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Article

Supplier replacement model in a one-level assembly system under lead-time uncertainty

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Abstract: Supplier selection/replacement strategies and optimized purchasing policies play a key role in efficient supply chain management in today's dynamic market. Here we study supplier replacement in a one-level assembly system (OLAS) producing one type of finished product. To assemble the product, we need to provide multi-type components, but assembly will be interrupted if any single component is missing, and incoming units will get hoarded until the missing component arrives. The assembly process can be interrupted by various sources of uncertainty, including delays in component deliveries. There is consequently a non-negligible risk that the assembly process may get stopped any moment. This brings inventory-related costs, which should be minimized. Here we consider discrete lead-time distributions to mimic industry-world reality. We present a model that takes into account not only optimal assignment of component order release dates but also replacement of a critical supplier. For a given unit, we model several alternative suppliers with alternative pricing and lead-time uncertainties, and we evaluate the impact on the total assembly system. For a more general case where several suppliers may be replaced, we propose a genetic algorithm.

Keywords: Assembly systems; replenishment; stochastic lead times; holding cost; backlogging cost; purchase cost; optimization.

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1. Introduction

Efficient replenishment is a major factor in inventory control. Inappropriate replenishment policy results in stock-outs or overstocks. If customer demands are unfulfilled, penalties may be imposed unless the order is shortened. However, overstocking drums up high inventory costs.

In the modern market, companies need to take every opportunity to reduce their costs and uncertainty to satisfy their customers. This makes supplier selection and replacement where needed key strategic considerations. Dynamic market changes demand selection of business associates who are logistically or otherwise capable of following changes in company requirements [1]. Bad supplier selection during a single period can have substantial negative impacts on a firm's long-term financial results [2]. Finding a suitable set of suppliers and effective replenishment are crucial problems in inventory control. Here, a one-level assembly system (OLAS) is comprised of n distinct components which are replenished from various suppliers with uncertain lead-times. Lead-time uncertainty counts as one of the most crucial matters in the production system. There are numerous circumstances that disrupt the assembly line, ranging from transport postponement to substandard quality and back to machine failure, etc. Lead-time uncertainty has even bigger impact in an assembly system through the assembly process. Consequently, it is important to have reliable information on the suppliers in order to reliably estimate planned lead times, anticipate orders, and negotiate delivery dates with customers.

Here, to be closer to real-world industrial planning methods, we consider a discrete temporal environment and integer decision variables. We consider the case of discrete lead-time distributions, as real-world industrial planning methods generally utilize the discrete temporal environment. A similar assembly system has already been studied in [3], [4] and [5], but the authors optimized planned lead times with a view to minimizing the expected total cost that includes the backlogging cost for finished products as well as the holding cost for components. In the research reported here, the model put forward diverges from ([3], [4] and [5] as follows: we propose several pricing and replenishment policies in which we suggest paying suppliers more if they agree to decrease their lead-time uncertainties. We developed the corresponding mathematical models and then ran numerical tests. The results show that the joint pricing and replenishment optimization approach has a huge effect.

The rest of this paper is organized as follows. Section 2 reviews previous research on replenishment planning under random lead times with a specific focus on assembly systems. Section 3 outlines a formulation of the supplier selection/replacement and replenishment optimization model for the considered OLAS. Section 4 presents a genetic algorithm for the case where several suppliers may be replaced. Section 5 reports the computational results, and Section 6 gives concluding remarks.

2. Related work

The literature counts several papers that study supply planning under uncertainty: most focus on demand uncertainty, and there has been little attempt to address the question of lead-time uncertainty. Here, we do not claim to provide an exhaustive review of the literature but a broad overview of important existing approaches in the field of supply planning under lead-time uncertainty. This first analysis of the literature confirms that most existing work focuses on one-customer demand planning and one-period planning with specific structures.

Week [6] was one the first to investigate one-demand planning for linear systems. He proved that the problem can be easily resolved using the well-known newsvendor model. [7] generalized it to the case of multi-stage linear systems, and [8] proved that the problem is equivalent to the one that calculates the best base-stock levels for serial inventory systems. [9] also dealt with multi-stage systems but with backlogging costs in intermediate stages. The author proposed a recursive procedure to calculate the expected total cost and then optimize planned lead times based on the convexity of the cost function.

The scholarship covering assembly systems under lead-time uncertainty counts several studies that consider one-customer demand planning. Yano [10] was among the first to study OLAS. The finished product is assembled from two components, where the lead time follows a stochastic Poisson distribution for the first component and a negative binomial distribution for the second component. As in [10], [11] developed a model to study OLAS under a known demand and uncertain lead times and to minimize the expected total cost. An exact method was proposed to obtain optimal order release dates. The main limitation of this method is that it is only valid for certain types of lead-time distributions (exponential, uniform and normal). Several other works have set out to demonstrate the convexity of the objective function for the case of OLAS [3] or to optimize it using newsboy formulae with specific assumptions on lead-time probability distributions [12,13]. For the case of a single stochastic demand, [14] studied an assembly system with one finished product and two components, each purchased from a given supplier, and optimized the ordered quantity purchased from each

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supplier.

[15] studied two-level assembly systems and developed a mathematical model to calculate the expected total cost, which is the sum of the inventory holding costs for components and the backlogging cost for the finished product. The authors assumed that the finished product is assembled on due date at the earliest. A GA was proposed to optimize this expected total cost and the related order release dates for the components at the second level of the bill of materials (BOM). [16], [17] and [18] made use of the same mathematical model with the same assumptions in a multi-objective context.

Later, [19] extended the model to allow assembly of the finished product before the due date if all the components at level 1 are available. A branch-and-bound procedure was introduced to optimize the planned lead times. The latest mathematical model provides the basis for a new way to integrate a maintenance plan that considers system deterioration [20,21]. For assembly systems with three levels in the BOM, [22] proposed a continuous modelling approach based on an approximate decomposition technique to optimize inventory and backlogging costs. The main limitation of these studies is that they are limited to assembly systems with less than three levels in the BOM. The past few years have seen a huge research effort to propose generalized models. To the best of our knowledge, only two studies have proposed mathematical formulations to model one-known-demand planning and multi-level assembly systems: [23] for the assemble-to-order environment and [24] for the configure-to-order environment. Even though these two studies express the dependency between levels and offer the potential to go beyond two levels in the nomenclature, there are still a number of unresolved questions over whether these approaches should be used for the case of multi-period planning where there is dependency between periods.

Lead-time uncertainty is covered in ample scholarship focusing on multi-period planning in a stochastic environment. Most models are limited to single-item replenishment and avoid order crossover despite it being a very real phenomenon in real-world replenishment planning and inventory control (see [6,25–28] and their related work for well-known models, and [29–32] for real examples).

For the case of assembly systems in a stochastic environment, research has tended to focus on multi-period planning with known constant demand rather than dynamic demand [33]. This is explained by the fact that order crossover is very hard to model for dynamic demand, even though it allows stock-level dependency between periods [29]. The choice of a known constant demand allows to take into account the dependency between periods, in which case at some point in time the backlog is covered by stocks of previous periods and vice versa [34]. A number of studies have built on this assumption to propose a one-period planning system that is equivalent to multi-period planning with a known constant demand and stochastic lead times. [35] was one of the first to propose an economic order quantity (EOQ) model for an OLAS but with two-period planning. Fujiwara et al. [36] were among the first to mathematically formulate the dependency between the inventories of components and develop an optimal (Q, r) policy for an OLAS. They assume that: (i) the finished product is composed of several types of components, (ii) the assembly capacity is unlimited, (iii) the demand is constant, and (iv) the probability distribution of procurement lead time for each component is given. Their proposed continuous model determines the optimal (Q, r) policy and minimizes the average total cost, which represents the sum of inventory, backlogging and setup costs.

A few years later, there was a surge in scholarship on multi-period planning and known-constant demand to provide generalized models, but they were always limited to OLAS. For example, in [4] and [37], the authors optimized the expected total cost composed of inventory and backlogging costs and gave the optimal safety stock and optimal safety times. However, a serious weakness in the proposed approach was that it is only valid if all component procurement lead times follow the same probability distribution and if all components have the same unit inventory holding cost. The optimization approach was generalized in [38] to consider procurement lead times that are independent but not necessary identically distributed in order to maximize customer service level for the finished product

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and minimize the expected total cost related to components. In continuation of this work, various approaches [5,39–42] have attempted to extend this model to study certain policies (L4L, POQ, EOQ, etc.) and include setup cost.

For the case of multi-period planning under uncertainty of both demand and lead times, Molinder [43] studied OLAS to develop a simulation model coupled with a hybrid algorithm. The model is based on simulated annealing to optimize order quantities and planned lead times. In [44], the authors proposed a stochastic linear programming model to study a multi-product problem with several OLAS under both demand and lead-time uncertainty. Both these models are very interesting, but the quality of the optimal solutions depends on the number of scenarios.

As can be seen in the body of work carried out to handle lead-time uncertainty in assembly system problems, all the papers concentrate on ordering policies by optimizing order quantities, order release dates and planned lead times [33], but to the best of our knowledge, no papers have attempted to co-optimize purchasing policies and pricing strategies even though both are crucial to successful supply chain management. Most of the literature on supplier diversification and responsive pricing has focused on the single-item purchasing case. Interested readers can refer to the-state-of-the-art surveys of [45–48] to obtain more details on existing models, and to the recent review paper [49] which provides an excellent overview of contributions to this issue.

In the field of inventory control for assembly systems in an uncertain environment, the issue of supplier diversification and supplier selection/replacement strategies has been under-researched. Under demand and lead-time uncertainty, [14] studied one-stochastic demand planning to assemble a finished product from two components and extended the model introduced in [50] to consider delivery of the required components by a joint supplier or two different suppliers. The authors demonstrated optimality of base-stock when one supplier is selected. A few years later, the same authors [51] replaced lead-time uncertainty by yield uncertainty and came up with propositions under which it can be economically beneficial for the company to multi-source rather than use a single supplier. Pan So [52] considered an ATO environment to study random yields and price-dependent demand in single-period planning. They modeled the assembly of a finished product from n components and analyzed the effects of uncertainty on the optimal solution, defined by the production quantities–product pricing dyad.

Here, we aim to investigate an OLAS under lead-time uncertainty and examine the benefit of paying suppliers an additional purchase cost (APC) in order to reduce the costs of uncertainty.

3. Problem description

In an OLAS with n different components replenished from n different suppliers, the uncertainty of component replenishment lead times causes a high level of component inventory and a backlog for the finished product. In this context, the producer may be ready to pay an additional purchasing cost (APC) if the supplier can decease the uncertainty of their lead times.

In this paper we consider a purchasing and replenishment optimization model for such systems, which was developed to measure the effects of such a policy and to optimize the total cost composed of purchasing, holding and backlogging costs. Our model is based on a well-known model for supply planning under lead-time uncertainties. It assumes that assembly system capacity is infinite and that demand for the finished product is known. For each component, the lead time may take several values with given corresponding probabilities. Models of this type have already been formulated in [40] and [5] for one-level multi-period problems and in [16] and [23] for multi-level one-period assembly systems with random lead times.

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Here, in a model of this type, we introduce additional decision variables dealing with purchasing policies. Every purchasing policy j for a given supplier i has a purchasing cost (PC_i^j). This paper uses the following notations (see Table 1).

Table 1. Notations

Parameters	
i	supplier index
j	purchase price index
h_i	unit stock cost of the product purchased from supplier i
b	unit backlog cost for one finished product
$H = b + \sum_{i=1}^{n} h_i$	global holding and backlog cost for one finished product
L_i^j	random lead time for supplier i under purchasing price j
u_i^j	maximum lead time i under purchase price j; i.e.,$1 \leq L_i^j \leq u_i^j$
Variables	
x_i^j	planned lead time for supplier i with purchase price j (planned lead time is equal to order release date if due date is equal to zero)
y_i^j	binary variable taking the value 1 if using purchase price j for supplier i
Functions	
PC_i^j	purchase cost under the purchase price j for supplier i
$E(L_i^j)$	expected lead-time value for supplier i under purchase price j
$F_i^j(x_i^j)$	cumulative distribution function of lead time for supplier i under purchase price j
$p_i^j(k)$	probability of having a lead time equal to k for supplier i under purchase price j

As the basic model, i.e. without taking into account PC, we will use the model proposed in [38] which consists in minimizing the expected cost composed of the sum of the component holding cost and backlogging cost of the finished product:

$$\min EC(X) = \sum_{i=1}^{n} h_i \left(x_i - E(L_i)\right) + H \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} F_i(x_i + k)\right) \tag{1}$$

s.t.

$$\begin{aligned} &1 \leq x_i \leq u_i && \forall i = 1, ..., n \\ &x_i \in \mathbb{N} && \forall i = 1, ..., n \end{aligned} \tag{2}$$

where:

$$X = (x_1, x_2, ..., x_i, ..., x_n) \tag{3}$$

To solve this non-linear optimization problem (1-3), a $B\&B$ was developed in [38]. We will extend this model by including the APC explained above. In other words, we integrate prices (purchase costs) as additional decision variables into this model. Our model with the new objective function (4) is expressed as follows:

$$\begin{aligned} \min EC(X, Y) = &\sum_{i=1}^{n} \sum_{j=0}^{u_i^0 - 1} PC_i^j \cdot y_i^j + \sum_{i=1}^{n} \sum_{j=0}^{u_i^0 - 1} y_i^j \cdot h_i \left(x_i^j - E(L_i^j)\right) \\ &+ H \cdot \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0 - 1} y_i^j \cdot F_i^j(x_i^j + k)\right)\right) \end{aligned} \tag{4}$$

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s.t.

$$\sum_{j=0}^{u_i^0-1} y_i^j = 1 \qquad \forall i = 1, ..., n \tag{5}$$

$$x_i^j \leq (u_i^0 - j) \cdot y_i^j \qquad \forall i = 1, ..., n, \forall j = 0, ..., u_i^0 - 1 \tag{6}$$

$$x_i^j \in \mathbb{N} \qquad \forall i = 1, ..., n \quad \forall j = 0, ..., u_i^0 - 1 \tag{7}$$

$$y_i^j \in \{0, 1\} \qquad \forall i = 1, ..., n \quad \forall j = 0, ..., u_i^0 - 1 \tag{8}$$

The objective function (4) represents the mathematical expectation of the total cost composed of purchase cost (noted $PC(Y)$), holding component costs ($HC(X, Y)$) and backlogging cost ($BC(X, Y)$):

$$PC = \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} PC_i^j \cdot y_i^j$$

$$HC = \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} \left(y_i^j \cdot h_i \left(x_i^j - E(L_i^j) \right) \right) +$$

$$\sum_{i=1}^{n} (h_i) \cdot \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j (x_i^j + k) \right) \right)$$

$$BC = b \cdot \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j (x_i^j + k) \right) \right)$$

Constraint (5) expresses that only one purchasing policy ($j = 0, ..., u_i^0 - 1$) is selected for each supplier ($i = 1, ..., n$). Constraint (6) makes each planned lead time x_i^j limited by an upper bound. This upper bound for the purchasing policy ($j = 0$) is equal to u_i^0.

The goal is to minimize (4) subject to (5–8). This minimization is fairly difficult because the objective function is not linear and because decision variables $X = (x_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)$ are integer and $Y = (y_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)$ are binary.

4. Genetic algorithm

For the general case where the costs and distributions are different, problem complexity makes it impossible to find an exact method. We Thus will develop a genetic algorithm (GA) somewhat similar to the one in [15] but with a new cost function (new fitness) integrating different purchasing policies. We will exploit the results obtained for the particular case in [53] to propose a heuristic algorithm for initial population generation and to reduce the search space. GAs work to very well-known principles inspired by process of natural selection [54,55]. A group of individuals (population of solutions) go through a reproduction phase during which the good solutions pass their genetic material to generations further down. Genetic diversity by operations such as mutations avoids a premature convergence to a local optimum (see Algorithm 1).

The following sub-sections present key elements of the genetic algorithm such as crossover and mutation operators along with the representation of the solution (or chromosome).

4.1. Chromosome representation of a solution

The chromosome representation must contain all the information necessary for a solution. If the representation fails to encode a possible solution, the genetic algorithm will naturally fail to find it.

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$pop_0 \leftarrow$ initial_population() ;
for $gen \leq \ Nb_{max}$ **do**
    $pop_{gen} \leftarrow pop_{gen-1}$;
    $parents \leftarrow reproduction\_selection(pop_{gen})$;
    $offspring \leftarrow crossover(parents)$;
    $pop_{temp} \leftarrow replacement\_selection(offspring + parents)$;
    $pop_{gen} \leftarrow mutation(pop_{temp})$;
    if $rapide\_convergence(pop_{gen})$ **then**
        $pop_{gen} \leftarrow perturbation(pop_{gen})$;
    end
end

Algorithm 1: General structure of a GA

In our algorithm, a chromosome has two chromatids that correspond to the two types of decision variables: purchasing policy (or price) j from supplier i, and order release date (x_i^j) (see Figure 1). The purchase release date depends on the purchasing policy.

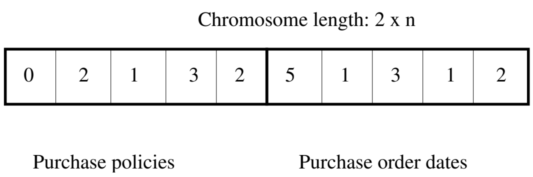

Figure 1. Chromosome representation

4.2. Initial population

An initial population of $size_{pop}$ should have an equilibrium between randomly-generated solutions designed to increase diversity and potentially good solutions needed to lead the algorithm towards better results. For that reason, our approach features two types of individuals in the initial population: 90% of the individuals are generated randomly, while the rest are obtained using a heuristic approach (see Algorithm 2).

- Randomly-generated solutions: a purchasing policy is chosen for each supplier following a uniform distribution. Once a purchasing policy is determined, order date is generated randomly according to the purchasing policy (order date should be less than or equal to u_i^j).
- Heuristic solutions take one of the suppliers (i) as the *seed* and all the others as *clones* of the seed to obtain an instance of the particular case explained in [53], (MIP_i). The corresponding linear model is solved with a commercial solver (CPLEX 12.6) to obtain sol_i which is added into the solution pool sol_pool.
- Finally, the solution pool is sorted according to the total cost and first $size_{pop} \times 10\%$ is included in the initial population.

4.3. Reproduction selection

Our GA uses a random selection method in which the probability of selection is uniformly distributed for each individual. This method determines $\frac{size_{pop}}{2}$ couples and with each individual can only appear in one couple. With a probability p_{co}, every couple undergoes crossover.

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foreach *supplier i* **do**
  foreach *supplier i′ such that* $i \neq i'$ **do**
    $APC_{i'} = APC_i$;
    $h_{i'} = h_i$;
  end
  $sol_i \leftarrow MIP_i$;
  $add(sol_i, sol\_pool)$;
  $Reinstore\_values(APC_{i'}, h_{i'})$;
end
$Sort(sol\_pool)$;
Choose first $size_{pop} \times 10\%$ solutions

Algorithm 2: Heuristic approach for initial population

4.4. Crossover operator

A special crossover operator is applied to each couple: two chromatids of the chromosome are cut at the same position. The purchasing policy and order date information for the same supplier(s) is transmitted to the *offspring*. Figure 2 gives an example of a single-point crossover on two chromatids.

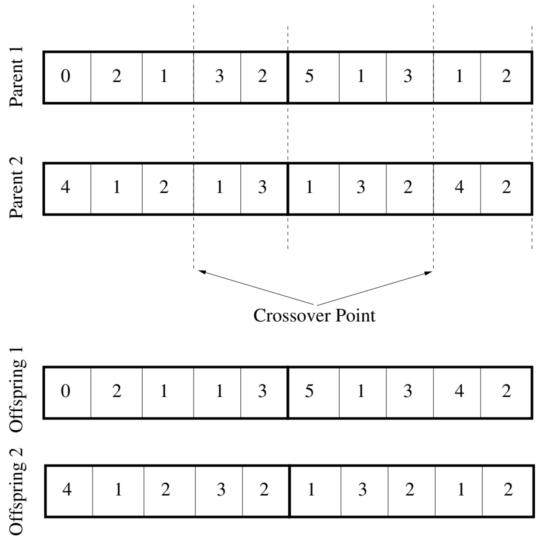

Figure 2. Single-point crossover on two chromatids

After the crossover, the offspring is added to the population, and so the population size increases by $size_{pop} \times p_{co}$ on average, because for each couple, two offspring are generated with a probability p_{co}. The best $size_{pop}$ solutions from this larger population are then kept. With this procedure, the offspring are not only in competition with their *parents* but also with other parents and offspring. The new set of $size_{pop}$ individuals is then subject to a mutation phase, before becoming the *next generation*.

4.5. Mutation operators

To introduce the right amount of diversity to the population, each individual undergoes mutation with a probability of p_m. We consider three types of mutations: randomly changing a supplier's purchasing policy (m_1), randomly changing a supplier's order release date (m_2), and permuting the purchasing policies and order release dates of two different suppliers (m_3).

For the mutation operators m_1 and m_2, we need to verify that the purchasing policy and order release date are suitable. After m_1 (respectively, m_2), if the order date (respectively, purchasing policy) is unsuitable, then another date is randomly attributed to the supplier concerned. This verification is not necessary for the third mutation operator, because the *genes* concerning purchase policies and order release dates are swapped at the same time.

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Each mutation can occur with the following probabilities:

$$p_{m_1} = p_{m_2} = \frac{p_{m_3}}{2} = \frac{p_m}{4}$$

If during $nbIterMax_{NoImprovement}$ iterations the best solution known so far is not improved, then the probability of mutation (p_m) is updated to 50%. As soon as the best solution is improved, pm retakes the original value.

4.6. Perturbation

If the current mutation operators are not enough to prevent a premature convergence, then a perturbation procedure is applied. Premature convergence is declared if 80% of a population has the same cost. The perturbation consists of destroying 90% of the solutions that have the same cost and replacing them with completely random but feasible solutions.

5. Experimental results

The GA was coded in C++ and numerical experiments were performed on an Intel Core i5-2520M processor at 2.50 GHz clock-speed and with 4GB of memory.
The solution approach was tested on a randomly-generated instance set (I). We created 10 instance families as a function of component numbers $[10, 20, \ldots, 100]$, and 100 test instances were created in each family. The input data for each instance were: unit component holding cost, unit finished product backlogging cost, additional purchasing cost per component per purchasing policy, and cumulative distribution function of lead times per component per purchasing policy.
After some preliminary tests, parameter values were assigned as follows: population size ($size_{pop}$) to 100, maximum number of generations (Nb_{max}) to 1000, maximum number of iterations without improvement ($nbIterMax_{NoImprovement}$) to 50, probability of crossover (p_{co}) to 90% and probability of mutation (p_m) to 10%.

In Tables 2-5, the first column gives the number of components (suppliers) for each instance family, the second column gives the average number of iterations where the best solution was found, and the third column gives the average gap between the best solution ($best_{sol_0}$) in the initial population and the best solution ($best_{sol_{1000}}$) found by the algorithm after 1000 generations ($gap = \frac{best_{sol_0} - best_{sol_{1000}}}{best_{sol_{1000}}} \times 100$). The next column provides the average gap between the best solution ($best_{sol_{1000}}$) found by the algorithm after 1000 generations and the best solution ($best_{sol*}$) among all versions of GA ($gap* = \frac{best_{sol_{1000}} - best_{sol*}}{best_{sol*}} \times 100$), i.e. the best known solution (BKS). Finally, the latest column reports the average time to execute the algorithm.

Table 2 reports the results of the genetic algorithm noted GA, i.e. without the complementary procedures described above (a heuristic to form the initial population and the perturbation procedure). It shows that even if there is a considerable improvement of the initial population, there is still a very big gap to the BKS. The average gap on all the instances is 87.06%. Table 3 reports the results of the genetic algorithm noted GA_P, i.e. with the perturbation procedure. It shows very little improvement, as the total average gap is no less than 86.14%. Table 4 shows that the genetic algorithm noted GA_H that includes the heuristic approach to construct the initial population vastly improves the quality of the solution, as the total average gap to the BKS on all instances is reduces to just 3.92%.

Finally, almost all the BKS were obtained with GA_{H+P} (Table 5), which achieves a total average gap of 0.67%. Tables 2-5 also show that even on the largest instances, the mean execution time of the GA_{H+P} is less than 1.6 seconds.

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Table 2. Results obtained by GA

Instance family	Average number of iterations	Mean gap (%)	Mean gap* from BKS (%)	CPU time (sec)
10	38.23	147.26	19.04	0.28
20	128.26	159.75	44.50	0.38
30	275.96	178.03	44.87	0.48
40	251.50	151.65	85.33	0.55
50	368.22	131.95	102.88	0.63
60	472.86	135.45	108.37	0.74
70	570.92	148.98	108.97	0.86
80	647.76	158.66	105.80	0.97
90	697.04	161.99	105.00	1.08
100	844.43	135.71	145.85	1.41

Table 3. Results obtained by GA_P

Instance family	Average number of iterations	Mean gap (%)	Mean gap* from BKS (%)	CPU time (sec)
10	148.06	162.23	11.09	0.31
20	172.08	160.86	44.01	0.46
30	306.38	178.86	44.44	0.59
40	251.00	151.65	85.33	0.65
50	385.06	132.32	102.62	0.73
60	533.07	135.62	108.24	0.88
70	593.01	148.96	108.99	0.99
80	698.12	158.73	105.74	1.11
90	728.58	161.89	105.09	1.27
100	837.98	135.69	145.88	1.59

Table 4. Results obtained by GA_H

Instance family	Average number of iterations	Mean gap (%)	Mean gap* from BKS (%)	CPU time (sec)
10	45.74	56.13	11.35	0.27
20	161.19	63.33	6.17	0.35
30	289.55	64.29	5.59	0.42
40	567.50	83.01	0.00	0.46
50	418.71	69.16	3.81	0.49
60	573.67	72.82	3.25	0.57
70	674.40	76.29	2.74	0.67
80	730.14	72.96	2.25	0.73
90	832.54	73.76	2.15	0.80
100	884.34	74.69	1.87	0.97

Table 5. Results obtained by GA_{H+P}

Instance family	Average number of iterations	Mean gap (%)	Mean gap* from BKS (%)	CPU time (sec)
10	115.50	66.33	3.93	0.32
20	279.74	72.60	0.06	0.43
30	422.37	72.65	0.00	0.53
40	295.50	83.01	0.00	0.60
50	609.03	75.06	0.03	0.63
60	716.57	77.94	0.10	0.73
70	781.95	80.60	0.18	0.83
80	835.20	75.93	0.44	0.92
90	892.58	76.28	0.64	1.01
100	904.72	75.70	1.28	1.19

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Figure 3 illustrates the evolution of the gap over iterations of GA_{H+p} on an instance with 100 components ($gap_{gen} = \frac{best_{sol_{gen}} - best_{sol_{5000}}}{best_{sol_{5000}}} \cdot 100$). In the first iteration, the gap is 86.81%, whereas at the 1000^{th} iteration the gap is reduced to 1.31%. The gap finally reaches 0.00% at iteration 2132. Ultimately, 98.48% of the improvement is achieved during the first 1000 iterations.

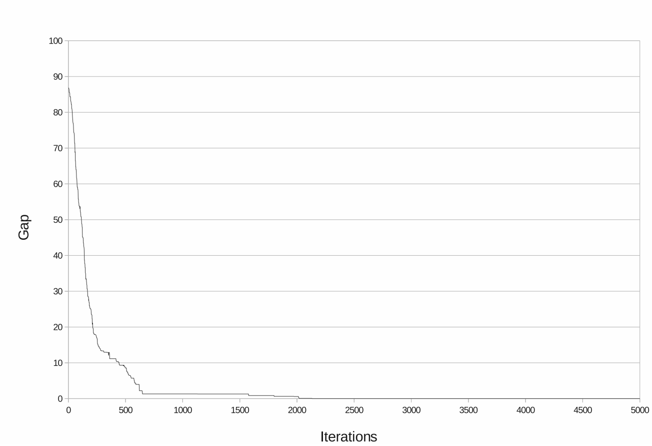

Figure 3. Evolution of the gap to the BKS over iterations

5.1. *Lower and upper bounds*

5.1.1. Lower bound

We can develop a lower bound on $EC(Z)$ from this model by decomposing it into n subproblems. In other words, instead of one final product with n components, we will consider n final products, each having only one component. Therefore, for each supplier i, we will solve:

$$\min EC_i(Z_i) = \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot APC^j + \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot s \cdot h$$
$$- \sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} h \cdot z_j^s \cdot E(L^j) + H \sum_{k \geq 0} \left(1 - \left(\sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} z_j^s \left(F^j(s+k)\right)\right)\right) \quad (9)$$

s.t.

$$\sum_{s=1}^{u_0} \sum_{j=0}^{u_0-1} z_j^s = 1 \quad (10)$$

The total cost (EC(Z)) of the problem (4-8) is greater than the sum of $EC_i(Z_i^*)$. Obviously, this solution is unfeasible, because the assembly process needs to synchronize the component flows, which then causes an additional cost. Therefore, this sum is a lower bound for $EC(Z)$.

The linear model (9-10) can be solved by a commercial solver (CPLEX 12.6).

5.1.2. Upper bound

We can compute the upper bound using two methods presented in [39]. However, [39] has no decision concerning purchasing policy (there is only one purchasing policy, i.e. a fixed price) and the authors only find the release (purchasing) dates that minimize some of the holding and backlogging costs under lead-time uncertainty. [39] proposed an exact method (BB) and a heuristic (beam search), which can be used to find upper bounds for the problem considered in this paper.

We can apply B&B for a small number of suppliers (up to 50) and the beam search approach for larger numbers. The tests show that the gap between the beam search and the optimal solution is

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less than 2.5%. The quality of the beam algorithm is very good, as the beam search found optimal solutions for 491 of the 500 tests executed. The optimal solutions found by B&B or the beam search developed in [39] are also optimal for our problem for policy 0 and feasible for other policies, so they can be used as upper bounds in our study.

Table 7 compares the results of our GA_{H+P} model with the upper and lower bounds. The table reports names of the instance families (corresponding to the number of suppliers) along with the mean gaps from the lower and upper bounds. As each family has 100 instances, the mean gaps in percentage format are calculated as follows:

$$\text{mean GAP}(GA_{H+P},\text{LB}) = \left(\frac{\sum_{k=1}^{K} \frac{GA^k_{H+P} - LB^k}{LB^k}}{K} \right) \times 100$$

$$\text{mean GAP}(GA_{H+P},\text{UB}) = \left(\frac{\sum_{k=1}^{K} \frac{UB^k - GA^k_{H+P}}{GA^k_{H+P}}}{K} \right) \times 100$$

where GA^k_{H+P}, LB^k and UB^k are the result of GA, lower bound and upper bound to the problem on instance k, respectively, and K=100 (instances by family).

Table 6. Gaps between the results of GA_{H+P} and upper and lower bounds

Instance family	mean GAP (GA_{H+P},LB) (%)	mean GAP (GA_{H+P},UB) (%)
10	10.89	145.89
20	6.33	181.06
30	6.00	183.49
40	8.16	185.18
50	8.77	194.95
60	10.24	194.31
70	12.00	192.26
80	14.34	192.57
90	10.97	191.04
100	10.35	190.74

The mean gap between the lower bound and the GA_{H+P} is less than 15%, and smaller gaps are observed for instances with 20 to 50 suppliers. However, the mean gap between the upper bound and the GA_{H+P} is far bigger, ranging between 145.89% and 194.95%. This result shows the huge impact of the policies for uncertainty reduction: even if each supplier has to be paid an additional purchasing cost to reduce their uncertainty, the total cost is still drastically minimized, so the solution without these policies gives a very bad upper bound.

5.2. Decisionmaker behavior analysis

In this section, we investigate the impact of the decisionmaker's behavior towards the uncertainty and risks.

If the decisionmaker is a risk-taking manager ($risk_{max}$), he/she will be reluctant to pay more to reduce the uncertainty, and hence will always choose APC^0, which is a policy with zero additional purchase cost but with maximum uncertainty. In this case, we have the model proposed in [39] that w already used as an upper bound in subsection 5.1. On the other hand, if the decisionmaker is risk-averse, he/she will be willing to pay the maximum APC^{u_0-1} ($risk_{min}$). In this case, the

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uncertainty disappears and the problem becomes deterministic. In the deterministic case, ordering just-in-time implies zero inventory holding and backlogging costs, leaving the only cost as $\sum_i APC_i^{u_0-1}$.

Table 7 compares our lead-time uncertainty-reducing pricing and inventory control model with these two strategies on three groups of scenarios. In the first group (G1), we considered a low average APC increase ($0 < \overline{\Delta APC} \leq \frac{H}{5n}$). In the second group (G2), we considered the case of an average APC increase between $\frac{2}{3}\frac{H}{n}$ and $\frac{H}{n}$. Finally in the third group (G3), we considered the case of an average APC increase greater than holding and backlogging costs ($2\frac{H}{n} \leq \overline{\Delta APC} \leq 5\frac{H}{n}$).

Table 7. Decisionmaker behavior analysis

Instance groupe	mean GAP $(GA_{H+P}, risk_{min})$(%)	mean GAP $(GA_{H+P}, risk_{max})$ (%)	mean GAP $(risk_{min}, risk_{max})$(%)
G1	0.57	7014.81	6993.03
G2	108.20	4.04	-49.68
G3	492.31	$1.78 \cdot 10^{-5}$	-83.17

Mean GAPs were computed as follows:

$$\text{mean GAP}(GA_{H+P}, risk_{min}) = \left(\frac{\sum_{k=1}^{K} \frac{risk_{min}^k - GA_{H+P}^k}{GA_{H+P}^k}}{K} \right) \times 100$$

$$\text{mean GAP}(GA_{H+P}, risk_{max}) = \left(\frac{\sum_{k=1}^{K} \frac{risk_{max}^k - GA_{H+P}^k}{GA_{H+P}^k}}{K} \right) \times 100$$

$$\text{mean GAP}(risk_{max}, risk_{min}) = \left(\frac{\sum_{k=1}^{K} \frac{risk_{max}^k - risk_{min}^k}{risk_{min}^k}}{K} \right) \times 100$$

Comparison of two simple strategies found that when the average APC increase is small, a risk-avoiding strategy is almost 7000% better on average than a risk-taking strategy. Similarly, if the average APC increase is very big, a risk-taking strategy is 83.17% better on average than a risk-avoiding strategy. Obviously, there is no reason to take risks when it is cheaper to pay to reduce the uncertainty. Nevertheless, when the cost of the reducing the uncertainty exceeds the inventory holding and backlogging costs caused by the uncertainty, then the decisionmaker should be open to taking risks.

Our genetic algorithm gives practically the same results with a $risk_{min}$ strategy for group G1 and with a $risk_{max}$ strategy for group G3. The most interesting case is when the average APC increase is close to the inventory holding and backlogging costs, where GA_{H+P} gave the best total cost and the gap to the $risk_{min}$ strategy was 108.2% whereas the gap to the $risk_{max}$ strategy was 4.04%. In other words, our GA_{H+P} adapts itself to changes in inventory holding, backlogging and additional purchase costs and chooses the best-suited strategy with regard to risks.

6. Conclusions

This paper deals with a pricing and replenishment problem for one-level assembly systems under component lead-time uncertainty. We focused on finding optimal values for the planned lead times (or order release dates) and purchasing prices. A linear model was developed, and a decomposition approach was proposed to calculate a lower bound while upper bounds were calculated using branch-and-bound and beam search algorithms developed for a particular case with fixed purchasing prices.

For the general case, we also proposed a genetic algorithm. Solution quality was improved by a heuristic to create initial solutions and a perturbation technique to diversify the search. The genetic algorithm was evaluated its results were compared against lower and upper bounds. Test results showed that having several policies with a higher purchasing price but lower uncertainty drastically improves the total cost. Comparisons with risk-taking and risk-avoiding behaviors showed that the proposed genetic algorithm adapts itself to the parameters of the supply chain and gives valuable insight to decisionmaker.

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Abbreviations

This manuscript uses the following abbreviations:

APC	Additional Purchase Cost
ATO	Assemble-To-Order
BKS	Best Known Solution
BOM	Bill Of Materials
CNRS	Centre National de la Recherche Scientifique
EOQ	Economic Order Quantity
GA	Genetic Algorithm
ICD	Institut Charles Delaunay
JIT	Just-in-time
L4L	Lot-for-Lot
LOSI	Laboratoire d'Optimisation des Systèmes Industriels
LS2N	Laboratoire des Sciences du Numérique de Nantes
OLAS	One-Level Assembly System
PC	Purchasing Cost
POQ	Periodic Order Quantity

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