\begin{aligned}&\left(1 - \lambda F(\tau) - \frac{\alpha}{N}(K_1+K_2)G(\tau) - \frac{\alpha\beta}{N}(K_1+K_3)H(\tau) - \gamma(K_1+K_4)J(\tau)\right) < 1, \\ &\left(1 + \frac{\alpha}{N}(K_1+K_2)G(\tau) + \frac{\alpha\beta}{N}(K_1+K_3)H(\tau) - \big((1-\phi)\delta + \phi\mu + \lambda\big)J_1(\tau)\right) < 1, \\ &\Big(1 + (1-\phi)\delta G_1(\tau) - (\sigma+\lambda)H_1(\tau)\Big) < 1,\end{aligned}$$

$$\Big(1+\phi\mu G_2(\tau)-(\rho+\lambda)H_2(\tau)\Big)<1,$$
$$\Big(1+\sigma G_3(\tau)+\rho H_3(\tau)-\lambda J_3(\tau)\Big)<1,$$
$$\Big(1+\kappa G_4(\tau)+\nu H_4(\tau)-\eta J_4(\tau)\Big)<1.$$

Therefore, nonlinear self mapping T has a fixed point. Next, we show that T satisfies all the conditions in Theorem 3.1. Let (3.12) to (3.13) holds then using $\varepsilon=(0,0,0,0,0,0)$,

$$\Gamma=\begin{cases}\Big(1-\lambda F(\tau)-\frac{\alpha}{N}(K_1+K_2)G(\tau)-\frac{\alpha\beta}{N}(K_1+K_3)H(\tau)-\gamma(K_1+K_4)J(\tau)\Big),\\ \Big(1+\frac{\alpha}{N}(K_1+K_2)G(\tau)+\frac{\alpha\beta}{N}(K_1+K_3)H(\tau)-\big((1-\phi)\delta+\phi\mu+\lambda\big)J_1(\tau)\Big),\\ \Big(1+(1-\phi)\delta G_1(\tau)-(\sigma+\lambda)H_1(\tau)\Big),\\ \Big(1+\phi\mu G_2(\tau)-(\rho+\lambda)H_2(\tau)\Big),\\ \Big(1+\sigma G_3(\tau)+\rho H_3(\tau)-\lambda J_3(\tau)\Big),\\ \Big(1+\kappa G_4(\tau)+\nu H_4(\tau)-\eta J_4(\tau)\Big).\end{cases}$$

Thus, each condition in Theorem 3.2 is satisfied by self map T. Hence, T is Picard $T-$ stable.

4 Data fitting and numerical simulations

Here, we study numerical simulations of the CF coronavirus model (1.2). For this, we consider few parametric values from the literature and remaining are estimated or fitted. We use the total initial population $N=481747192$ which we consider 35% of total population of India [34], since as per report of WHO 35% of Indian population lives in towns and urban part of India. We have $N=S(0)+E(0)+I(0)+A(0)+R(0)+Q(0)$, $E_0=\theta_2=1724266$, $I_0=\theta_3=745$, $A_0=\theta_4=413$, $R_0=\theta_5=66$, initial susceptible population is determined to be $S(0)=\theta_1=480021700=N-(E(0)+I(0)+A(0))-R(0)$ and $Q(0)=10,00,000$. The life expectancy in India for the year 2019 is 69.50, so we estimate natural mortality rate $\lambda=\frac{1}{69.50}$ per year. The birth rate is estimated as $\Delta=\lambda\times N=6931614.27$ and consider this is to be the limited population in the absence of infection.

The feasible area of the model (1.2) is given as

$$\chi=\left\{\big((S(t),E(t),I(t),A(t),R(t)\big)\in R_+^5/N\leq\frac{\Delta}{\lambda},Q\in R_+\right\}\tag{4.1}$$

From the explanation given in [11] about the basic reproduction number R_0 that interprets as the expected number of secondary infections which obtains from a single

Description of Parameter and Notation		Value	Reference
Birth rate	Δ	6931614.27	Estimated
Contact rate	α	0.25	[35],Estimated
Natural mortality rate	λ	$\frac{1}{69.50}$	[36]
Transmission rate	β	0.5944	Fitted
Incubation period	δ	0.0047876	Fitted
Incubation period	μ	0.05	Fitted
The proportion of asymptomatic infection	ϕ	0.01243	Fitted
Disease transmission coefficient	γ	0.1231×10^{-7}	Fitted
Recovery or removal or rate of I	σ	0.09871	[11]
Recovery or removal or rate of A	ρ	0.854302	[11]
Contribution of the virus to Q by I	κ	0.000398	[11]
Contribution of the virus to Q by A	ν	0.001	Fitted
Removing rate of virus from Q	η	0.01	Fitted

Table 1: Estimated and fitted parametric values used in the Coronavirus model (1.2)

infected individual into an otherwise susceptible population. Moreover, it states that the disease free equilibrium(DER) of the model (1.2) given as,

$$E_0 = \left(S^0, 0, 0, 0, 0, 0\right) = \left(\frac{\Delta}{\lambda}, 0, 0, 0, 0, 0\right) \tag{4.2}$$

To evaluate R_0 of the model (1.2), we use the computational part given in [37]. The matrices F and V are given as

$$F = \begin{bmatrix} 0 & \alpha & \beta\alpha & \frac{\gamma\Delta}{\lambda} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad V = \begin{bmatrix} \phi\mu + (1-\phi)\delta + \lambda & 0 & 0 & 0 \\ (\phi-1)\delta & \lambda+\sigma & 0 & 0 \\ -\phi\mu & 0 & \rho+\lambda & 0 \\ 0 & -\kappa & -\nu & \eta \end{bmatrix}$$

Using spectral radius the required basic reproduction number R_0 is given as

$$R_0 = \frac{\mu\phi(\lambda+\sigma)(\alpha\beta\eta\lambda + \gamma\Delta\nu) + \delta(1-\phi)(\lambda+\rho)(\alpha\eta\lambda + \gamma\Delta\kappa)}{\eta\lambda(\lambda+\rho)(\lambda+\sigma)(\phi(\mu-\delta)+\delta+\lambda)}$$

By using set of values given in and estimating some parameters the basic reproduction number is estimated as $R_0 = 2.02283$ for the model (1.2).

Theorem 4.1 ([11]) *The disease free equilibrium(DFE) E_0 of the system (1.2) is locally asymptotically stable if $R_0 < 1$.*

By applying ILTM successively upto four terms we get approximate solution of the fractional coronavirus model (1.2) in series form as given below

$$S(t) = 4.84583 * 10^8 + 5.73198 * 10^6\tau - 447.123\tau^2 + 0.0418646\tau^3 + 4.89747 * 10^-6t^6\tau^3 + t(3.44501 * 10^6 - 5.78046 * 10^6\tau + 770.281\tau^2 - 0.0992709\tau^3) + t^3(-0.601247$$

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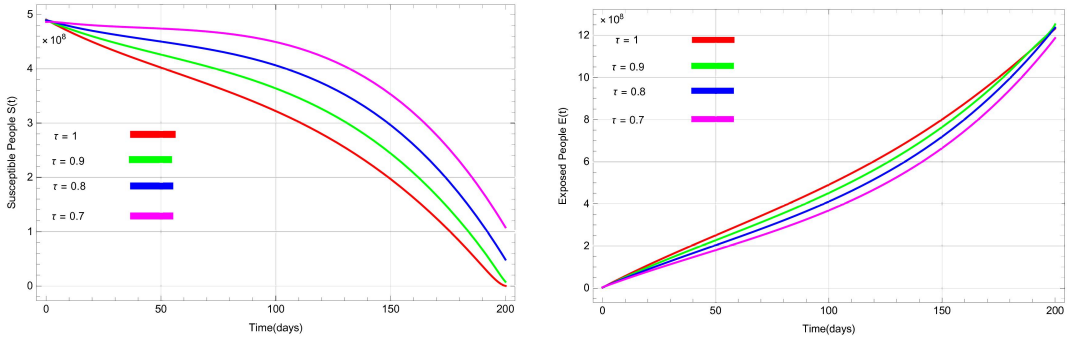
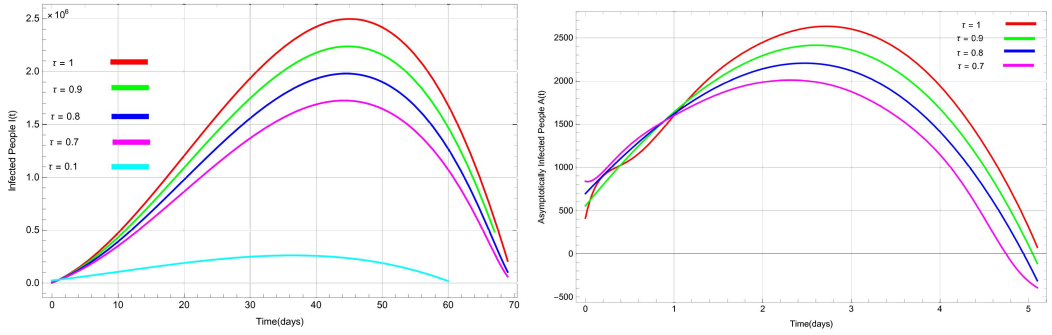
$$
\begin{aligned}
&- 123.99\tau - 41.5649\tau^2 - 0.0144025\tau^3) + t^4(-0.232179\tau + 0.969849\tau^2 - 0.00643553\tau^3)\\
&+t^5(-0.00267283\tau^2 - 0.000198446\tau^3) + t^2(-6185.1 + 24718.7\tau - 329.938\tau^2 + 0.0822512\tau^3).\\
E(t) &= -4.897472194774953^{*\wedge}\text{-}6\tau^3t^6 + (0.000198446\tau + 0.00267283)\tau^2t^5 + \tau\big(0.00643553\tau^2\\
&- 0.989308\tau + 0.232179\big)t^4 + \left(0.0144025\tau^3 + 41.1757\tau^2 + 142.277\tau + 0.601247\right)t^3\\
&- 0.0418646\tau^3 + \left(-0.0822512\tau^3 + 329.12\tau^2 - 28356.7\tau + 3123.34\right)t^2 + 445.021\tau^2 +\\
&\left(0.0992709\tau^3 - 767.479\tau^2 + 5.75594 \times 10^6\tau - 23771.2\right)t - 5.70029 \times 10^6\tau + 7.47211 \times 10^6.\\
I(t) &= 26603.1. - 25859.9\tau + \left(-2.47656\tau^2 + 20715.4\tau + 5143.31\right)t + 1.85742\tau^2\\
&+ 0.0171984\tau^2t^4 + (0.343967\tau - 43.6678)\tau t^3 + \left(0.722331\tau^2 + 2703.28\tau - 131.107\right)t^2\\
A(t) &= 1832.83 - 1420.08\tau + 0.244155\tau^2 + \left(-0.32554\tau^2 + 1706.66\tau - 286.744\right)t\\
&+ 0.0022607\tau^2t^4 + (0.0452139\tau - 29.0178)\tau t^3 + \left(0.0949492\tau^2 - 56.2375\tau - 87.0668\right)t^2.\\
R(t) &= 4478.42 - 4412.42\tau + 54.3988\tau t^3 + (990.893\tau + 163.196)t^2 + (2104.24\tau + 2308.18)t.\\
Q(t) &= 990099 - 98.7453\tau + 0.00129197\tau t^3 + 2.66833\tau t^2 + 0.00387591t^2 + 93.4009\tau t + 5.3444t.
\end{aligned}
$$



(a) (b)

Figure 3: Dynamical behavior of a) suspected population S(t) and b) exposed population E(t) for various values of τ with respect to time(days)



(a) (b)

Figure 4: Dynamical behavior of a) infected(symptomatic) population I(t) and b) asymptotic population A(t) for various values of τ with respect to time(days)

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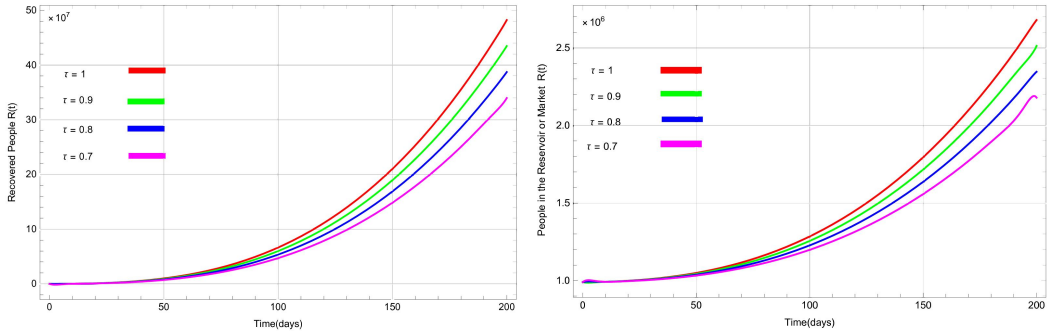

(a) (b)

Figure 5: Dynamical behavior of a) recovered population R(t) and b) reservoir population Q(t) for various values of τ with respect to time(days)

The simulations reveal that a difference in the esteem influences the dynamics of the epidemic. The non-integer order has a notable effect on the dynamics of the epidemic. The Figures 3 to 5 shows the behavior of susceptible people S(t), exposed people E(t), infected people I(t), asymptotic infected people A(t), recovered people R(t) and People in reservoir Q(t) versus time t in days for distinct values of τ respectively. Figure 3a demonstrates that susceptible people decreases rapidly and converges to zero as the value of τ decreases. The graph in Figure 3b of exposed people shows that as the value of τ goes down the rate of increment also reduces. Figure 4a shows that the infected population increases sharply in the first few days and then decreases to zero after reaching its peak with non-integer values of τ. It also shows that at a very slow pace ($\tau = 0.1$) the number of infected peoples is significantly low.

The asymptotic infected people A(t) increases initially and then converges to zero for various values of τ as shown in Figure 4b. Likewise, it can be seen in Figure 5a that the people are recovered or removed(death) very rapidly with a change of τ. Also, Figure 5b shows People in the Market or reservoir decreases directly with fractional values of τ. It is noted from Figures 3 to 5 that there is a remarkable difference at various estimations of τ and this model depends continuously on the time-fractional derivative.

5 Discussion

This model consists of many parameters, hence several limitations arise in this study. Firstly, we have not used the detailed data of the COVID-19 for the estimation whereas we have utilized the data from literature [11]. Besides, the parameters of population versatility were not from a precise data set. There are uncertainties in all parameters of our model and these would translate into uncertainties in forecasts and estimates. Increasing the capacity of testing people for COVID-19 can lead to getting lucid information about the number of asymptomatic cases. This will enhance the

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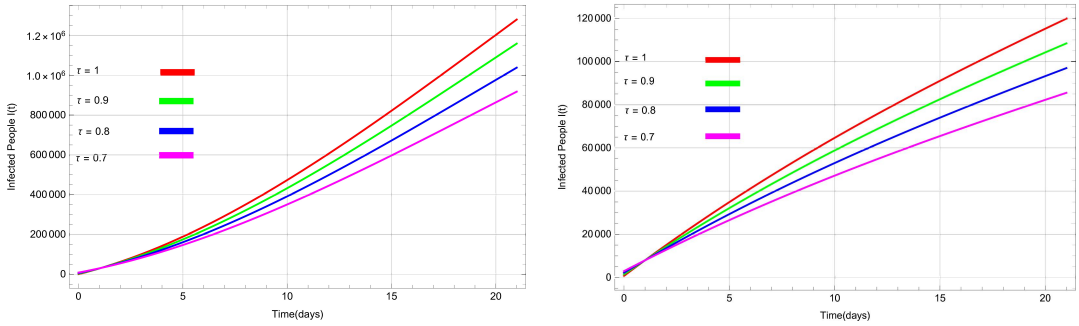
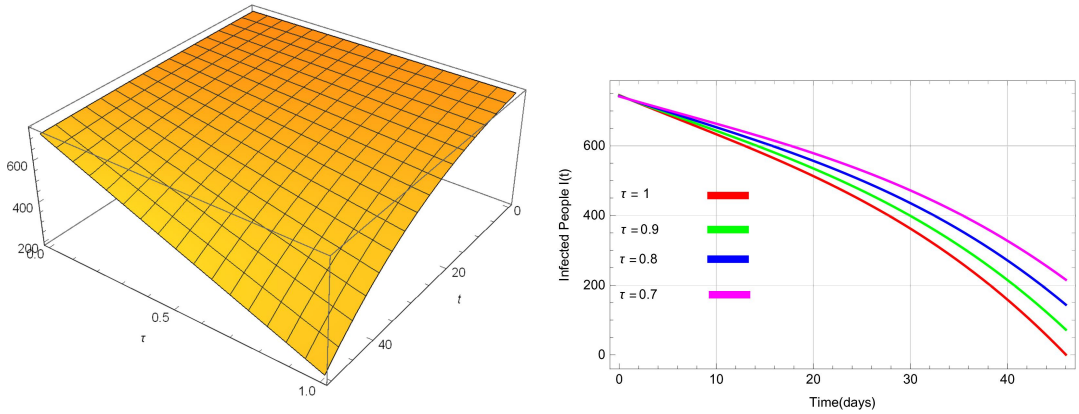
accuracy of estimation and further progress of COVID-19. The government of India has imposed 21 days nationwide lockdown from 25 March 2020 and asking people to stay at home, restrict population movement which can help to limit transmission of the virus. The following figures demonstrate the impact of social distancing.



(a) (b)

Figure 6: Dynamical behavior of the infected population I(t) for various values of τ where a) Without lockdown b)With lockdown up to 21 days



(a) (b)

Figure 7: Dynamical behavior of the infected population I(t) where a) surface for $0 < \tau \leq 1$ and $0 \leq t \leq 50$ b)complete lock-down up to 47 days for various values of τ

Figure 6a shows the increase in infected people after 21 days without any lock-down or social distancing whereas Figure 6b shows the situation after complete lock-down and following social distancing (Q=0). From figure 6b, it is observed that 21 days lockdown rapidly reduces the number of infectious population with different rate of change given by τ as compared to figure 6a. Figure 7a shows surface of infected people with respect to time 't' $(0 \leq t \leq 50)$ and τ $(0 < \tau \leq 1)$. It is also noted from figure 7b that continuous lockdown till 45 to 50 days will decrease the infected cases near to zero in India. Hence, without any vaccine till date, complete lockdown and physical distancing other than the household ones proves to be an effective measure at this moment to control the outbreak of COVID-19 in India.

This model becomes highly reliable when real-time and actual estimates of transmission structures are available.

6 Conclusions

It has been observed from the present work that infectious diseases like coronavirus can be effectively modeled with nonlocal Caputo-Fabrizio fractional operator. Moreover, by implementing the Banach fixed point theory, the stability criteria for steady solutions and existence have been verified. Approximate solutions and graphical demonstration by using iterative Laplace transform method of the CF fractional coronavirus model have presented. It has been observed that memory features in CF derivative explore hidden dynamics of the infection in the mathematical models of infectious disease which is not possible to find with integer type of derivatives.

To the early end of the COVID-19 pandemic and in the absence of some sure treatment like a vaccine, preventive measures to reduce the spread of the virus such as social distancing, decreasing number of contacts of susceptible population, mitigation, containment and suppression against the infection recommending self-quarantine of entire populations living in affected areas are crucial with the participation of the public, along with the policy of reducing the transmission period by finding and isolating patients as quickly as possible through efforts by the quarantine authorities would benefit at large towards control of COVID-19.

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