The Impact of Annihilation of Magnetic Monopole on Inflation, Dark Energy and Dark Matter

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Over abundance of magnetic monopoles predicted by the Grand Unification Theories is inconsistence with current astronomical observation. The inflationary hypotheses with a vacuum energy deriving the exponential expansion can explain the two long-standing problems, flatness and horizon of the universe; and somehow suppress the abundance of monopoles. However, the dynamical scalar field has considerable uncertainties, which even leads to inflationary models separating from particle physics. This paper makes a small change, the early universe undergoes free expansion rather than an adiabatic one widely adopted, In such a case, the annihilation of abundant magnetic monopoles is just required in driving the inflation, so that the three long-term problems are automatically solved. On the other hand, the relic mass of failed annihilation of monopoles is responsible for the dark matter at present epoch. And ongoing annihilation on stars and compact objects corresponds to the dark energy. As monopole relics may exist in the form of monopole anti-monopole pairs, a new strategy of direct search is proposed.

Introduction

The order-disorder transition around the critical temperature occurs in ferromagnets. Similar effect is expected in the early universe. According to Grand Unification Theories (GUTs), a spontaneous symmetry breaking of a higher symmetry, such as $SU(5)$, to a subgroup leaves an unbroken U(1) of lower symmetry. With such symmetry breaking, it is possible to have non-trivial topologies for the gauge orientation of the vacuum expectation value of the Higgs field. One such an example of a non-trivial topology corresponds to a magnetic monopole (MM) (*1, 2*). The course of a phase transition and the dynamics of the phase transition is thus intimately related to the rate of MM production.

However, GUTs predict a number density of the superheavy MMs of n_m which is of the order, or greater than the number density of baryons, n_b. This corresponds to a total mass of MMs of 16 order of magnitude greater than that of baryons, which is intolerably large.

The first inflationary model by Guth (*3*), so called old inflation, is based on a scalar field theory undergoing a first order phase transition. The scalar field was initially trapped in a local minimum of some potential, and then leaked through the potential barrier and rolled towards a true minimum of the potential. The transition from initial "false vacuum" phase to the lower energy "true vacuum" phase, so called the super-cooling, corresponds to a difference in energy density which drive the exponential expansion of the universe.

The process cannot be simultaneous everywhere, small bubbles of true vacuum are inevitable, which would be carried apart by the expanding phase too quickly for them to coalesce and produce a large bubble. The resultant universe would be highly inhomogeneous and anisotropic, oppositing to what is observed in cosmic microwave background.

The successor to the old inflation was new inflation (*4, 5*). This is again a theory based on a scalar field. The field is originally in the false vacuum state, but as the temperature lowers it begins to roll down into one of the two degenerate minima. There is no potential barrier, so the phase transition is the second order. Problem of such inflationary models is that they suffer from severe fine-tuning problem and requires very specific field, ϕ. One of the most popular inflationary models is the chaotic inflation (*6*), which is based on a scalar field, but it does not require any phase transition. In such cases, the cosmology is in fact separated from the standard model of particle physics.

However, there are still considerable uncertainties in these models, including axion of much light mass comparing with MM (*7, 8*). To drive the exponential expansion of the universe, the energy discrepancy between “false vacuum” phase and “true vacuum” phase must satisfy a number of specific assumptions on the scalar field and effective potential.

If the scalar field is only responsible for the generation of MMs, then it is much simpler and easier to achieve. This paper shows that once the annihilation of MMs is under the free expansion rather than adiabatic one widely used, the annihilation of MMs becomes so efficient that the rapid expansion at early universe can be driven by it.

The free expansion prevents the universe from energy exchange with surroundings, a problem existing in the scenario of adiabatic expansion. Such an annihilation naturally solve the puzzle of over abundant MMs and other two long-term problems. Moreover, it sheds new light on the dark matter (DM), dark energy (DE), and the ultra-high cosmic rays (UHECRs).

All these can be accomplished by changing of polytropic index from the adiabatic one of $\gamma = 1.333$, to the free expansion one of $\gamma = 1.156$. As a result, the inflation required in cosmology can still be understood in the context of the standard model of particle physics.

The annihilation of MMs

The rate of annihilation of MMs (*9*) was derived by assuming an adiabatic expansion. A simple modification can make it applicable to the free expansion, containing only two free parameters, the polytropic index and the number of MMs, which can satisfy the requirements of cosmology evolution from the early universe to current epoch.

The annihilation of MMs can be investigated by the Boltzmann equation, in which the rate of change in the abundance of a given particle is the difference between the rates for producing and eliminating that species. The annihilation of MMs varies the density, n (with M and $\bar{M}$ densities are assumed equal), and relates with the scale factor of the universe, a,

$$dn/dt = -Dn^2 - (3\dot{a}/a)n \tag{1}$$

where D is the velocity averaged product of the cross-section and the velocity. In the case of adiabatic expansion (corresponding to $\beta = 1$ in the relationship of, $Ta^\beta = const$, with T denoting the temperature), the expansion rate of the universe is read (*9*)

$$\beta\dot{a}/a = -\dot{T}/T = T^2/(Cm_P) \tag{2}$$

where $m_P = 1.2 \times 10^{19} GeV$ is the Planck mass, and $C = 0.6N^{-1/2}$, where N is the effective number of spin degree of freedom due to particles light compared with the temperature. In fact, Equation (2) can be applied to other polytropic process which corresponds to the parameter, β, differing from 1, which will be shown later.

If the process of annihilation can be performed in the case of thermodynamic equilibrium (it does in the case of free expansion as shown later), then D relates with the temperature by a power law, and thus Equation (1) and Equation (2) can be integrated,

$$r(T) = \frac{1}{B\hbar^2}[\frac{4\pi}{\hbar^2}]^2\frac{m}{(\beta Cm_p)} = \frac{n}{T^3} \tag{3}$$

where $B = \frac{3}{4\pi^2}\zeta(3)\sum_i(\frac{\hbar q_i}{4\pi})^2$, with $\zeta(3) = 1.202$ is the Riemann zeta function. Equation (3) can be rewritten as $n = r(T)T^3$. The annihilation cross section is of $\langle\sigma v\rangle = \hbar^2/(BT^2) \propto T^{-2}$ (*9*). Together with the number density of MMs given by Equation (3), the energy release of such an annihilation can be given

$$|\dot{E}_m| = M_m n^2 \langle \sigma v \rangle = M_m (\frac{m}{Cm_p})^2 \frac{(4\pi)^{13}\pi^3}{3^3\zeta(3)^3} \frac{\sigma T^4}{\hbar c q^6 \beta^2} = 4 \times 10^{45} \frac{T^4}{\beta^2} (ergs^{-1}cm^{-3}), \quad (4)$$

where M_m is the mass of a single MM and $q^2 = 137\pi\mu_0\hbar c$ is the square of the magnetic charge. Notice that in Equation (4), the number density of MMs and the temperature decrease with time, so that $\dot{E}_m < 0$.

The radiation of the annihilation as shown in Equation (4) in unit of $ergs^{-1}cm^{-3}$, can be integrated over time and volume, under the relationship of scale factor, time and temperature of the universe(subscript P denotes Planck value) of,

$$\frac{a}{a_P} = \frac{t}{t_P} = (\frac{T_P}{T})^\omega \quad (5)$$

the validity of this relationship will be discussed in the last section. The Planck temperature, time, and scale factor, are $T_P = 10^{32}K$, $T_P = 10^{-43}s$ and $a_P = 10^{-32}cm$ respectively. By Equation (5), the elements of volume and time can be written, $dV = 3a^2da$ and $dt = -\omega T_P^\omega T_P T^{-\omega-1}dT$ respectively, so that the resultant integration of Equation (4) yields,

$$E_m = -\frac{4 \times 10^{45}}{\beta^2} \int_{a_i}^{a_f} \int_{t_i}^{t_f} T^4 dt dV = \frac{4 \times 10^{45}}{\beta^3} \frac{t_P a_P^3}{(4-\omega)} T_f^4 (\frac{T_P}{T_f})^{4\omega} \quad (6)$$

The E_m is the overall energy release of annihilation of MMs during the inflation, which corresponds to an energy density of, $\rho = E_m/V \approx E_m$ (with the final volume of the inflation of $\sim 1cm^3$). How large should E_m be ? The flatness problem is avoided if the inflation stretches the curvature (by enlarging the scale factor) tremendously, which corresponds to the lower bound of the number of $e-$foldings of scale factor, a_f/a_i to be (with $h = 0.7$) (*10*)

$$e^N > \frac{[\rho]^{1/4}}{0.037heV}. \quad (7)$$

This demands an energy density of order of, $\rho = (2 \times 10^{16}GeV)^4 = 4 \times 10^{103}erg/cm^{-3}$. In such a case, the value of e^N would have to be at least 8×10^{26}, so that $N > 62$.

By Equation (6), E_m depends on two values, the temperature at the final state of inflation, T_f, and the index ω. These two parameters are constrained by three factors. The first is the equivalence between E_m/V of Equation (6) and the energy density, ρ, of Equation (7). The second is that the $e-$foldings of scale factor ($a_f/a_i \sim e^N$) of Equation (7) shall satisfy the condition of free expansion $TV^{1/3\omega} = const$ (where $V = a^3$). Finally, the temperature and volume are subjected to the relationship of Equation (5).

Then we have $T_f = 10^{17}K$ and $\omega = 2.138$ in Equation (6), which corresponds to a polytropic index of $\gamma = 1.156$ (with the relationship $\omega = \frac{1}{3(\gamma-1)}$). Furthermore, $\beta = 1/\omega$ in Equation (2)-Equation (4) can be obtained.

The energy release of $E_m \sim 4 \times 10^{103}(erg)$ required in solving the flatness problem corresponds to a total number of annihilation of MMs of $N_m \sim 4 \times 10^{88}$ during the inflation (with each MM of mass of $M_m \sim 1 \times 10^{16}GeV$).

The free expansion of the early universe

How the universe expand ? The ideal process is the free expansion, an irreversible process without work done and free of heat exchange with the surroundings. Because the universe expansion is actually an expansion of space-time carrying matter and energy; there is nothing outside the universe except the universe itself. With the heat reservoir by the energy release of the annihilation of MMs, the universe undergoes an asymptotic isothermal expansion, which is infinitely close to the free expansion.

Normally the entropy of such a free expansion can be calculated as if work done in a reversible, isothermal process, AB, or an adiabatic expansion, AC, plus a constant volume process, CB, between the same end points, as shown in the pressure-volume (p-V) diagram of Fig1. For the process of ACB, the work done to the surroundings during the adiabatic expansion from

A to C is given,

$$W = C_V T_A[1 - (\frac{V_A}{V_C})^{\gamma-1}] \tag{8}$$

where C_V is the heat capacity, p_k and V_k are pressure and volume at certain state respectively. Apparently, the work done W is at expense of internal energy U (dropping of temperature), so that $\Delta U = W$.

In the second part of process ACB, heat flowing into the internal energy by a process of constant volume from C to B,

$$Q = C_V[T_B - T_C] = C_V T_B[1 - (\frac{V_A}{V_C})^{\gamma-1}] \tag{9}$$

With $T_A = T_B$ as shown in Fig1, the work done in Equation (8) equals heat absorption of Equation (9), which results in, $Q = W = \Delta U$. In other words, the process of constant volume from C to B gets the system reheated, so that the loss of internal energy in the process AC is compensated.

In the case of the free expansion of the early universe, the work done required in volume change given by W of Equation (8) equals the heat absorption, Q, of Equation (9). In practice, the heat absorption is supplied by the annihilation of MMs, so that $Q \sim E_m \approx 4 \times 10^{103} erg$. And the work done from A to B in Fig1 is given by,

$$W = \frac{1}{\gamma - 1}(p_A V_A - p_C V_C) \approx \frac{p_A V_A}{\gamma - 1} = \frac{1}{\gamma - 1} g^\star N_m k_B T_i \approx 2 \times 10^{103}, \tag{10}$$

where the effective number of degrees of freedom is estimated, $g^\star = 100$, and the initial temperature of inflation is estimated of $T_i = 10^{28} K$. Therefore, even in such a simple estimation, the magnitude of work done required in powering a rapid expansion of the universe, W, is well consistent with the supply energy, Q, from the annihilation of MMs.

The ratio between the initial temperature of Equation (10),$T_i = 10^{28} K$; and the final temperature of inflation of Equation (6), $T_f = 10^{17} K (10^4 GeV)$ is of, $T_i/T_f \sim 10^{11}$, which corresponds to the ratio of scale factor of $a_f/a_i \sim 10^{24-25}$ under the temperature and volume relationship of $TV^{\gamma-1} = const$. Such a ratio is about one order of magnitude less than $a_f/a_i \sim 10^{26}$ (*10*). Considering these are simple estimations, the energy release of MMs, Q; the work done, W, required in a rapid expansion of a system of a number of $N_m = 10^{88}$ MMs; as well as the temperature and volume change (T_i/T_f and a_i/a_f respectively) during the expansion are basically consistent with each other.

As $T_i/T_f \approx 10^{-64}V_f/V_i$, the temperature variation during the inflation is negligible compared with the volume change. Thus the polytropic process of AB' is infinitively close to the ideal isothermal process, AB, representing the irreversible process of free expansion, so that the temperature can be safely treated as unchanged during the inflation. On the other hand, the situation of energy exchange with surroundings of the universe is avoided.

Notice that Equation (2)-Equation (4), corresponding to the annihilation of MMs are derived in the case of adiabatic expansion with $\gamma = 1.333$ (*9*), which can be applied to the case of free expansion with a polytropic index of $\gamma = 1.156$, by simply changing the parameter, β from $\beta = 1$ to $\beta = 0.468$ in Equation (2)-Equation (4).

Vacuum energy or radiation of annihilation of MMs

As shown in the last section, the annihilation of MMs can be so powerful that the resultant free expansion of the early universe is sufficient to account for the flatness problem. Moreover, the asymptotic isothermal process automatically solves the causality (horizon) problem. Here physical property of free expansion is further compared with that of vacuum field.

Firstly, can the energy release of the annihilation corresponds to a negative pressure as the vacuum field predicted by the inflationary theories do? The expansion of de Sitter universe can be written,

$$(\frac{\ddot{a}}{a}) = -\frac{4}{3}\pi G(\rho + 3p - 2\rho_\Lambda) \tag{11}$$

As shown in Equation (11), a negative energy density (e.g., the vacuum energy), ρ_Λ counters the gravity of the universe and resulting in an accelerated expansion $\ddot{a} > 0$ of the universe; in contrast, a positive energy and pressure of perfect fluid, ρ and p in Equation (11) behaviors in the opposite ways.

Then the question becomes: does the energy release of MM annihilation counter the gravity ? Yes, the interaction of the annihilation induced photons with MMs results in a force or pressure resisting the gravity, which is analogy to the case of Eddington limit in astrophysics, obtained by the balance between the inward gravitational attraction and the outward pressure of radiation. The collision of photons with charge particles in the derivation of Eddington limit is equivalent to the collision of photons with magnetic charge of MMs.

The equivalence between the heat required in the free expansion and the energy release of annihilation, $Q = E_m$, can be rewritten as, $TdS = dE_m$ (where S is the entropy of the system of idea gas) in the case of constant temperature. As the process AB' satisfying $T \approx const$, the corresponding variation of Helmholtz free energy is of (defined $F \equiv U - TS$)

$$dF = dU - TdS = dW - dE_m = 0 \tag{12}$$

Equation (12) corresponds to the condition for equilibrium in a system in thermal contact with a reservoir while free of energy exchange with surroundings.

The free energy satisfying Equation (12) corresponds to the minimum of free energy, $F = 0$. In such a case the system undergoes free expansion powered by the energy release of the annihilation of MMs. In the context of inflationary models, the universe is endowed with an enormous vacuum energy during the evolution from ϕ evolve to ϕ_0 (where ϕ_0 is the order parameter when F is at the true minimum).

In contrast, the scenario of annihilation of MMs induced inflation occurs at the true minimum, $F = 0$. As the temperature of the universe dropped below the critical temperature,

T_c, occurs the spontaneous symmetry breaking and MMs are generated and distributed in the bubbles. The universe experiences the first order phase transition which leaks the "false vacuum" phase to the lower energy "true vacuum" phase. Once the free energy reached the true minimum at $F(\phi_0) = 0$, the temperature equals that of the isothermal curve, AB', as shown in Fig1. In such a case, the universe is powered by the radiation due to the annihilation of MMs, $\rho_\Lambda = E_m/V$, resulting in a rapid expansion of the universe along the curve AB'. As the annihilation of MMs is the result of the symmetric breaking of GUTs, it can be seen as latent heat.

What about the rate of expansion in the context of annihilation dominated inflation? Differentiation Equation (8) or Equation (9), we have

$$\dot{E} = \frac{C_V T}{V_A^{\gamma-1}}(1-\gamma)V^{\gamma-2}\dot{V} = 3(1-\gamma)C_V T_A H \tag{13}$$

where $H = \dot{a}/a$ is the Hubble constant, $\dot{E} \approx \dot{E}_m a^3$ (with $\dot{E}_m$ is given by Equation(4)); and the heat capacity can be obtained by, $C_V = dE_m/dT$ with E_m given by Equation (6). Then Hubble constant can be obtained from Equation(13),

$$H = \frac{4 \times 10^{45}\omega^3}{C_V} T^{4-3\omega} T_A^{3\omega-1} V_A \tag{14}$$

Integrate the Hubble constant, H, over the time interval during which the expansion can be described by a constant index ω,

$$\int_{t_A}^{t_B} H dt \approx \frac{\omega - 4}{\omega^2(4-4\omega)}(\frac{a_A}{a_P})^3(\frac{T_A}{T_P})^{3\omega} = N \tag{15}$$

where $N \approx 61$ can be obtained in the case of the initial temperature, $T_A = T_i \sim 10^{28.7}$K, which is a little higher than the value of $T_i \sim 10^{28}$K. This actually gets e-folding of the scale factor, $ln(a_f/a_i) = \int_{t_i}^{t_f} H dt \approx 61$, through the thermal dynamic derived Hubble constant of Equation (14), which is close to that via energy density of Equation (7).

Consequently, once the energy release of MMs is sufficient to that required to solve the flatness problem, the initial and final temperature, volume, time, the polytropic index, and expansion rate (denoted by the Hubble constant) are consistent with each other. Thus the inflation of the universe can have a simple and self-consistent interpretation in the context of a free expansion of ideal gas.

The annihilation induces collision of bubbles can be affected by two processes. The energy release of annihilation of MMs in a bubble can lead to a bubble expansion, so that MMs are transferred initially to the wall of bubbles. This energy can be thermalized only when the bubble's wall undergo many collisions, which is similar to Guth's (*3*).

The other possibility is that annihilation of MMs is not significant in a bubble before its collision with other ones. It is the bubble collision that induces major annihilation of MMs at the collision region between two bubbles, and thus drives the free expansion of the universe. Both of the two circumstances tend to reproduce walls and strings resembling the result of merging of galaxies observed in astronomy.

This provides seeds of inhomogeneities of the universe. As such structures are formed in the case that MMs are annihilated in an asymptotic isothermal environment, inhomogeneity violating causality as shown in Guth(1980,1981) is automatically avoided.

The clumps generated in the annihilation of MMs can contribute to the growth of primordial perturbation or leave a texture like signature in the cosmic microwave background (*11*). As a result, the gravitational radiation originating in annihilation of MMs induced bubble collision deserves further investigation. On the other hand, the free expansion of polytropic index of $\gamma = 1.156$ in the early universe comes to an end when the number density of MMs is substantially reduced. Then the universe has to expand at other polytropic index γ at reduced energy reservoir of expansion. This avoids the problem of no ending and inhomogeneity in Guth's model (*3*).

Association with DM and DE

The MMs generated by phase transition are annihilated significantly during the inflation, the total number of which can be reduced up to 25 order of magnitude from the original one, $\sim 4\times10^{88}$ as discussed below. The number of relics MMs at current epoch is constrained by the observational bound on the total mass of DM. A typical mass of galaxy is of, $E_g \sim 10^{11} M_\odot$, and the mass of the universe at current epoch is estimated, $M_u \sim 10^{11} M_g \approx 10^{55} g$. As a MM corresponds to a mass of $M_m = 2.5\times10^{-8} g$, the baryon mass of current universe corresponds to a number of MMs of $N'_m \sim 10^{55}/(2.5\times10^{-8}) \approx 4\times10^{62}$. If the MMs is responsible for the DM, which is five times of the baryons, the required number of MMs is of $N_m = 5N'_m \sim 2\times10^{63}$.

In other words, the number of relic MMs cannot exceed $N_m \sim 2\times10^{63}$ in order to consist with the current astronomical observation on DM.This requires a drop of number of MMs for approximately 25 order of magnitudes from the original number of $\sim 4\times10^{88}$ at the beginning of the inflation.

The number density of MMs at current epoch is of $n_m \sim 2\times10^{63}/(10^{28\times3}) = 2\times10^{-21} cm^{-3}$, which is compatible with the baryon density, $n_b \sim 10^{15} n_m \sim 10^{-6} cm^{-3}$. Such a number density of MMs is also consistent with the Parker limit of MM flux, inferred from coherence size of existing galactic magnetic field (*12*).

In fact, supermassive particles as a candidate of DM have been investigated extensively (*13–15*), reference herein. Many superheavy models of DM (*15*), supposing new interactions rather than the standard model of particle physics to interpret DM. On the contrary, remnant MMs surviving to current epoch automatically corresponds to a mass to account for DM, so that DM can still be understood in the context of the standard model of particle physics. Moreover, DM of this paper is closely related with DE.

The observation of distant supernovae suggests accelerated expansion of the universe (*16,* *17*). The existence of DE, with a negative pressure repulsive to gravity (similar to the case of inflation) becomes necessary, which can be represented by the cosmology constant Λ, first introduced by Einstein (*18*). The equivalent density of the dark energy is given,

$$\rho_\Lambda = \Lambda c^2/8\pi G \tag{16}$$

Under the standard model, the observation requires the density of Equation (16) to be, $\rho_\Lambda = 10^{-8} ergcm^{-3}$. Assuming that the dark energy is driven by the annihilation of MMs, with a negative pressure as discussed in Section(), then energy density in a radius of current epoch of the universe, $R_c \sim 10^{28} cm$, is given,

$$\rho_\Lambda \sim \frac{g^\star \bar{N}_m M_m}{R_c^3} \tag{17}$$

Satisfying Equation (17) demands, $g^\star \bar{N}_m \sim 2 \times 10^{62}$. Thus the number of MMs participating annihilation is between $\bar{N}_m \sim 2 \times 10^{60}$ with $g^\star \sim 10^2$, and $\bar{N}_m \sim 2 \times 10^{61}$ with $g^\star \sim 10^1$, which corresponds to 0.1% to 1% of the total number of MMs allowed by the bound of DM.

Suppose a number of MMs of $\sim 10^{61}$ embedded in stars are annihilated at the time interval of $\sim 10^7$years (life time of a star). Annihilating all of them, $\sim 10^{63}$, would take billions of years.

Consequently, both the inflation and the accelerated expansion of the current universe can be interpreted by the annihilation of MMs repulsing the gravity of the universe, the former with a number of $\sim 10^{88}$ and the latter with a number of $\sim 10^{60-61}$. The fine tuning problem in those two processes are naturally avoided.

Discussion and search of MMs

The relationship among scale factor, time and temperature given by Equation (5) corresponds to a free expansion of the universe, which deviates from the widely used radiation predominated case, $T \propto a \propto t^{1/2}$, under the assumption of adiabatic expansion. Interestingly, both the relations are compatible with that of the standard cosmology.

By Equation (5), the temperature of $T = 10^{12}$K corresponds to a scale factor of $a = 10^{10.8}$cm. Evolving to the current epoch of $T_c = 10^0 K$ and $a_c = 10^{28}$, we get, $\omega = 1.44$ and hence polytropic index $\gamma = 1.23$ by Equation (5). Such a polytropic index is between that of the inflation $\gamma = 1.156$ (flat) and the adiabatic $\gamma = 1.333$ (steep), due to the reduced fuel, MMs.

On the other hand, the zero-order Einstein can be expressed as $(\dot{a}/a)^2 \propto \rho_\Lambda$, in which the energy density can be written as, $\rho_\Lambda \approx \dot{E}_m dt$ where $\dot{E}_m \propto T^4$ as shown in Equation (4), so that the relationship between time and temperature of current universe can be estimated, $t \propto T^{-1.33}$ corresponding a polytropic index of $\gamma = 1.25$, which is close to the result, $\gamma = 1.23$, inferred from Equation (5) as discussed above.

Furthermore, with MM numbers of $\sim 10^{61}$, scale factor of $a \approx 10^{10.8}$ and temperature of $T \approx 10^{12}$K, the dimensionless ratio between the number density and temperature of Equation (3) is of $r = N_m a^{-3} T^{-3} \sim 10^{-10}$, which is at the criteria of annihilation (*9*). Such a ratio is much less than that of the starting of the inflation, $r \sim 10^{74}$.

This means that the annihilation of MMs not only undergoes throughout the inflation period with temperature from $T \sim 10^{28}$ to $T \sim 10^{17}$ (of polytropic index $\gamma = 1.156$); but also extends from the beginning of leptogenesis of temperature of $T \sim 10^{17}$ to the quark-hadron transition of temperature, $T \sim 10^{12}$. Note that the annihilation of MMs at the second stage is much less than that of the first. Nevertheless, the MMs decoupled before starting of primordial nucleosynthesis of temperature, $T \sim 10^{10}$K.

Interestingly, after the emergence of stars, the annihilation of MMs can continue as long as the core temperature of a star is sufficient to trigger the annihilation of MMs. Putting the same $a = 10^{10.8}$ and $T = 10^{12}$K into Equation (6), the total energy release of annihilation undergoing in all stars is of $E_m \sim 10^{74}$erg, which corresponds to a number of MMs of $N_m \sim 10^{61}$ and a polytropic index of $\gamma \approx 1.22$. This is again compatible with the free expansion of the current universe.

Therefore, parameters inferred from the relation of Equation (5) , the criteria of annihilation of Equation (3) and the energy release Equation (6) are consistent each other in the context of the universe driving by the annihilation of MMs.

The annihilation of MMs from universe temperature of $T \sim 10^{28}$K to $T \sim 10^{12}$K may yield several by products: a) light particles of number of $\sim 10^{19}$; b) photons of number density of $n_\gamma \sim 10^{10} n_m$ (*19*)(reference herein); and c) monopolium, a $M\bar{M}$ bound state confined by their strong magnetic force thought to be produced at the end of inflation (*20, 21*).

The structured regions of a monopole predicted by GUTs ranges from the electro-weak unification region of $10^{-16}cm$ to fermion-anti-fermion condensation of $10^{-13}cm$ (*22, 23*). Coincidently, at the separation of $r \approx 10^{-13}cm$, the binding energy of a $M\bar{M}$ pair is of $g^2/r \sim 0.8GeV$, which is compatible with the decoupling energy of MMs, $T \sim 10^{12}$K (0.1GeV). Hence, it is conceivable that most of MMs (10^{63}) surviving to current epoch exist in the form of monopolium.

Such a $M\bar{M}$ pair has a lower energy level and is thus more stable than individual ones. Moreover, once a $M\bar{M}$ pair is formed, it can anti-aligned with another $M\bar{M}$ pair, in which the external magnetic field is further cancelled out comparing with a single $M\bar{M}$ pair. Such a monopolium pair is not only stable but also "dark". Because its external magnetic field is essentially canceled out except in the very vicinity region, $r \approx 10^{-13}cm$, to the core. And due to monopolium pairs correspond to much lower number density compared with baryons, the chance of their interacting with particles is extremely rare.

Although such a configuration appears very dark, it can get annihilated when the environment temperature exceeds their binding energy, e.g., during the supernova explosion.

A monopolium pair is much heavier than a proton, which is more difficult to get accelerated comparing with electric charged particles. A possible way of detection is that monopolium samples are fixed and collided with beams of particles, protons or electrons. When the energy of the collision is sufficient to break the binding of monopoliums of $\sim 1GeV$, occurs two consequences. One is the annihilation of monopolium with radiation energy of $\sim 10^{16}GeV$, which can be constrained by its characteristic spectrum; the other is breaking up of monopolium pairs into individual MMs or monopoliums, which can be captured and further tested by inductive searches (*22, 24*).

The existence of relics of MMs in the form of monopolium can provide seed of magnetic field in stars, compact objects and galaxies, etc. A brief history of early investigation on MM, as well as the effects of annihilation induced free expansion are shown in Fig2.

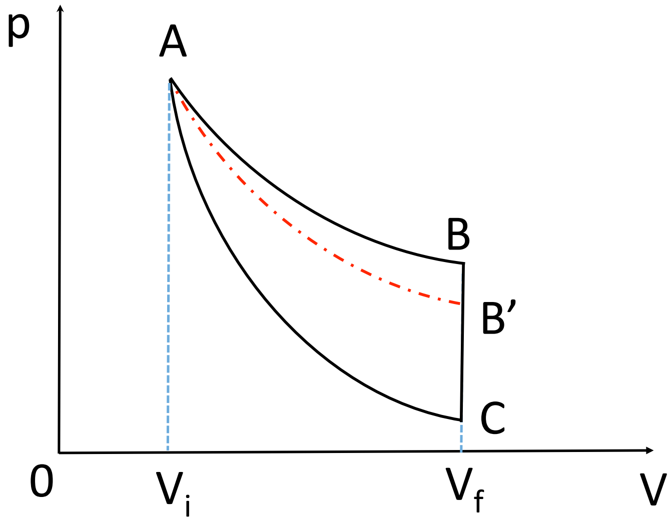

Figure 1: The ideal free expansion from state A to B can be represented by the isothermal process, AB, which is equivalent to an adiabatic expansion, AC, plus a constant volume process, CB. The Universe expands along the curve of the polytropic process, AB', infinitively close to the ideal isothermal process, AB. At different stages of universe evolution the polytropic index takes different values

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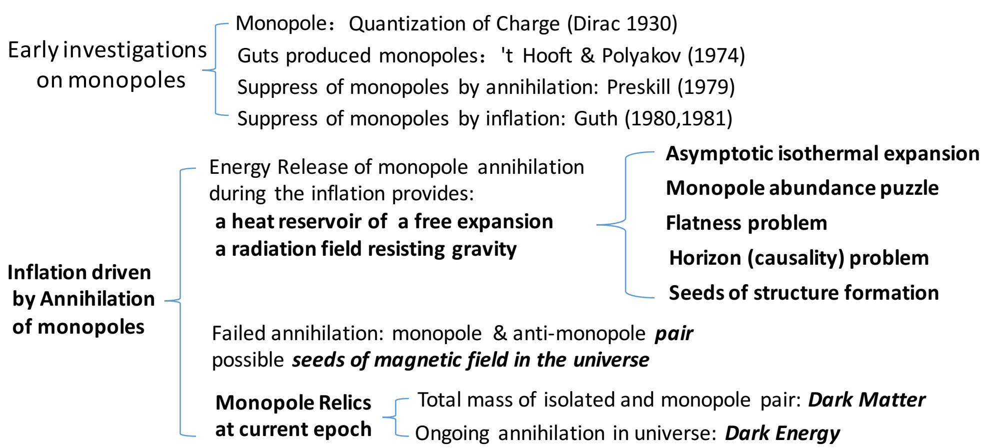

Figure 2: A brief history of early investigation on MM, and a summary of effects of annihilation induced free expansion.

References

1. G. 't Hooft, "Magnetic monopoles in unified gauge theories," *Nuclear Physics B*, vol. 79, pp. 276–284, Sep 1974.
2. A. M. Polyakov, "Particle spectrum in quantum field theory," *ZhETF Pisma Redaktsiiu*, vol. 20, p. 430, Sep 1974.
3. A. H. Guth, "Inflationary universe: A possible solution to the horizon and flatness problems," *Physical Review D*, vol. 23, pp. 347–356, Jan 1981.
4. A. D. Linde, "A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems," *Physics Letters B*, vol. 108, pp. 389–393, Feb 1982.
5. A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, "Reheating an Inflationary Universe," *Physical Review Letters*.
6. A. D. Linde, "Chaotic inflation," *Physics Letters B*, vol. 129, pp. 177–181, Sep 1983.
7. R. D. Peccei and H. R. Quinn, "CP conservation in the presence of pseudoparticles," *Physical Review Letters*, vol. 38, pp. 1440–1443, Jun 1977.
8. M. B. Wise, H. Georgi, and S. L. Glashow, "SU(5) and the Invisible Axion," *Physical Review Letters*, vol. 47, pp. 402–404, Aug 1981.
9. J. P. Preskill, "Cosmological production of superheavy magnetic monopoles," *Physical Review Letters*, vol. 43, pp. 1365–1368, Nov 1979.
10. S. Weinberg, *Cosmology*. 2008.

11. M. Cruz, N. Turok, P. Vielva, E. Martínez-González, and M. Hobson, "A Cosmic Microwave Background Feature Consistent with a Cosmic Texture," *Science*, vol. 318, p. 1612, Dec 2007.

12. M. S. Turner, E. N. Parker, and T. J. Bogdan, "Magnetic monopoles and the survival of galactic magnetic fields," *Physical Review D*, vol. 26, pp. 1296–1305, Sep 1982.

13. D. J. H. Chung, E. W. Kolb, and A. Riotto, "Superheavy dark matter," *Physical Review D*, vol. 59, p. 023501, Jan 1999.

14. I. F. Albuquerque and L. Baudis, "Direct Detection Constraints on Superheavy Dark Matter," *Physical Review Letters*, vol. 90, p. 221301, Jun 2003.

15. G. Bertone, D. Hooper, and J. Silk, "Particle dark matter: evidence, candidates and constraints.," *Physics Report*, vol. 405, pp. 279–390, Jan 2004.

16. A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, B. Leibundgut, M. M. Phillips, D. Reiss, B. P. Schmidt, R. A. Schommer, R. C. Smith, J. Spyromilio, C. Stubbs, N. B. Suntzeff, and J. Tonry, "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *The Astrophysical Journal*, vol. 116, pp. 1009–1038, Sep 1998.

17. S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R. S. Ellis, M. Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, W. J. Couch, and T. S. C. Project, "Measurements of Ω and Λ from 42 High-Redshift Supernovae," *The Astrophysical Journal*, vol. 517, pp. 565–586, Jun 1999.

18. A. Einstein, “Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie,” *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin*, pp. 142–152, Jan 1917.

19. P. Coles and F. Lucchin, *Cosmology: The Origin and Evolution of Cosmic Structure, Second Edition*. 2002.

20. C. T. Hill, “Monopolonium,” *Nuclear Physics B*, vol. 224, pp. 469–490, Sep 1983.

21. L. N. Epele, H. Fanchiotti, C. A. García Canal, and V. Vento, “Monopolium: the key to monopoles,” *European Physical Journal C*, vol. 56, pp. 87–95, Jul 2008.

22. J. Carrigan, R. A. and W. P. Trower, “Magnetic monopoles,” *Nature*, vol. 305, pp. 673–678, Oct 1983.

23. J. Preskill, “Magnetic Monopoles,” *Annual Review of Nuclear and Particle Science*, vol. 34, pp. 461–530, Dec 1984.

24. R. Dusad, F. K. K. Kirschner, J. C. Hoke, B. Roberts, A. Eyal, F. Flicker, G. M. Luke, S. J. Blundell, and J. C. S. Davis, “Magnetic Monopole Noise,” *arXiv e-prints*, p. arXiv:1901.10044, Jan 2019.