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Article

Further robust dissipativity analysis of uncertain stochastic generalized neural networks with Markovian jump parameters

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Abstract: This paper analyzes the robust dissipativity of uncertain stochastic generalized neural networks (USGNNs) with Markovian jumping parameters and time-varying delays. In practical applications most of the systems refer to uncertainties, hence, the norm-bounded parameter uncertainties and stochastic disturbance are considered. Then, by constructing an appropriate Lyapunov-Krasovskii functional (LKF) and by employing integral inequalities LMI-based sufficient conditions of the considered systems are established. Numerical simulations are given to show the merit of the presented results.

Keywords: Dissipativity analysis; Generalized neural networks; Markovian jump parameters; Stochastic disturbance.

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1. Introduction

In the last ten years, neural networks (NNs) have been gaining considerable attention because of their important applications in optimization, signal processing, image processing, associative memories and so forth [1]-[5]. Mainly, generalized neural networks (GNNs) has been considered an energetic research model because of their fitting mathematical model capacity and many powerful results were published concerning the stability of GNNs [15], [16], [35], [36]. On the other hand, time delay is the inherent feature of many physical processes including chemical processes, nuclear reactors, which is major source of instability and poor performance [6]-[12]. Therefore, many efficient approaches and important results have been reported regarding various dynamics of NNs with time-delays [13]-[39].

On the other side, the Markovian jumping neural network (MJNN) has recently been received significant research interest, since MJNN extremely useful model for understanding its dynamics when the NNs incorporate abrupt changes in their structure. To prove this matter, many effective methods have been developed [10]-[16]. On the other hand, the stochastic effects are certainly present in all neural systems. Therefore, the study of stochastic NNs is not only interesting but also essential, because the existence of certain stochastic inputs may affect the behavior of the system [17]-[20]. The stability of stochastic nonlinear systems has recently become an important field of research, and considerable efforts have been devoted to Markovian jumping stochastic NNs, several stability conditions were published recently [21]-[25]. For example, the stability of stochastic static NNs has been investigated in [13] with Markovian switching. On the basis of Lyapunov functional, the exponential stability of stochastic NNs with Markovian jump parameters was studied in [40]. Some other results regarding the proposed problem can be found [22]-[24]. On the other hand, when practical systems are modelled, uncertainties of system parameters are often included. Therefore, many systems refer to uncertainties in practical applications.

From the practical viewpoint, it is important to investigate NNs with uncertain parameters [18], [19], [23], [24], [28], [40].

Undoubtedly, dissipative behavior is certainly essential for control and engineering; thus, dissipativity analysis of USGNNs presents a theoretical challenge, which has gained growing attention [26]-[28]. For instance, in [29], three types of neuron activation functions are discussed for global dissipativity of delayed recurrent NNs, i.e. bounded, Lipschitz-continuous and monotonous non-decreasing. In [30], the authors analyzed the global dissipativity of NNs with both time-varying delays and unbounded delays. Meanwhile, $(\mathbf{Q},\mathbf{S},\mathbf{R})$ dissipativity theory contains multi-dynamic behaviors, which lead to effective known results by changing the weight matrices in their structure and discovered their applications in the field of control and engineering [27]-[32]. Similar studies on the dissipativity of various NNs can be found in [28]-[33]. To the best of the authors' knowledge, the problem of robust dissipativity of USGNNs with norm-bounded uncertainties has been not fully investigated and remains a good challenge.

Motivated by these mentioned shortcomings for the existing results, in this paper we aim to establish robust dissipativity and stability for USGNNs with Markovian jumping parameters. On the basis of Lyapunov functional method, an appropriate LKF is constructed with more delay information and the derivative of LKF has been estimating by new integral inequalities, which has great support to reduce the conservatism of the obtained results. By employing Ito's formula and some analytic techniques, robust dissipativity and stability conditions are derived in terms of simplified LMI. Numerical simulations are also given to prove the merits of the presented results. The layout of the paper is as follows, the problem is formally defined in the next section. The main results of this paper are presented in section 3. Numerical examples are presented in section 4. Finally, section 5 concludes this paper.

List of symbols: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denotes the n-dimensional Euclidean space and the set of $m\times n$ real matrices, respectively. The superscript $\mathcal{X}^T$ represents the transpose of $\mathcal{X}$. $\mathcal{P}>0$ means that $\mathcal{P}$ is the symmetric positive definite matrix. $\mathrm{tr}\{\mathcal{D}\}$ denotes the trace of matrix $\mathcal{D}$, $\star$ denotes the elements below the main diagonal of a symmetric block matrix. $(\Omega,\mathscr{F},\mathscr{P})$ is complete probability space with a natural filtration. I_n represents the identity matrix with appropriate dimensions. diag$\{.\}$ denotes the block diagonal matrix. $\mathcal{L}_2[0,\infty)$ is the space of an n-dimensional square integral vector function on $[0,\infty)$. $\mathbb{E}\{\cdot\}$ denote the mathematical expectation.

2. Problem description and preliminaries

Let $\{\mathfrak{e}(t),t\geq 0\}$ be a right-continuous Markovian process on $(\Omega,\mathscr{F},\mathscr{P})$ and taking values in a finite state space $S=\{1,2,...,N\}$ with generator $\Pi=[\pi_{xy}]_{N\times N}$ given by

$$\Pr\{\mathfrak{e}(t+\triangle t)=y|\mathfrak{e}(t)=x\}=\begin{cases}\pi_{xy}\triangle t+o(\triangle t), & \text{if } x\neq y,\\ 1+\pi_{xx}\triangle t+o(\triangle t), & \text{if } x=y,\end{cases} \tag{1}$$

where $\triangle t>0$ and $\lim_{\triangle t\to 0}\frac{o(\triangle t)}{\triangle t}=0$, $\pi_{xy}\geq 0$ is the transition rate from x to y if $x\neq y$ while $\pi_{xx}=-\sum_{y=1,y\neq x}^{N}\pi_{xy}$.
We consider the following GNNs with MJPs

$$\begin{cases}\dot{\mathfrak{p}}(t)= & -\mathcal{D}(\mathfrak{e}(t))\mathfrak{p}(t)+\mathcal{A}(\mathfrak{e}(t))\mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t))+\mathcal{B}(\mathfrak{e}(t))\mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t-\mathfrak{r}(t)))+\mathfrak{u}(t)\\ \mathfrak{q}(t)= & \mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t)),\end{cases} \tag{2}$$

where $\mathfrak{p}(t)=[\mathfrak{p}_1(t),\mathfrak{p}_2(t),...,\mathfrak{p}_n(t)]^T\in\mathbb{R}^n$ is the state vector; $\mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(\cdot))=[\mathfrak{g}_1(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}_1(\cdot)), \mathfrak{g}_2(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}_2(\cdot)),...,\mathfrak{g}_n(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}_n(\cdot))]^T\in\mathbb{R}^n$ is the nonlinear neuron activation function; $\mathfrak{u}(t)=[\mathfrak{u}_1(t),....,\mathfrak{u}_n(t)]^T\in\mathbb{R}^n$ is the external disturbance which belongs to $\mathcal{L}_2[0,\infty)$; $\mathfrak{q}(t)$=$[\mathfrak{q}_1(t),...,\mathfrak{q}_n(t)]^T\in\mathbb{R}^n$ is the output vector; $\mathfrak{r}(t)$ corresponds to the transmission delay; $\mathcal{D}(\mathfrak{e}(t)), \mathcal{A}(\mathfrak{e}(t)), \mathcal{B}(\mathfrak{e}(t))$ and $\mathcal{W}(\mathfrak{e}(t))$ are matrix functions of $\mathfrak{e}(t)$ and for each $\mathfrak{e}(t) \in S$,

$$\mathcal{D}(\mathfrak{e}(t)) = \begin{bmatrix} d_1(\mathfrak{e}(t)) & & & \\ & . & & \\ & & . & \\ & & & . & \\ & & & & d_n(\mathfrak{e}(t)) \end{bmatrix} \in \mathbb{R}^n, \quad \mathcal{A}(\mathfrak{e}(t)) = \begin{bmatrix} a_{11}(\mathfrak{e}(t)) & . & . & . & a_{1n}(\mathfrak{e}(t)) \\ . & & . & & . \\ . & & & . & . \\ . & & & . & . \\ a_{n1}(\mathfrak{e}(t)) & . & . & . & a_{nn}(\mathfrak{e}(t)) \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$\mathcal{B}(\mathfrak{e}(t)) = \begin{bmatrix} b_{11}(\mathfrak{e}(t)) & . & . & . & b_{1n}(\mathfrak{e}(t)) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ b_{n1}(\mathfrak{e}(t)) & . & . & . & b_{nn}(\mathfrak{e}(t)) \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad \mathcal{W}(\mathfrak{e}(t)) = \begin{bmatrix} w_{11}(\mathfrak{e}(t)) & . & . & . & w_{1n}(\mathfrak{e}(t)) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ w_{n1}(\mathfrak{e}(t)) & . & . & . & w_{nn}(\mathfrak{e}(t)) \end{bmatrix} \in \mathbb{R}^{n\times n}.$$

$(\mathcal{A}_1)$: $\mathfrak{r}(t)$ is a known time-varying delay of the system which satisfies

$$0 \leq \mathfrak{r}(t) \leq \mathfrak{r}, \; \dot{\mathfrak{r}}(t) \leq \mu. \tag{3}$$

where $\mathfrak{r}$ and μ are constants.
$(\mathcal{A}_2)$: For all $\zeta_1, \zeta_2 \in \mathbb{R}$, $\zeta_1 \neq \zeta_2$, the neuron activation function $\mathfrak{g}(\cdot)$ is continuous and bounded which satisfies

$$[\mathfrak{g}(\zeta_1) - \mathfrak{g}(\zeta_2) - \Delta_1(\zeta_1 - \zeta_2)]^T[\mathfrak{g}(\zeta_1) - \mathfrak{g}(\zeta_2) - \Delta_2(\zeta_1 - \zeta_2)] \leq \; 0, \tag{4}$$

where Δ_1 and Δ_2 are known constant matrices.
The initial condition of (2) is defined by $\mathfrak{p}(t) = \phi(t)$ on $-\mathfrak{r} \leq t \leq 0$ in $\phi \in \mathcal{C}([-\mathfrak{r}, 0]; \mathbb{R}^n)$.
As mentioned earlier, it is often the case in practice that the NNs is disturbed by environmental noises that affect the stability of the equilibrium. Motivated by this we express a stochastic system whose consequent parts are a set of stochastic Markovian jump GNNs with time-varying delays:

$$\begin{cases} d\mathfrak{p}(t) = & [-\mathcal{D}(\mathfrak{e}(t))\mathfrak{p}(t) + \mathcal{A}(\mathfrak{e}(t))\mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t)) + \mathcal{B}(\mathfrak{e}(t))\mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t - \mathfrak{r}(t))) + \mathfrak{u}(t)]dt \\ & +\sigma(t, \mathfrak{e}(t), \mathfrak{p}(t), \mathfrak{p}(t - \mathfrak{r}(t)))d\omega(t), \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}(\mathfrak{e}(t))\mathfrak{p}(t)), \end{cases} \tag{5}$$

where $\sigma(t, \mathfrak{e}(t), \mathfrak{p}(t), \mathfrak{p}(t - \mathfrak{r}(t)))$ is the stochastic perturbation. $\omega(t) = [\omega_1(t), ..., \omega_m(t)]^T \in \mathbb{R}^m$ is the Brownian motion m-space on $(\Omega, \mathscr{F}, \mathscr{P})$.
For the purpose of simplicity, let $\mathfrak{e}(t) = x$ $(x \in S)$. Then $\mathcal{D}(\mathfrak{e}(t)) = \mathcal{D}_x$, $\mathcal{A}(\mathfrak{e}(t)) = \mathcal{A}_x, \mathcal{B}(\mathfrak{e}(t)) = \mathcal{B}_x$ and $\mathcal{W}(\mathfrak{e}(t)) = \mathcal{W}_x$. The system (5) becomes

$$\begin{cases} d\mathfrak{p}(t) = & [-\mathcal{D}_x\mathfrak{p}(t) + \mathcal{A}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)) + \mathcal{B}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t - \mathfrak{r}(t))) + \mathfrak{u}(t)]dt \\ & +\sigma(t, x, \mathfrak{p}(t), \mathfrak{p}(t - \mathfrak{r}(t)))d\omega(t) \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)). \end{cases} \tag{6}$$

For the sake of convenience, the following abbreviations are adopted in the sequel:

$$\begin{cases} \varphi(t) \triangleq & -\mathcal{D}_x\mathfrak{p}(t) + \mathcal{A}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)) + \mathcal{B}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t - \mathfrak{r}(t))) + \mathfrak{u}(t) \\ \sigma(t) \triangleq & \sigma(t, x, \mathfrak{p}(t), \mathfrak{p}(t - \mathfrak{r}(t))). \end{cases} \tag{7}$$

The system (6) reads as

$$\begin{cases} d\mathfrak{p}(t) = & \varphi(t)dt + \sigma(t)d\omega(t) \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)). \end{cases} \tag{8}$$

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$(\mathcal{A}_3)$: There exist matrices $\mathcal{L}_{1x} > 0, \mathcal{L}_{2x} > 0$ such that for all $x \in S$,

$$\text{tr}\{\sigma^T(t)\sigma(t)\} \leq \mathfrak{p}^T(t)\mathcal{L}_{1x}\mathfrak{p}(t) + \mathfrak{p}^T(t-\mathfrak{r}(t))\mathcal{L}_{2x}\mathfrak{p}(t-\mathfrak{r}(t)). \tag{9}$$

For a general stochastic system $d\mathfrak{p}(t) = \varphi(t)dt + \sigma d\omega(t)$, where $\omega(t)$ is an n dimensional Brownian motion defined on $(\Omega, \mathscr{F}, \mathscr{P})$, $\varphi(\cdot), \sigma(\cdot) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$. An operator $\mathcal{L}$ is defined by

$$\begin{aligned}\mathcal{L}\mathbb{V}(t,\mathfrak{p}(t),x) =& \mathbb{V}_t(t,\mathfrak{p}(t),x) + \mathbb{V}_{\mathfrak{p}}(t,\mathfrak{p}(t),x)[-\mathcal{D}_x\mathfrak{p}(t) + \mathcal{A}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)) + \mathcal{B}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t-\mathfrak{r}(t))) + \mathfrak{u}(t)] \\ &+ \frac{1}{2}\text{tr}[\sigma^T(t,\mathfrak{p}(t),\mathfrak{p}(t-\mathfrak{r}(t)),i)\mathbb{V}_{\mathfrak{p}\mathfrak{p}}(t,\mathfrak{p}(t),x)\sigma(t,\mathfrak{p}(t),\mathfrak{p}(t-\mathfrak{r}(t)),x)] \\ &+ \sum_{yy=1}^{N} \pi_{xy}\mathbb{V}(t,\mathfrak{p}(t),y),\end{aligned}$$

where

$$\begin{aligned}\mathbb{V}_t(t,\mathfrak{p}(t),x) &= \frac{\partial\mathbb{V}(t,\mathfrak{p}(t),x)}{\partial t}, \\ \mathbb{V}_q(t,\mathfrak{p}(t),x) &= \left(\frac{\partial\mathbb{V}(t,\mathfrak{p}(t),x)}{\partial\mathfrak{p}_1}, \ldots, \frac{\partial\mathbb{V}(t,\mathfrak{p}(t),i)}{\partial\mathfrak{p}_n}\right), \\ \mathbb{V}_{\mathfrak{p}\mathfrak{p}}(t,\mathfrak{p}(t),x) &= \left(\frac{\partial^2\mathbb{V}(t,\mathfrak{p}(t),x)}{\partial\mathfrak{p}_x\partial\mathfrak{p}_y}\right)_{n\times n}.\end{aligned}$$

Definition 1. The NN (6) is said to be mean-square stable if for any $\varepsilon > 0$ there exists a scalar $\upsilon(\varepsilon) > 0$ such that $\mathbb{E}\{\|\mathfrak{p}(t)\|^2\} < \varepsilon$, $t > 0$, whenever $\sup_{-\mathfrak{r}\leq t\leq 0} \mathbb{E}\{\|\phi(t)\|^2\} < \upsilon(\varepsilon)$. In addition, if $\lim_{t\to\infty}\mathbb{E}\{\|\mathfrak{p}(t)\|^2\} = 0$, for any initial condition, the NN (6) is called mean-square asymptotically stable.

Definition 2. The NN (6) is said to be strictly $(\mathbf{Q},\mathbf{S},\mathbf{R}) - \gamma-$ dissipative if, for $\gamma > 0$ and under zero initial condition, the following inequality is satisfied:

$$\mathbb{E}\{\mathbf{G}(\mathfrak{u},\mathfrak{q},t_d)\} \geq \mathbb{E}\{\gamma\langle\mathfrak{u},\mathfrak{u}\rangle_{t_d}\}, \ \forall t_d \geq 0. \tag{10}$$

Remark 3. *The energy supply function $\mathcal{G}(\mathfrak{u},\mathfrak{q},t_d)$ can be express as follows*

$$\mathbf{G}(\mathfrak{u},\mathfrak{q},t_d) = \langle\mathfrak{q},\mathbf{Q}\mathfrak{q}\rangle_{t_d} + 2\langle\mathfrak{q},\mathbf{S}\mathfrak{u}\rangle_{t_d} + \langle\mathfrak{u},\mathbf{R}\mathfrak{u}\rangle_{t_d}, \ \forall t_d \geq 0, \tag{11}$$

where $\mathbf{Q},\mathbf{S},\mathbf{R} \in \mathbb{R}^{n\times n}$ with $\mathbf{Q},\mathbf{R}$ are symmetric. The notations $\langle\mathfrak{q},\mathbf{Q}\mathfrak{q}\rangle_{t_d}$, $\langle\mathfrak{q},\mathbf{S}\mathfrak{u}\rangle_{t_d}$ and $\langle\mathfrak{u},\mathbf{R}\mathfrak{u}\rangle_{t_d}$ are represents $\int_0^{t_d}\mathfrak{q}^T(t)\mathbf{Q}\mathfrak{q}(t)dt$, $\int_0^{t_d}\mathfrak{q}^T(t)\mathbf{S}\mathfrak{u}(t)dt$ and $\int_0^{t_d}\mathfrak{u}^T(t)\mathbf{R}\mathfrak{u}(t)dt$, respectively.
Hence, the relation (10) can be written in the following dissipativity condition:

$$\mathbb{J}_{\gamma,t_d} = \int_0^{t_d}\mathbb{E}\left\{\begin{bmatrix}\mathfrak{q}(t)\\ \mathfrak{u}(t)\end{bmatrix}^T\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \star & \mathbf{R}-\gamma I\end{bmatrix}\begin{bmatrix}\mathfrak{q}(t)\\ \mathfrak{u}(t)\end{bmatrix}\right\}dt. \tag{12}$$

Definition 4. The NN (6) is said to be passive if there exists a scalar $\gamma > 0$ such that for all $t_d \geq 0$

$$2\int_0^{t_d}\mathbb{E}\{\mathfrak{q}^T(t)\mathfrak{u}(t)\}dt \geq -\gamma\int_0^{t_d}\mathbb{E}\{\mathfrak{u}^T(t)\mathfrak{u}(t)\}dt. \tag{13}$$

holds under all solution with $\mathfrak{p}(0) = 0$.

Lemma 5. *[37] For a given matrix $\mathcal{W} = \mathcal{W}^T > 0$, given scalars $\mathfrak{s}_1$ and $\mathfrak{s}_2$ satisfying $\mathfrak{s}_1 < \mathfrak{s}_2$, the following inequality holds for all continuously differentiable function ϑ in $[\mathfrak{s}_1,\mathfrak{s}_2] \to \mathbb{R}^n$:*

$$\int_{\mathfrak{s}_1}^{\mathfrak{s}_2}\vartheta^T(\mathfrak{z}_1)\mathcal{W}\vartheta(\mathfrak{z}_1)d\mathfrak{z}_1 \geq \frac{1}{(\mathfrak{s}_2-\mathfrak{s}_1)}\varpi_1^T\Theta_1\varpi_1,$$

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where

$$\varpi_1 = \left[\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \vartheta^T(\mathfrak{z}_1) d\mathfrak{z}_1 \quad \int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \vartheta^T(\mathfrak{z}_2) d\mathfrak{z}_2 d\mathfrak{z}_1 \quad \int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \int_{\mathfrak{s}_1}^{\mathfrak{z}_2} \vartheta^T(\mathfrak{z}_3) d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1 \right]^T,$$

$$\Theta_1 = \begin{bmatrix} 9\mathcal{W} & \frac{-36}{(\mathfrak{s}_2-\mathfrak{s}_1)}\mathcal{W} & \frac{60}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{W} \\ \frac{-36}{(\mathfrak{s}_2-\mathfrak{s}_1)}\mathcal{W}^T & \frac{192}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{W} & \frac{-360}{(\mathfrak{s}_2-\mathfrak{s}_1)^3}\mathcal{W} \\ \frac{60}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{W}^T & \frac{-360}{(\mathfrak{s}_2-\mathfrak{s}_1)^3}\mathcal{W}^T & \frac{720}{(\mathfrak{s}_2-\mathfrak{s}_1)^4}\mathcal{W} \end{bmatrix}.$$

Lemma 6. *[38] For a given matrix $\mathcal{R} = \mathcal{R}^T > 0$, given scalars $\mathfrak{s}_1$ and $\mathfrak{s}_2$ satisfying $\mathfrak{s}_1 < \mathfrak{s}_2$, the following inequality holds for all continuously differentiable function ϑ in $[\mathfrak{s}_1, \mathfrak{s}_2] \to \mathbb{R}^n$:*

$$\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \vartheta^T(\mathfrak{z}_2) \mathcal{R} \vartheta(\mathfrak{z}_2) d\mathfrak{z}_2 d\mathfrak{z}_1 \geq \frac{2}{(\mathfrak{s}_2 - \mathfrak{s}_1)^2} \varpi_2^T \Theta_2 \varpi_2,$$

where

$$\varpi_2 = \left[\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \vartheta^T(\mathfrak{z}_2) d\mathfrak{z}_2 d\mathfrak{z}_1 \quad \int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \int_{\mathfrak{s}_1}^{\mathfrak{z}_2} \vartheta^T(\mathfrak{z}_2) d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1 \quad \int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_1} \int_{\mathfrak{s}_1}^{\mathfrak{z}_2} \int_{\mathfrak{s}_1}^{\mathfrak{z}_3} \vartheta^T(\mathfrak{z}_4) d\mathfrak{z}_4 d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1 \right]^T,$$

$$\Theta_2 = \begin{bmatrix} 6\mathcal{R} & -\frac{30}{(\mathfrak{s}_2-\mathfrak{s}_1)}\mathcal{R} & \frac{60}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{R} \\ -\frac{30}{(\mathfrak{s}_2-\mathfrak{s}_1)}\mathcal{R} & \frac{210}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{R} & -\frac{480}{(\mathfrak{s}_2-\mathfrak{s}_1)^3}\mathcal{R} \\ \frac{60}{(\mathfrak{s}_2-\mathfrak{s}_1)^2}\mathcal{R} & -\frac{480}{(\mathfrak{s}_2-\mathfrak{s}_1)^3}\mathcal{R} & \frac{1200}{(\mathfrak{s}_2-\mathfrak{s}_1)^4}\mathcal{R} \end{bmatrix}.$$

Lemma 7. *[39] Let $\mathcal{M} \in \mathbb{R}^{n\times n}$ be a positive-definite matrix, vector function $\vartheta : [\mathfrak{s}_1, \mathfrak{s}_2] \to \mathbb{R}^n$, with scalars $\mathfrak{s}_1 < \mathfrak{s}_2$, then*

$$\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \vartheta^T(\mathfrak{z}_1) \mathcal{M} \vartheta(\mathfrak{z}_1) d\mathfrak{z}_1 \geq \frac{1}{(\mathfrak{s}_2 - \mathfrak{s}_1)} \left[\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \vartheta(\mathfrak{z}_1) d\mathfrak{z}_1 \right]^T \mathcal{M} \left[\int_{\mathfrak{s}_1}^{\mathfrak{s}_2} \vartheta(\mathfrak{z}_1) d\mathfrak{z}_1 \right].$$

Lemma 8. *[40] Let $\Theta = \Theta^T$, $\mathcal{J}_1$ and $\mathcal{J}_2$ be real matrices, $\mathcal{F}(t)$ satisfies $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$. Then $\Theta + (\mathcal{J}_1 \mathcal{F}(t) \mathcal{J}_2) + (\mathcal{J}_1 \mathcal{F}(t) \mathcal{J}_2)^T < 0$, iff there exist a scalar $\varepsilon > 0$ such that $\Theta + \varepsilon^{-1} \mathcal{J}_1 \mathcal{J}_1^T + \varepsilon \mathcal{J}_2^T \mathcal{J}_2$ or equivalently*

$$\begin{bmatrix} \Theta & \mathcal{J}_1 & \varepsilon \mathcal{J}_2 \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{bmatrix} < 0.$$

3. Main results

For the notation clearness, the following abbreviations are adopted in the subsequent parts:

$$\begin{aligned} \mathfrak{p}_t &\triangleq \mathfrak{p}(t), \\ \mathfrak{p}_{\mathfrak{r}(t)} &\triangleq \mathfrak{p}(t - \mathfrak{r}(t)), \\ \mathfrak{p}_{\mathfrak{r}} &\triangleq \mathfrak{p}(t - \mathfrak{r}), \\ \mathfrak{g}_t &\triangleq \mathfrak{g}(\mathcal{W}_x \mathfrak{p}(t)), \\ \mathfrak{g}_{\mathfrak{r}(t)} &\triangleq \mathfrak{g}(\mathcal{W}_x \mathfrak{p}(t - \mathfrak{r}(t))), \\ \mathfrak{q}_t &\triangleq \mathfrak{q}(t), \\ \mathfrak{u}_t &\triangleq \mathfrak{u}(t), \\ \varphi_t &\triangleq \varphi(t), \end{aligned}$$

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$$
\begin{aligned}
\varphi_{\mathfrak{r}(t)} &\triangleq \int_{t-\mathfrak{r}}^{t} \varphi(\mathfrak{z}_1)d\mathfrak{z}_1,\\
\chi_1 &\triangleq \int_{t-\mathfrak{r}}^{t} \mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\\
\chi_2 &\triangleq \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1} \mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_2\\
\chi_3 &\triangleq \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2} \mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\\
\chi_4 &\triangleq \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_3} \mathfrak{p}(\mathfrak{z}_4)d\mathfrak{z}_4 d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\\
\xi_{(t)} &\triangleq [\mathfrak{p}_t^T \ \varphi_t^T \ \mathfrak{p}_{\mathfrak{r}(t)}^T \ \mathfrak{p}_{\mathfrak{r}}^T \ \mathfrak{g}_t^T \ \mathfrak{g}_{\mathfrak{r}(t)}^T \ \varphi_{\mathfrak{r}(t)}^T \ \chi_1^T \ \chi_2^T \ \chi_3^T \ \chi_4^T \ \mathfrak{u}_t^T]^T.
\end{aligned}
$$

3.1. Dissipativity analysis

In this subsection, several sufficient conditions for the dissipativity analysis of the considered system model (6) will be established by using LKF and LMI method.

Theorem 9. *For given scalars* $\mathfrak{r}$ *and* μ, *the NNs* (6) *is* $(\mathbf{Q},\mathbf{S},\mathbf{R})-\gamma$ *dissipative, if there exist matrices* $\mathcal{P}_x(x \in S) > 0, \mathcal{Q} > 0, \mathcal{R} > 0, \mathcal{S} > 0, \mathcal{U} > 0$, *diagonal matrices* $\mathcal{H}_1 > 0, \mathcal{H}_2 > 0$, *any matrices* $\mathcal{G}_1, \mathcal{G}_2$ *and scalar* $\gamma > 0$ *such that the following LMIs holds for all* $(x \in S)$:

$$
\mathcal{P}_x \leq \delta_x I, \tag{14}
$$

$$
\Theta_1 = (\Theta_{i,j})_{12\times 12} < 0, \tag{15}
$$

where
$\Theta_{1,1,x} \triangleq \sum_{y=1}^{N} \pi_{xy}\mathcal{P}_y + \mathcal{Q} + \mathcal{R} + \mathfrak{r}^2\mathcal{U} + \frac{\mathfrak{r}^4}{4}\mathcal{V} + \delta_x\mathcal{L}_{1x} - \mathcal{G}_1\mathcal{D}_x - (\mathcal{G}_1\mathcal{D}_x)^T - \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\Theta_{1,2,x} \triangleq \mathcal{P}_x - \mathcal{G}_1 - (\mathcal{G}_2\mathcal{D}_x)^T$, $\Theta_{1,5,x} \triangleq \mathcal{G}_1\mathcal{A}_x + \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_2$, $\Theta_{1,6,x} \triangleq \mathcal{G}_1\mathcal{B}_x$, $\Theta_{1,12,x} \triangleq \mathcal{G}_1$, $\Theta_{2,2,x} \triangleq \mathfrak{r}\mathcal{W} - \mathcal{G}_2 - \mathcal{G}_2^T$, $\Theta_{2,5,x} \triangleq \mathcal{G}_2\mathcal{A}_x$, $\Theta_{2,6,x} \triangleq \mathcal{G}_2\mathcal{B}_x$, $\Theta_{2,12,x} \triangleq \mathcal{G}_2$, $\Theta_{3,3,x} \triangleq -(1-\mu)\mathcal{Q} + \delta_x\mathcal{L}_{2x} - \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\Theta_{3,6,x} \triangleq \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_2$, $\Theta_{4,4,x} \triangleq -\mathcal{R}$, $\Theta_{5,5,x} \triangleq \mathcal{S} - \mathcal{H}_1 - \mathbf{Q}$, $\Theta_{5,12,x} \triangleq -\mathbf{S}$, $\Theta_{6,6,x} \triangleq -(1-\mu)\mathcal{S} - \mathcal{H}_2$, $\Theta_{7,7,x} \triangleq -\frac{1}{\mathfrak{r}}\mathcal{W}$, $\Theta_{8,8,x} \triangleq -9\mathcal{U}$, $\Theta_{8,9,x} \triangleq \frac{36}{\mathfrak{r}}\mathcal{U}$, $\Theta_{8,10,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{U}$, $\Theta_{9,9,x} \triangleq -\frac{192}{\mathfrak{r}^2}\mathcal{U} - 6\mathcal{V}$, $\Theta_{9,10,x} \triangleq \frac{360}{\mathfrak{r}^3}\mathcal{U} + \frac{30}{\mathfrak{r}}\mathcal{V}$, $\Theta_{9,11,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{V}$, $\Theta_{10,10,x} \triangleq -\frac{720}{\mathfrak{r}^4}\mathcal{U} - \frac{210}{\mathfrak{r}^2}\mathcal{V}$, $\Theta_{10,11,x} \triangleq \frac{480}{\mathfrak{r}^3}\mathcal{V}$, $\Theta_{11,11,x} \triangleq -\frac{1200}{\mathfrak{r}^4}\mathcal{V}$, $\Theta_{12,12,x} \triangleq -\mathbf{R} + \gamma I$.

Proof: Let us consider the LKF candidate for the NNs (6)

$$
\mathbb{V}(t,\mathfrak{p}_t,x) = \sum_{i=1}^{4} \mathbb{V}_i(t,\mathfrak{p}_t,x), \tag{16}
$$

where

$$
\begin{aligned}
\mathbb{V}_1(t,\mathfrak{p}_t,x) &= \mathfrak{p}_t^T \mathcal{P}_x \mathfrak{p}_t,\\
\mathbb{V}_2(t,\mathfrak{p}_t,x) &= \int_{t-\mathfrak{r}(t)}^{t} \mathfrak{p}^T(\mathfrak{z}_1)\mathcal{Q}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\\
&\quad + \int_{t-\mathfrak{r}}^{t} \mathfrak{p}^T(\mathfrak{z}_1)\mathcal{R}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\\
&\quad + \int_{t-\mathfrak{r}(t)}^{t} \mathfrak{g}^T(\mathcal{W}_x\mathfrak{p}(\mathfrak{z}_1))\mathcal{S}g(\mathcal{W}_x\mathfrak{p}(\mathfrak{z}_1))d\mathfrak{z}_1,
\end{aligned}
$$

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$$
\begin{aligned}
\mathbb{V}_3(t,\mathfrak{p}_t,x) &= \mathfrak{r}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}^T(\mathfrak{z}_2)\,\mathcal{U}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1 \\
&\quad +\frac{\mathfrak{r}^2}{2}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\mathfrak{p}^T(\mathfrak{z}_3)\,\mathcal{V}\mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1 \\
\mathbb{V}_4(t,\mathfrak{p}_t,x) &= \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\varphi^T(\mathfrak{z}_2)\,\mathcal{W}\varphi(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1.
\end{aligned}
$$

Let $\mathcal{L}$ be the weak infinitesimal random process. Then $\mathbb{V}(t,\mathfrak{p}_t,x)$ can be calculated by Ito's formula that

$$
d\mathbb{V}(t,\mathfrak{p}_t,x) = \mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x)dt + \{\sigma(t,x,\mathfrak{p}_t,\mathfrak{p}_{\mathfrak{r}(t)}\}d\omega(t), \tag{17}
$$

where

$$
\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) = \sum_{i=1}^{4}\mathcal{L}\mathbb{V}_i(t,\mathfrak{p}_t,x). \tag{18}
$$

Now we calculate $\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x)$ along the solutions of the NNs (6), one has

$$
\mathcal{L}\mathbb{V}_1(t,\mathfrak{p}_t,x) = 2\mathfrak{p}_t^T\mathcal{P}_x\varphi_t + \mathfrak{p}_t^T\left(\sum_{y=1}^{N}\pi_{xy}\mathcal{P}_y\right)\mathfrak{p}_t + \mathrm{tr}\{\sigma^T(t)\mathcal{P}_x\sigma(t)\}, \tag{19}
$$

$$
\begin{aligned}
\mathcal{L}\mathbb{V}_2(t,\mathfrak{p}_t,x) &= \mathfrak{p}_t^T\mathcal{Q}\mathfrak{p}_t - (1-\dot{\mathfrak{r}}(t))\mathfrak{p}_{\mathfrak{r}(t)}^T\mathcal{Q}\mathfrak{p}_{\mathfrak{r}(t)} + \mathfrak{p}_t^T\mathcal{R}\mathfrak{p}_t - \mathfrak{p}_{\mathfrak{r}}^T\mathcal{R}\mathfrak{p}_{\mathfrak{r}} \\
&\quad + \mathfrak{g}_t^T\mathcal{S}\mathfrak{g}_t - (1-\dot{\mathfrak{r}}(t))\mathfrak{g}_{\mathfrak{r}(t)}^T\mathcal{S}\mathfrak{g}_{\mathfrak{r}(t)}, \\
&\leq \mathfrak{p}_t^T\mathcal{Q}\mathfrak{p}_t - (1-\mu)\mathfrak{p}_{\mathfrak{r}(t)}^T\mathcal{Q}\mathfrak{p}_{\mathfrak{r}(t)} + \mathfrak{p}_t^T\mathcal{R}\mathfrak{p}_t - \mathfrak{p}_{\mathfrak{r}}^T\mathcal{R}\mathfrak{p}_{\mathfrak{r}} \\
&\quad + \mathfrak{g}_t^T\mathcal{S}\mathfrak{g}_t - (1-\mu)\mathfrak{g}_{\mathfrak{r}(t)}^T\mathcal{S}\mathfrak{g}_{\mathfrak{r}(t)},
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\mathcal{L}\mathbb{V}_3(t,\mathfrak{p}_t,x) &= \mathfrak{r}^2\mathfrak{p}_t^T\mathcal{U}\mathfrak{p}_t - \mathfrak{r}\int_{t-\mathfrak{r}}^{t}\mathfrak{p}^T(\mathfrak{z}_1)\,\mathcal{U}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1 + \frac{\mathfrak{r}^4}{4}\mathfrak{p}_t^T\mathcal{V}\mathfrak{p}_t \\
&\quad - \frac{\mathfrak{r}^2}{2}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}^T(\mathfrak{z}_2)\,\mathcal{V}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1
\end{aligned} \tag{21}
$$

$$
\mathcal{L}\mathbb{V}_4(t,\mathfrak{p}_t,x) = \mathfrak{r}\varphi_t^T\mathcal{W}\varphi_t - \int_{t-\mathfrak{r}}^{t}\varphi^T(\mathfrak{z}_1)\,\mathcal{W}\varphi(\mathfrak{z}_1)d\mathfrak{z}_1. \tag{22}
$$

By using lemma (5), (6) and (7) the integral term in (21)-(22) can be estimate that

$$
\begin{aligned}
-\mathfrak{r}\int_{t-\mathfrak{r}}^{t}\mathfrak{p}^T(\mathfrak{z}_1)\,\mathcal{U}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1 \leq &-\begin{bmatrix}\int_{t-\mathfrak{r}}^{t}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\end{bmatrix}^T \\
&\begin{bmatrix}9\mathcal{U} & -\frac{36}{\mathfrak{r}}\mathcal{U} & \frac{60}{\mathfrak{r}^2}\mathcal{U}\\ * & \frac{192}{\mathfrak{r}^2}\mathcal{U} & -\frac{360}{\mathfrak{r}^3}\mathcal{U}\\ * & * & \frac{720}{\mathfrak{r}^4}\mathcal{U}\end{bmatrix}\begin{bmatrix}\int_{t-\mathfrak{r}}^{t}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\end{bmatrix},
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
-\frac{\mathfrak{r}^2}{2}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}^T(\mathfrak{z}_2)\,\mathcal{V}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1 \leq &-\begin{bmatrix}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_3}\mathfrak{p}(\mathfrak{z}_4)d\mathfrak{z}_4 d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\end{bmatrix}^T \\
&\begin{bmatrix}6\mathcal{V} & -\frac{30}{\mathfrak{r}}\mathcal{V} & \frac{60}{\mathfrak{r}^2}\mathcal{V}\\ * & \frac{210}{\mathfrak{r}^2}\mathcal{V} & -\frac{480}{\mathfrak{r}^3}\mathcal{V}\\ * & * & \frac{1200}{\mathfrak{r}^4}\mathcal{V}\end{bmatrix}\begin{bmatrix}\int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\mathfrak{p}(\mathfrak{z}_2)d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\mathfrak{p}(\mathfrak{z}_3)d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\\ \int_{-\mathfrak{r}}^{0}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_1}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_2}\int_{t-\mathfrak{r}}^{t+\mathfrak{z}_3}\mathfrak{p}(\mathfrak{z}_4)d\mathfrak{z}_4 d\mathfrak{z}_3 d\mathfrak{z}_2 d\mathfrak{z}_1\end{bmatrix},
\end{aligned} \tag{24}
$$

$$
-\int_{t-\mathfrak{r}}^{t}\varphi^T(\mathfrak{z}_1)\,\mathcal{W}\varphi(\mathfrak{z}_1)d\mathfrak{z}_1 \leq \frac{1}{\mathfrak{r}}\left[\int_{t-\mathfrak{r}}^{t}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\right]^T\mathcal{W}\left[\int_{t-\mathfrak{r}}^{t}\mathfrak{p}(\mathfrak{z}_1)d\mathfrak{z}_1\right]. \tag{25}
$$

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From (9) and (14), we can get

$$\begin{aligned} \mathrm{tr}\{\sigma^T(t)\mathcal{P}_x\sigma(t)\} &\le \delta_x \mathrm{tr}\{\sigma^T(t)\sigma(t)\} \\ &\le \mathfrak{p}_t^T \delta_x \mathcal{L}_{1x}\mathfrak{p}_t + \mathfrak{p}_{\mathfrak{r}(t)}^T \delta_x \mathcal{L}_{2x}\mathfrak{p}_{\mathfrak{r}(t)}. \end{aligned} \tag{26}$$

For any constant matrix $\mathcal{G}_1, \mathcal{G}_2$ with suitable dimension, the subsequent condition holds,

$$2[\mathfrak{p}_t\mathcal{G}_1 + \varphi_t\mathcal{G}_2]^T[-\varphi_t - \mathcal{D}_x\mathfrak{p}_t + \mathcal{A}_x\mathfrak{g}_t + \mathcal{B}_x\mathfrak{g}_{\mathfrak{r}(t)} + \mathfrak{u}_t] = 0. \tag{27}$$

In addition, from (4) the following inequalities can be obtained

$$(\mathfrak{g}_t - \Delta_1\mathcal{W}_x\mathfrak{p}_t)^T(\mathfrak{g}_t - \Delta_2\mathcal{W}_x\mathfrak{p}_t) \le 0, \tag{28}$$
$$(\mathfrak{g}_{\mathfrak{r}(t)} - \Delta_1\mathcal{W}_x\mathfrak{p}_{\mathfrak{r}(t)})^T(\mathfrak{g}_{\mathfrak{r}(t)} - \Delta_2\mathcal{W}_x\mathfrak{p}_{\mathfrak{r}(t)}) \le 0. \tag{29}$$

There exist positive diagonal matrices $\mathcal{H}_1, \mathcal{H}_2$, we have

$$0 \le -\mathcal{H}_1 \begin{bmatrix} \mathcal{W}_x\mathfrak{p}_t \\ \mathfrak{g}_t \end{bmatrix}^T \begin{bmatrix} \mathcal{K}_1 & -\mathcal{K}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \mathcal{W}_x\mathfrak{p}_t \\ \mathfrak{g}_t \end{bmatrix}, \tag{30}$$
$$0 \le -\mathcal{H}_2 \begin{bmatrix} \mathcal{W}_x\mathfrak{p}_{\mathfrak{r}(t)} \\ \mathfrak{g}_{\mathfrak{r}(t)} \end{bmatrix}^T \begin{bmatrix} \mathcal{K}_1 & -\mathcal{K}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \mathcal{W}_x\mathfrak{p}_{\mathfrak{r}(t)} \\ \mathfrak{g}_{\mathfrak{r}(t)} \end{bmatrix}. \tag{31}$$

where

$$\mathcal{K}_1 = \frac{\Delta_1^T\Delta_2 + \Delta_2^T\Delta_1}{2}, \ \mathcal{K}_2 = \frac{\Delta_1^T + \Delta_2^T}{2}. \tag{32}$$

Combining from (19)-(31), one can obtain

$$\begin{aligned} &\mathbb{E}\{\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) - \mathfrak{q}_t^T\mathbf{Q}\mathfrak{q}_t - 2\mathfrak{q}_t^T\mathbf{S}\mathfrak{u}_t - \mathfrak{u}_t^T(\mathbf{R}-\gamma I)\mathfrak{u}_t\} \le 2\mathfrak{p}_t^T\mathcal{P}_x\varphi_t + \mathfrak{p}_t^T\left(\sum_{y=1}^N \pi_{xy}\mathcal{P}_y\right)\mathfrak{p}_t + \mathfrak{p}_t^T\delta_x\mathcal{L}_{1x}\mathfrak{p}_t \\ &+ \mathfrak{p}_{\mathfrak{r}(t)}^T\delta_x\mathcal{L}_{2x}\mathfrak{p}_{\mathfrak{r}(t)} + \mathfrak{p}_t^T\mathcal{Q}\mathfrak{p}_t - (1-\mu)\mathfrak{p}_{\mathfrak{r}(t)}^T\mathcal{Q}\mathfrak{p}_{\mathfrak{r}(t)} + \mathfrak{p}_t^T\mathcal{R}\mathfrak{p}_t - \mathfrak{p}_{\mathfrak{r}}^T\mathcal{R}\mathfrak{p}_{\mathfrak{r}} + \mathfrak{g}_t^T\mathcal{S}\mathfrak{g}_t \\ &- (1-\mu)\mathfrak{g}_{\mathfrak{r}(t)}^T\mathcal{S}\mathfrak{g}_{\mathfrak{r}(t)} + \mathfrak{r}^2\mathfrak{p}_t^T\mathcal{U}\mathfrak{p}_t - \chi_1^T(9\mathcal{U})\chi_1 + \chi_1^T(\frac{36}{\mathfrak{r}}\mathcal{U})\chi_2 - \chi_1^T(\frac{60}{\mathfrak{r}^2}\mathcal{U})\chi_3 \\ &- \chi_2^T(\frac{192}{\mathfrak{r}^2}\mathcal{U})\chi_2 + \chi_2^T(\frac{360}{\mathfrak{r}^3}\mathcal{U})\chi_3 - \chi_3^T(\frac{720}{\mathfrak{r}^4}\mathcal{U})\chi_3 + \frac{\mathfrak{r}^4}{4}\mathfrak{p}_t^T\mathcal{V}\mathfrak{p}_t - \chi_2^T(6\mathcal{V})\chi_2 \\ &+ \chi_2^T(\frac{30}{\mathfrak{r}}\mathcal{V})\chi_3 - \chi_2^T(\frac{60}{\mathfrak{r}^2}\mathcal{V})\chi_4 - \chi_3^T(\frac{210}{\mathfrak{r}^2}\mathcal{V})\chi_3 + \chi_3^T(\frac{480}{\mathfrak{r}^3}\mathcal{V})\chi_4 - \chi_4^T(\frac{1200}{\mathfrak{r}^4}\mathcal{V})\chi_4 \\ &+ \mathfrak{r}\varphi_t^T\mathcal{W}\varphi_t - \varphi_{\mathfrak{r}(t)}^T(\frac{1}{\mathfrak{r}}\mathcal{W})\varphi_{\mathfrak{r}(t)} - 2\mathfrak{p}_t^T(\mathcal{G}_1)\varphi_t - 2\mathfrak{p}_t^T(\mathcal{G}_1\mathcal{D}_x)\mathfrak{p}_t + 2\mathfrak{p}_t^T(\mathcal{G}_1\mathcal{A}_x)\mathfrak{g}_t \\ &+ 2\mathfrak{p}_t^T(\mathcal{G}_1\mathcal{B}_x)\mathfrak{g}_{\mathfrak{r}(t)} + 2\mathfrak{p}_t^T(\mathcal{G}_1)\mathfrak{u}_t - 2\varphi_t^T(\mathcal{G}_2)\varphi_t - 2\mathfrak{p}_t^T(\mathcal{G}_2\mathcal{D}_x)^T\varphi_t + 2\varphi_t^T(\mathcal{G}_2\mathcal{A}_x)\mathfrak{g}_t \\ &+ 2\varphi_t^T(\mathcal{G}_2\mathcal{B}_x)\mathfrak{g}_{\mathfrak{r}(t)} + 2\varphi_t^T(\mathcal{G}_2)\mathfrak{u}_t - \mathfrak{p}_t^T(\mathcal{W}_x^T\mathcal{H}_1\mathcal{K}_1\mathcal{W}_x)\mathfrak{p}_t + \mathfrak{p}_t^T(\mathcal{W}_x^T\mathcal{H}_1\mathcal{K}_2)\mathfrak{g}_t \\ &- \mathfrak{g}_t^T(\mathcal{H}_1)\mathfrak{g}_t - \mathfrak{p}_{\mathfrak{r}(t)}^T(\mathcal{W}_x^T\mathcal{H}_2\mathcal{K}_1\mathcal{W}_x)\mathfrak{p}_{\mathfrak{r}(t)} + \mathfrak{p}_{\mathfrak{r}(t)}^T(\mathcal{W}_x^T\mathcal{H}_2\mathcal{K}_2)\mathfrak{g}_{\mathfrak{r}(t)} \\ &- \mathfrak{g}_{\mathfrak{r}(t)}^T(\mathcal{H}_2)\mathfrak{g}_{\mathfrak{r}(t)} - \mathfrak{q}_t^T\mathbf{Q}\mathfrak{q}_t - 2\mathfrak{q}_t^T\mathbf{S}\mathfrak{u}_t - \mathfrak{u}_t^T(\mathbf{R}-\gamma I)\mathfrak{u}_t, \end{aligned} \tag{33}$$

which is equivalent to

$$\mathbb{E}\{\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) - \mathfrak{q}_t^T\mathbf{Q}\mathfrak{q}_t - 2\mathfrak{q}_t^T\mathbf{S}\mathfrak{u}_t - \mathfrak{u}_t^T(\mathbf{R}-\gamma I)\mathfrak{u}_t\} \le \mathbb{E}\{\xi_{(t)}^T\Theta\xi_{(t)}\}. \tag{34}$$

where Θ are defined in (15) and $\xi_{(t)}$ defined in the main results.
Suppose $\Theta < 0$, it is easy to get

$$\mathbb{E}\{\mathfrak{q}_t^T \mathbf{Q} \mathfrak{q}_t - 2\mathfrak{q}_t^T \mathbf{S} \mathfrak{u}_t - \mathfrak{u}_t^T \mathbf{R} \mathfrak{u}_t\} \geq \mathbb{E}\{\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) + \gamma \mathfrak{u}_t^T \mathfrak{u}_t\}. \tag{35}$$

Integrating (35) from 0 to t_d, under zero initial conditions we obtain

$$\mathbb{E}\{\mathbf{G}(\mathfrak{q},\mathfrak{u},t_d)\} \geq \mathbb{E}\{\gamma\langle \mathfrak{u},\mathfrak{u}\rangle_{t_d} + \mathbb{V}(t_d,\mathfrak{p}_{(t_d)},x) - \mathbb{V}(0,\mathfrak{p}_{(0)},x)\} \geq \mathbb{E}\{\gamma\langle \mathfrak{u},\mathfrak{u}\rangle_{t_d}\}, \tag{36}$$

for all $t_d \geq 0$. Therefore, the NNs (6) is strictly $(\mathbf{Q},\mathbf{S},\mathbf{R}) - \gamma -$ dissipative in the sense of Definition (2). This completes the proof ■

Remark 10. *It ought to be mentioned that the network models are normally affected by stochastic disturbance. Therefore, numerous researchers have taken different types of NNs to the investigation of the stability with stochastic inputs. For example, local-field NNs [12], Cohen-Grossberg NNs [**?**], Markovian switching static NNs [13]. From the above conversation, it is easy to realize, without considering the generalized NNs, some stability results have been discussed in [12]-[13]. Its should be noted that the proposed problem in this paper has been considered a general form of the system model. Therefore, the results obtained in this literature is more general than [12]-[13].*

Remark 11. *It ought to be specified that the* $(\mathbf{Q},\mathbf{S},\mathbf{R}) - \gamma -$ *dissipativity contains multi-dynamic behaviors by setting the weight matrices in their structure. For example, choose* $\mathbf{Q} = 0, \mathbf{S} = \mathbf{I}$ *and* $\mathbf{R} = 2\gamma I$*, then* (10) *turns to the following passivity condition* $2\mathbb{E}\{\langle \mathfrak{q},\mathbf{S}\mathfrak{u}\rangle_{t_d}\} \geq -\gamma\mathbb{E}\{\langle \mathfrak{u},\mathfrak{u}\rangle_{t_d}\}$.

Corollary 12. *For given scalars* $\mathfrak{r}$ *and* μ*, the NNs* (6) *is passive, if there exist matrices* $\mathcal{P}_x(x \in S) > 0, \mathcal{Q} > 0, \mathcal{R} > 0, \mathcal{S} > 0, \mathcal{U} > 0$*, diagonal matrices* $\mathcal{H}_1 > 0, \mathcal{H}_2 > 0$*, any matrices* $\mathcal{G}_1, \mathcal{G}_2$ *and scalar* $\gamma > 0$ *such that the following LMIs holds for all* $(x \in S)$*:*

$$\mathcal{P}_x \leq \delta_x I, \tag{37}$$
$$\Theta_2 = (\bar{\Theta}_{i,j})_{12\times 12} < 0, \tag{38}$$

where
$\bar{\Theta}_{1,1,x} \triangleq \sum_{y=1}^{N} \pi_{xy}\mathcal{P}_y + \mathcal{Q} + \mathcal{R} + \mathfrak{r}^2\mathcal{U} + \frac{\mathfrak{r}^4}{4}\mathcal{V} + \delta_x\mathcal{L}_{1x} - \mathcal{G}_1\mathcal{D}_x - (\mathcal{G}_1\mathcal{D}_x)^T - \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\bar{\Theta}_{1,2,x} \triangleq \mathcal{P}_x - \mathcal{G}_1 - (\mathcal{G}_2\mathcal{D}_x)^T$, $\bar{\Theta}_{1,5,x} \triangleq \mathcal{G}_1\mathcal{A}_x + \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_2$, $\bar{\Theta}_{1,6,x} \triangleq \mathcal{G}_1\mathcal{B}_x$, $\bar{\Theta}_{1,12,x} \triangleq \mathcal{P}_x + \mathcal{G}_1$, $\bar{\Theta}_{2,2,x} \triangleq \mathfrak{r}\mathcal{W} - \mathcal{G}_2 - \mathcal{G}_2^T$, $\bar{\Theta}_{2,5,x} \triangleq \mathcal{G}_2\mathcal{A}_x$, $\bar{\Theta}_{2,6,x} \triangleq \mathcal{G}_2\mathcal{B}_x$, $\bar{\Theta}_{2,12,x} \triangleq \mathcal{G}_2$, $\bar{\Theta}_{3,3,x} \triangleq -(1-\mu)\mathcal{Q} + \delta_x\mathcal{L}_{2x} - \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\bar{\Theta}_{3,6,x} \triangleq \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_2$, $\bar{\Theta}_{4,4,x} \triangleq -\mathcal{R}$, $\bar{\Theta}_{5,5,x} \triangleq \mathcal{S} - \mathcal{H}_1$, $\bar{\Theta}_{5,12,x} \triangleq -I$, $\bar{\Theta}_{6,6,x} \triangleq -(1-\mu)\mathcal{S} - \mathcal{H}_2$, $\bar{\Theta}_{7,7,x} \triangleq -\frac{1}{\mathfrak{r}}\mathcal{W}$, $\bar{\Theta}_{8,8,x} \triangleq -9\mathcal{U}$, $\bar{\Theta}_{8,9,x} \triangleq \frac{36}{\mathfrak{r}}\mathcal{U}$, $\bar{\Theta}_{8,10,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{U}$, $\bar{\Theta}_{9,9,x} \triangleq -\frac{192}{\mathfrak{r}^2}\mathcal{U} - 6\mathcal{V}$, $\bar{\Theta}_{9,10,x} \triangleq \frac{360}{\mathfrak{r}^3}\mathcal{U} + \frac{30}{\mathfrak{r}}\mathcal{V}$, $\bar{\Theta}_{9,11,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{V}$, $\bar{\Theta}_{10,10,x} \triangleq -\frac{720}{\mathfrak{r}^4}\mathcal{U} - \frac{210}{\mathfrak{r}^2}\mathcal{V}$, $\bar{\Theta}_{10,11,x} \triangleq \frac{480}{\mathfrak{r}^3}\mathcal{V}$, $\bar{\Theta}_{11,11,x} \triangleq -\frac{1200}{\mathfrak{r}^4}\mathcal{V}$, $\bar{\Theta}_{12,12,x} \triangleq -\gamma I$.

Proof: Consider the similar LKF (16) and define the following passivity condition for the system (6).

$$2\int_0^{t_d} \mathbb{E}\{\mathfrak{q}_t^T \mathfrak{u}_t\}dt \geq -\gamma\int_0^{t_d} \mathbb{E}\{\mathfrak{u}_t^T \mathfrak{u}_t\}dt. \tag{39}$$

The following proof can be obtained by Theorem (9), we get

$$\mathbb{E}\{\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) - 2\mathfrak{q}_t^T \mathfrak{u}_t - \mathfrak{u}_t^T \gamma \mathfrak{u}_t\} \leq \mathbb{E}\{\xi_{(t)}^T \bar{\Theta} \xi_{(t)}\}. \tag{40}$$

Hence $\bar{\Theta} < 0$ holds, then (40) implies that

$$\mathbb{E}\{\mathcal{L}\mathbb{V}(t,\mathfrak{p}_t,x) - 2\mathfrak{q}_t^T \mathfrak{u}_t - \mathfrak{u}_t^T \gamma \mathfrak{u}_t\} \leq 0. \tag{41}$$

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Integrating (41) from 0 to t_d, under zero initial conditions we obtain

$$2\int_0^{t_d}\mathbb{E}\{\mathfrak{q}_t^T\mathfrak{u}_t\}dt \geq \mathbb{E}\{\mathbb{V}(t_d,\mathfrak{p}_{(t_d)},x)-\mathbb{V}(0,\mathfrak{p}_{(0)},x)-\int_0^{t_d}\mathfrak{u}_t^T(\gamma I)\mathfrak{u}_t dt\} \geq -\int_0^{t_d}\mathbb{E}\{\mathfrak{u}_t^T(\gamma I)\mathfrak{u}_t dt\} \quad (42)$$

for all $t_d \geq 0$. Therefore, the NNs (6) is passive in the sense of Definition (4). This completes the proof ■

Remark 13. *When we consider $\mathfrak{u}_t = 0$, then NNs (6) turns to*

$$\begin{cases} d\mathfrak{p}(t) = & [-\mathcal{D}_x\mathfrak{p}(t)+\mathcal{A}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t))+\mathcal{B}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t-\mathfrak{r}(t)))]dt \\ & +\sigma(t,x,\mathfrak{p}(t),\mathfrak{p}(t-\mathfrak{r}(t)))d\omega(t) \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)). \end{cases} \quad (43)$$

Based on Theorem (9), the following Corollary (14) can be obtained.

Corollary 14. *For given scalars $\mathfrak{r}$ and μ, the NNs (43) is globally asymptotically stable in the mean square, if there exist matrices $\mathcal{P}_x(x\in S)>0, \mathcal{Q}>0, \mathcal{R}>0, \mathcal{S}>0, \mathcal{U}>0$, diagonal matrices $\mathcal{H}_1>0, \mathcal{H}_2>0$, any matrices $\mathcal{G}_1,\mathcal{G}_2$ such that the following LMIs holds for all $(x\in S)$:*

$$\mathcal{P}_x \leq \delta_x I, \quad (44)$$
$$\Theta_3 = (\breve{\Theta}_{i,j})_{11\times 11} < 0, \quad (45)$$

where
$\breve{\Theta}_{1,1,x} \triangleq \sum_{y=1}^{N}\pi_{xy}\mathcal{P}_y + \mathcal{Q}+\mathcal{R}+\mathfrak{r}^2\mathcal{U}+\frac{\mathfrak{r}^4}{4}\mathcal{V}+\delta_x\mathcal{L}_{1x}-\mathcal{G}_1\mathcal{D}_x-(\mathcal{G}_1\mathcal{D}_x)^T-\mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\breve{\Theta}_{1,2,x} \triangleq \mathcal{P}_x-\mathcal{G}_1-(\mathcal{G}_2\mathcal{D}_x)^T$, $\breve{\Theta}_{1,5,x} \triangleq \mathcal{G}_1\mathcal{A}_x+\mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_2$, $\breve{\Theta}_{1,6,x} \triangleq \mathcal{G}_1\mathcal{B}_x$, $\breve{\Theta}_{2,2,x} \triangleq \mathfrak{r}\mathcal{W}-\mathcal{G}_2-\mathcal{G}_2^T$, $\breve{\Theta}_{2,5,x} \triangleq \mathcal{G}_2\mathcal{A}_x$, $\breve{\Theta}_{2,6,x} \triangleq \mathcal{G}_2\mathcal{B}_x$, $\breve{\Theta}_{3,3,x} \triangleq -(1-\mu)\mathcal{Q}+\delta_x\mathcal{L}_{2x}-\mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\breve{\Theta}_{3,6,x} \triangleq \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_2$, $\breve{\Theta}_{4,4,x} \triangleq -\mathcal{R}$, $\breve{\Theta}_{5,5,x} \triangleq \mathcal{S}-\mathcal{H}_1$, $\breve{\Theta}_{6,6,x} \triangleq -(1-\mu)\mathcal{S}-\mathcal{H}_2$, $\breve{\Theta}_{7,7,x} \triangleq -\frac{1}{\mathfrak{r}}\mathcal{W}$, $\breve{\Theta}_{8,8,x} \triangleq -9\mathcal{U}$, $\breve{\Theta}_{8,9,x} \triangleq \frac{36}{\mathfrak{r}}\mathcal{U}$, $\breve{\Theta}_{8,10,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{U}$, $\breve{\Theta}_{9,9,x} \triangleq -\frac{192}{\mathfrak{r}^2}\mathcal{U}-6\mathcal{V}$, $\breve{\Theta}_{9,10,x} \triangleq \frac{360}{\mathfrak{r}^3}\mathcal{U}+\frac{30}{\mathfrak{r}}\mathcal{V}$, $\breve{\Theta}_{9,11,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{V}$, $\breve{\Theta}_{10,10,x} \triangleq -\frac{720}{\mathfrak{r}^4}\mathcal{U}-\frac{210}{\mathfrak{r}^2}\mathcal{V}$, $\breve{\Theta}_{10,11,x} \triangleq \frac{480}{\mathfrak{r}^3}\mathcal{V}$, $\breve{\Theta}_{11,11,x} \triangleq -\frac{1200}{\mathfrak{r}^4}\mathcal{V}$.

Remark 15. *Suppose there has no stochastic disturbance and $\mathfrak{u}_t = 0$, then NNs (6) turns to*

$$\begin{cases} d\mathfrak{p}(t) = & [-\mathcal{D}_x\mathfrak{p}(t)+\mathcal{A}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t))+\mathcal{B}_x\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t-\mathfrak{r}(t)))]dt \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)). \end{cases} \quad (46)$$

Based on Theorem (9), the following Corollary (16) can be obtained.

Corollary 16. *For given scalars $\mathfrak{r}$ and μ, the NNs (46) is globally asymptotically stable, if there exist matrices $\mathcal{P}_x(x\in S)>0, \mathcal{Q}>0, \mathcal{R}>0, \mathcal{S}>0, \mathcal{U}>0$, diagonal matrices $\mathcal{H}_1>0, \mathcal{H}_2>0$, any matrices $\mathcal{G}_1,\mathcal{G}_2$ such that the following LMIs holds for all $(x\in S)$:*

$$\Theta_4 = (\tilde{\Theta}_{i,j})_{11\times 11} < 0, \quad (47)$$

where
$\tilde{\Theta}_{1,1,x} \triangleq \sum_{y=1}^{N}\pi_{xy}\mathcal{P}_y + \mathcal{Q}+\mathcal{R}+\mathfrak{r}^2\mathcal{U}+\frac{\mathfrak{r}^4}{4}\mathcal{V}+\delta_x\mathcal{L}_{1x}-\mathcal{G}_1\mathcal{D}_x-(\mathcal{G}_1\mathcal{D}_x)^T-\mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\tilde{\Theta}_{1,2,x} \triangleq \mathcal{P}_x-\mathcal{G}_1-(\mathcal{G}_2\mathcal{D}_x)^T$, $\tilde{\Theta}_{1,5,x} \triangleq \mathcal{G}_1\mathcal{A}_x+\mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_2$, $\tilde{\Theta}_{1,6,x} \triangleq \mathcal{G}_1\mathcal{B}_x$, $\tilde{\Theta}_{2,2,x} \triangleq \mathfrak{r}\mathcal{W}-\mathcal{G}_2-\mathcal{G}_2^T$, $\tilde{\Theta}_{2,5,x} \triangleq \mathcal{G}_2\mathcal{A}_x$, $\tilde{\Theta}_{2,6,x} \triangleq \mathcal{G}_2\mathcal{B}_x$, $\tilde{\Theta}_{3,3,x} \triangleq -(1-\mu)\mathcal{Q}+\delta_x\mathcal{L}_{2x}-\mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\tilde{\Theta}_{3,6,x} \triangleq \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_2$, $\tilde{\Theta}_{4,4,x} \triangleq -\mathcal{R}$, $\tilde{\Theta}_{5,5,x} \triangleq \mathcal{S}-\mathcal{H}_1$, $\tilde{\Theta}_{6,6,x} \triangleq -(1-\mu)\mathcal{S}-\mathcal{H}_2$, $\tilde{\Theta}_{7,7,x} \triangleq -\frac{1}{\mathfrak{r}}\mathcal{W}$, $\tilde{\Theta}_{8,8,x} \triangleq -9\mathcal{U}$, $\tilde{\Theta}_{8,9,x} \triangleq \frac{36}{\mathfrak{r}}\mathcal{U}$,

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$\tilde{\Theta}_{8,10,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{U}$, $\tilde{\Theta}_{9,9,x} \triangleq -\frac{192}{\mathfrak{r}^2}\mathcal{U} - 6\mathcal{V}$, $\tilde{\Theta}_{9,10,x} \triangleq \frac{360}{\mathfrak{r}^3}\mathcal{U} + \frac{30}{\mathfrak{r}}\mathcal{V}$, $\tilde{\Theta}_{9,11,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{V}$, $\tilde{\Theta}_{10,10,x} \triangleq -\frac{720}{\mathfrak{r}^4}\mathcal{U} - \frac{210}{\mathfrak{r}^2}\mathcal{V}$, $\tilde{\Theta}_{10,11,x} \triangleq \frac{480}{\mathfrak{r}^3}\mathcal{V}$, $\tilde{\Theta}_{11,11,x} \triangleq -\frac{1200}{\mathfrak{r}^4}\mathcal{V}$.

3.2. Robust dissipativity analysis

In this subsection, we extend the previous dissipativity condition to robust dissipativity analysis of the following uncertain NNs:

$$\begin{cases} d\mathfrak{p}(t) = & [-(\mathcal{D}_x + \Delta\mathcal{D}_x(t))\mathfrak{p}(t) + (\mathcal{A}_x + \Delta\mathcal{A}_x(t))\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)) + (\mathcal{B}_x + \Delta\mathcal{B}_x(t))\mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t-\mathfrak{r}(t))) \\ & +\mathfrak{u}(t)]dt + \sigma(t,x,\mathfrak{p}(t),\mathfrak{p}(t-\mathfrak{r}(t)))d\omega(t) \\ \mathfrak{q}(t) = & \mathfrak{g}(\mathcal{W}_x\mathfrak{p}(t)). \end{cases} \tag{48}$$

where $\Delta\mathcal{D}_x(t), \Delta\mathcal{D}_x(t)$ and $\Delta\mathcal{D}_x(t)$ are the time-varying parameter uncertainties, which are assumed to be of the form

$$[\Delta\mathcal{D}_x(t)\ \Delta\mathcal{A}_x(t)\ \Delta\mathcal{B}_x(t)] = \mathcal{H}_x\mathcal{F}_x(t)[\mathcal{E}_{1x}\ \mathcal{E}_{2x}\ \mathcal{E}_{3x}], \tag{49}$$

where $\mathcal{H}, \mathcal{E}_{1x}, \mathcal{E}_{2x}$ and $\mathcal{E}_{3x}$ are known real constant matrices, and $\mathcal{F}_x(t)$ is an unknown time-varying matrix function satisfying $\mathcal{F}_x(t)^T\mathcal{F}_x(t) \leq I$.
Based on Theorem (9) the following Theorem (17) can be derived.

Theorem 17. *For given scalars $\mathfrak{r}$ and μ, the NNs (48) is $(\mathbf{Q},\mathbf{S},\mathbf{R}) - \gamma$ dissipative, if there exist matrices $\mathcal{P}_x(x \in S) > 0, \mathcal{Q} > 0, \mathcal{R} > 0, \mathcal{S} > 0, \mathcal{U} > 0$, diagonal matrices $\mathcal{H}_1 > 0, \mathcal{H}_2 > 0$, any matrices $\mathcal{G}_1, \mathcal{G}_2$ and scalar $\gamma > 0$ such that the following LMIs holds for all $(x \in S)$:*

$$\mathcal{P}_x \leq \delta_x I, \tag{50}$$

$$\Theta_5 = \begin{bmatrix} \widehat{\Theta} & \Gamma_1 & \varepsilon\Gamma_2 \\ \maltese & -\varepsilon I & 0 \\ \maltese & \maltese & -\varepsilon I \end{bmatrix} < 0. \tag{51}$$

where
$\widehat{\Theta}_{1,1,x} \triangleq \sum_{y=1}^{N}\pi_{xy}\mathcal{P}_y + \mathcal{Q} + \mathcal{R} + \mathfrak{r}^2\mathcal{U} + \frac{\mathfrak{r}^4}{4}\mathcal{V} + \delta_x\mathcal{L}_{1x} - \mathcal{G}_1\mathcal{D}_x - (\mathcal{G}_1\mathcal{D}_x)^T - \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\widehat{\Theta}_{1,2,x} \triangleq \mathcal{P}_x - \mathcal{G}_1 - (\mathcal{G}_2\mathcal{D}_x)^T$, $\widehat{\Theta}_{1,5,x} \triangleq \mathcal{G}_1\mathcal{A}_x + \mathcal{H}_1\mathcal{W}_x^T\mathcal{K}_2$, $\widehat{\Theta}_{1,6,x} \triangleq \mathcal{G}_1\mathcal{B}_x$, $\widehat{\Theta}_{1,12,x} \triangleq \mathcal{P}_x + \mathcal{G}_1$, $\widehat{\Theta}_{2,2,x} \triangleq \mathfrak{r}\mathcal{W} - \mathcal{G}_2 - \mathcal{G}_2^T$, $\widehat{\Theta}_{2,5,x} \triangleq \mathcal{G}_2\mathcal{A}_x$, $\widehat{\Theta}_{2,6,x} \triangleq \mathcal{G}_2\mathcal{B}_x$, $\widehat{\Theta}_{2,12,x} \triangleq \mathcal{G}_2$, $\widehat{\Theta}_{3,3,x} \triangleq -(1-\mu)\mathcal{Q} + \delta_x\mathcal{L}_{2x} - \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_1\mathcal{W}_x$, $\widehat{\Theta}_{3,6,x} \triangleq \mathcal{H}_2\mathcal{W}_x^T\mathcal{K}_2$, $\widehat{\Theta}_{4,4,x} \triangleq -\mathcal{R}$, $\widehat{\Theta}_{5,5,x} \triangleq \mathcal{S} - \mathcal{H}_1 - \mathbf{Q}$, $\widehat{\Theta}_{5,12,x} \triangleq -\mathbf{S}$, $\widehat{\Theta}_{6,6,x} \triangleq -(1-\mu)\mathcal{S} - \mathcal{H}_2$, $\widehat{\Theta}_{7,7,x} \triangleq -\frac{1}{\mathfrak{r}}\mathcal{W}$, $\widehat{\Theta}_{8,8,x} \triangleq -9\mathcal{U}$, $\widehat{\Theta}_{8,9,x} \triangleq \frac{36}{\mathfrak{r}}\mathcal{U}$, $\widehat{\Theta}_{8,10,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{U}$, $\widehat{\Theta}_{9,9,x} \triangleq -\frac{192}{\mathfrak{r}^2}\mathcal{U} - 6\mathcal{V}$, $\widehat{\Theta}_{9,10,x} \triangleq \frac{360}{\mathfrak{r}^3}\mathcal{U} + \frac{30}{\mathfrak{r}}\mathcal{V}$, $\widehat{\Theta}_{9,11,x} \triangleq -\frac{60}{\mathfrak{r}^2}\mathcal{V}$, $\widehat{\Theta}_{10,10,x} \triangleq -\frac{720}{\mathfrak{r}^4}\mathcal{U} - \frac{210}{\mathfrak{r}^2}\mathcal{V}$, $\widehat{\Theta}_{10,11,x} \triangleq \frac{480}{\mathfrak{r}^3}\mathcal{V}$, $\widehat{\Theta}_{11,11,x} \triangleq -\frac{1200}{\mathfrak{r}^4}\mathcal{V}$, $\widehat{\Theta}_{12,12,x} \triangleq -\mathbf{R} + \gamma I$. $\Gamma_1 = [\mathcal{G}_{1x}^T\mathcal{H}_x^T\ \ \mathcal{G}_{2x}^T\mathcal{H}_x^T\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$,
$\Gamma_2 = [\mathcal{E}_{1x}^T\ \ 0\ 0\ 0\ \ \mathcal{E}_{2x}^T\ \ \mathcal{E}_{3x}^T\ \ 0\ 0\ 0\ 0\ 0\ 0]^T$.

Proof: Replacing $\mathcal{D}_x, \mathcal{A}_x, \mathcal{B}_x$, in LMI (15) with $(\mathcal{D}_x + \Delta\mathcal{D}_x(t)), (\mathcal{A}_x + \Delta\mathcal{A}_x(t)), (\mathcal{B}_x + \Delta\mathcal{B}_x(t))$ yields

$$\widehat{\Theta} + (\Gamma_1\mathcal{F}(t)\Gamma_2) + (\Gamma_1\mathcal{F}(t)\Gamma_2)^T < 0. \tag{52}$$

By using Lemma (8) there exists a scalar $\varepsilon > 0$, such that

$$\widehat{\Theta} + \varepsilon^{-1}\Gamma_1\Gamma_1^T + \varepsilon\Gamma_2^T\Gamma_2 < 0. \tag{53}$$

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where

$$\Gamma_1 = [\mathcal{G}_{1x}^T\mathcal{H}_x^T \;\; \mathcal{G}_{2x}^T\mathcal{H}_x^T \;\; 0\;0\;0\;0\;0\;0\;0\;0\;0\;0]^T,$$
$$\Gamma_2 = [\mathcal{E}_{1x}^T \;\; 0\;0\;0 \;\; \mathcal{E}_{2x}^T \;\; \mathcal{E}_{3x}^T \;\; 0\;0\;0\;0\;0\;0]^T.$$

By Schur complement lemma, inequality (53) is equivalent to inequalities (51). It completes the proof.

4. Illustrative examples

In this section, three numerical examples and their simulations are presented to illustrate the effectiveness of the obtained results.

Example 1. Consider the NNs (6) with the following two modes:

$$\mathcal{D}_1 = \begin{bmatrix} 2.3 & 0 \\ 0 & 0.9 \end{bmatrix}, \; \mathcal{A}_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & -0.2 \end{bmatrix}, \; \mathcal{B}_1 = \begin{bmatrix} 0.2 & -0.3 \\ 0.4 & 0.2 \end{bmatrix},$$
$$\mathcal{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \mathcal{L}_{11} = \begin{bmatrix} 0.22 & 0 \\ 0 & 0.22 \end{bmatrix}, \; \mathcal{L}_{21} = \begin{bmatrix} 0.18 & 0 \\ 0 & 0.18 \end{bmatrix},$$
$$\mathcal{D}_2 = \begin{bmatrix} 1.9 & 0 \\ 0 & 2 \end{bmatrix}, \; \mathcal{A}_2 = \begin{bmatrix} 0.3 & 0.5 \\ -0.2 & 0.1 \end{bmatrix}, \; \mathcal{B}_2 = \begin{bmatrix} 0.3 & 0.2 \\ -0.3 & 0.5 \end{bmatrix},$$
$$\mathcal{W}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \mathcal{L}_{12} = \begin{bmatrix} 0.20 & 0 \\ 0 & 0.20 \end{bmatrix}, \; \mathcal{L}_{22} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}.$$

Moreover we take,

$$\mathcal{Q} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}, \; \mathcal{S} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.5 \end{bmatrix}, \; \mathcal{R} = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}.$$

Let $\Pi = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$ and $\mathfrak{r}(t) = 0.2 + 0.1sint$ which satisfies $\mathfrak{r} = 0.3$, $\mu = 0.2$. Further, choose $\mathfrak{g}_i(\mathfrak{p}_i(t)) = tanh(\mathfrak{p}_i(t))$, $i = 1,2$, then we can get $\Delta_1 = 0$, $\Delta_2 = I$, from (32) we have $\mathcal{K}_1 = 0$, $\mathcal{K}_2 = -0.5I$. LMIs (14)-(15) can be solved by MATLAB. The initial values are chosen as $\mathfrak{p}(0) = [0.3, -0.6]^T$, when taking $\mathfrak{u}(t) = 0.01e^{-t}sin(0.02t)$ $t > 0$, the following simulation results can be obtained. Figure 1 shows that time responses of the system (6), Figure 2 depicts the transient response of state of the system (6) and Figure 3 describes the Markovian switching signal.

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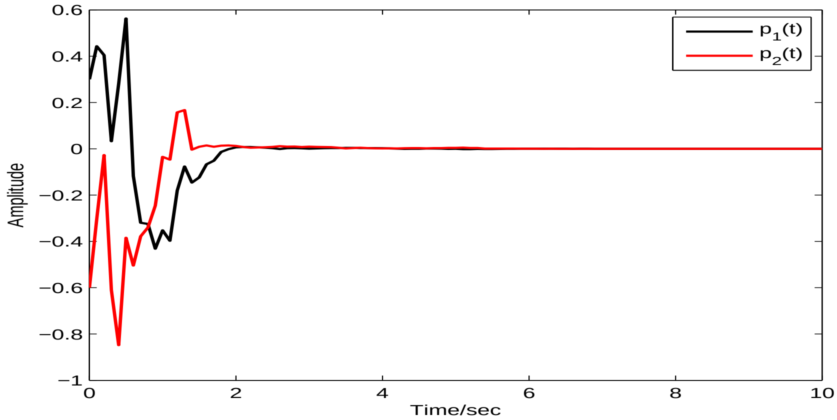

Figure 1. Time responses of the NNs (6) in Example 1.

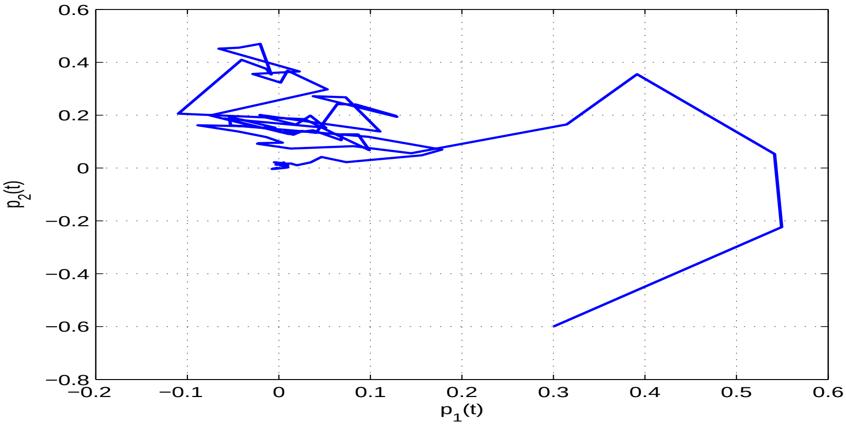

Figure 2. Transient response of state variables $\mathfrak{p}_1(t)$, $\mathfrak{p}_2(t)$ in Example 1.

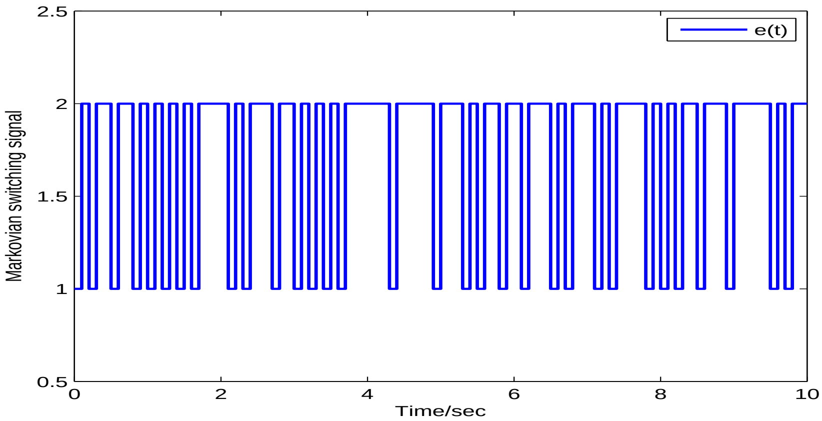

Figure 3. Markovian switching signal $\mathfrak{e}(t)$ in Example 1.

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Example 2. Consider the GNNs (46) with $x = 1$ and the following parameters:

$$\mathcal{A} = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3894 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$\mathcal{D} = diag\{1.2769, 0.6231, 0.9230, 0.4480\}, \ \mathcal{W} = diag\{1, 1, 1, 1\}.$$

Take $\mathfrak{g}_i(\mathfrak{p}_i(t)) = 0.3\, tanh(\mathfrak{p}_i(t)),\ i = 1, 2, 3, 4$, then we can get $\Delta_1 = 0,\ \Delta_2 = diag\{0.1137, 0.1279, 0.7994, 0.2368\}$, from (32) we have $\mathcal{K}_1 = 0,\ \mathcal{K}_2 = diag\{-\frac{0.1137}{2}, -\frac{0.1279}{2}, -\frac{0.7994}{2}, -\frac{0.2368}{2}\}$. LMI (47) can be solved by MATLAB. For various μ, the maximum permissible delay limit $\mathfrak{r}$ is listed in Table 1. From Table 1, it is easy to realize that the obtained result in this paper is less conservative than those results discussed in [34], [35], [36]. For the simulation purpose, take $\mathfrak{r}(t) = 2.6272 + 0.2 sint$ which satisfies $\mathfrak{r} = 2.8272$. Under the initial conditions $\mathfrak{p}(0) = [1.5, 1, -1.6, 0.5]^T$ the following simulation can be obtained. The time responses of the NNs (46) with $x = 1$ is given in Figure 4. According to the Corollary (16) the GNNs (46) is globally asymptotically stable.

Table 1: The maximum permissible delay limit $\mathfrak{r}$ for different μ.

Methods	μ	0.1	0.5	0.9
[34]	$\mathfrak{r}$	3.8739	2.7821	2.3279
[35]	$\mathfrak{r}$	4.1903	3.0779	2.8268
[36]	$\mathfrak{r}$	4.1919	3.0790	2.8271
Corollary (16)	$\mathfrak{r}$	4.1920	3.0791	2.8272

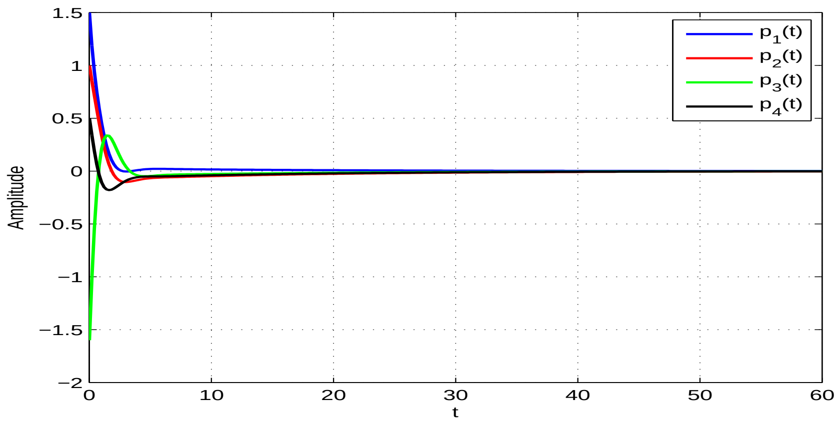

Figure 4. Time responses of the GNNs (46) in Example 2.

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Example 3. Consider the NNs (48) with the following two modes:

$$\mathcal{D}_1 = \begin{bmatrix} 2.3 & 0 \\ 0 & 0.9 \end{bmatrix}, \ \mathcal{A}_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & -0.2 \end{bmatrix}, \ \mathcal{B}_1 = \begin{bmatrix} 0.2 & -0.3 \\ 0.4 & 0.2 \end{bmatrix}, \ \mathcal{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathcal{L}_{11} = \begin{bmatrix} 0.22 & 0 \\ 0 & 0.22 \end{bmatrix}, \ \mathcal{L}_{21} = \begin{bmatrix} 0.18 & 0 \\ 0 & 0.18 \end{bmatrix}, \ \mathcal{H}_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$\mathcal{E}_{11} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \ \mathcal{E}_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \ \mathcal{E}_{31} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\mathcal{D}_2 = \begin{bmatrix} 1.9 & 0 \\ 0 & 2 \end{bmatrix}, \ \mathcal{A}_2 = \begin{bmatrix} 0.3 & 0.5 \\ -0.2 & 0.1 \end{bmatrix}, \ \mathcal{B}_2 = \begin{bmatrix} 0.3 & 0.2 \\ -0.3 & 0.5 \end{bmatrix}, \ \mathcal{W}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathcal{L}_{12} = \begin{bmatrix} 0.20 & 0 \\ 0 & 0.20 \end{bmatrix}, \ \mathcal{L}_{22} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, \ \mathcal{H}_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$\mathcal{E}_{12} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \ \mathcal{E}_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \ \mathcal{E}_{32} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

Moreover we take,

$$\mathcal{Q} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \mathcal{S} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \mathcal{R} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.$$

Let $\Pi = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$ and $\mathfrak{r}(t) = 0.2 + 0.1sint$ which satisfies $\mathfrak{r} = 0.3$, $\mu = 0.2$. Further, choose $\mathfrak{g}_i(\mathfrak{p}_i(t)) = tanh(\mathfrak{p}_i(t))$, $i = 1,2$, then we can get $\Delta_1 = 0$, $\Delta_2 = I$, from (32) we have $\mathcal{K}_1 = 0$, $\mathcal{K}_2 = -0.5I$. LMIs (50)-(51) can be solved by MATLAB. The initial values are chosen as $\mathfrak{p}(0) = [0.3, -0.6]^T$, when taking $\mathfrak{u}(t) = 0.01e^{-t}sin(0.02t)$ $t > 0$, the following simulation results can be obtained. Figure 5 shows that time responses of the system (48), Figure 6 depicts the state of the system (48) under given initial values and and Figure 7 describes the Markovian switching signal.

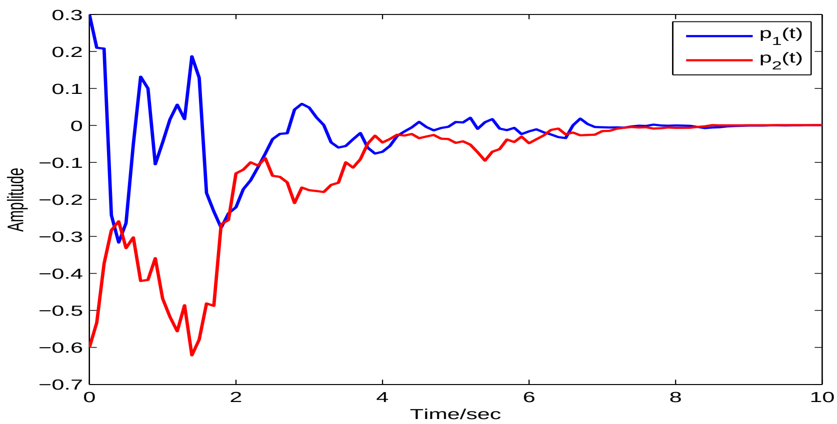

Figure 5. Time responses of the NNs (48) in Example 3.

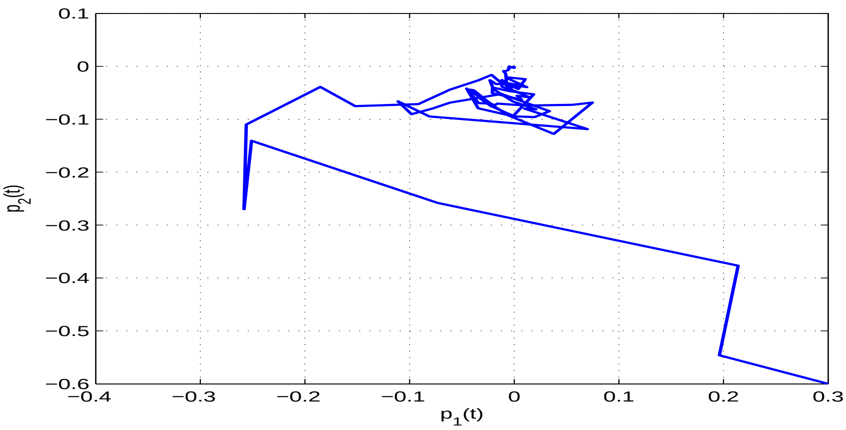

Figure 6. Transient response of state variables $\mathfrak{p}_1(t)$, $\mathfrak{p}_2(t)$ in Example 3.

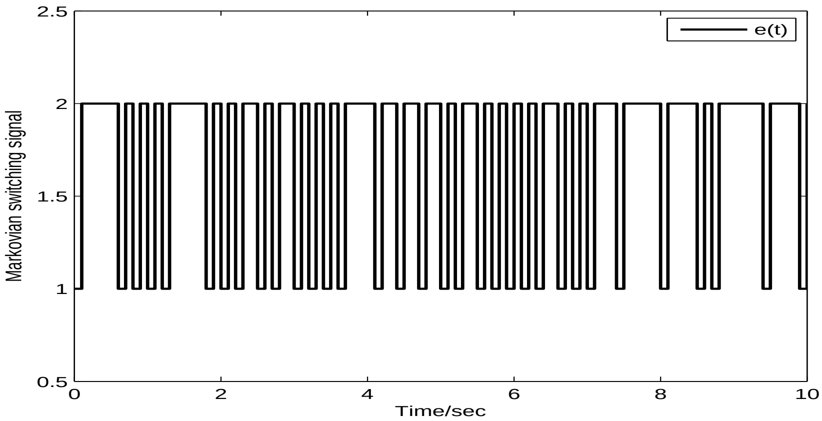

Figure 7. Markovian switching signal $\mathfrak{e}(t)$ in Example 3.

5. Conclusion

In this article, the problem of dissipativity and stability of USGNNs with Markovian jumping parameters has been investigated. In order to handle this problem easily, an appropriate LKF is constructed and by employing effective integral inequalities, sector bound activation function, Itô's formula and some analytical techniques, several LMI-based sufficient conditions are derived, whose feasible solution can be verified by MATLAB. Finally, three numerical examples and their simulations are discussed to demonstrate the feasibility and effectiveness of the obtained analytical results. Moreover, it is possible to investigate include the stability and synchronization analysis of various NNs including complex-valued NNs. Thus, in the future work, we will extend the present results to the investigations of stability and synchronization analysis of stochastic complex-valued discrete-time NNs. This will occur in the near future.

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