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Peer-reviewed version available at *Materials* **2020**, *13*, 1890; doi:10.3390/ma13081890

Negative Effective Mass in Plasmonic Systems

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Abstract

We report the negative effective mass metamaterials based on the electro-mechanical coupling exploiting plasma oscillations of a free electron gas. The negative mass appears as a result of vibration of a metallic particle with a frequency of ω which is close the frequency of the plasma oscillations of the electron gas m_2 relatively to the ionic lattice m_1. The plasma oscillations are represented with the elastic spring $k_2 = \omega_p^2 m_2$, where ω_p is the plasma frequency. Thus, the metallic particle vibrated with the external frequency ω is described by the effective mass $m_{eff} = m_1 + \frac{m_2\omega_p^2}{\omega_p^2-\omega^2}$, which is negative when the frequency ω approaches ω_p from above. The idea is exemplified with two conducting metals, namely Au and Li.

Keywords: metamaterials; negative effective mass; plasma oscillations; low frequency plasmons.

Introduction

Metamaterials are recently developed artificial materials demonstrating properties that is not found in naturally occurring materials [1-2]. The domain of metamaterials covers a broad diversity of fields in physics and engineering: electromagnetics, acoustics, mechanics and thermodynamics. All "natural" materials have positive electrical permittivity, magnetic permeability and an index of refraction [1-3]. In metamaterials the index of refraction and magnetic permittivity may be negative. Moreover, they may be tuned in a broad range of values [4].

The relatively new kind of metamaterials are acoustic ones [5-10]. Acoustic metamaterial, in which both the effective density and bulk modulus are simultaneously negative, in the true and strict sense of an effective medium have been reported [5]. Acoustic metamaterials demonstrating the negative Poisson's ratio were discussed [10]. Acoustic metamaterials demonstrate a potential as perfect absorbers of mechanical vibrations [11] and also as materials enabling focusing of ultrasound [7]. The present paper introduces the negative effective mass metamaterials based on the mechano-electromagnetic coupling. The idea of the negative effective mass (density) acoustic

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metamaterials was demonstrated and discussed in refs. 12-13. We propose to exploit the plasma oscillations of the electron gas [14] for the development of the metamaterials with the negative effective mass (density) [13,15].

1. Results and discussion

1.1. Negative effective mass and plasma oscillations in metals

The mechanical model giving rise to the negative effective mass effect is depicted in **Figure 1**. A core with mass m_2 is connected internally through the spring with k_2 to a shell with mass m_1. The system is subjected to the external sinusoidal force $F = \hat{F}sin\omega t$. If we solve the equations of motion for the masses m_1 and m_2 and replace the entire system with a single effective mass m_{eff} we obtain [12-13, 15]:

$$m_{eff} = m_1 + \frac{m_2\omega_0^2}{\omega_0^2-\omega^2} \quad , \tag{1}$$

where $\omega_0 = \sqrt{\frac{k_2}{m_2}}$. Obviously, when the frequency ω approaches ω_0 from above the effective mass m_{eff} will be negative [12-13, 15]. Now consider the electro-mechanical analogy of the aforementioned model, giving rise to the negative effective mass. Consider cubic metal particle, seen as ionic lattice m_1 containing the Drude-Lorenz free electrons gas possessing the total mass of $m_2 = m_e nV$, where $m_e = 9.1 \times 10^{-31}$kg is the mass of electron, n is the concentration (number density) of the electron gas and V is the volume of the particle [14,16,17]. Electron gas is free to oscillate with the plasma frequency $\omega_p = \sqrt{\frac{ne^2}{m_e\varepsilon_0}}$ [14-15].

Expose the entire metal particle to the external sinusoidal force $F = \hat{F}sin\omega t$. The effective mechanical scheme of the metallic particle is shown in **Figure 1B** (the right sketch) and it exactly coincides with that giving rise to the negative effective mass, supplied in this case by:

$$m_{eff} = m_1 + \frac{m_2\omega_p^2}{\omega_p^2-\omega^2} \quad , \tag{2}$$

where m_1 is the mass of the ionic lattice, m_2 is the total mass of the electronic gas and $k_2 = \omega_p^2 m_2$; it is seen that it may be negative when the frequency ω approaches ω_p from above. Considering $\frac{m_2}{m_1} \ll 1$ yields:

$$\frac{m_{eff}}{m_1+m_2} \cong \frac{m_{eff}}{m_1} \cong 1 + \frac{m_2}{m_1}\frac{\omega_p^2}{\omega_p^2-\omega^2} \tag{3}$$

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It is clear from Eq. 3 that the effective dimensionless mass $\frac{m_{eff}}{m_1+m_2} \cong \frac{m_{eff}}{m_1}$ depends only on the ratio $\frac{m_2}{m_1}$; thus, it is independent on the metallic particles' size. Thus for the purposes of calculation m_2 is taken as the mass of electron m_e, and m_1 is the mass of the atom of metal (see **Table 1**). The dependence of the dimensionless mass $m_{eff}/(m_1 + m_2)$ on the dimensionless frequency ω/ω_p for two model metals Li and Au is plotted in **Figure 2** (the data relevant to these metals is supplied in **Table 1**). The macro-scale values of the "plasma spring" constant $k_2 \cong 10^2 \frac{N}{m}$ are noteworthy. The dependencies of the effective mass m_{eff} on the difference $\Delta\omega = \omega - \omega_p$ calculated for Li and Au are presented in **Figures 3-4**.

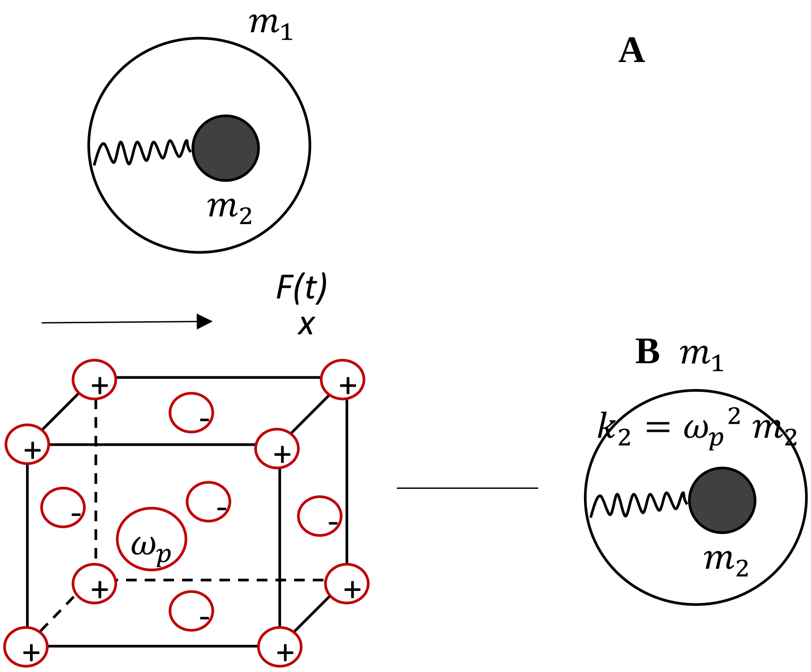

Figure 1. A. Core with mass m_2 is connected internally through the spring with k_2 to a shell with mass m_1. The system is subjected to the sinusoidal force $F(t) = \hat{F} sin\omega t$. **B.** Free electrons gas m_2 is embedded into the ionic lattice m_1; ω_p is the plasma frequency (the left sketch). The equivalent mechanical scheme of the system (right sketch).

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Table 1. Material constants used in calculations.

Metal	m_1, kg	m_2, kg	n, m^{-3}	ω_p, Hz	$k_2 = \omega_p^2 m_2$ N/m
Li	1.17×10^{-26}	9.1×10^{-31}	4.7×10^{28}	1.0×10^{16}	90.0
Au	3.27×10^{-25}	9.1×10^{-31}	5.9×10^{28}	1.3×10^{16}	152.1

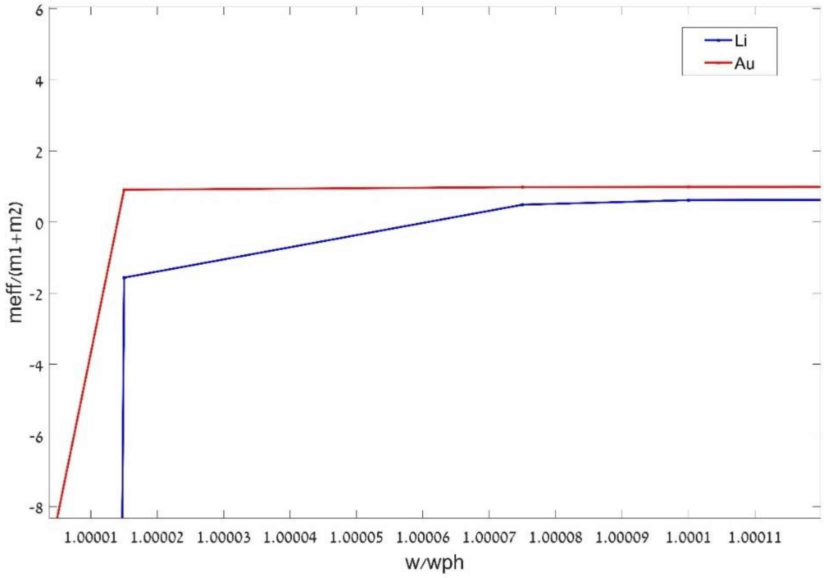

Figure 2. The dependence of the dimensionless mass $m_{eff}/(m_1+m_2)$ on the ratio ω/ω_p is plotted; the red line corresponds to Au; the blue line corresponds to Li.

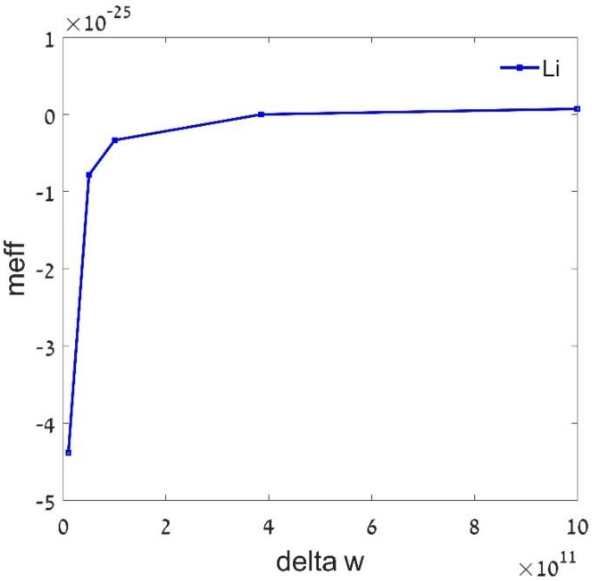

Figure 3. The dependence of the effective mass calculated for Li on the $\Delta\omega = \omega - \omega_p$.

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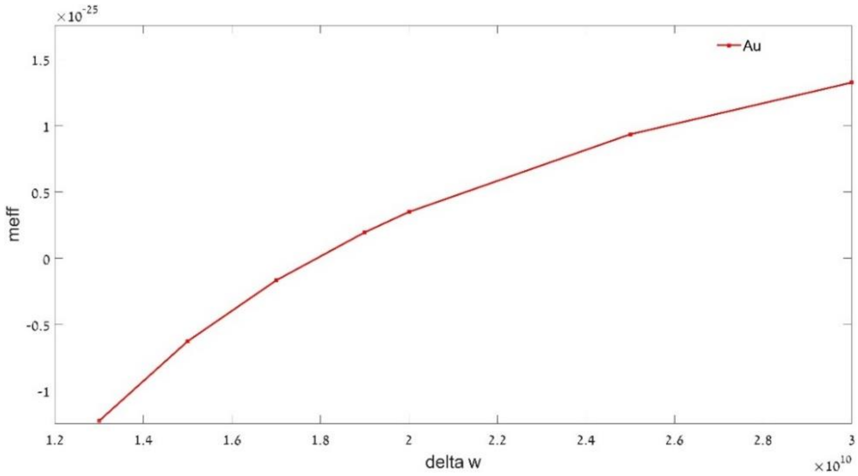

Figure 4. The dependence of the effective mass calculated for Au on the $\Delta\omega = \omega - \omega_p$.

1.2. Negative mass and low frequency plasmons in 1D metallic meso-structures

The plasma oscillations shown in **Figure 1** will demonstrate the negative mass in the vicinity of the plasma frequency which is on the order of magnitude of $\omega_p \cong 10^{16}$Hz , which is very high. However, this frequency may be decreased very strongly for meso-structures built of thin metallic wires, as demonstrated in Ref. 18. Depression of the plasma frequency into the far infrared and even GHZ band becomes possible due to the mutual inductance appearing in the periodic arrays built of thin metallic wires [18]. Consider 1D lattice built of the metallic wires with diameter *2r* connected with springs k_1 as depicted in **Figure 5**.The effective (pseudo) density of electrons in the metamaterial lattice shown in **Figure 5** is given by [18]:

$$\tilde{n} \cong \pi n \frac{r^2}{a^2}, \qquad (4)$$

where *n* is the concentration of the free electron gas supplied in **Table 1** for Li and Au.

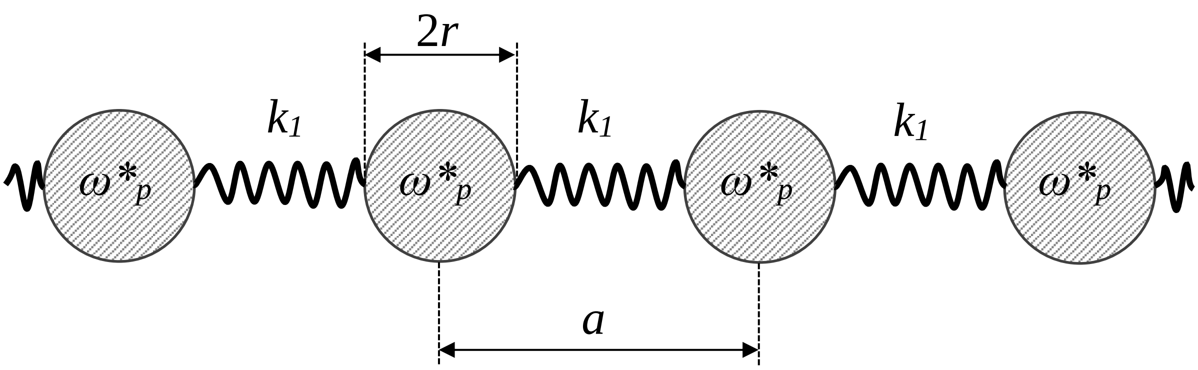

Figure 5. 1D lattice built of metallic wires $2r$ connected with springs k_1. The separation between wires is a.

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The pseudo-mass of electrons in such matrices is given by [20]:

$$\tilde{m} = \frac{\mu_0 r^2 e^2 n}{2} ln\frac{a}{r}, \tag{5}$$

where *n* is the concentration of the free electron gas supplied in **Table 1**. The value expressed by Eq. 5 is called in ref. 18 as the "effective mass"; however, in our paper the notion of the "effective mass" is already ascribed to the mass of the vibrated element, given by Eq. 1. Thus, we call the value expressed by Eq. 6 the "pseudo-mass", and the effective density of electrons expressed by Eq. 4 we label as the "pseudo-density". Assuming $r = 1.0 \times 10^{-6}$m; $a = 5.0 \times 10^{-3}$m we estimate $\tilde{m}_{Li} \cong 6.4 \times 10^{-27}$ kg; $\tilde{m}_{Au} \cong 8.1 \times 10^{-27}$kg. Eqs. 4-5 enable calculation of the effective pseudo-plasma frequencies ω_p^* for Au and Li according to Eq. 6:

$$\omega_p^* = \sqrt{\frac{\tilde{n}e^2}{\varepsilon_0 \tilde{m}}} \tag{6}$$

Substituting aforementioned numerical parameters yields for effective the plasma frequencies of the lattices built from Au and Li wires $\omega_p^{*Au} = 4.6 \times 10^{10}$ Hz; $\omega_p^{*Li} = 5.2 \times 10^{10}$ Hz. The spring constants k_2 corresponding to aforementioned plasma frequencies are already small and equal k_2(Li)=2.4×10^{-9}N/m, k_2(Au)=1.9×10^{-9}N/m. The optical and acoustical branches of the longitudinal modes propagation in the 1D lattice, depicted in **Figure 5**, should be elucidated.

It should be emphasized that the ensembles of metallic wires, shown schematically in **Figure 5**, will not demonstrate simultaneously the negative mass (density) and the negative refraction effects [18-19]. This is due to the fact that the negative refraction becomes possible below the plasma frequency ω_p [18-19]; contrastingly, the effect of the negative mass in our model emerges when the frequency ω approaches ω_p from above; thus, creating of the material demonstrating simultaneously the negative density and dielectric constant remains challenging.

Conclusions

We conclude that exploiting the plasma oscillations of the electron gas relatively to the ion lattice gives rise to the negative effective mass phenomenon. The effect takes place when a metallic particle is vibrated with the external frequency ω approaching the plasma frequency $\omega_p = \sqrt{\frac{ne^2}{m_e \varepsilon_0}}$ from above. In this case the effective mass of the particle $m_{eff} = m_1 + \frac{m_2 \omega_p^2}{\omega_p^2 - \omega^2}$, where m_1 is the mass of the ionic lattice, and m_2 is the mass of the electron gas, becomes negative [12, 13, 15].

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The plasma oscillations may be phenomenologically represented with the ideal spring $k_2 = \omega_p^2 m_2$. Just macro-scaled values of $k_2 \cong 10^2 \frac{N}{m}$ for typical metals (namely Li and Au) are noteworthy. The effects due to the negative effective mass become possible in the nearest vicinity of the plasma frequencies, inherent for typical metals which are high, namely $\omega_p \sim 10^{16} Hz$. The dimensionless effective mass of the particle $\frac{m_{eff}}{m_1+m_2} \cong \frac{m_{eff}}{m_1} \cong 1 + \frac{m_2}{m_1}\frac{\omega_p^2}{\omega_p^2-\omega^2}$ does not depend on the size of the metallic particle. The plasma frequency may be decreased markedly for the low frequency plasmons predicted for the metallic mesostructures [18], enabling manufacturing metamaterials, demonstrating the effective negative density.

Acknowledgements:

The authors are indebted to Mrs. Yelena Bormashenko for her kind help in preparing this work.

Funding:

No external funding was obtained for this work.

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