On the relationship between the cosmological background field and the Higgs field

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Summary
It is shown that gravity and quantum physics can be unified upon the basis of a quark description in terms of a recently discovered third type Dirac particle. It requires the awareness of a polarisable second elementary dipole moment next to the angular moment (spin) and the awareness of an (unbroken) omnipresent energetic cosmological background field. The unification has been made explicit by relating the two major gravitational constants of nature (the gravitational constant and Milgrom's acceleration constant) with the two major nuclear constants of nature (the weak interaction boson and the Higgs boson).

Keywords: Cosmological Constant; Higgs field; Milgrom's constant; Dirac particle; unification

1. Introduction

Present-day theory of quantum physics as well as present-day theory of gravity rely upon the presence of an omni-present energetic background field. In quantum physics, this field is known as the "Higgs field". It is required for explaining the origin of mass. It has an axiomatic definition, conceived in 1964,[1]. In gravity, the existence of the background field is required to explain the accelerated expansion of the universe, known since 1998, [2]. This cosmological background field has been defined on the basis of Einstein's Cosmological Constant [3]. It is also known as "dark energy". It would be odd if two different energetic background fields would exist next to each other. More logical would be if the Higgs field and the cosmological background field would be the same. This is the issue that will be discussed in this article. In [4,5,6,7,8] it has been argued that if the cosmological background field would consist of energetic uniformly distributed polarisable vacuum particles, the dark energy would give an explanation for the dark matter problem as well, because the polarization would evoke a gravitational equivalent of the well-known Debije effect [9]. With the difference, though, that the central force from a gravitational nucleus is enhanced just opposite to the suppression of the Coulomb force from an electrically charged nucleus in an ionized plasma. This picture fits to the Higgs field of nuclear particle physics as well, albeit that the energetic background particles would show the true Debije effect in the sense that they would exponentially suppress a central nuclear force such as required to explain the short range of nuclear forces. It corresponds more or less with the common view that mass less force carrying particles are retarded by a surrounding field of energy, thereby gaining mass [10,11].

The modeling of the omni-present background energy by energetic vacuum particles, which we shall denote as *darks* for short, requires a model for its elementary constituent. This element must be a source of energy, and must be force feeling as well. In those aspects it resembles an electron, which is ultimately the source of electromagnetic energy, and which is sensitive to the fields spread by other electrons. However, where the *dark* in the

cosmological background field must be polarisable under the gravitational potential, an electron is non-polarisable under an electric potential. The electric dipole moment of an electron is zero, while a dark should have a non-zero gravitational dipole moment. In [8, 12,13] the suggestion has been made that these particles could be of the particular Dirac type as theorized back in 1937 by Ettore Majorana [14]. There is, however, no convincing argument why a Majorana particle would have a dipole moment that is polarisable in a scalar potential field. It is recognized, though, that Dirac's theory contains some heuristic elements. Recently, the author of this article found a third type Dirac particle, next to the electron type and the Majorana type [15]. This *third* has the unique property that, unlike the electron type, it possesses a dipole moment that is polarisable in a scalar potential field. It is my aim to show in this article that this *third* matches with the *dark*. In the next paragraph first a summary will be given of the *third*. Thereafter a view will be given on the cosmological background energy. The viability of this view will be proven by a calculation of Milgrom's empirical acceleration constant of dark matter. After that, it will be shown that the novel Dirac particle applies to quarks as well, ending up in a model for the nuclear domain in which the common Lagrangian description of the Higgs field is harmonized with a nuclear energetic background field with similar characteristics as the cosmological one. Finally it is shown in verifiable formulae how these fields are related.

2. Summary of the *third*

The canonic formulation of Dirac's particle equation reads as [16,17],

$$(i\hbar\gamma^{\mu}\partial_{\mu}\psi - \beta m_0 c\psi) = 0.$$

where β is the 4 x 4 unity matrix and where the 4 x 4 gamma matrices have the properties,

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu; \text{ and } \gamma_0^2 = 1; \gamma_i^2 = -1. \tag{1}$$

As usual, c is the vacuum light velocity, $\hbar$ is the reduced Planck constant and m_0 is the rest mass of the particle. Where the canonical set is given by,

$$\gamma_0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}; \gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix}; \gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix}; \gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix}; \beta = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \tag{2a}$$

the γ-set of the third type has been found as,

$$\gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}; \gamma_1 = \begin{bmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{bmatrix}; \gamma_2 = \begin{bmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{bmatrix}; \gamma_3 = \begin{bmatrix} i\sigma_3 & 0 \\ 0 & -i\sigma_3 \end{bmatrix}; \beta = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \tag{2b}$$

where σ_i are the Pauli matrices.

Although the wave equation of the electron type and that of the "third" are hardly different, there is a major difference in an important property. Both have *two* dipole moments. A first

one, to be indicated in this text as the *first dipole moment*, is associated with the elementary angular momentum $\hbar$. The second one, to be indicated as the *second dipole moment* is associated with the vector $\hbar/c$. These dipole moments show up in the calculation of the excess energy of the particle in motion subject to a vector potential $A(A_0, A_x, A_y, A_z)$. In the canonic case (2a) we have,

$$\Delta E = \frac{e\hbar}{2m_0} \begin{bmatrix} \overline{\sigma} \cdot \mathbf{B} & 0 \\ 0 & \overline{\sigma} \cdot \mathbf{B} \end{bmatrix} + \frac{e\hbar}{2m_0 c} \begin{bmatrix} 0 & i\overline{\sigma} \cdot \mathbf{E} \\ i\overline{\sigma} \cdot \mathbf{E} & 0 \end{bmatrix}$$

where $\overline{\sigma}$ is the Pauli vector, defined by

$$\overline{\sigma} = \sigma_1 \mathbf{i} + \sigma_2 \mathbf{j} + \sigma_3 \mathbf{k} , \tag{3}$$

where ($\mathbf{i}, \mathbf{j}, \mathbf{k}$) are the spatial unit vectors and where $\mathbf{B}$ and $\mathbf{E}$ are field vectors derived from the vector potential. The redundancy in (3) allows writing it as,

$$\Delta E = \frac{e}{2m_0} (\overline{\sigma} \ \hbar \cdot \mathbf{B} + i\overline{\sigma} \ \textit{ħ/c} \cdot \mathbf{E}), \tag{4}$$

The electron has a real first dipole moment ($e\hbar/2m_0$), known as the magnetic dipole moment, and an imaginary second dipole moment ($ie\hbar/2m_0 c$), known as the anomalous electric dipole moment. The spin vector $S = \overline{\sigma}/2$ has an eigen value $|S| = 1/2$. In the case that the Dirac particle is of the third type as defined by (2b), we have [15],

$$\Delta E = \frac{e}{2m_0} (\overline{\sigma} \ \hbar \cdot \mathbf{B} \pm \overline{\sigma} \ \textit{ħ/c} \cdot \mathbf{E}), \tag{5}$$

The generic third type Dirac particle has two real dipole moments, generically without identifying it as an electromagnetic one, to the amounts of $\overline{\sigma}\hbar$, respectively $\overline{\sigma}\hbar/c$. If the *dark* would be of the electron type, it would not be polarisable in a gravitational field, because such a field is Coulomb-like and is unable to polarize an imaginary second dipole moment. If, however, the *dark* is a third type, its second dipole moment can be polarized under influence of a scalar potential field. This field is not necessarily the electromagnetic one. The coupling factor is not necessarily the elementary electric charge. If the field is just a static one, eq. (5) can be written as,

$$\Delta E = -\frac{g\overline{\sigma}}{2m_0} (\ \textit{ħ/c} \cdot \nabla A_0) , \tag{6}$$

where g is a generic coupling factor. Hence, taking into account that the eigen value of the spin vector with the state variable $\overline{\sigma}$ is $|S| = |\overline{\sigma}/2| = 1/2$, the dipole moment p of a single particle in a gravity field (where $g = m_0$), is given by,

$$p = \frac{\hbar}{2c} . \tag{7}$$

Hence, the third type is a candidate for being the elementary constituent of the cosmological background energy. Further profiling of this constituent will be given in the next paragraph.

3. The cosmological background field

The presence of an omnipresent background field is imposed by the vacuum solution of Einstein's Field Equation with Einstein's (cosmological) constant Λ, [18]. As discussed in [19], a positive value of Λ, under the weak limit constraint and under particular constraints for the spatial validity range, results into a modification of Poisson's equation, such that,

$$\nabla^2\Phi + \lambda^2\Phi = -\frac{4\pi GM}{c^2}\delta^3(r), \tag{8}$$

where Φ is the gravity potential, G the gravitational constant, M the mass of a pointlike baryonic source, $\delta(r)$ Dirac's delta function and where λ is related with Einstein's constant Λ, such that

$$\lambda^2 = 2\Lambda. \tag{9}$$

This view on Einstein's Λ is different from the common perception that regards it as a constant of nature, while, like extensively motivated and analysed in [19], it is basically a integration constant - its covariant derivative is zero - in Einstein's Field Equation. How to arrive from the Einsteinean Field equation to the simple format (8) requires an explanation, because the canonical view is different.

Let us start by considering the gravitational wave equation as a consequence of the weak field limit of the Einsteinean Field Equation. The equation reads as,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{10}$$

where $T_{\mu\nu}$ is the stress-energy tensor, which describes the energy and the momenta of the source(s) and where $R_{\mu\nu}$ and R are respectively the so-called Ricci tensor and the Ricci scalar, which can be calculated if the metric tensor components $g_{\mu\nu}$ are known [20,18]. In the case that a particle under consideration is subject to a central force only, the time-space condition shows a spherical symmetric isotropy. This allows to read the metric elements g_{ij} from a simple line element that can be written as

$$ds^2 = g_{tt}(r,t)dq_0^2 + g_{rr}(r,t)dr^2 + r^2\sin^2\vartheta d\varphi^2 + r^2 d\vartheta^2, \tag{11}$$

where $q_0 = ict$ and $i = \sqrt{-1}$.

It means that the number of metric elements g_{ij} reduce to a few, and that only two of them are time and radial dependent. A generalization of Schwarzschild's solution of Einstein's equation for empty space and $\Lambda = 0$, relates the metric components as [18,20,21],

$$g_{rr} g_{tt} = 1. \tag{12}$$

Solving Einstein's equation under adoption of a massive source with pointlike distribution $T_{00} = Mc^2 \delta^3(r)$, results in a wave equation with the format

$$-\frac{\partial^2 g_{tt}}{c^2 \partial t^2} + \frac{1}{r} \frac{\partial^2 (r g_{tt})}{\partial r^2} = -\frac{8\pi GM}{c^2} g_{rr} \delta^3(r) \tag{13}$$

Its stationary solution [22] under the weak field limit

$$g_{tt}(r,t) = 1 + h_\varphi(r,t), \text{ where } \left| h_\varphi(r,t) \right| << 1, \tag{14}$$

is the well-known Newtonian potential,

$$\Phi = -\frac{MG}{r}, \text{ where } h_\varphi = \frac{2\Phi}{c^2}. \tag{15}$$

With inclusion of Einstein's Λ, the wave equation is modified to,

$$-\frac{\partial^2 g_{tt}}{c^2 \partial t^2} + \frac{1}{r} \frac{\partial^2 (r g_{tt})}{\partial r^2} + 2\Lambda = -g_{rr} \frac{8\pi G T_{tt}}{c^2}. \tag{16}$$

If T_{tt} were a pointlike source $T_{tt} = Mc^2 \delta^3(r)$, the static solution of this equation would be provided by the Schwarzschild-de Sitter metric [23,24,25,26], given by

$$g_{tt}(r) = \frac{1}{g_{rr}(r)} = 1 - \frac{R_S}{r} - \frac{\Lambda}{3} r^2, \quad \text{with} \quad R_S = \frac{2MG}{c^2}. \tag{17}$$

Obviously, we meet a problem here, because we cannot separate a weak field Φ (= gravitational potential) from the metric, because we cannot identify an $r-$domain that justifies the adoption of the constraint (14). However, given the fact that a viable wave function can be obtained for $\Lambda = 0$, one might expect that it must be possible to obtain a valid wave equation for a weak field Φ showing a gradual move from $\Lambda = 0$ to $\Lambda \neq 0$. The way out from the problem is the consideration that (16) should be valid both for vacuum with a massive source as well as for vacuum without a source. Hence, where an empty space with $\Lambda = 0$ corresponds with virtual sources $T_{\mu\mu} = 0$, the vacuum with $\Lambda \neq 0$ is a fluidal space with virtual sources $T_{\mu\mu} = -p\Lambda$, with $g_{\mu\mu} = (1,1,r^2 \sin^2 \vartheta, r^2)$, where $p = c^4/8\pi G$ [27,28]. This particular stress-energy tensor with equal diagonal elements corresponds with the one for a perfect fluid in thermodynamic equilibrium [22]. Inserting a massive source in this fluid will curve the vacuum to $g_{\mu\mu} = (g_{tt}, g_{rr}, r^2 \sin^2 \vartheta, r^2)$.

Therefore, inclusion of the cosmological constant Λ implies that, under absence of massive sources, Einstein's equation can be satisfied if empty space is given up and is replaced by a space that behaves as a perfect liquid in thermodynamic equilibrium. If in this fluid a massive pointlike source is inserted, the resulting wave equation is a modification of (16), such that

$$-\frac{\partial^2 g_{tt}}{c^2 \partial t^2} + \frac{1}{r}\frac{\partial^2 (rg_{tt})}{\partial r^2} + 2\Lambda = -g_{rr}\frac{8\pi GM}{c^2}\delta^3(r) + \Lambda g_{rr}, \tag{18}$$

It will be clear that the curving of space-time as a consequence of inserting a massive pointlike source in empty space, like assumed under the Schwarzschild-de Sitter condition, is different from the curving of space-time in the case of inserting a massive source in a fluidal vacuum. Adopting the weak field constraint (14) with $h_\varphi(r,t) > 1$ on (18) yields the wave equation,

$$-\frac{\partial^2}{c^2 \partial t^2}(r\Phi) + \frac{\partial^2}{\partial r^2}(r\Phi) + \lambda^2 (r\Phi) = -r\frac{8\pi GM}{c^2}\delta^3(r), \tag{19}$$

where $\lambda^2 = 2\Lambda$,

which can be written as (8).

The striking feature of (8) is the $+$ sign associated with λ^2 for a positive value of Λ. If it were a $-$ sign, the equation would be similar to Debije's equation for the potential of an electric pointlike charge in an ionic plasma [9]. As is well known, the solution of such equation is a shielded Coulomb field, i.e., an electric field with an exponential decay. In the gravitational equivalent (with the $+$ sign) the near field is enhanced ("anti-screened") [8], because masses are attracting, while electric charges with the same polarity are repelling. It is useful to note, particularly for the remainder of this article, that without additional constraints, the same background energy in Einstein's Field equation (with $\Lambda > 1$), allows the anti-screened solution as well as the screened solution. The anti-screened condition shows up under positive space-time curving ($h_\varphi(r,t) > 1$), while the screened condition shows up under negative space-time curving ($h_\varphi(r,t) < 1$).

Under positive space-time curving, (8) can be solved by

$$\Phi = \Phi_0 \frac{\cos \lambda r + \sin \lambda r}{\lambda r}. \tag{20}$$

This solution gives a fit with Milgrom's empirical enhanced gravity law for, [19],

$$\lambda^2 = \frac{2}{5}\frac{a_0}{MG}, \tag{21}$$

where a_0 is Milgrom's acceleration constant. This relates the anti-decay parameter λ with the mass of a galaxy. Because $\lambda^2 = 2\Lambda$ as shown by (9) and because a_0 has appeared being a

constant, Einstein's Λ is an integration constant with a value that is galaxy dependent. While at the level of the whole universe, Λ may be considered as an invariant cosmological constant, it will not be the case at the level of galaxies.

The next step in this analysis is the assessment of the particle density of the *darks*. To do so, eq. (8) is rewritten as,

$$\nabla^2 \Phi = -4\pi G \rho(r), \text{ where}$$

$$\rho(r) = \frac{M}{c^2}\delta^3(r) + \rho_D(r); \ \rho_D(r) = \frac{\lambda^2}{4\pi G}\Phi(r). \tag{22}$$

In Debije's theory of electric dipoles [6,9,29],

$$\rho_D(r) = -\nabla \cdot \mathbf{P_g}. \tag{23}$$

The vector $\mathbf{P_g}$ is the dipole density. From (23),

$$\rho_D = \frac{1}{r^2}\frac{d}{dr}\{r^2 P_g(r)\}. \tag{24}$$

Assuming that in the static condition the space fluid is eventually fully polarized by the field of the pointlike source, $P_g(r)$ is a constant P_{g0}. Hence, from (24),

$$\rho_D(r) = 2\frac{P_{g0}}{r}. \tag{25}$$

Taking into account that to first order,

$$\Phi(r) = \frac{MG}{r}, \tag{26}$$

we have from (25) and (26),

$$\rho_D(r) = \frac{2P_{g0}}{MG}\Phi(r). \tag{27}$$

Hence, from (25-27),

$$P_{g0} = \frac{a_0}{20\pi G}. \tag{29}$$

The volume density N/m^3 of the *darks* is of course the same as the dipole density (29). Hence,

$$N / \mathrm{m}^3 = \frac{a_0}{20\pi G}.$$
(30)

From (29) and (30), we may calculate the amount N_g of darks in the spatial volume V enclosed by the event horizon of the universe, under consideration of the dipole moment of a single *dark* given by (7),

$$N_g = \frac{P_g V}{p} \; ; \; V = \frac{4}{3}\pi (c t_H)^3 \; ; \; p = \frac{\hbar}{2c} \, ,$$
(31)

where $t_H (\approx 13.8$ Gyear) represents the Hubble time scale.

Not all of these gravitational dipoles are baryonic. In terms of the Lamda-CDM nomenclature , the baryonic share is expressed as Ω_B in the relationship

$$1 = \Omega_m + \Omega_\Lambda = (\Omega_B + \Omega_D) + \Omega_\Lambda \, ,$$
(32)

where $\Omega_m, \Omega_\Lambda, \Omega_B, \Omega_D$, respectively are the relative matter density, the relative dark energy matter density, the relative baryonic matter density and the relative dark matter density [3,30]. Where the matter distribution between the matter density Ω_m (= 0.259) and dark energy density Ω_Λ (= 0.741) is largely understood as a consequence from the Friedmann equations [30] that evolve from Einstein's Field Equation under the Friedmann-Lemairtre-Robertson-Walker (FLRW) metric [31], the distribution between the baryonic matter density Ω_B (= 0.0486) and dark matter density Ω_D (= 0.210) is empirically established from observation. The quoted values are those as established by the Planck Collaboration [32].

Taking this account, the amount of baryonic darks in a spatial volume V equal to the size of the universe amounts to,

$$N_B = \frac{P_g}{\Omega_B p} V = \frac{a_0}{20\pi G} \frac{2c}{g\Omega_B \hbar} (\frac{4}{3}\pi c^3 t_H^3) \, .$$
(33)

Entropy
Because the dipoles of the darks can only assume two quantized values (bits), this number represents the total information content of the universe. Like shown by Verlinde [33,34], the information content can be established as well from quite a different viewpoint. The Bekenstein-Hawking expression for the entropy of a black hole is a first ingredient for calculation. It reads as [33,34],

$$S_H = k_B \frac{c^3}{4G\hbar} A,$$
(34)

where c is the vacuum light velocity, G the gravitational constant, $\hbar$ Planck's (reduced) constant, A the black hole's peripheral area and k_B is Boltzmann's constant. The peripheral area of a spherical black hole is determined by its Schwarzschild radius as,

$$A = 4\pi R_S^2 , \ R_S = \frac{2MG}{c^2} , \tag{35}$$

where M is the baryonic mass of the black hole. Boltzmann's constant shows up as a consequence of the thermodynamic definition of entropy. In that definition S_H is not dimensionless, because of the thermodynamic interpretation of entropy as a measure for the unrest of molecules due to temperature, which relates the increase ΔS of entropy with an increase molecular energy ΔE due to temperature T, such as expressed by the thermodynamic definition,

$$\Delta E = T\Delta S . \tag{36}$$

Boltzmann's famous conjecture connects entropy with information, by stating

$$S_B = k_B \log(\text{\# microstates}). \tag{37}$$

This conjecture expresses the expectation that entropy can be expressed in terms of the total number of states that can be assumed by an assembly of molecules. Boltzmann's constant shows up to correct for dimensionality. I would like to emphasize here that (34) and (37) are different definitions for the entropy S, and not necessarily identical. Knowing that (34) has been derived from (36) and accepting Boltzmann's conjecture, we would have,

$$\frac{c^3}{4G\hbar} A = \log(\text{\#microstates}). \tag{38}$$

Both sides of this expression are dimensionless. Omitting Boltzmann's constant makes entropy a dimensionless measure of information, which, of course, is very appealing. At this point, I wish to elaborate on a subtlety, which has been shown by Verlinde. According to Boltzmann's conjecture, an elementary step ΔS in entropy would imply $\Delta S = k_B$. Verlinde has proven, however, that an elementary step in entropy from the Hawking-Bekenstein entropy implies $\Delta S = 2\pi k_B$. If not, the Hawking-Bekenstein's formula would violate Newton's gravity law [35]. Because Boltzmann's expression is a conjecture without proof, the problem can be settled by modifying the dimensionless expression of entropy (38) into,

$$S = \frac{1}{2\pi} \frac{c^3}{4G\hbar} A = \log(\text{\#microstates}). \tag{39}$$

The second ingredient is the well known observation that the event horizon ct_H of the visible universe equals Schwarzschild radius of the critical mass enclosed within that horizon (t_H is the Hubble time scale), thereby allowing to conceive the visible universe as a virtual

black hole [35]. Hence, from (39), the entropy within the event horizon of the universe can be established as

$$ S = \frac{c^3}{8\pi G \hbar} 4\pi (c t_H)^2 \; . \tag{40} $$

Equating (40) with (33) gives,

$$ a_0 = \frac{15}{4} \Omega_B \frac{c}{t_H} \; . \tag{41} $$

From (41), we find, as in [37], with $\Omega_B = 0.0486$, $c = 3 \times 10^8$ m/s and Hubble time scale $t_H = 13.8$ Gyear, the result $a_0 \approx 1.25 \times 10^{-10}$ m/s^2, which corresponds extremely well with observational evidence [36]. This relationship has been derived in a preprint before [37] in an analysis without invoking the entropy derivation as used in this text. Moreover, in [37] the cosmological model is more general in the sense that the simple spherical model that fits so well for galaxies, is extended to a non-spherical model of the universe with distributed baryonic matter.

Note: Accepting the Hubble time scale $t_H = 13.8$ Gyear and Milgrom's acceleration constant $a_0 \approx 1.25 \times 10^{-10}$ m/s^2 as the primary cosmological parameters gives a simple two-parameter description of the Lamda-CDM model expressed by (32). As noted before, the relationship between the matter density Ω_m (= 0.259) and dark density Ω_Λ (= 0.741) is theoretically determined by the Friedmann equations [31]. The relationship between the total matter density Ω_m (= 0.259) and the baryonic matter density $\Omega_B = 0.0486$ is determined by (41) from t_H and a_0. The dark matter density is determined as $\Omega_D = \Omega_m - \Omega_B = 0.210$. The contribution of radiation is neglected as a second order effect. In this picture, the energy of the visible universe is totally built up by the darks. The baryonic mass consists of clustered darks, the dark matter consists of polarized darks and the dark energy consists of unpolarized darks beyond the reach of the baryonic mass composed by the clustered darks.

This derivation of a numerical value of Milgrom's acceleration constant that culminates in the simple expression (41) is pretty convincing for the viability of the view presented in this paragraph. The view, though, is unconventional. Let me summarize a few issues. An important one is the view that Einstein's Λ is not a constant of nature. Instead, it is just an integration constant in his Field Equation. Hence, dependent on the scope of the cosmological system under consideration. In the case of galaxies, its value depends on the mass content of the galaxy. Only at the level of the universe, it may be regarded as the true Cosmological Constant. A second issue is the abandon of the Schwarzschild-de Sitter metric as the solution of Einstein's Field Equation for a spherical cosmological system. The justification is the consideration that the classical Schwarzschild-de Sitter solution is single sourced, ignoring a energetic background fluid. It has been shown in this paragraph that

accepting an energetic bias allows the derivation of a rather simple extension of Poisson's equation for the gravitational potential, albeit valid within a certain spatial range only. Within the scope of this text, this range will not be specified. Details of its calculation can be found in [19]. A third issue is the hypothesis that the cosmological background energy is built up by polarisable vacuum particles, in this text named as *darks* identified as third type Dirac particles.

4. Profiling a quark as a third type Dirac particle

Let us proceed trying to set up a similar model of the nuclear background energy similar to the cosmological background energy. To do so, let us suppose, as usual by the way, that a quark is a Dirac particle [38]. In this text, however under the assumption that a quark is a Dirac particle of the third type, the second dipole moment of which is polarisable. Hence, we may conceive its potential field as the sum of a far field from the monopole and a near field from the dipole moment. The second dipole moment $m_p d = \hbar / 2c$, where m_p and d are unknown quantities, creates a near field potential field $\Phi_{GN}(x)$ along the dipole axis x such that,

$$\Phi_{GN}(x) = \frac{Gm_p d}{x^2} \rightarrow \Phi_{GN}(x) = \frac{\hbar}{2c}\frac{\lambda^2 G}{(\lambda x)^2} \rightarrow \Phi_{GN}(x) = \Phi'_0 \frac{1}{(\lambda x)^2} \; ; \; \Phi'_0 = \frac{\hbar}{2c}G\lambda^2 . \qquad (42)$$

Note that Φ'_0 is expressed in energy per unit of mass (hence signed by ') Note that there is no particular reason yet to identify the λ in (42) as the Λ-related one in (8,9). Apart from the near field $\Phi_{GN}(x)$, the quark spreads a far field $\Phi_{GF}(r)$. The far field is the result of the (monopole) mass $m_{qu} (\neq m_p)$ of a bare quark. Under absence of any mass generation mechanism, only the eigen value $\hbar/2$ of the quark's elementary angular momentum is left as the manifestation of this bare mass. As will be discussed later in this text, the bare mass is different from the quark's constituting mass. The latter is the result of the mass generation mechanism that will be explained later in this text as well. Interpreting the angular momentum as a virtual rotation with light speed at a fictitious radius $r_0 = 1/g_m\lambda$, we have

$$\frac{\hbar}{2} = \frac{m_{qu}c}{g_{m\lambda}\lambda} \rightarrow m_{qu} = g_m \frac{\hbar}{2}\frac{\lambda}{c} . \qquad (43)$$

The quantity g_m is an unknown *gyrometric* constant. Hence, from classical field theory,

$$\Phi_{GF}(r) = \frac{m_{qu}G}{r} = g_m \frac{\hbar}{2}\frac{\lambda}{c}\frac{G}{r} = g_m \frac{\hbar}{2}\frac{G\lambda^2}{c}\frac{1}{\lambda r} , \qquad (44)$$

and, under consideration of Φ_0 as defined in (42),

$$\Phi_{GF}(r) = g_m \frac{\hbar}{2} \frac{G\lambda^2}{c} \frac{1}{\lambda r} = \Phi_0' \frac{g_m}{(\lambda r)} \; . \tag{45}$$

Hence, the potential field of the quark along the axis set up between the dipole axis can be expressed as an energy $\Phi(\lambda x)$ such that

$$\Phi(\lambda x) = \Phi_0' \{ \frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x} \} \; ; \quad \Phi_0' = \frac{\hbar}{2c} G\lambda^2 \; . \tag{46a}$$

Other nuclear particles may couple to the field of a quark. The quantities Φ_0 and λ are subject to the relationship given by (42), leaving some individual freedom and can therefore be different for different quark flavours. Similarly as the field of darks, the quark field is influenced by the energetic background field. However, where the gravitational field is enhanced under polarization of the background particles, the nuclear field is suppressed. This can be understood as a consequence of the different influence on the polarization of the background particles from nuclear bare masses as compared to gravitational masses evoked by a mass generation mechanism. Taking the background polarization into account, the effective potential field of the quark along the axis set up between the dipole axis can be written as,

$$\Phi(\lambda x) = \Phi_0 \exp(-\lambda x) \{ \frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x} \} ; \quad \Phi_0 = m_0 \frac{\hbar}{2c} G\lambda^2 \; , \tag{46b}$$

Where m_0 is the unknown mass of the bare quark and where λ is identified as the decay parameter of the field due to the background energy.

As I wish to show now, the quark's profile (46) resembles the one that can be derived from the heuristic Lagrangian of the Higgs field conceived in the Standard Model of particle physics. In its most simple representation, this field Φ is characterized by its Lagrangian density $U_H(\Phi)$. This density is heuristically defined as [1],

$$U_H(\Phi) = \mu_N^2 \frac{\Phi^2}{2} - \lambda_N^2 \frac{\Phi^4}{4} \; , \tag{47}$$

where μ_N and λ_N are characteristic real constants. The justification for this format is the simple fact that many predictions from the theoretical elaboration of this underlying axiom of the Standard Model are in agreement with experimental evidence and understood from the hypothesis of the Systematic Symmetry Breaking (SSB) of the classical field potential. It is instructive to compare this heuristically conceived energetic background field with the background field around an electric pointlike charge in an ionized plasma [9], where the background field has the format

$$U_{DB} = \lambda_{DB}^2 \frac{\Phi^2}{2}. \tag{48}$$

Note that λ_{DB} is more like μ_N rather than like λ_N. The energetic field Φ flowing from the pointlike charge in an ionized plasma is influenced by this background field and can be derived by the overall Lagrangian density L with the generic format

$$\mathrm{L} = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho\Phi, \tag{49}$$

where $U(\Phi)$ is the potential energy of the background field and where $\rho\Phi$ is the source term. By application of the Euler-Lagrange equation, the potential Φ of the pointlike source of this (Debije) field can be derived as,

$$\Phi_{DB} = \Phi_0 \frac{\exp(-\lambda_{DB} r)}{\lambda_{DB} r}, \tag{50}$$

with $\Phi_0 = Q\lambda/4\pi\varepsilon_0$, where Q is the electric charge and ε_0 the vacuum electric permittivity.

Supposing that a quark is an energetic pointlike source, we may try to establish its potential from (47) in a similar way, thereby expecting it being influenced by the energetic background field. From the difference between (48) and (47) it appears that the background field seems to be broken. In present state particle theory the broken format is accepted as an axiom.

Unfortunately the particular format of this (broken) field prevents deriving an analytical solution $\Phi(r)$ of from (49) subject to (47). However, a numerical procedure allows deriving a two-parameter expression for $\Phi(r)$ that closely approximates a true analytical solution. The result is [42],

$$\Phi(r) = \Phi_0 \frac{\exp[-\lambda r]}{\lambda r}\left(\frac{\exp[-\lambda r]}{\lambda r} - 1\right) \text{ with } \frac{1}{2}\mu_c^2 = 1.06\lambda^2 \text{ and } \frac{1}{4}\lambda_c^2 = 32.3\frac{\lambda^2}{\Phi_0^2}. \tag{51}$$

This result can be written as the sum of a far field and a near field, such that,

$$\Phi(r) = \Phi_F(r) + \Phi_N(r) \text{ with } \Phi_F(r) = -\Phi_0 \frac{\exp(-\lambda r)}{\lambda r} \text{ and } \Phi_N(r) = \Phi_0 \frac{\exp(-2\lambda r)}{(\lambda r)^2}. \tag{52}$$

Such a quark would show the characteristics as shown in Figure 1. It would imply that a quark would be repelled by any other quark under influence of the far field, but attracted by the near field, thereby giving rise to mesons as stable two-quark junctions and baryons as three-quark junctions.
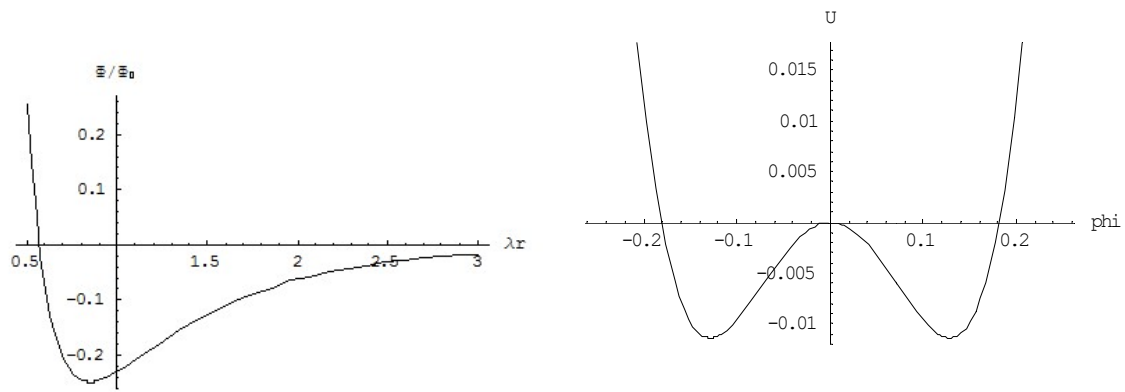
Figure 1. (Left) The quark's scalar field Φ / Φ_0 as a function of the normalized radius λx ; (Right) The Higgs field $U(\Phi)$ retrieved from the spatial expression.

Unfortunately, this radial symmetric solution is not viable, because the near field violates the renormalization constraint. The problem can be solved by a slight modification of (52) to

$$\Phi(\lambda x) = \Phi_0 \, \exp(-\lambda x)\{\frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x}\}, \qquad (53)$$

which corresponds with the a potential function along the axis between the dipole poles of a Dirac particle polarisable in a scalar potential field under the influence of a shielding background, like shown by (46). Under the condition that $g_m \approx 1.76$, this potential function is almost indistinguishable from the numerically obtained solution (51) from the Higgs potential, albeit that near $x = 0$ the functions are deviating. In fact, the deviation is not relevant as can be understood as follows: If two bare quarks, more particularly a quark and an antiquark, each producing a generic flow of energy of non-baryonic nature, are interacting, a quasi-stable bond is created, thereby producing the archetype meson, i.e. the pion. In that condition, the spacing $2d'_{min} = 2d\lambda$ between quark and antiquark is determined by the condition retrieved from (51) as [39],

$$\exp(-d'_{min})/ d'_{min} = \frac{1}{2} \to d'_{min} \approx 0.853, \qquad (54)$$

which is hardly affected by the difference between the formats (51) and (46). This gives an interesting view on the mass generation mechanism, somewhat different from the canonical one, but not necessarily in conflict. Let us stick close to the Standard Model and adopt the two-parameter field potential obtained from the numerical solution retrieved from the Higgs potential [39], albeit under the restriction of validity along a single axis. Hence, considering the definition of Φ_0 ,

$$\Phi(\lambda x) = \Phi_0 \frac{\exp(-\lambda x)}{\lambda x} \{\frac{\exp(-\lambda x)}{\lambda x} - 1\}; \quad \Phi_0 = m_0 \frac{\hbar}{2c} G\lambda^2 . \qquad (55)$$

The potential function is characterized by a strength parameter Φ_0 and a spatial range parameter λ. The renormalization issue disappears because of the consideration that the near field is due to the quark's second dipole moment.

The bond can be modelled as a quantum mechanical oscillator with a center of baryonic mass. Something similar happens between the three quarks in a baryon that create a three-body quantum mechanical oscillator. The vibration energy of the oscillator is the dominant part of the rest mass of the pion. The contribution of the bare masses of the quarks to the baryonic mass is negligible. All quarks show the same potential function. Their Φ_0 and λ values are related with those of the quarks in the archetype meson (pion) and are subject to a constant value of the ratio Φ_0 / λ. It means that, similarly as in the case of darks, λ is a variable that get a particular value in a particular scope. Details can be found in [39], as well as the proofs that the decay and spatial range parameter λ in the scope of the pion is a measure of the energetic mass equivalence of the Higgs boson, i.e. $m'_H \approx 2\lambda(\hbar c)$ and that its numerical value of about 126.5 GeV can be determined by theory.

The assignment of a classical potential field to a quark is unconventional. In this paragraph the field has been derived from the view that the quark is a Dirac particle of a particular type that up to recently was unknown. It has been shown in this paragraph that the derived potential function is consistent with a numerical solution from the heuristic Higgs Lagrangian. In that respect the unconventional view on the quark as expressed by (46) is not in conflict with the Standard Model of particle physics. The intriguing feature is the mass generation mechanism associated with this potential.

5. Relating cosmological properties with nuclear properties

From the analyses made so far, it is fair to conclude that quarks in a meson and the cosmological darks are Dirac particles. In that sense they are similar to electrons. However, where the field of an electron is not affected in vacuum, the field of a quark is shielded by an energetic background field while the field of a dark is enhanced by such a field. To enable a proper comparison between the three particle types, a generic force F will be defined as the spatial derivative of a generic potential Φ in units of energy, such that for electrons, darks and quarks, respectively,

$$F = \frac{\partial}{\partial y}\Phi = e\frac{\partial}{\partial y}\Phi_{\mathrm{e}}$$

$$F = \frac{\partial}{\partial y}\Phi = m_0\frac{\partial}{\partial y}\Phi_{\mathrm{G}}$$

(56)

$$F = \frac{\partial}{\partial y}\Phi = g\frac{\partial}{\partial y}\Phi_{\mathrm{qu}}$$

where q, m_0 and g, are the coupling factors of, respectively, an electron, a massive particle and a quark to respectively, an electric potential Φ_{e}, a gravitational potential Φ_{G} and a

nuclear potential Φ_{qu}. Electroweak unification relates the nuclear coupling factor g with the electromagnetic coupling factor e by the fine structure relationship $e^2 = 4\pi\varepsilon_0 \hbar c g^2$. Where these potential fields are specific and have specific dimensionalities, they are all derived from a generic potential Φ in the dimension of energy.

A dark feels a force from another dark as,

$$F = m_D \frac{\partial}{\partial x}\Phi_{\mathrm{G}}; \quad \Phi_{\mathrm{G}} = -m_D G\lambda \frac{\cos\lambda x + \sin\lambda x}{\lambda x}. \tag{57a}$$

Recognizing that quarks and darks have common roots as third type Dirac particles, it follows that a quark feels a force from another quark as,

$$F = g\frac{\partial}{\partial x}\Phi_{qu}; \quad \Phi_{qu} = m_0 \frac{\hbar}{2c}G\lambda^2 \frac{\exp(-\lambda x)}{\lambda x}\{\frac{\exp(-\lambda x)}{\lambda x} - 1\}, \tag{57b}$$

Let us use these relationships for relating the strength of the nuclear force with the strength of the gravitational force. Modeling the quark similarly as a dark allows balancing the two force expressions. Let us do so for the unshielded far field, i.e.

$$F = g\frac{\partial}{\partial x}(m_0 \frac{\hbar}{2c}G\lambda^2 \frac{1}{\lambda x}) = m_0 \frac{\partial}{\partial x}(m_0 G\lambda \frac{1}{\lambda x}) \rightarrow m_0 = g\frac{\hbar}{2c}\lambda, \tag{58}$$

where m_0 is the undefined bare mass of a quark. It relates the λ variable of the quark with its mass. In view of the remark made before that (in the scope of a pion) λ is a measure for the Higgs boson $m'_H \approx 2\lambda(\hbar c)$, the bare quark mass is unrealistically high. It has, however, to be taken into account that the Higgs boson flies about light speed. Hence, the relationship between the Higgs boson mass and the lab frame value of the bare quark as expressed by (58) is subject to a relativistic correction. It will be done later in this text, which will reveal $m'_0 \approx 5$ MeV. This value corresponds with the data from the Particle Data Group [40].

Let us proceed by considering the spatial parameter λ_D of a dark. As has been specified in (21), It is related with its rest mass

$$\lambda_D^2 = \frac{2a_0}{5m_D G}, \tag{59}$$

where m_D is the rest mass of a dark. Recognizing a common origin of the quark and the dark as a third type Dirac particle, imposes the following invariance,

$$m_D\lambda_D^2 = m_0\lambda^2 = \frac{2}{5}\frac{a_0}{G}. \tag{60}$$

This is a basic formula for the unification of gravity with quantum physics, which needs somewhat more justification apart from the logic exposed by (60). It has to do with a slight

difference between the role of λ_D of darks and the role of λ of quarks. The source dark is an elementary energetic particle surrounded by background darks enhancing the energetic flow from the source quark. The λ_D is a measure for the enhancement. The source quark is an elementary energetic particle surrounded by background particles, hypothetically the background darks, suppressing the energetic flow from the source quark. This makes the semantics of λ_D somewhat different from those of λ. One might guess that the sign ambiguity on the second dipole moment as shown in (5) makes the quark and the dark different in their behavior of their dipole polarization in a scalar potential field. Hence, the unification formula (58) may seem at best a conjecture that needs an ultimate justification from an observable result. There is, however, somewhat more. Because the dimensionality of $m\lambda^2$ is mass per square meter, the left hand part of (60) represents gravitational massive energy per unit of area and the middle term represents nuclear energy per unit of area. The projection of this property on any spherical boundary justifies the conclusion that (60) expresses not more and not less than that the total gravitational energy within the horizon of the visible universe equals the total nuclear energy. We shall proceed now by accepting the conjecture (60).

From (58-60),

$$\frac{a_0}{G} = \frac{5}{2}g\frac{\hbar}{2c}\lambda^3 \,. \tag{61}$$

The remaining issue for relating the relationship between cosmological quantities with nuclear quantities is establishing a value for the quark's quantity λ. This is possible by considering that the spacing between the quarks in the center of frame of a pion (flying at about light speed) is just the half wavelength of a weak interaction boson $\hbar\omega_W$. The half wave length of the weak interaction boson $\hbar\omega_W$ is about equal to the spacing $2d_{\min}$ between the two quarks. Hence,

$$2d_{\min} \approx \frac{1}{2}cT_W \approx \frac{1}{2}\hbar c\frac{2\pi}{\hbar\omega_W} \,, \tag{62}$$

where T_W is the period of the weak interaction boson. This boson is the carrier of two forces, namely a repulsion from the far field potential and an attraction from the near field potential evoked by the dipole moments of the two quarks. If there were no attraction, the boson would just be the carrier of the far field. Let us denote this far field boson as $m'_H = \hbar\omega_W / \alpha$, where α is a characteristic proportionality constant of order 1. The far field boson would bridge the distance $2d_{\min}$ as

$$2d_{\min} \approx \frac{\lambda}{2} = \frac{1}{2}\hbar c\frac{2\pi}{m'_H} \,. \tag{63}$$

The energetic value of the weak interaction boson($\hbar\omega_W \approx 80.4$ GeV) is related via a characteristic value $\alpha \approx 0.69$ with the energy of the far field boson ($m'_H \approx 127$ GeV), which

can now be identified as the Higgs boson. The proof that $\alpha \approx 0.69$ indeed, can be found in [39].

These relationships (62-63) hold in the center of mass frame of the pion. The magnitude of the normalized spacing $2d'_{min}$ in the archetype meson is known from the equilibrium state between the quark and the antiquark, as can be determined from the quark's potential function. This equilibrium state is obtained for [39],

$$\exp(-d'_{min})/d'_{min} = \frac{1}{2} \rightarrow d'_{min} \approx 0.853 \,;\; d'_{min} = d_{min}\lambda \tag{64}$$

The λ value in this expression is a center of mass frame value and therefore not the same as the lab frame quantity λ_0. Considering that in the lab frame the energy of the weak interaction boson is the rest mass of the meson $\hbar\omega_\pi$, we have

$$\frac{\hbar\omega_W}{\hbar\omega_\pi} = \frac{\lambda}{\lambda_0}\,. \tag{65}$$

Note: In this view on the pion mass, the quarks are conceived as bare quarks and not as constituent quarks. In the bare quark model, the mass is due to the binding energy from the interaction bosons.

Hence, from (62-65),

$$\lambda_0 = \frac{2}{\pi}\frac{\hbar\omega_\pi}{\hbar c}d'_{min}\,. \tag{66}$$

The equations (62), (65) and (66) establish the relationship between gravity and quantum physics as,

$$\frac{a_0}{G} = \frac{5}{2}g\frac{\hbar}{2c}\lambda_0^3\,;\; \lambda_0 = \frac{2}{\pi}\frac{\hbar\omega_\pi}{\hbar c}d'_{min}\,. \tag{67}$$

With G = 6.67 10^{-11} kg^{-1} m^3 s^{-2}, $\hbar\omega_\pi \approx 140$ MeV (pion), , the calculated value of Milgrom's acceleration constant is 1.44 10^{-10} m/s^2. This corresponds fairly with the present known value of Milgrom's acceleration constant (1.25 10^{-10} m/s^2). In view of the high sensitivity for λ, this is rather satisfying. It is fair to consider this result as the proof for the conjecture formulated by (58) that the total gravitational massive energy within the horizon of the visible horizon is equal to the total nuclear energy.

The viability of this result can be supported from a calculation of the mass value of a bare quark. From (58) and (65), we have under consideration that $m'_H = \hbar\omega_W/\alpha = 2\lambda\hbar c$,

$$m_0 = g\frac{\hbar}{2c}\lambda = g\frac{\hbar}{4c^2}(2\lambda c) = g\frac{1}{4c^2}m'_H \rightarrow m_0 c^2 = \frac{g}{4}(\frac{140}{0.69}) \approx 5\,\text{MeV}. \tag{68}$$

This value corresponds with the data from the Particle Data Group [40].

The derived relationship between Milgrom's acceleration constant a_0 and the rest mass $m_\pi = \hbar\omega_\pi / c^2$ relates a major cosmological property with a major nuclear property. It is not yet quite the same as the aim claimed in the title of this article. This aim would imply the derivation of an expression that relates the Cosmological Constant with the value of the Higgs boson. However, it has been argued in this text before that the Cosmological Constant is the value of Einstein's Λ at the level at the universe. This value can be calculated from the relationships (9) and (21) by inserting the critical mass value of the universe within the visible horizon. This means that Milgrom's acceleration constant a_0 is good enough, or maybe even better, as the characteristic quantity for the cosmological background energy. Something similar is true for the Higgs field. As argued in this article, the rest mass of the pion is the lab frame value of the weak interaction boson $m'_W = \hbar\omega_W$ that flies at about light speed. This value ($\approx$ 80.4 GeV) is related via a characteristic value $\alpha \approx 0.69$ with the energy of the Higgs boson ($m'_H \approx$ 127 GeV).

As stated at the end of paragraph 4 in this text, this quark model gives a simple explanation on the mass generation mechanism, thereby still recognizing the role of the Higgs boson. There is, however, no particular reason to adopt the QCD gluons of the color force as defined in the standard model of particle physics, because the gluing is a result from the polarisable dipole moment of the quark. How to harmonize the view as developed in this text with QCD remains a challenge beyond the scope of this article.

6. Conclusion

In this article, *quarks*, as the elementary nuclear particles, and *darks* as elementary constituents of the cosmological background energy, have been described as Dirac particles of a particular kind, dubbed as *thirds*, being subject to a classical potential field. This has been possible by recognizing that these particular Dirac particles show a real valued polarisable second elementary dipole moment next to the well known elementary angular moment. It has been shown that the theory, based upon this correspondence, has resulted in the view that gravity and quantum physics can be unified. The viability of the theory is proven by the derivation of two verifiable relationships. The first one is the calculated value of Milgrom's acceleration constant a_0 from the baryonic content Ω_B (= 0.0486) of the universe,

$$a_0 = \frac{15}{4}\Omega_B \frac{c}{t_H}, \tag{68}$$

where t_H ($\approx$ 13.8 Gyear), which gives $a_0 = 1.25 \; 10^{-10}$ m/s^2. The second one is the relationship between Milgrom's acceleration constant as characteristic cosmological quantity with the energetic equivalent $\hbar\omega_\pi$ of the pion mass as the characteristic nuclear quantity,

$$\frac{a_0}{G} = \frac{5}{2} g \frac{\hbar}{2c} \lambda^3 ; \quad \lambda = \frac{2}{\pi} \frac{\hbar \omega_\pi}{\hbar c} d'_{min} ; \quad \exp(-d'_{min})/d'_{min} = \frac{1}{2}, \tag{69}$$

with constants of nature, $G, \hbar, c$ and g ($g^2 = 1/137$). Reasons have been given why a_0 and $\hbar \omega_\pi$ are preferred as characteristic quantities above the Cosmological Constant (Einstein's Λ at the level of the universe) for cosmology and the Higgs boson for the nuclear domain.

The calculated value of Milgrom's acceleration constant from (67) amounts to 1.44 10^{-10} m/s^2. It is slightly different from the one expressed by (66), which is equal to the known empirical one (1.25 10^{-10} m/s^2). In view of the high sensitivity for λ, this is rather satisfying. All this supports the view that it is possible to unify gravity and quantum physics on the basis of the recognition of the polarisability Dirac's second dipole moment. It proves the expectation that the total gravitational massive energy within the horizon of the visible horizon is equal to the total nuclear energy.

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