Quantum Cognitive Triad.
Semantic geometry of context representation

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Abstract

The paper describes an algorithm for cognitive representation of triples of related behavioral contexts two of which correspond to mutually exclusive states of some binary situational factor while uncertainty of this factor is the third context. The contexts are mapped to vector states in the two-dimensional quantum Hilbert space describing a dichotomic decision alternative in relation to which the contexts are subjectively recognized. The obtained triad of quantum cognitive representations functions as a minimal carrier of semantic relations between the contexts, which are quantified by phase relations between the corresponding quantum representation states. The described quantum model of subjective semantics supports interpretable vector calculus which is geometrically visualized in the Bloch sphere view of quantum cognitive states.

1 Introduction

1.1 Progress of psychology

Since antiquity, psychology made a progress in understanding of human nature, including basic motivations of behavioral and cognitive activity [Freud, 1923, Adler, 1923], traits of perception and thinking [James, 1890, Kahneman, 2011], classification of personalities [Jung, 1921, Bukalov et al., 1999] and systematization of unconscious cognition [Lacan, 1998, Hopwood, 2014]. Although useful in many ways, the kind of descriptions used in the these works has a problem which casts doubts on scientific status of the psychology field as a whole: linguistic categories it is based upon have no quantitative expression and as such are subject to voluntary interpretation [Ferguson, 2012,

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Morf, 2018]; as Mendeleev put is, *Science begins when one starts to quantify.* Although this problem is partially recognized (see e.g. consideration of a subfield of emotion modeling [Ortony and Turner, 1990]), no resolution to this annoying situation is currently visible [Falikman, 2018]. Even when a certain agreement on the terms and procedures is reached, repeatability of experimental observations, as compared to exact sciences, is strikingly low [Ferguson, 2012, Camerer et al., 2018].

1.2   Classical-like behaviorism

This problem of quantification is solved in the approach developed by Pavlov, Sechenov, Watson and Fechner, who renounced introspective study of their own «consciousness» in favor of modeling their neighbors' behavior. The plan was to formalize the psychological study with the same scientific method which turned natural philosophy, medieval astrology and alchemy to their contemporary counterparts. Seemingly reasonable, this strategy did not produce for psychology a reliable theoretical structure comparable to physics or chemistry. Generally, the problem is that classical behaviorist models view a human being as a mechanical automaton programmed to execute a set of stimulus-response scripts. No room for creativity and free will in these models leaves higher psychological functions on the ground of verbal descriptions criticized above.

Generalization of a standard Stimulus-Response structure may be attempted by supplementing it with a decision-making agency; an example of this approach is a Stimulus-Organism-Response scheme [Young, 2016]. But within classical methodology, this approach makes sense only if internal machinery of an «Organism» is specified. Expectedly, this leads to the mechanistic picture of life, again throwing baby out with the bathwater.

Classical methodology of science   is similar to childish way knowing things: it aims to decompose an object into «elementary» parts and then, starting from a heap of those pieces, strives to assemble the whole thing back. But in practice the latter process depends on ability to recover relations between the components, which may be lost or destroyed in the former one. Effective for analysis of complex inert systems like fine electronics and megapolis infrastructure, when applied to living systems this approach faces difficulties at modeling even a single living cell [Breuer et al., 2019]; behavior of a microscopic worm, whose three hundred neurons with several thousand couplings were perfectly known three decades ago, is still not understood [Cook et al., 2019, Larson et al., 2018]. Acknowledging contemporary progress of neuronal mapping [Glasser et al., 2016], one is bound to recognize

that the course to study human behavior by inspecting activity of 100 billion neurons in its nervous system is far from practical results [Stix, 2013].

1.3   Advent of the Quantum

Historical context   Fundamental reasons for difficulties encountered by methodology of classical science is human-centered studies were suggested in the end of 19-seventies [Gurevich and Feygenberg, 1977, Orlov, 1981, Orlov, 1982], when nuclear weaponry together with transistor and laser technologies came to life. These previously unthinkable capabilities displayed the power of quantum physics – a novel branch of science, conceptual structure of which is radically different from anything known in natural science before [Wheeler, 1989], at the same time being perfectly expressed in mathematical language producing quantitative models of fabulous precision. This newly certified natural science appeared as an alternative to the methodology of classical physics, not available to founders of classical behaviorism.

Progress   Discontent about classical human-centered science both on individual and collective level [Ferguson, 2012, Bouchaud, 2008] motivated search for new paradigms for behavioral modeling. In this role quantum theory showed efficiency in areas problematic for classical approach, including irrational preference, contextual decision making and game equilibria, modeling of concepts in natural language, collective cognitive and behavioral excitations and more. Review of these and other topics is provided in monographs [Khrennikov, 2010, Busemeyer and Bruza, 2012, Haven and Khrennikov, 2013, Asano et al., 2015b] and review articles [Khrennikov, 2015, Busemeyer and Wang, 2015, Asano et al., 2015a].

Quantum language   Quantum theory exploits correspondence established between phenomena of human behavior and mathematically expressed concepts of quantum theory, most productive of which are state space, superposition, entanglement, observable and measurement. In short, quantum models consider a human individual in particular behavioral context as an instance of a physical system prepared in a particular quantum state, encoding available decision alternatives and propensities of their realizations. These propensities are quantified by complex-valued amplitudes, which constitutes the key difference with classical measure of uncertainty in terms of real-valued probabilities.

The phase problem Phase dimension of complex-valued amplitudes, crucial in many quantum models of cognition and behavior, is used as an additional fitting parameter, meaning of which is usually undisclosed. Accompanied by the fact that phase parameters have no straight measurement procedure [Lynch, 1995], this situation renders quantum models of cognition and behavior to the role of a post factum fitting apparatus useless for making predictions. With this so-called phase problem resolved the whole field of quantum cognition would be taken to radically different level of scientific and practical value.

This paper describes a quantum model of decision making such that interpretation of the complex-valued structure of the quantum state space suggests itself. Methodology used to design the model is outlined in the Sect. 2. The model is explained in Sect. 3 and related to the experimental data in Sect. 4. Sect. 5 provides interpretation of the result.

2 Methodology: Quantum Behaviorism

Instead of breaking things apart, quantum methodology aims to model, at least probabilistically, only behavior of the whole system in a particular experiment, while internal mechanism of its behavior may remain unknown. Compared to classical criterion of scientific knowledge, this amounts to significant decrease in ambition. Such retreat was not accepted easily; physicists were pushed to this humble stance after decades of fruitless fight for classical understanding of what is going on in quantum labs around the world [Wiseman, 2015]. This seemingly weak position, however, constitutes the core advantage of quantum methodology in application to humanities.

2.1 Basic notions

The Black Box in Context and Experiment In quantum models, a decision-making subject is considered as a black box - a device, revealing itself to the outside exclusively through observable behavior. It contains all of complexity characterizing the decision making agency, whether is a human being, social system or individual electron in the physical experiment. As in physical experiments, an ensemble of similarly prepared black boxes is exposed to the set of behavioral situations, or contexts, which provide the black box with necessary information. The contexts strictly define the spectrum of available behavioral alternatives, corresponding to possible experimental outcomes in physics.

As a quantum experiment actively changes state of a system, and the outcome is fundamentally probabilistic (see *The nature of uncertainty*), each black box is subjected to a carefully designed sequence of contexts. As estimation of the decision probabilities requires an ensemble of identically staged experiments, quantum modeling always is of statistical nature.

Cognitive space All behavioral contexts are accommodated in multidimensional vector space spanning the basis of mutually exclusive decision alternatives. This space of subjective representations is referred to as cognitive space, in which each context is represented by a particular vector object called cognitive state. The cognitive state generates the observed decision probabilities in agreement with the rules of quantum theory, thus representing the behavioral regularities of a subject group in a given context.

In that way, unlimited number of behavioral contexts can be represented in the same vector space without increasing its dimensionality. This is of clear computational advantage [Khrennikov, 2019]: instead of analyzing each new decision situation anew, it is represented as a composition of related decision contexts which are already learned.

Cognitive representations of contexts are composed as superpositions of corresponding vectors weighted with complex-valued coefficients. Except the complex-valued structure, this is the simplest possible algebra of context representations. In technical terms it is referred to as linear algebra of complex-valued (Hilbert) vector space.

The nature of uncertainty This linear algebra of cognitive space expresses logic of probabilistic events which differs from Boolean algebra of classical logic inherited by classical probability theory [Birkhoff and Neumann, 1936, Kolmogorov, 1956]. Compared to the the latter, quantum probability describes uncertainty of fundamentally different nature, which is the basis of «paradoxes» pervading quantum physics [Holland, 2000, Merali, 2015].

Quantum probability describes uncertainty that in current state of the system is not resolved; it may become resolved in the future if the system transfers to a state in which the considered alternative takes definite value [Peres, 1978, Gabora and Aerts, 2005]. This is a quality in which quantum «measurement» is fundamentally different from the classical one; the latter merely removes subjective ignorance of the experimenter by rewriting information from one carrier to another, while the former records an objective change of the system invoked by its interaction with experimental context [Bell, 1990].

Free will and creativity   Having fundamental uncertainty at its core, quantum methodology does not seek to put a living system in a condition where its behavior is predetermined. Not because it is impossible: between jump and landing, center of mass of a human body follows the mechanical laws, not differing in its behavior from a bag of sand; but because such approach automatically limits the study to procrustean bed of classical methodology.

Non-predeterminativeness of nature accounted by quantum uncertainty is a crucial ingredient missing in the classical behaviorism. Even when the latter turns to probabilistic view, the underlying Boolean algebra of events allows it to capture only those phenomena which have a predetermined course; in such models the subject, effectively, has already taken every possible decision and thereby follows a fixed table of stimulus-response pairs. This entails a worldview in which nothing really happens since no new information is created.

Quantum methodology of science provides a way out of this dead static models of nature [Briegel, 2012, Stapp, 2017]. While consensus on the nature of creativity and free will is not reached, quantum models of human behavior reserves space for phenomena of this kind.

Experimental precautions   Necessary for quantum modeling to succeed is a clear distinction between a context controlled by the experimenter, a part of nature included in the black box, and rest of the universe referred to as an environment. After the delineation has been made, opening of the black box is prohibited. It should be carefully shielded both from the environment and the experimenter. This idea is very unnatural for classical psychologist who aims to control a living system as fully as possible by subjecting it to intricate «preliminary» examinations. Quantum methodology recognizes such procedures as active manipulation sequences preparing the subject in exotic behavioral states never encountered in practical situations.

Being aware of numerous intricacies of human psyche, quantum experiment does not seek to differentiate between thoughts, motives, emotions, moods and tempers. These factors are out of control, but not neglected; included in the black box, they are free to affect the observed probabilistic behavior and find reflection in quantum models.

2.2   Quantum sociophysics

The idea of quantum cognition appeared in close sync with a plan to model social, political and economic phenomena with methods of statistical physics

[Galam, 2012b], which engendered fields of science known today as socio-physics and econophysics [Chakrabarti et al., 2006, Galam, 2012a]. Methodology of behavioral modeling outlined above thus contributes to this broader domain of science. By suggesting quantitative account for non-classical behavioral phenomena of life, quantum sociophysics closes a major loophole in scientific unity of Nature advocated by Zipf [Zipf, 1942].

Classical limit of the quantum Living organisms are seen as amplifiers of quantum uncertainty (in the sense articulated above) [Davies, 2004, Maldonado and Gómez-Cruz, 2014, Wendt, 2015] which at macroscopic level take form of behavioral phenomena referred to as creativity and free will [Stapp, 2017]. Parallel to physics [Bohm and Hiley, 1993, chapter 8], these fundamentally «quantum» (individual) agents of social phenomena in certain aggregated configurations display predetermined behavior described by sociophysical models based upon classical algebra of events and probability calculus as e.g. in hydrodynamical account of crowd motion [Henderson, 1974]. Classical algebra of events and probability calculus represent limiting cases of their quantum counterparts [Warmuth and Kuzmin, 2010].

Outside of special conditions of this kind social phenomena elude classical descriptions. As information connectivity of humanity establishes global cognitive and behavioral coherence [Grandpierre, 1997, Morales et al., 2017], social phenomena move from classical to quantum (wave-like [Orefice et al., 2009]) domain [Bouchaud, 2008, Plikynas, 2010, Haven, 2015, Khrennikov, 2018]. Methodology of quantum modeling is therefore expected to form the basis for a new-age science of human-centered phenomena of both individual and collective scale [Widdows and Bruza, 2007].

Quantization of complexity Whereas an elementary particle, an atom, a living cell, complex organism, social system and the whole planet Earth include each other in terms of compositional structure, many aspects of their behavior are described by models of similar complexity. Absence of predetermination, discreteness and complementarity which motivated birth of quantum theory are also not unique to the microworld and manifest at other levels including individual organisms and social systems [Atmanspacher et al., 2002, Wendt, 2015]. This phenomenon of complexity renormalization seems to be accounted by methodology of quantum behaviorism not worried about internal complexity of the black box. This allows one to build simple models of complex systems thereby crossing a complexity barrier dividing phenomena of life from mechanistically predetermined processes [Kitto, 2008].

Parallel to physics, structures of global cognitive entanglement may be

seen as novel phases of social matter foreseen by de Chardin and Vernadsky [Levit, 2000], synchronized behavior of which is attributed to a «global superogranism» [Heylighen, 2007]. As a novel form of life, the latter is expected to amplify quantum uncertainty to a new scale generating individual behavior of the whole humanity. Observed from outside, the system will start to display coherent behavior in response to external contexts. Models of quantum sociophysics are expected to apply.

3 Model: The Qubit

3.1 Mathematical basics

Considered is a situation when an individual subject resolves a binary alternative, mutually exclusive and complementary outcomes of which are encoded by numbers 0 and 1, identified with orthogonal vectors $|0\rangle$ and $|1\rangle$. These vectors form the basis of a two-dimensional space of possible cognitive states of a subject with respect to the alternative under consideration.

Any of these cognitive states can be represented by vector

$$|\Psi\rangle = c_0 |0\rangle + c_1 |1\rangle, \tag{1}$$

in which c_0 and c_1 are complex amplitudes describing propensities of a subject to take decisions 0 and 1. Amplitudes c_0 and c_1 define probabilities of corresponding decisions as [Nielsen and Chuang, 2010]

$$p[i] = |\langle i|\Psi\rangle|^2 = |c_1|^2, \quad i = 0, 1. \tag{2}$$

Mutual exclusivity and complementarity of outcomes 0 and 1 corresponds to the condition

$$1 = p[0] + p[1] = |c_0|^2 + |c_1|^2 = \langle\Psi|\Psi\rangle, \tag{3}$$

expressing normalization of vector $|\Psi\rangle$ to unit length. This allows to parametrize $|\Psi\rangle$ as

$$|\Psi\rangle = e^{i\Phi}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right), \tag{4}$$

where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$ are defined such that

$$\cos\frac{\theta}{2} = |c_0| = \sqrt{p[0]}, \quad \sin\frac{\theta}{2} = |c_1| = \sqrt{p[1]}, \tag{5}$$
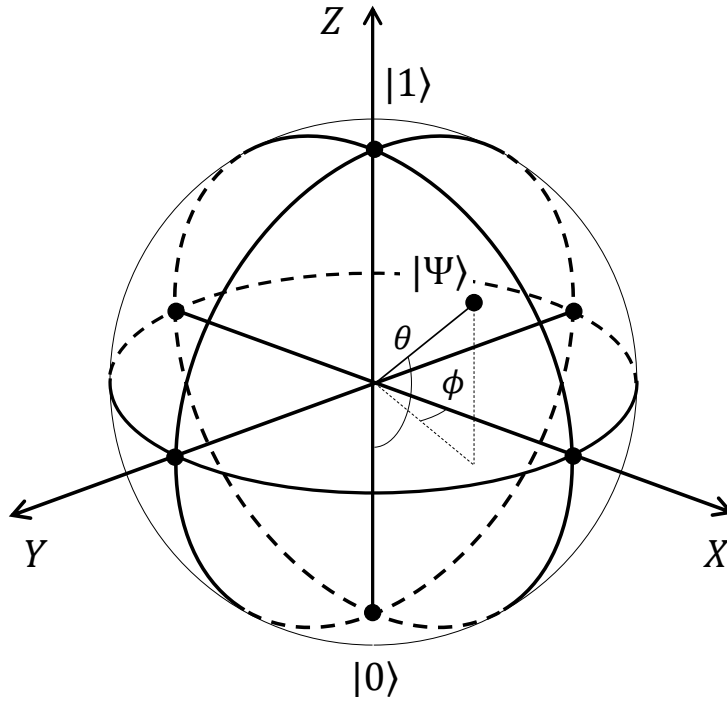
$$e^{i\phi} = \frac{c_1}{c_0}, \tag{6}$$

Figure 1: The Bloch sphere: geometrical representation of the quantum uncertainty of choice between alternatives 0 and 1 expressed by qubit state (4) with the global phase Φ ignored. Polar angle θ accounts for the probability of observing the outcomes if the experiment would be performed. Azimuthal phase ϕ has no classical analogy.

and Φ is a global phase factor. Setting the latter aside, state $|\Psi\rangle$ is represented it by a point on the unit sphere uniquely defined by θ and ϕ being polar and azimuthal angles as shown in the Figure 1. In that geometric representation, projection of the state point to Z axis divides the diameter $0 - 1$ proportionally to probabilities $p[0]$ and $p[1]$ since

$$\frac{1 - \cos\theta}{1 + \cos\theta} = \left(\frac{\sin\theta/2}{\cos\theta/2}\right)^2 . \tag{7}$$

Azimuthal phase ϕ describes rotation of vector $|\Psi\rangle$ around Z axis without changing its Z component, thus having no apparent connection with observable probabilities. Instead, it describes how state $|\Psi\rangle$ combines with other states of this kind. This property of azimuthal phase is central for the model developed below.

3.2 Setup and Model

The alternatives 0 and 1 is considered in three different contexts a, b and c, each preparing the subject in a particular cognitive state of the form (4). Contexts a and b are defined by mutually exclusive states of some two-state situation factor (true/false, white/black, etc.). Certainty about either of these options is reflected by subjects to cognitive states

$$|\Psi_a\rangle = \cos\frac{\theta_a}{2}|0\rangle + \sin\frac{\theta_a}{2}|1\rangle, \tag{8}$$

$$|\Psi_b\rangle = \cos\frac{\theta_b}{2}|0\rangle + e^{i\phi_b}\sin\frac{\theta_b}{2}|1\rangle, \tag{9}$$

where zero azimuth is identified with state $|\Psi_a\rangle$, so that ϕ_b is azimuthal phase of state $|\Psi_b\rangle$ relative to $|\Psi_a\rangle$.

Polar angles θ_a and θ_b are related to the measurable probabilities of decisions 0 and 1 as prescribed by (5)

$$p_a[0] = \left(\cos\frac{\theta_a}{2}\right)^2, \quad p_a[1] = 1 - p_a[0] = \left(\sin\frac{\theta_a}{2}\right)^2, \tag{10}$$

$$p_b[0] = \left(\cos\frac{\theta_b}{2}\right)^2, \quad p_b[1] = 1 - p_b[0] = \left(\sin\frac{\theta_b}{2}\right)^2. \tag{11}$$

In the third context c subjects are uninformed about state of the situation factor defining contexts a and b, considering them as equiprobable. This is reflected by subjects in cognitive state

$$|\Psi_c\rangle = N\left(|\Psi_a\rangle + e^{ix}|\Psi_b\rangle\right), \tag{12}$$

which is superposition of states (8) and (9) discriminated by the phase factor e^{ix} and equally amplified by normalization constant N. This cognitive state produces decision probabilities

$$p_c[0] = N^2\left|\cos\frac{\theta_a}{2} + e^{ix}\cos\frac{\theta_b}{2}\right|^2 =$$
$$= N^2\left(p_a[0] + p_b[0] + 2\sqrt{p_a[0]p_b[0]}\cos x\right), \tag{13}$$

$$p_c[1] = N^2\left|\sin\frac{\theta_a}{2} + e^{i(\phi_b+x)}\sin\frac{\theta_b}{2}\right|^2 =$$
$$= N^2\left(p_a[1] + p_b[1] + 2\sqrt{p_a[1]p_b[1]}\cos(\phi_b + x)\right). \tag{14}$$

Unit sum of these values amounts to normalization

$$1 = \langle\Psi_c|\Psi_c\rangle = p_c[0] + p_c[1] =$$
$$= 2N^2\left(1 + \sqrt{p_a[0]p_b[0]}\cos x + \sqrt{p_a[1]p_b[1]}\cos(\phi_b + x)\right). \tag{15}$$

Fitting procedure   Given experimentally measured values p_a, p_b and p_c, fitting the above model to these data goes in following steps:

- polar angles θ_a and θ_b are determined from (10) and (11);

- unknown phases x and ϕ_b are found as solution of any two of equations (13), (14) and (15);

- thus determined $|\Psi_c\rangle$ (12) can also be presented in polar form

$$|\Psi_c\rangle = \cos\frac{\theta_c}{2}|0\rangle + e^{i\phi_c}\sin\frac{\theta_c}{2}|1\rangle, \qquad (16)$$

where θ_c and ϕ_c are determined from (12), (8) and (9) as prescribed by (5) and (6).

Result of this fitting procedure crucially depends on the predefined value of the normalization constant N, possible values of which are limited by condition that equations (13) and (14) are solvable in real x and ϕ_b for the given triple of experimental probabilities.

3.3   Cognitive triad

Consider a hypothetical behavior such that decision probabilities in all three contexts are equal: $p_a[1] = p_b[1] = p_c[1] = p$. For any p equations (13) and (14) then reduce to

$$\cos x = \cos(\phi_b + x) = \frac{1}{2N^2} - 1, \qquad (17)$$

which resolves in two ways

$$x = \pm\arccos\left(\frac{1}{2N^2} - 1\right) \quad \text{and} \quad \phi_b = 0 \qquad (18)$$

$$\text{or}$$

$$x = \pm\arccos\left(\frac{1}{2N^2} - 1\right) \quad \text{and} \quad \phi_b = -2x. \qquad (19)$$

According to definitions (12) and (16), solution (18) leads to $\phi_c = 0$ so that contexts a, b and c are all mapped to a single cognitive state

$$|\Psi_a\rangle = |\Psi_b\rangle = |\Psi_c\rangle = \sqrt{1-p}\,|0\rangle + \sqrt{p}\,|1\rangle. \qquad (20)$$

Solution (19) entails a qualitatively different cognitive structure in which states $|\Psi_a\rangle$, $|\Psi_b\rangle$ and $|\Psi_c\rangle$ are distinct even though they produce indistinguishable decision probabilities. Arrangement of these states depends on the
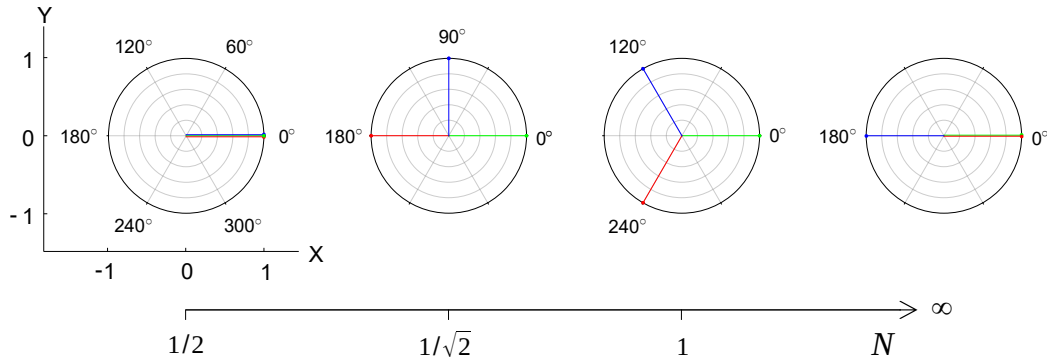
Figure 2: Layout of the cognitive states $|\Psi_a\rangle$ (green), $|\Psi_b\rangle$ (red) and $|\Psi_c\rangle$ (blue) resulting from solution (19) for different values of normalization constant N in special case $p_a[1] = p_b[1] = p_c[1]$. Projection to the azimuthal plane XY is shown.

normalization constant N, possible values of which due to $|\cos x| \leq 1$ (17) lie between $1/2$ and ∞. Starting with degenerate case (20) corresponding to $N = 1/2$, increase of N leads to rotation of state $|\Psi_b\rangle$ according to (19), whereas $|\Psi_c\rangle$ is oriented halfway between $|\Psi_a\rangle$ and $|\Psi_b\rangle$ as shown in the Figure 2. The circle is completed in the limit $N \to \infty$, when $|\Psi_c\rangle$ opposes coinciding pair $|\Psi_a\rangle$ and $|\Psi_b\rangle$.

Discrimination of contexts   Principal difference solution (19) compared to (18) is that it gives the subject ability to recognize difference between different contexts which boosts his behavioral efficiency.

The best discrimination of the three contexts a, b and c is achieved when they are represented by equidistant cognitive states as shown in the third panel of the Figure 2. This representation corresponds to the normalization constant $N = 1$, which means that cognitive states in (12) combine in full amplitudes without any suppression or amplification; in neurophysiological cognitive machinery [Busemeyer et al., 2017, Khrennikov et al., 2018, Khrennikov and Asano, 2020] this is arguably the simplest possible option. This triangular structure of cognitive representations of contexts is referred to as cognitive triad.

In cognitive triad mutually exclusive contexts a and b are, in general, recognized neither as orthogonal ($\langle\Psi_a|\Psi_b\rangle \neq 0$) nor opposite in the azimuthal plane, which ; exception is the rectangular cognitive structure generated by $N = 1/\sqrt{2}$ and shown in second panel of the Figure 2. This may reflect the fact that the factor of the environment making these contexts exclusive may have different degrees of importance to the considered decision. For example,
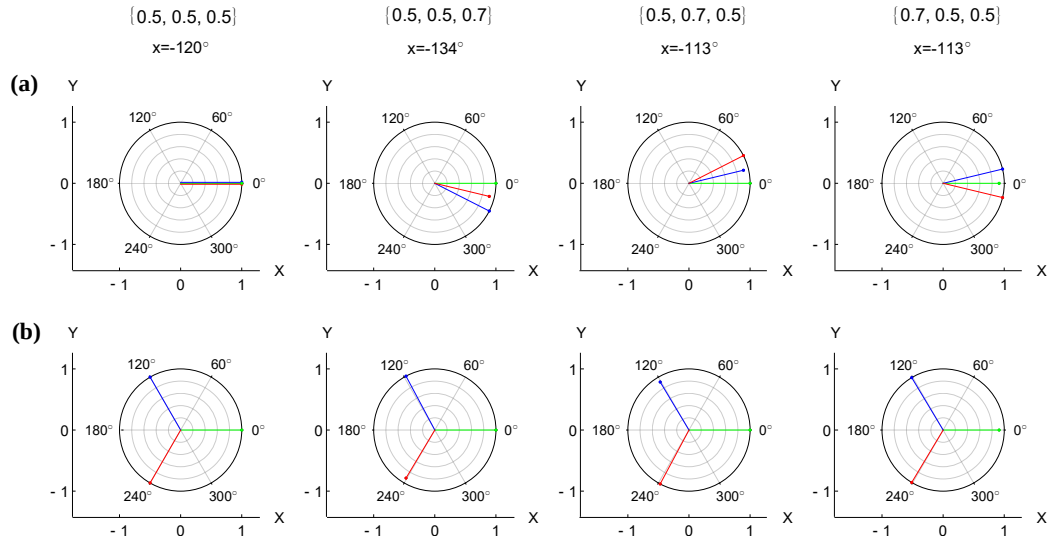
Figure 3: Cognitive representations $|\Psi_a\rangle$, $|\Psi_b\rangle$ and $|\Psi_c\rangle$ of three contexts a (green), b (red) and c (blue) producing decision probabilities $p_a[1]$, $p_b[1]$ and $p_c[1]$ indicated on top, projected to the azimuthal plane of the Bloch sphere. Direction of the projected vector is an azimuthal phase of the corresponding state (Figure 1). (a): solution of type (18). (b): solution of type (19). Indicated value of the combination phase x (12) is the same in both solutions.

charge but not color of the battery is important for decision engaged with its energetic performance. In this situation contexts «charged», «uncharged» and «unknown charge» have to be subjectively distinguished well, while contexts «white», «black» and «unknown color» need not be distinguished at all. This preference surely reverses for decision concerned not by performance but by visual appearance of the battery.

Cognitive stability   Solutions (18) and (19) also differ in another aspect, namely, their ability to account for variative behavior based on the stable cognitive structure.

In case of solution 18 cognitive degeneracy (20) is strongly lifted when probability data deviate from the identity $p_a[1] = p_b[1] = p_c[1] = p$. The disturbance $p_i[1] \to p + \Delta$ splits states $|\Psi_a\rangle$, $|\Psi_b\rangle$ and $|\Psi_c\rangle$ by amount of the first order in Δ as shown in the Figure 3(a).

Contrariwise, the same disturbance of probabilities causes almost no change in the cognitive states produced by (19); modification of behavioral probabilities in this case is realized by tuning of only the combination phase x, which is identical in both solutions. This value x thus parallels

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the path interference phase in standard quantum models of decision making [Khrennikov, 2009].

Phase stability of the quantum cognitive triad allows an individual to have a stable subjective representation of gross behavioral contexts, at the same time preserving vital ability to adjust decision probabilities according to the details of a particular situation. On physiological level this property relieves an individual from the need to reconfigure his neuronal system each time a particular decision has to be made. Instead, a single phase relation realized e.g. by temporal delay between the neuronal oscillation modes [de Barros and Oas, 2017] has to be handled. In this manner a stable yet flexible cognitive model of an individual activity, surely favored by evolution [Gabora and Aerts, 2009], can function.

4 Experiment: the two-stage gamble

4.1 Data

General behavioral structure accounted by the above model is realized in the so called two-stage gambling experiment devised by Tversky and Kahneman [Tversky et al., 1992]. By design, subjects are exposed to the decision to play or not to play in a gamble, such that lose of 1 monetary unit and win of 2 monetary units is defined by outcome of the fair coin tossing. Decisions «to play» and «not to play» are mutually exclusive and complementary alternatives denoted above by 0 and 1.

Contexts a and b are defined by the «won» and «lost» outcomes of previous round of the same game, representing mutually exclusive states of a two-valued situation factor. Third context c is a condition of uncertainty of the previous round, outcomes of which have equal probabilities of 0.5 in absence of biasing information. Outcome of the experiment is a triple of statistical probabilities $p_a[1]$, $p_b[1]$ and $p_c[1]$, calculated as the number of subjects who decided to play divided by total number of respondents in each context.

In total 32 existing experiments of this type reported in [Tversky et al., 1992, Kuhberger et al., 2001, Lambdin and Burdsal, 2007, Surov et al., 2019, Broekaert et al., 2020] are used to test the model described in Sect. 3. Raw data constituting 32 probability triples $p_a[1]$, $p_b[1]$ and $p_c[1]$ are listed in Table 1.

Table 1: Existing experimental data on the two-stage gambling task. The first three columns show measured statistical probabilities to play the game in three contexts in which the previous round is won, lost or unknown with 50/50 chance. Experiments 1-4: ref. [Tversky et al., 1992]; 5-8: ref. [Kuhberger et al., 2001]; 9-11: ref. [Lambdin and Burdsal, 2007]; 12: ref. [Surov et al., 2019]; 13-32: ref. [Broekaert et al., 2020]. In the latter group, experiments 13-17 correspond to the between subjects setup; 18-22: within subjects setup with random order of contexts; 23-27: within subjects setup with «Known»→«Unknown» order of contexts; 28-32: within subjects setup with «Unknown»→«Known» order of contexts. In each of the pentads 13-17, 18-22, 23-27, 28-32 experiments are arranged by increasing payoff parameter $\{0.5, 1, 2, 3, 4\}$.

No.	1	2	3	4	5	6	7	8	9	10	11	12
«Won» $p_a[1]$	.69	.75	.69	.71	.60	.83	.80	.68	.64	.53	.73	.30
«Lost» $p_b[1]$	.57	.69	.59	.56	.47	.70	.37	.32	.47	.38	.49	.24
«Unknown» $p_c[1]$	.38	.73	.35	.84	.47	.62	.43	.38	.38	.24	.60	.17

No.	13	14	15	16	17	18	19	20	21	22
«Won» $p_a[1]$	.82	.75	.65	.58	.56	.75	.72	.64	.57	.55
«Lost» $p_b[1]$	.92	.89	.87	.85	.85	.86	.83	.81	.80	.77
«Unknown» $p_c[1]$	.87	.86	.87	.85	.81	.89	.86	.84	.84	.85

No.	23	24	25	26	27	28	29	30	31	32
«Won» $p_a[1]$	.69	.61	.56	.45	.42	.68	.62	.52	.48	.43
«Lost» $p_b[1]$	.74	.65	.60	.59	.53	.66	.65	.51	.51	.41
«Unknown» $p_c[1]$	.64	.59	.48	.41	.37	.78	.75	.58	.63	.52

4.2   Modeling

The modeling is performed in the symmetrical triad mode resulting from neutral normalization $N = 1$ in (12) as discussed in Sect. 3.3. The fitting procedure described in Sect. 3.2 is successfully accomplished for all 32 probability triples. For each triple, the fitting determines polar angles θ_a, θ_b, θ_c and azimuthal phases ϕ_b, ϕ_c together with the combination phase x. These values are shown in the Figure 4.
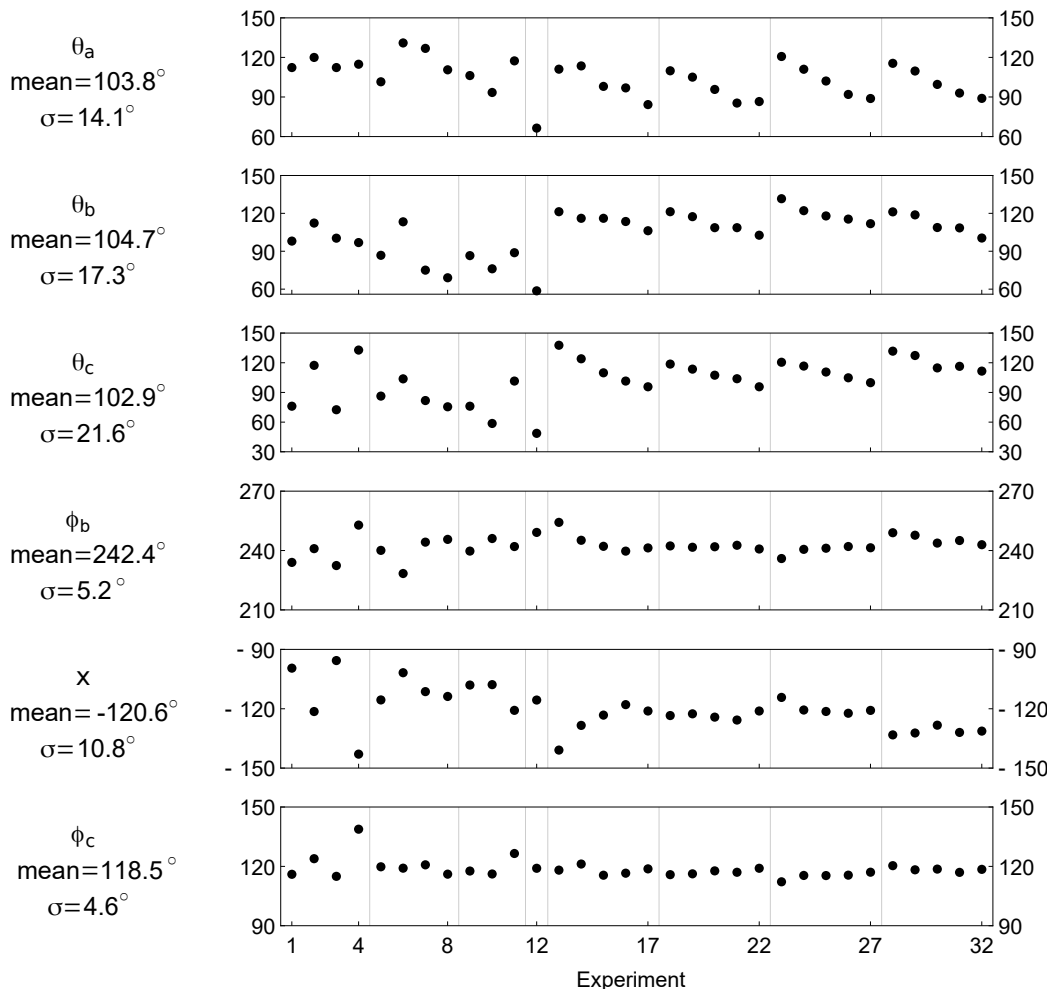

Figure 4: Polar angles θ_a, θ_b, θ_c, azimuthal phases ϕ_b, ϕ_c and combination phase x fitted for each experiment from the Table 1 in symmetrical cognitive triad mode (third panel of the Figure 2).

Polar angles of $\theta_a = 103.8 \pm 14.1°$, $\theta_b = 104.7 \pm 17.3°$ and $\theta_c = 102.9 \pm 21.6°$ individually encode probabilities p_a, p_b and p_c, reflecting regularities and
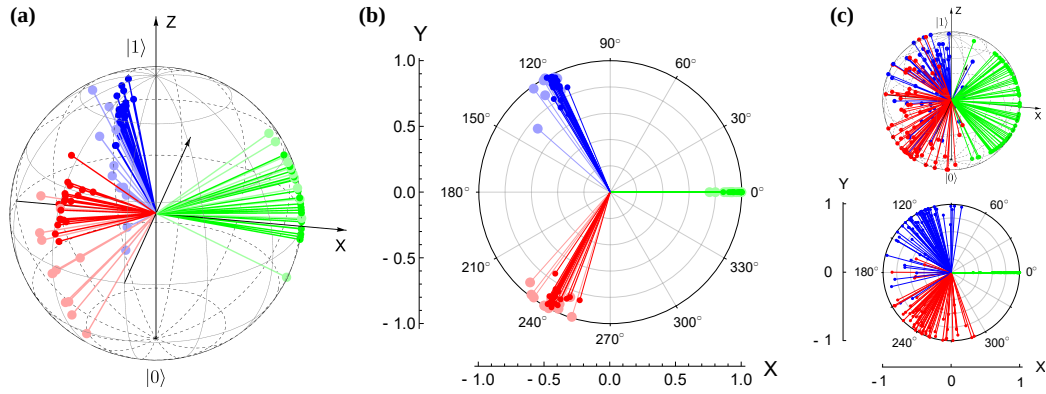
Figure 5: Cognitive triad model for three contexts of the two-stage gambling experiments in the Bloch sphere representation. (a) Three dimensional view of the cognitive states Ψ_a («won» context, green), Ψ_b («lost» context, red) and Ψ_c («unknown» context, blue) on the Bloch sphere. (b) Projection of Bloch state vectors to the azimuthal XY plane, where vector orientation visualizes azimuthal phases ϕ_b and ϕ_c. (c): the same for randomly generated probability triples.

variance of these values via (10) and (11).

Azimuthal phases ϕ_b, ϕ_c and combination phase x show regular distributions

$$\phi_b = 242.4 \pm 5.2° \approx \frac{2\pi}{3}, \tag{21}$$

$$x = -120.6 \pm 10.8° \approx \frac{2\pi}{3}, \tag{22}$$

$$\phi_c = 118.5 \pm 4.6°, \approx -\frac{2\pi}{3}. \tag{23}$$

Due to phase stability of cognitive triad, these values represent slightly perturbed baseline solution (19).

Modeling results are visualized in the Figure 5, showing cognitive states $|\Psi_a\rangle$, $|\Psi_b\rangle$ and $|\Psi_c\rangle$ in the Bloch sphere representation introduced in the Figure 1. Panel (a) shows full three dimensional view of the cognitive state vectors on the Bloch sphere. Panel (b) shows projection of Bloch vectors to the azimuthal XY plane, with phases ϕ_b and ϕ_c being directions of the projected vectors.

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5 Interpretation: semantic geometry

5.1 Azimuthal phases

Function   When a single behavioral context is considered in isolation (Sect. 3.1), azimuthal phase ϕ has no connection to probabilistic data, defined solely by polar angle θ, and therefore is redundant. Azimuthal phases are needed when behavior in different contexts has to be modeled without increase of dimensionality of the cognitive representation space. Then, mapping each context to a particular value of azimuthal phase constitutes an additional degree of freedom to relate and combine different contexts. The obtained organization of cognitive representations allows to optimize system's behavior in versatile environment.

The two-stage gambling setup constitutes the minimal case for this mapping algorithm, when two cognitive representations are combined to obtain the third. When cognitive state space is restricted to two dimensions, the decision probabilities observed in three different contexts require involvement of the complex-valued structure expressed in azimuthal phases ϕ_b, ϕ_c and the combinational phase x. This parallels quantum physical situation where the complex-valued structure of the qubit state space is necessary to account for transition probabilities observed in quantum physics for the case of «spin-1/2» system and three binary observables [Accardi and Fedullo, 1982, Khrennikov, 2003].

Phase stability   Variance of azimuthal phases ϕ_b and ϕ_c (21), (23) which amounts to approximately 5° is higher than might be expected from the Figure 3; this is because the latter is limited to specific probability triples chosen for illustration. It is nevertheless radically lower than what is obtained for probability triples generated randomly as shown in the Figure 5 (c), and also 2-3 times lower than variance of the polar angles θ_a, θ_b and θ_c shown in the Figure 4. Moreover, Figure 4 shows that azimuthal phases ϕ_b and ϕ_c have no significant trace of regular patterns exhibited by polar angles θ_a, θ_b and θ_c in series of experiments 13-32.

These features indicate that near-constant behavior of ϕ_b and ϕ_c is neither imposed by the modeling scheme, nor can it be an incidental byproduct of overall homogeneity in probability data. The observed sharpness of the azimuthal phases thus parallels the phase stability reported in [Surov et al., 2019]. Now, however, the stability refers to a particular structure in the space of subjective context representations - the cognitive triad (Sect. 3.3). This develops the idea of quantum phase stability by supplementing it with quantitative geometrical meaning.
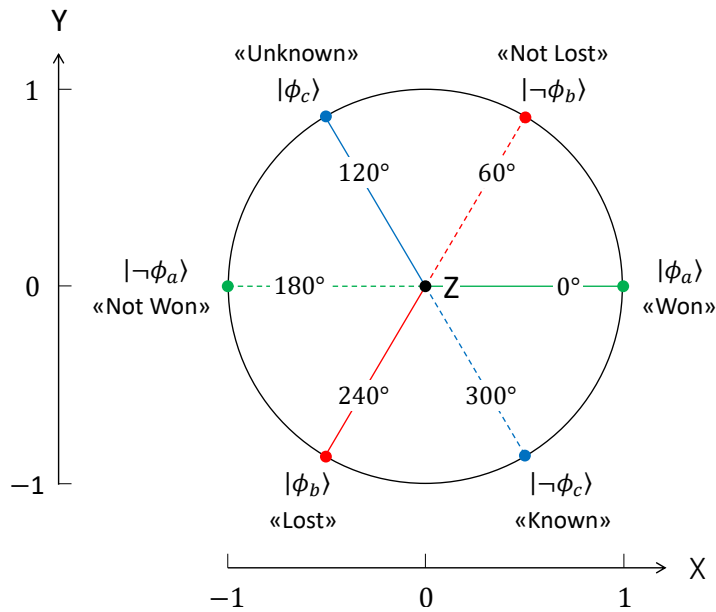
Figure 6: Representation of contexts defined by state of the previous gamble in relation to the decision to play or not to play second time. Two cognitive triads represent triple of contexts «won», «lost» and «unknown» and their negations in the azimuthal plane of the Bloch sphere.

5.2   Semantics in the azimuthal plane

The phase dimension of cognitive representation space is used by subject to differentiate between behavioral contexts by their relation to the considered decision alternative. Azimuthal phase being coordinate of this dimension reflects the meaningful part of contextual information - that part which is subjectively relevant for decision making of an organism [Gershenson, 2012, Kolchinsky and Wolpert, 2018, De Jesus, 2018, Galofaro et al., 2018]. The phase dimension of the qubit model presented above thus can be viewed as a simplest one-dimensional «semantic space», where ranges of the azimuthal phase certain meanings of a context relative to the considered decision alternative [Gärdenfors, 2014].

In the two-stage gambling example, the present model maps behavioral contexts «lost» and «unknown» having distinctively different meanings to azimuthal phase ranges (21) and (23) relative to zero phase identified with the meaning of the «won» context. Let these semantic bands be associated with ill-normalized vector projections of the cognitive states $|\Psi_a\rangle$, $|\Psi_b\rangle$ and

$|\Psi_c\rangle$ to the azimuthal plane in phase-symmetrized form

$$
\begin{aligned}
&|\phi_a\rangle = |0\rangle + |1\rangle, && |\neg\phi_a\rangle = -i\,|0\rangle + i\,|1\rangle, \\
&|\phi_b\rangle = e^{-i\frac{2\pi}{3}}\,|0\rangle + e^{i\frac{2\pi}{3}}\,|1\rangle, && |\neg\phi_b\rangle = e^{-i\frac{\pi}{6}}\,|0\rangle + e^{i\frac{\pi}{6}}\,|1\rangle, &&(24) \\
&|\phi_c\rangle = e^{-i\frac{\pi}{3}}\,|0\rangle + e^{i\frac{\pi}{3}}\,|1\rangle, && |\neg\phi_c\rangle = e^{-i\frac{5\pi}{6}}\,|0\rangle + e^{i\frac{5\pi}{6}}\,|1\rangle.
\end{aligned}
$$

where $|\neg\phi_i\rangle$ denotes negation of the $|\phi_i\rangle$ such that $\langle\neg\phi_i|\phi_i\rangle = 0$. The following vector identities then hold:

$$
\begin{aligned}
&|\phi_a\rangle + |\phi_b\rangle = |\phi_c\rangle, && |\phi_a\rangle - |\phi_b\rangle = \sqrt{3}\,|\neg\phi_c\rangle \\
&|\phi_c\rangle - |\phi_b\rangle = |\phi_a\rangle, && |\phi_c\rangle + |\phi_b\rangle = \sqrt{3}\,|\neg\phi_a\rangle, &&(25) \\
&|\phi_c\rangle - |\phi_a\rangle = |\phi_b\rangle, && |\phi_c\rangle + |\phi_a\rangle = \sqrt{3}\,|\neg\phi_b\rangle.
\end{aligned}
$$

These semantic relations between cognitive representations of contexts (24) are shown in the Figure 6.

The first column of (25) trivially reads that «unknown» context c symmetrically resolves into «won» a or «lost» b, whereas this uncertainty without one of the components gives the remaining alternative. The second column of (25) is also interpretable in the linguistic terms, with negations $|\neg\phi_a\rangle$ and $|\neg\phi_b\rangle$ corresponding to «not won» and «not lost» outcomes of previous gamble. State $|\neg\phi_c\rangle$ literally expressing «known» outcome of the gamble reflects what is subjectively common between «won» and «lost» contexts with respect to the considered decision.

6 Outlook

6.1 Quantum modeling of semantics

Current view of quantum models Quantum modeling is currently viewed as an instrument for quantitative explanation of probabilistic regularities of behavioral compromising expectations of Boolean logic; the need for quantum modeling is then indicated by violation of behavioral rationality identified with provisions of Boolean logic. Two-stage gambling, for example, is considered as a case for quantum modeling due to a so-called disjunction effect, violation of logical distributivity and the sure thing principle [Tversky et al., 1992]. In practice such violations are not necessarily observed [Kuhberger et al., 2001, Lambdin and Burdsal, 2007, Broekaert et al., 2020]; as seen from the Figure 4, polar angles θ_a, θ_b and θ_c have similar mean values, so that according to (5) averaged probabilities p_a, p_b and p_c are also nearly equal, thus obeying the law of total probability expressing the distributive axiom of Boolean logic [Khrennikov, 2010].

Quantum model as semantic representation   Absence of irrationality in behavioral data, however, does not prevent the above model from mapping behavioral contexts to their cognitive representations, as articulated in Sect. 3.3 where a degenerate probability triple $p_a[1] = p_b[1] = p_c[1]$ is taken as an example. This is a mode of application of quantum models for semantic representation of behavioral data [Bruza and Cole, 2005, Bruza, 2008, Aerts et al., 2018, Busemeyer and Wang, 2018, Bruza, 2018] which is essentially different from explanation of non-classical probability patterns.

In this semantic representation mode, this work emphasizes the role of azimuthal phases of quantum states. In the qubit model considered above this azimuthal phase dimension is recognized as a simplest one-dimensional semantic space of circular topology [Gärdenfors, 2014]. Behavioral contexts identified with definite linguistic meanings are mapped to particular domains in this space, supporting interpretable vector-logical calculus. This technique may contribute to semantic-statistical analysis fusing qualitative and quantitative descriptions anticipated by Brower [Brower, 1949]. Generalization for multiple triads accommodated in adjacent quantum-cognitive registers may be considered [Chiara et al., 2016].

Based upon quantum methodology of behavioral modeling and crucially dependent on the complex-valued structure of cognitive Hilbert space, the method semantic mapping shown above has no parallel in behavioral models based on classical algebra of events and probability calculus. Behaviorism, in its quantum version, thus seems to meet the expectation as a basis for a quantitative science of meaning [DeGrandpre, 2000].

6.2   Reflexivity quantified

The above results contribute to the question about adequacy of quantification of psychic phenomena. This possibility, identified with objective mode of description, contradicts subjective and reflexive nature of psychic processes [Tafreshi et al., 2016]; it is then concluded that numerical models are fundamentally incapable of expressing meaning and semantics [Brower, 1949]. Psychic phenomena of this kind are then addressed with qualitative methods [Madill and Gough, 2008] prone to the repeatability problems noted in the Introduction.

Quantum method of behavioral-semantic modeling contravenes the above identification of quantitativeness and objectivity: in quantum approach contexts are represented not by themselves, but by their relation to the particular decision alternative in cognition of a subject; as shown in Sect. 2, this subjective relativity lies at the very core of quantum modeling methodology. Objectivity is therefore not a primordial quality of numbers, but

merely a feature of Boolean logic and classical probability calculus. Quantification of reflexivity, recognized as a necessary for adequate life science [Gough and Madill, 2012, Lepskiy, 2018], just asks for another algebra of event representation. One possibility for that, a linear algebra of complex-valued Hilbert space, is examined above.

6.3   Triad: a carrier of meaning

Inspection of a single context (situation, alternative) is of no use; it is always difference between the two contexts that matters in practice. And inspection itself implies a reference frame, which determines in what respect the two contexts are compared. This subjective «point of view» is the third in a minimal bundle of three contexts supporting a meaning-based cognition of humans [Kuznetsov, 2013].

This is the reason for a triad cognitive structure considered in this work. As noted in Sect. 5.1, it is third context which does not fit in the real vector space asking for the complex-valued structure of quantum theory; the number of three observables employed in the Leggett-Garg and Bell inequalities designed to identify exclusively quantum behavior of physical systems [Bell, 1964, Leggett and Garg, 1985] seem to reflect the same point.

Triad versus dyad   If ternary representation structure is indeed a minimal carrier of meaning, it should be favored in comparison with binary one. This expectation is supported by finding in social studies, where cognitive perception and real behavioral data of social interaction are found to be in much better agreement at triadic compared to the dyadic level of analysis [Killworth and Russell, 1979]. An intuition for that is when a reference context of the triad is dropped, a subjective, reflexive, semantic account of data becomes impossible; what remains is objectified correlation analysis unsuitable for quantification of psychic processes. A subjective context then becomes an uncontrolled confounder spoiling the observed binary relation.

Same mechanism qualifies for phase stability phenomenon observed in the two-stage gambling data, where a quantum phase parameter is much less sensitive to context variation than individual probability values [Surov et al., 2019]. This quantum interference phase, accounting for behavioral probabilities for all of the contexts in a triple, captures semantic relations between them which are much less sensitive to occasional situation factors. The cognitive triad model described above provides an explicit mechanism for this stability of cognitive representation structure, at the same time supporting behavioral flexibility as shown in Sect. 3.3.

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Another aspect of triadic stability is robustness of cognitive system. If contexts are represented in binary association structure then damage of either component is irretrievable and fatal to the whole dyad. In triad, in contrast, any single component can be recovered as superposition of the other two as sketched by equations (25) based on the semantic relations recorded in stable quantum phase values.

Triad and higher adics   In terms of meaning as it is outlined in the beginning of this subsection, quartic structures seem to be expressible through triadic ones [Mertz, 1979]. On the other hand, the number of possible relations in quartic and higher-adic structures quickly grows beyond the limits of attention [Holland and Leinhardt, 1975]. Given that, until a new kinds of meaning inherent to these higher adicities [Sorkin, 1994] come to action, practical efficiency is likely to favor cognition in terms of triadic structure described above.

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