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Article

Optimal Phase Load Balancing in Low Voltage Distribution Networks using a Smart Meter Data-based Algorithm

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Abstract: In the electric distribution systems, the "Smart Grid" concept is implemented to encourage energy savings and integration of the innovative technologies, helping the Distribution Network Operators (DNOs) in choosing the investment plans which to lead the optimal operation of the networks and increasing the energy efficiency. In this context, a new phase load balancing algorithm was proposed to be implemented in the low voltage distribution networks with hybrid structures of the consumption points (switchable and non-switchable consumers). It can work in both operation modes (on-line and off-line), uploading information from different databases of the DNO which contain: the consumers' characteristics, the real loads of the consumers integrated into the Smart Metering System (SMS), and the typical load profiles for the consumers non-integrated in the SMS. The algorithm was tested in a real network, having a hybrid structure of the consumption points, on a time interval by 24 hours. The obtained results were analyzed and compared with other algorithms from the heuristic (Minimum Count of Loads Adjustment algorithm) and the metaheuristic (Particle Swarm Optimization and Genetic Algorithms) categories. The best performances were provided by the proposed algorithm, such that the unbalance coefficient resulted in the smallest value (1.0017). The phase load balancing led to the following technical effects: decreasing the average current in the neutral conductor with 94% and for the energy losses with 61.75 %, and increasing the minimum value of the phase voltage at the farthest pillar with the 7.14 %, compared to the unbalanced case.

Keywords: phase load balancing; smart meters; dynamic optimization, on-line implementation; low voltage electric distribution networks

1. Introduction

The three-phase electric distribution networks (EDN) are designed and built to operate in symmetrical regimes, balanced on all phases so that all the elements (lines, transformers, and not least the distributed generation sources) have the identical electrical parameters. In these situations, there are symmetrical current and voltage systems in each node of the system, namely the magnitudes of the voltage and currents on each phase are equal with a phase shift by 120 degrees. But, such an ideal system of the currents and voltages is practically impossible in the real operation conditions of the EDN due to the emergence of imbalances created mainly by the constructive conditions of some network elements (lines and transformers) or the supply of the single-phase (1-

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P) consumers. Thus, the operating regimes become asymmetric (unbalanced), which causes the loss of symmetry in the voltage and current systems [1].

The main causes of the imbalances in an EDN can be grouped into the following categories [2]:

- Constructive imbalances due, on the one hand, to the spatial arrangement of the conductors of the three phases of the electrical lines in the same plane (differences of the phase impedances), and on the other hand, to the arrangement of the windings of power transformers on the three columns of the ferromagnetic core (willing in the same plane).
- Functional imbalances created by 1-P consumers. These consumers are connected, either between two phases of the network or between a phase and the neutral point. Most 1-P consumers are represented by domestic and tertiary consumers, which are connected to the low-voltage (LV) network, requesting small values of the absorbed power (up to 100 kVA). In this category, 1-P industrial consumers can be added. They have high values of the absorbed powers and are connected to electric medium voltage networks (welding installations, with powers between 100 kVA and 3 MVA, 1-P arc furnaces, electric stations that supply power the railway traction network). It can be stated that the current unbalances are presented at all voltage levels of the electric distribution networks (low and medium voltage) causing various issues, including voltage unbalances [3].

The issues caused by current unbalance and the effects on the voltage unbalance are presented in Figure 1.

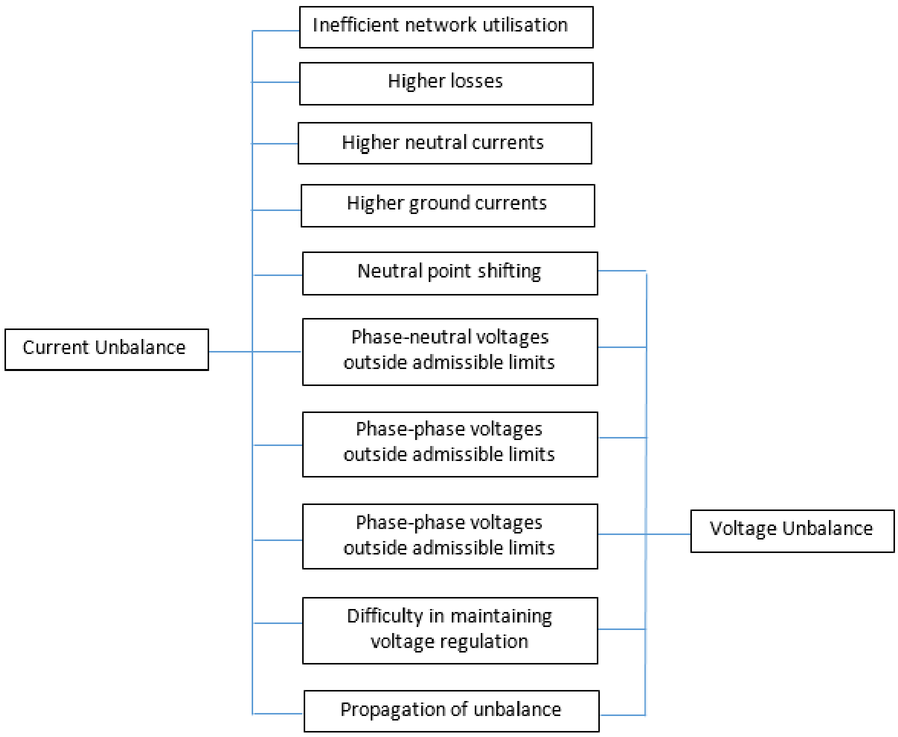

Figure 1. The issues caused by the current unbalances [3]

It can observe that the current and voltage unbalances cannot be separated. In this context, the voltage unbalances could cause a current unbalances, which can be translated into economic and technical losses for both partners (consumers and DNO). From the point of view of the attenuation measures, the current unbalances can be easier solved by the DNO than the voltage unbalances. The main advantage of current balancing refers to the minimization of the current flow in the neutral conductor with the benefits of decreasing the total losses in the EDN [1].

Several phase load balancing (PLB) mechanisms are found in the literature. For example, the PLB problem at the first feeder near the substation was solved through a branch and bound algorithm [4]. The aforementioned approach uses real data of customer power demand in the different time

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periods in order to minimize the value of unbalance factor and to find the optimal three-phase load balance in EDN. In [5], the PLB approach considers the re-phasing of the customers for the reduction of unbalance level in the EDN. Other approaches consider different automatic three-phase load balancing devices [6-8]. The solutions for the PLB model were obtained using various techniques and technical measures, such as hierarchical Petri nets [9], low voltage (LV) feeder reconfiguration [10-11], or switching the consumers on the three phases [12-14]. In [15-17] the studied problem is solved with particular metaheuristic algorithms. A PLB mechanism at the three-phase power transformer level was proposed in [18]. A particular approach is developed in [19] and [20] wherein a three-phase real EDN the PLB solution was obtained by optimal placement of the decentralized and autonomous battery storage systems. A different formulation of the PLB optimization problem is presented in [21] following the implementation of a commutation system, with two-phase thyristor parallel contactor structure, or based on power-line communication (PLC) and Supervisory Control and Data Acquisition (SCADA) technologies in [22-23], and not least based on Smart Meters [24-25]. Another category of the published papers [26-28] regards the PLB problem at the active distribution networks (smart grids) level, using heuristic or metaheuristic methods. Moreover, an automatic phase load balancing device [29], a shunt passive compensator [30], or a controlled active filter [31] were proposed. Also, a controller was proposed in [31] to switch the connected 1-P loads from one phase to another based on an algorithm with a minimum count of loads adjustment.

To highlight the originality of the proposed algorithm, in Table 1 a brief description of the literature is presented, considering the four main characteristics: network type, the location of PLB operation, the used algorithm and the operation mode. Other many papers from the literature indeed solve the PLB problem, but they coincide with those presented in Table 1. The objective functions identified are referred to the minimization of unbalance factor at the pillar level or supply point (electric distribution substation).

Table 1. A comparative state of the art between proposed method and the literature.

Number of Reference	Type of network		Location of PLB		Type of Algorithm	Operation mode	
	Real	Fictive (test)	Pillar (P) / Consumer (C)	Supply point		On -line	Off-line
[4], [27]	Yes	Yes	No	Yes	Heuristic	No	Yes
[5], [17], [28]	Yes	No	No	Yes	Metaheuristic	No	Yes
[6], [21], [24]	No	Yes	No	No	Experimental	No	Yes
[7], [8], [26]	No	Yes	No	Yes	Heuristic	No	Yes
[9], [10]	Yes	No	No	Yes	Heuristic	No	Yes
[12], [13]	No	Yes	Yes	No	Metaheuristic	No	Yes
[14], [29]	No	Yes	Yes	No	Experimental	Yes	No
[15], [16]	No	Yes	No	Yes	Metaheuristic	No	Yes
[18], [32]	No	No	No	Yes	Heuristic	No	Yes
[19], [20]	Yes	No	Yes	No	Heuristic	No	Yes
[23]	No	Yes	No	No	Heuristic	Yes	No
[30], [31]	No	Yes	Yes	Yes	Metaheuristic	No	Yes
Proposed approach	Yes	No	Yes	Yes	Heuristic	Yes	Yes

Regardless of the algorithm used and the locations where the PLB is done, each consumer should have a smart system integrated in the SMS, which to contain in its structure a smart meter and an automatic phase load balancing device (APBD) [27], Figure 2.

Currently, the technical solution is developed by the producers to be implemented by the DNO in the EDN with a high unbalanced degree [33], [34]. The solution is introduced by the DNOs only on base of a feasibility analysis which to justify the investment.

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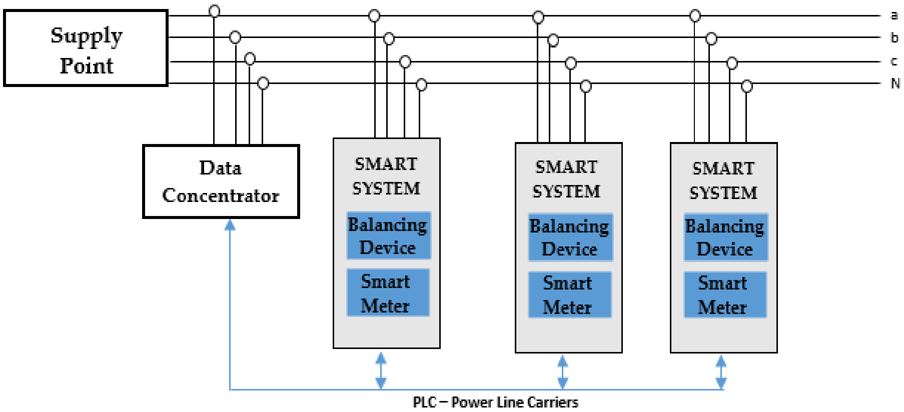

Figure 2. The structure of a smart system installed at the consumers integrated in the SMS

The analysis should identify in each stage the associated cost to implement the PLB. The main stages refer to the identification of a feasible technology, the planning of assembly at consumers, the commissioning of the system, the integration in the SMS, the testing the communication with data concentrator from the supply point, and not least the maintenance plan [27].

Compared to the approaches from the literature, the proposed algorithm has the following advantages:

- It can be implemented in the EDNs with hybrid structures of the consumption points (switchable and non-switchable consumers).
- It can work in both operation modes (on-line and off-line), uploading information from different databases of the DNO. The consumers' characteristics (connecting pillar, allocated phase, consumption sector and class, integration in the SMS, identification number of the meter) are extracted based on the identification number of the supply point. The value of consumption and operating status of phase load balancing device (PLBD) are uploaded from the database of the SMS if the meter is integrated, or from the typical load profiles (TLPs) database if the consumer has a standard energy meter (non-integrated in the SMS).
- The convergence is rapid due to the fast recognition of the EDN topology with the help of a structure vectors based-algorithm. The optimal solutions for PLB are found at the level of each pillar such that the global solution obtained the level of the supply point will be also optimal.

The paper has a structure organized as following: Section 2 details the stages of proposed PLB algorithm, accompanied by the implementation procedure, Section 3 presents the results obtained in the case of a real EDN belonging a DNO from the north-eastern of Romania and a comparison with other three algorithms (heuristic and metaheuristic) to demonstrate the accuracy of the proposed algorithm, and Section 4 highlights the conclusions and the future works.

2. The proposed PLB algorithm

The proposed algorithm can be implemented at the level of data concentrator from the supply point (electric distribution substation) to work in the on-line mode or to the Decision-Making Central Level (DMCL) of DNO for the off-line work mode helping to identify the EDNs with a high unbalanced degree and to determine the optimal solutions to decrease it. The on-line implementation in the soft architecture of the data concentrator from the supply point (SP) involves the installation of a smart system at the level of each consumer, as indicated in Figure 2. But, the algorithm can be also implemented in the EDN with measurement structures which include standard meters together with smart meters.

The PLB algorithm has the following stages:

Stage 1. Identification of topology for the EDN.

The topology is identified using a two structure vectors-based algorithm [35]. The algorithm leads to the systematizationof the topology, grouping the branches into vicinity levels in relation to

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the supply point (the electric distribution substation). As example, for an EDN with 9 nodes and 8 branches presented in Figure 3, the branches are grouped in three vicinity levels, starting from the supply point (SP): Level 1 - 1 branch (B2); Level 2 - 2 branches (B3 and B4), and Level 3 - 5 branches (B5, B6, B7, B8, and B9). Each branch is recognized based on the input and end nodes (pillars), being numbered in relation to the end node. The input and end nodes of branches are recorded in the vectors B_i and B_e, respectively, considered as input data of the algorithm.

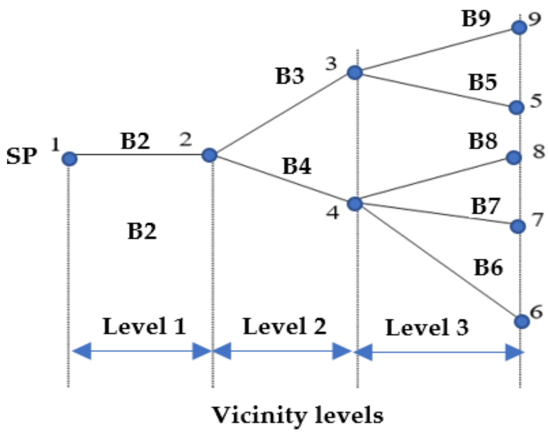


Figure 3. Grouping the branches into vicinity levels

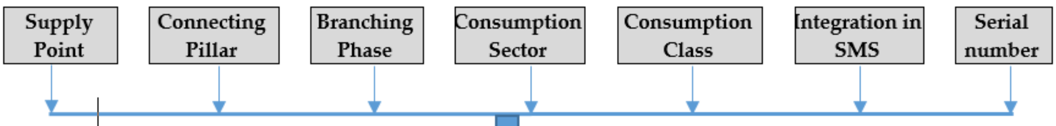
Considering these aspects, the topology of the EDN can be described using two integer vectors, TV1 and TV2. The vector TV1 contains the number of branches from each vicinity level and the vector TV2 includes all branches in the order of the vicinity levels. The elements of vectors TV1 and TV2 are presented in Table 2.

Table 2. The elements of topology vectors

TV1	L1	L2	L3
TV2	B1	B2,B3	B5, B6,B7,B8,B9

Stage 2. Upload the input data sequence

The algorithm upload from the database of the DNO a data sequence that will be stored in the input vectors. This input data sequence is formed by the following fields (see Figure 4):



2002814771	3	a	1	4	1	111069
----	-	-	-	-	-	-
2002814771	4	c	1	3	0	111075

Figure 4. The input data sequence of the algorithm

- *Supply point*: Each electric distribution substation has an identification number that allows the algorithm to allocate correct from the database all consumers supplied from this point.
- *Connecting pillar*: To identify the position of each consumer in the network, the connecting pillar is recorded in the database. Also, this information is very important in the calculus of a steady-state regime with the aim of evaluating the performance of the PLB measure through reducing the power/energy losses and improving the voltage level at the consumers. The vector associated with this field is noted with *CP*, having the size (N_Cx1), where N_C represents the total number of consumers from the EDN.

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- *Branching Phase*: Each 1-P consumer is allocated by the DNO at one of the phases *ph* = {*a*, *b*, *c*}, and the 3-phase consumers are connected at all three phases *ph* = {*abc*}. The records regarding this information are found in the vector *PB* with the size (N_C×1).
- *Consumption Sector*. The information is used to assign the consumer to a consumption sector: domestic, non-domestic, commercial, and industrial. The records for this information have the following identification numbers: 1 (domestic), 2 (non-domestic), 3 (commercial), and 4 (industrial) included in the vector *CS* with the size (N_C×1).
- *Consumption class*. More consumption classes are allocated to each consumption sector by the DNO. As an example, a Romanian DNO was made a classification in five consumption classes for the consumers from the domestic sector [35]: < 400 kWh (first class), range [400 kWh, 1250 kWh] (second class), range [1250 kWh, 2500 kWh] (third class), range [2500 kWh, 3500 kWh] (the fourth class), and range [2500 kWh, 3500 kWh] (the fifth class). This information is loaded in the vector *CC*, having the size (N_C×1).
- *Integration in SMS*. Currently, not all consumers from the LV distribution networks are integrated into the Smart Metering System. In this case, the value 1 (if it is integrated) and 0 (otherwise) will be recorded in the database. If the meter is integrated into the SMS, it can communicate to the central system information about the currents or active and reactive powers which will record them in the database (see Figure 5).

SERIAL_NUMBER	SAMPLE	Hour 00	Hour 01	Hour 02	Hour 03	Hour 04	Hour 05	Hour 06	Hour 07	Hour 08	Hour 09
100915300017409	27.12.2017	0.339	0.355	0.336	0.343	0.341	0.642	0.483	0.262	0.127	0.254
100915300018517	27.12.2017	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.003
100915300018857	27.12.2017	0.201	0.166	0.209	0.178	0.171	0.205	0.615	0.795	0.212	0.152

SERIAL_NUMBER	Hour 10	Hour 11	Hour 12	Hour 13	Hour 14	Hour 15	Hour 16	Hour 17	Hour 18	Hour 19	Hour 20	Hour 21	Hour 22	Hour 23
100915300017409	0.104	0.082	0.061	0.101	0.173	0.153	0.299	0.418	0.360	0.433	0.250	0.102	0.122	0.095
100915300018517	0.003	0.003	0.004	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.004
100915300018857	0.277	0.231	0.263	0.073	0.056	0.041	0.146	0.599	0.452	0.300	0.376	0.301	0.264	0.175

Figure 5. The sequence of the power actives recorded in the database

If the consumer has a PLBD then the central system will be communicated both its operating status and connection phase. Thus, it will be classified by the algorithm in the category of switchable consumers, recording the value 1 in the database. Otherwise, even if the consumer is integrated into the SMS and PLBD is faulty (value is 0), or has a standard meter, it cannot be allocated on other phase and will be classified in the set of non-switchable consumers, recording the value 0 in the database. The algorithm will record these values in the vectors *INT* (for integration mode) and *BS* (for the PBLD status), having the size (N_C×1). Also, for the non-switchable consumers due to the missing data from the consumption point, the algorithm will use the hourly values from the typical load profiles (TLPs) allocated in function by *Consumption Sector* (vector *CS*) and *Consumption class* (vector *CC*).

Concerning the TLPs, these are defined by the DNO to all consumers which are not integrated in the SMS and are determined for each consumption sector (domestic, non-domestic, commercial, and industrial) having common characteristics regarding the consumption classes, days (weekend or working), and seasons (springer, summer, autumn, or winter). Finally, each consumer will have an assigned TLP, depending on the above characteristics. The profiling process to obtain the TLPs is presented in [35].

The values of the hourly loads for all consumers are recorded in the matrix *IC*, with the size (N_c×*H*).

- *Serial number*. Each consumer is recognized in the database through the serial number of meter installed (smart or standard). The information is recorded in the vector *SN*, having the size (N_c×1).

Stage 3. The PLB procedure

The PLB procedure is characterized by a dynamic process having as main objective the minimization of unbalance degree (as close to 1) at the level of each pillar by allocation to other phases (e.g., phase *a* on phases *b* or *c*) of the switchable consumers (with PLBD installed). If an optimal solution is obtained in each iteration, corresponding to each pillar, then the global solution (at the SP level) will be optimal.

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To evaluate if an EDN is in an unbalanced regime, an unbalance coefficient is calculated. If for the voltage unbalance, there are formulas proposed by the IEEE (The Institute of Electrical and Electronics Engineers) and NEMA (The National Equipment Manufacturer's Association) standards [36], for the current unbalance there is no widespread agreement. Thus, the negative and positive sequence components of the current can be used to evaluate the current unbalance [3]. This approach requires the decomposition of the current system into instantaneous positive, negative, and zero sequence components using phasor representation, which is not always possible. Easy evaluation of current unbalance in a node (pillar) of the EDN can be made based on an unbalance coefficient calculated based on the effective values of phase currents [27]. The value of this coefficient must be less than 1.1, agreed by the DNOs.

$$UC = \frac{1}{3}\cdot\left(\left(\frac{I_a}{I_{average}}\right)^2 + \left(\frac{I_b}{I_{average}}\right)^2 + \left(\frac{I_c}{I_{average}}\right)^2\right) \tag{1}$$

where: UC – the unbalance coefficient; I_a, I_b, I_c – the currents on the phases a, b, and c; $I_{average}$ – the average value of the phase currents.

The mechanism of the proposed algorithm is explained in Figure 6 for the case of 2 pillars with 2 and 3 consumers, respectively, connected. The initial phase of the switchable consumers has a yellow colour, and the optimal phase has a red colour. If the yellow colour is missing for a certain consumer, then the optimal phase is identical to the initial phase. The blue colour was used to highlight the phase of non-switchable consumers.

The analysis of obtained results highlighted that a final value of UC very close by 1 (1.006) can be reached starting from an initial high value (1.636), switching only 2 consumers (2 and 5) on other phases.

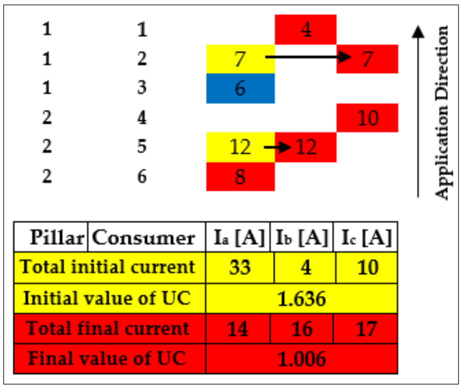

Figure 6. The mechanism of proposed algorithm and the obtained results

The minimization of unbalance coefficient (UC), at each hour h = 1, …, H, and each pillar p = 1,…, Np represents the objective of the PLB problem:

$$\min(UC^{(p),h}),\ p = 1,\dots,N_p;\ h = 1,\dots,H \tag{2}$$

where:

$$UC^{(p),h} = \frac{1}{3}\cdot\left(\left(\frac{I_a^{(p),h}}{I_{average}^{(p),h}}\right)^2 + \left(\frac{I_b^{(p),h}}{I_{average}^{(p),h}}\right)^2 + \left(\frac{I_c^{(p),h}}{I_{average}^{(p),h}}\right)^2\right) \tag{3}$$

$$I_{average}^{(p),h} = \frac{1}{3}\left(I_a^{(p),h} + I_b^{(p),h} + I_c^{(p),h}\right) \tag{4}$$

$$I_a^{(p),h} = I_{a,ns}^{(p),h} + I_{a,s}^{(p),h} + I_a^{(d),h};\ p = 1,\dots,N_p;\ p \neq d \tag{5}$$

$$I_b^{(p),h} = I_{b,ns}^{(p),h} + I_{b,s}^{(p),h} + I_b^{(d),h};\ p = 1,\dots,N_p;\ p \neq d \tag{6}$$

$$I_c^{(p),h} = I_{c,ns}^{(p),h} + I_{c,s}^{(p),h} + I_c^{(d),h};\ p = 1,\dots,N_p;\ p \neq d \tag{7}$$

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$$I_{a,ns}^{(p),h} = \left(\sum_{j=1}^{N_{a,ns}^{(p),h}} I_{a,ns,j}^{(p),h} \right) \quad (8)$$

$$I_{b,ns}^{(p),h} = \left(\sum_{k=1}^{N_{b,ns}^{(p),h}} I_{b,ns,k}^{(p),h} \right) \quad (9)$$

$$I_{a,ns}^{(p),h} = \left(\sum_{l=1}^{N_{c,ns}^{(p),h}} I_{c,ns,l}^{(p),h} \right) \quad (10)$$

$$I_{a,s}^{(p),h} = \left(\sum_{m=1}^{N_{a,s}^{(p),h}} I_{a,s,m}^{(p),h} \right) \quad (11)$$

$$I_{b,s}^{(p),h} = \left(\sum_{n=1}^{N_{b,ns}^{(p),h}} I_{b,s,n}^{(p),h} \right) \quad (12)$$

$$I_{c,s}^{(p),h} = \left(\sum_{o=1}^{N_{c,ns}^{(p),h}} I_{c,s,o}^{(p),h} \right) \quad (13)$$

$$N_{C,ns}^{(p)} = N_{a,ns}^{(p),h} + N_{b,ns}^{(p),h} + N_{c,ns}^{(p),h} \quad (14)$$

$$N_{C,s}^{(p)} = N_{a,s}^{(p),h} + N_{b,s}^{(p),h} + N_{c,s}^{(p),h} \quad (15)$$

$$N_{C}^{(p),h} = N_{C,ns}^{(p),h} + N_{C,s}^{(p),h} \quad (16)$$

where: $UC^{(p),h}$ – the unbalance coefficient calculated at the pillar p and hour h ; $I_a^{(p),h}$, $I_b^{(p),h}$, $I_c^{(p),h}$ – the currents on the phases a, b, and c, at the pillar p and hour h; $I_{average}^{(p),h}$ – the average value of the phase currents, at the pillar p and hour h; $I_{a,ns}^{(p),h}$, $I_{b,ns}^{(p),h}$, $I_{c,ns}^{(p),h}$ – the total current of the non-switchable consumers on the phases a, b, and c, at the pillar p and hour h; $I_{a,s}^{(p),h}$, $I_{b,s}^{(p),h}$, $I_{c,s}^{(p),h}$ – the total current of the switchable consumers on the phases a, b, and c, at the pillar p and hour h; $I_{a,s}^{(d),h}$, $I_{b,s}^{(d),h}$, $I_{c,s}^{(d),h}$ - the currents on the phases a, b, and c, at the pillar d (located downstream by pillar p), and hour h; $I_{a,ns,j}^{(p),h}$ – the current of the non-switchable consumer j connected on the phase a, at the pillar p, and hour h; $I_{b,ns,k}^{(p),h}$ – the current of the non-switchable consumer k connected on the phase b, at the pillar p, and hour h; $I_{c,ns,l}^{(p),h}$ – the current of the non-switchable consumer l connected on the phase c, at the pillar p, and hour h; $I_{a,s,m}^{(p),h}$ – the current of the switchable consumer m connected on the phase a, at the pillar p, and hour h; $I_{b,s,n}^{(p),h}$ – the current of the switchable consumer n connected on the phase b, at the pillar p, and hour h; $I_{c,s,o}^{(p),h}$ – the current of the switchable consumer o connected on the phase c, at the pillar p, and hour h; $N_{a,ns}^{(p),h}$, $N_{b,ns}^{(p),h}$, $N_{c,ns}^{(p),h}$ – the number of the non-switchable consumers connected on the phases a, b, and c, at the pillar p, and hour h; $N_{a,s}^{(p),h}$, $N_{b,s}^{(p),h}$, $N_{c,s}^{(p),h}$ – the number of the switchable consumers connected on the phases a, b, and c, at the pillar p, and hour h; $N_{C,ns}^{(p),h}$ – the total number of the non-switchable consumers connected at the pillar p, and hour h; $N_{C,s}^{(p),h}$ – the total number of the switchable consumers connected at the pillar p, and hour h; $N_C^{(p),h}$ – the total number of the consumers connected at the pillar p, and hour h; N_p – the total number of the pillars; H – the analysed time period.

The implementation procedure of the mathematical model (2) – (16) is presented in Table 3.

Table 3. The implementation procedure of the proposed PLB algorithm

Steps of PLB algorithm based on the smart meter data
Step 1. Identification of the topology for the EDN based on the vectors TV1 and TV2, built with the vectors B_i and B_e which contain the input and end nodes (pillars) assigned each branch.
Step 2. Upload the input data sequence from the database of the DNO corresponding to the SP of EDN: Store the information in the vectors: *CP*, *BP*, *CS*, *CC*, *INT*, *BS*, and *SN*. Determine the number of consumers supplied: *NC* = length (*SN*);

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Initialize the matrices $IC \in \mathbb{R}^{*(N_c \times H)}$, I_a, I_b, and $I_c \in \mathbb{R}^{*(N_p \times H)}$, and $UC \in \mathbb{R}^{*(N_p \times H)}$;
for each hour h, $h = 1 \ldots H$
Set initial consumer index: $i = 0$;
while $i \leq N_c$
Increase consumer index: $i = i + 1$;
if $INT\ (i, h) = 1$
if $BS\ (i, h) = 1$
Update $IC\ (i, h)$ with the value recorded on the line $SN(i)$ and column h of the consumption matrix loaded from the SMS;
else
Send a warning message to the central system on the failure/missing communication of PLBD to be repaired as soon as possible;
Update $IC\ (i, h)$ with the assigned value from the TLP depending the records from the vectors $CS\ (i)$ and $CC\ (i)$, day (weekend or working), and season (springer, summer, autumn, or winter);
else
Update $IC\ (i, h)$ with the assigned value from the TLP depending the records from the vectors $CS\ (i)$ and $CC\ (i)$, day (weekend or working), and season (springer, summer, autumn, or winter);

Step 3. The PLB sequence at the level of each pillar:
Set initial pillar index: $p = Np$;
while $(p \geq 1)$ and $(p \leq N_p)$
Initialize the vector *index*;
Find the index corresponding to pillar p in vector CP, and store in vector *index*;
Determine the number of consumers connected at the pillar p: n_p = length (*index*);
Initialize the sums of phase currents corresponding to:
switchable consumers: $I_{as} = 0$, $I_{bs} = 0$, $I_{cs} = 0$;
non-switchable consumers: $I_{ans} = 0$, $I_{bns} = 0$, $I_{cns} = 0$;
all consumers: $I_{ap} = 0$, $I_{bp} = 0$, $I_{cp} = 0$;
Set initial consumer index: $j = 0$;
while $j \leq n_p$
Increase consumer index: $j = j+1$;
if $(INT(index\ (j)) = 0)$ and $(BP\ (index\ (j)) = \{a\})$
Update sum of current to non-switchable consumers on the phase a:
$I_{ans} = I_{ans} + IC\ (index\ (j))$;
if $BP\ (index\ (j)) = \{b\})$
Update sum of current to non-switchable consumers on the phase *b:*
$I_{bns} = I_{bns} + IC\ (index(j))$;
else
Update sum of current to non-switchable consumers on the phase *c:*
$I_{cns} = I_{cns} + IC\ (index\ (j))$;
if $(INT(index\ (j)) = 1)$ and $(BS\ (index\ (j)) = 0)$
Changing the category of consumer j from switchable in non-switchable;
if $(BP\ (index\ (j)) = \{a\})$
Update sum of current to non-switchable consumers on the phase a:
$I_{ans} = I_{ans} + IC\ (index\ (j))$;
if $BP\ (index\ (j)) = \{b\})$
Update sum of current to non-switchable consumers on the phase *b:* $I_{bns} = I_{bns} + IC\ (index\ (j))$;
else
Update sum of current to non-switchable consumers on

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the phase *c*: $I_{cns} = I_{cns} + IC\ (index\ (j))$;
if ($INT(index\ (j)) = 1$) and ($BS\ (index\ (j)) = 1$)
Assigning the consumer *j* on each of the three phases:
case *Combination 1 – allocation of the consumer j on the phase a*
Compute the fictive sum of phase currents to switchable consumers:
$I_{asf1} = I_{as} + IC\ (index\ (j))$; $I_{bsf1} = I_{bs}$; $I_{csf1} = I_{cs}$;
Compute the fictive sum of the phase currents to all consumers:
$I_{apf1} = I_{ans} + I_{asf1}$; $I_{bpf1} = I_{bns} + I_{bsf1}$; $I_{cpf1} = I_{cns} + I_{csf1}$;
Compute the average value of the phase currents, $I_{average1}$ (rel. (3))
Compute the UC_1 (rel. (2));
case *Combination 2 – allocation of the consumer j on the phase b*
Compute the fictive sum of phase currents to switchable consumers:
$I_{asf2} = I_{as}$; $I_{bsf2} = I_{bs} + IC\ (index\ (j))$; $I_{csf2} = I_{cs}$;
Compute the fictive sum of the phase currents to all consumers:
$I_{apf2} = I_{ans} + I_{asf2}$; $I_{bpf2} = I_{bns} + I_{bsf2}$; $I_{cpf2} = I_{cns} + I_{csf2}$;
Compute the average value of the phase currents, $I_{average2}$, (rel. (3));
Compute the UC_2 (rel. (2));
case *Combination 3 – allocation of the consumer j on the phase c*
Compute the fictive sum of phase current to switchable consumers:
$I_{asf3} = I_{as}$; $I_{bsf3} = I_{bs}$; $I_{csf3} = I_{cs} + IC\ (index\ (j))$;
Compute the fictive sum of the phase currents of all consumers:
$I_{apf3} = I_{ans} + I_{asf3}$; $I_{bpf3} = I_{bns} + I_{bsf3}$; $I_{cpf3} = I_{cns} + I_{csf3}$;
Compute the average value of the phase currents, $I_{average3}$ (rel. (3));
Compute the UC_3 (rel. (2));
Determine the minimum value of *UC*: min (UC_1, UC_2, UC_3);
Store the number of combination with UC_{min}, CO_{min}, corresponding to one of the three phase:
if $CO_{min} = 1$
Update in the vector *PB* the phase *a*: $PB\ (index\ (j)) = \{a\}$;
Update the sum of phase currents to switchable consumers:
$I_{as} = I_{asf1}$; $I_{bs} = I_{bsf1}$; $I_{cs} = I_{csf1}$;
Update the sum of phase currents to all consumers:
$I_{ap} = I_{apf1}$; $I_{bp} = I_{bpf1}$; $I_{cp} = I_{cpf1}$;
if $CO_{min} = 2$
Update in the vector *PB* the phase *b*: $PB(index\ (j)) = \{b\}$;
Update the sum of phase currents to switchable consumers:
$I_{as} = I_{as2}$; $I_{bs} = I_{bsf2}$; $I_{cs} = I_{csf2}$;
Update the sum of phase currents to all consumers:
$I_{ap} = I_{apf2}$; $I_{bp} = I_{bpf2}$; $I_{cp} = I_{cpf2}$;
else
Update in the vector *PB* the phase *c*: $PB(index\ (j)) = \{c\}$;
Update the sum of phase currents to switchable consumers:
$I_{as} = I_{as3}$; $I_{bs} = I_{bsf3}$; $I_{cs} = I_{csf3}$;
Update the sum of phase currents to all consumers:
$I_{ap} = I_{apf3}$; $I_{bp} = I_{bpf3}$; $I_{cp} = I_{cpf3}$;
Update the value of unbalanced coefficient $UC\ (p, h) = UC_{min}$;
Update the value of phase currents $I_a\ (p, h) = I_{ap}$, $I_b\ (p, h) = I_{bp}$, and $I_c\ (p, h) = I_{cp}$;
Decrease pillar index: $p = p - 1$;
According with the new allocations from vector *PB* the central system emits the instructions at each PLBD;
Increase hour index: $h = h + 1$;
Print results: *UC*, I_a, I_b, I_c.

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3. Case study

The proposed PLB algorithm was tested in the case of a real LV EDN from a rural area, located in the north-eastern of Romania. The structure of the network is presented in Figure 7.

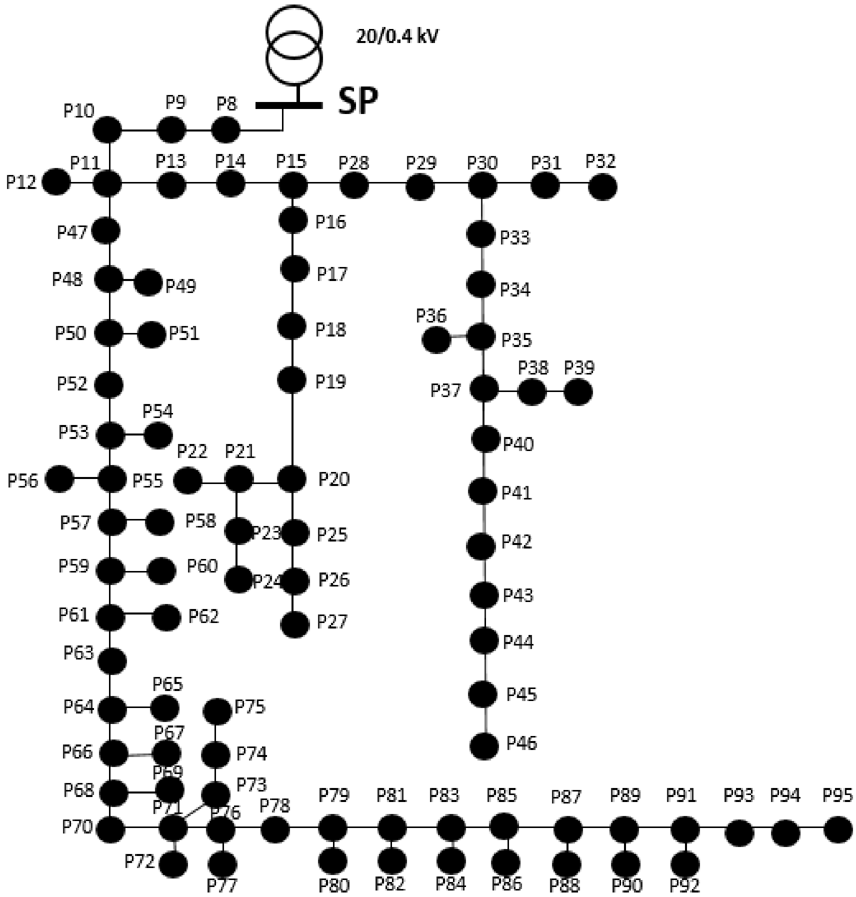

Figure 7. The structure of analysed EDN

Table 4. The technical characteristics of the branches

Branch	Type conductor	Cross-section of phase conductors [mm²]	Cross-section of neutral conductor [mm²]	Length [km]	r_0 [Ω/km]	x_0 [Ω/km]
SP-11	Classic	50	50	0.160	0.61	0.298
11-15	Classic	50	50	0.160	0.61	0.298
11-95	Classic	50	50	1.960	0.61	0.298
15-27	Classic	35	35	0.480	0.871	0.055
15-39	Classic	35	35	0.480	0.871	0.055
37-46	Classic	25	25	0.280	1.235	0.319
Total		50	50	2.280	0.61	0.298
		35	35	0.960	0.871	0.055
		25	25	0.280	1.235	0.319
		Total		3.520		

The EDN is supplied from a point (SP) through a power transformer 20/0.4 kV. The numbering of pillars is real, given by the DNO from this distribution area, which means that the first pillar is Pillar 8. The distance between two successive pillars is 0.04 km, stipulated in Romanian technical

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normative [37]. The technical characteristics of the branches are presented in Table 4, where r_0 and x_0 represent the specific resistance and reactance.

From the database of the DNO were uploaded the information about characteristics of the consumers from this EDN based on the identification number of the SP. The format of the input data was presented in Section 2 (see Figure 4). The characteristics of the consumers are presented synthetically in Table 5. Detailed information regarding the connected pillars, the branching phase, and the consumption sector are given in Table A1, Appendix A.

Table 5. Synthesis on the characteristics of the consumers from the analysed EDN

Consumer' type		**Initial phase**				**Consumption Sector**			
1-P	**3-P**	**a**	**b**	**c**	**abc**	**I**	**II**	**III**	**IV**
161	2	42	72	47	2	161	2	-	-

It can observe that the vast majority of consumers (98.8 %) have a 1-P branching with the following initial allocation: 25.8 % on phase *a*, 44.2 % on phase *b*, 28.8 % on phase *c*, and 1.2 % have a 3-P branching. Regarding the consumption sector, 98.8 % belong to domestic sector and only 1.2 % are from the non-domestic sector.

From all consumers, 114 1-P consumers (70.8 %) are integrated into the SMS with the possibility to have PLBD installed. They will be considered from the switchable consumers' category in our algorithm. The algorithm imports for each consumer i, i =1,…, N_c, according to the serial number of meter recorded in the vector *SN*, the hourly load from the database of SMS for the analysed period *H*. In our case study, the period *H* corresponds to a winter working day with hourly records h, h =1,...,24. The other 47 1-P consumers are considered as non-switchable consumers due to the standard meters, non-integrated in the SMS. For these consumers, the algorithm uses TLPs according to the information stored in the vectors CS (consumption sector) and *CC* (consumption class).

The phase currents (I_a, I_b, and I_c) and neutral current (I_0) in the SP (on the 0.4 kV side) were determined considering all load profiles, using a calculation algorithm of the steady-state regime for the LV unbalanced distribution networks [35] (see Table 6 and Figure 8).

The analysis of the obtained results highlights a high difference between phase currents and an important current in the neutral conductor (exceeds the current on the phases *a* and *b*), which leads to an unbalanced degree beyond the threshold (1.1) imposed by the DNO. The *UC* is in the range [1.17, 1.35], having an average value of 1.26.

Also, this current unbalance leads to higher power/energy losses due to current flows in the neutral conductor and significant voltage unbalance, as shown in Table 7 and Figures 9. The losses in the neutral conductor represent an important percent (37 %) of the total energy losses such that the PLB measure must be implemented.

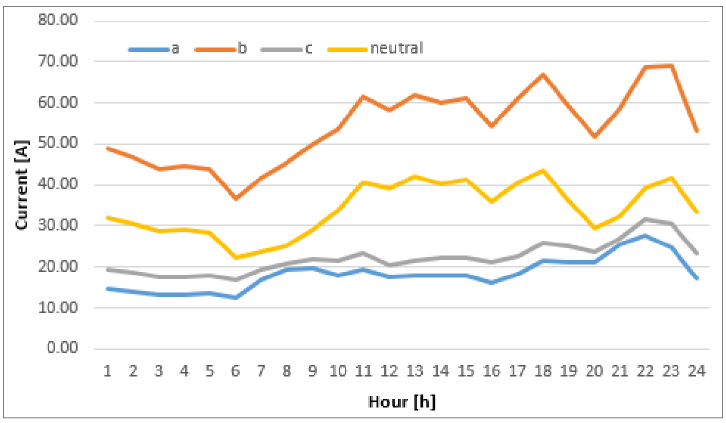

Figure 8. The currents in the conductors of the first branch, SP – Pillar 8, initial case

Table 6. The currents in the conductors of the first branch, SP-Pillar 8, initial case

Hour	I_a [A]	I_b [A]	I_c [A]	I_0 [A]	UC
1	14.77	48.71	19.47	31.84	1.29
2	14.01	46.55	18.64	30.49	1.30
3	13.24	43.81	17.73	28.58	1.29
4	13.36	44.40	17.45	29.20	1.30
5	13.55	43.94	17.99	28.43	1.28
6	12.38	36.47	16.98	22.15	1.23
7	16.73	41.58	19.49	23.59	1.18
8	19.53	45.17	20.93	24.97	1.17
9	19.69	49.91	21.88	29.18	1.20
10	18.05	53.57	21.70	33.83	1.26
11	19.21	61.57	23.16	40.52	1.30
12	17.44	58.17	20.53	39.28	1.33
13	17.94	61.76	21.40	42.20	1.35
14	17.87	60.11	22.35	40.18	1.32
15	17.91	61.07	22.21	41.18	1.33
16	15.99	54.16	21.22	35.84	1.31
17	18.38	61.07	22.53	40.77	1.32
18	21.55	66.87	25.80	43.34	1.29
19	21.31	59.27	25.14	36.19	1.23
20	21.27	51.86	23.77	29.41	1.18
21	25.66	58.78	27.08	32.43	1.17
22	27.69	68.53	31.57	39.04	1.19
23	24.83	69.17	30.67	41.72	1.22
24	17.12	53.18	23.17	33.45	1.26

Table 7. The energy losses calculated in the initial case, [kWh]

Hour	Main Conductors				Branching Conductors				Total
	a	b	c	Neutral	a	b	c	Neutral	
1	0.03	0.54	0.11	0.43	0.003	0.014	0.001	0.011	1.14
2	0.03	0.49	0.10	0.39	0.003	0.013	0.001	0.011	1.04
3	0.02	0.43	0.09	0.35	0.002	0.011	0.001	0.009	0.92
4	0.02	0.44	0.09	0.35	0.002	0.012	0.001	0.010	0.93
5	0.02	0.44	0.09	0.35	0.002	0.011	0.001	0.010	0.93
6	0.02	0.31	0.08	0.25	0.002	0.006	0.001	0.006	0.67
7	0.04	0.41	0.11	0.32	0.005	0.007	0.001	0.008	0.90
8	0.05	0.50	0.12	0.38	0.007	0.009	0.001	0.011	1.08
9	0.05	0.59	0.14	0.46	0.007	0.012	0.001	0.013	1.27
10	0.04	0.66	0.13	0.52	0.005	0.017	0.001	0.015	1.40
11	0.05	0.87	0.15	0.68	0.006	0.025	0.001	0.021	1.81
12	0.04	0.77	0.12	0.60	0.005	0.025	0.001	0.020	1.58
13	0.04	0.86	0.13	0.68	0.005	0.029	0.001	0.023	1.77
14	0.04	0.82	0.14	0.65	0.005	0.025	0.001	0.020	1.71
15	0.04	0.85	0.14	0.67	0.005	0.026	0.001	0.021	1.76
16	0.04	0.67	0.13	0.53	0.004	0.019	0.001	0.015	1.40
17	0.05	0.85	0.15	0.67	0.005	0.026	0.001	0.021	1.76

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18	0.06	1.04	0.19	0.82	0.007	0.028	0.002	0.024	2.17
19	0.06	0.84	0.18	0.66	0.007	0.017	0.002	0.017	1.78
20	0.06	0.66	0.16	0.51	0.007	0.011	0.001	0.013	1.43
21	0.09	0.87	0.21	0.68	0.012	0.014	0.002	0.018	1.89
22	0.10	1.18	0.29	0.93	0.012	0.019	0.002	0.022	2.55
23	0.08	1.17	0.27	0.93	0.009	0.021	0.002	0.021	2.51
24	0.04	0.66	0.15	0.53	0.004	0.014	0.001	0.012	1.42
Total	**1.13**	**16.93**	**3.48**	**13.34**	**0.130**	**0.408**	**0.028**	**0.370**	**35.81**

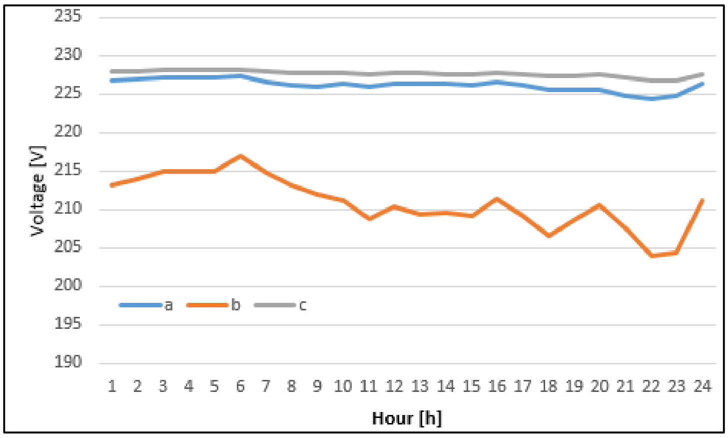

Figure 9. Exemplification of the voltage unbalance at the farthest pillar (P95)

Table 8. The currents in the conductors of the first branch, SP-Pillar 8, calculated with the data obtained using the proposed algorithm

Hour	I_a [A]	I_b [A]	I_c [A]	I_0 [A]	UC
1	27.56	27.50	27.82	0.30	1.0000
2	26.25	26.53	26.37	0.24	1.0000
3	24.88	25.03	24.82	0.19	1.0000
4	25.29	24.88	24.99	0.36	1.0000
5	25.22	25.01	25.21	0.21	1.0000
6	21.47	22.69	21.65	1.14	1.0006
7	24.77	24.68	28.31	3.58	1.0042
8	31.90	26.76	26.93	5.06	1.0070
9	28.83	29.06	33.54	4.59	1.0050
10	30.66	30.78	31.81	1.10	1.0003
11	34.76	34.55	34.53	0.22	1.0000
12	32.61	31.65	31.78	0.91	1.0002
13	33.25	34.50	33.23	1.26	1.0003
14	33.91	33.04	33.29	0.77	1.0001
15	33.49	34.20	33.40	0.76	1.0001
16	30.88	30.23	30.18	0.68	1.0001
17	33.72	34.38	33.77	0.64	1.0001
18	38.43	37.96	37.71	0.63	1.0001
19	37.69	34.07	33.87	3.72	1.0025
20	30.67	30.70	35.48	4.79	1.0049
21	34.87	41.56	35.03	6.61	1.0070
22	40.63	46.86	40.21	6.46	1.0051

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23	39.94	40.25	44.37	4.29	1.0024
24	31.96	30.73	30.71	1.24	1.0004

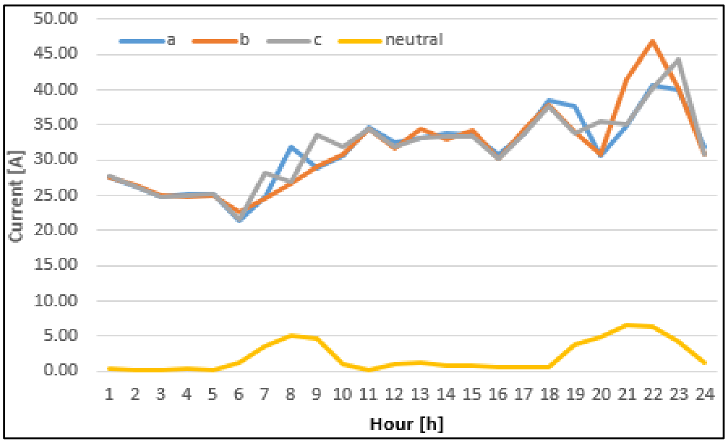

Figure 10. The currents in the conductors of the first branch, SP-Pillar 8, with the proposed algorithm

Table 9. The energy losses calculated with the data obtained using the proposed algorithm, [kWh]

Hour	Main Conductors				Branching Conductors				Total
	a	b	c	Neutral	a	b	c	Neutral	
1	0.12	0.13	0.14	0.01	0.00	0.01	0.01	0.01	0.43
2	0.11	0.13	0.12	0.01	0.00	0.01	0.01	0.01	0.39
3	0.10	0.12	0.10	0.01	0.00	0.01	0.00	0.01	0.35
4	0.12	0.10	0.10	0.01	0.01	0.01	0.00	0.01	0.35
5	0.11	0.11	0.10	0.01	0.01	0.00	0.00	0.01	0.35
6	0.08	0.08	0.08	0.00	0.00	0.00	0.00	0.01	0.26
7	0.11	0.11	0.13	0.01	0.00	0.00	0.01	0.01	0.37
8	0.17	0.12	0.13	0.01	0.01	0.00	0.00	0.01	0.45
9	0.17	0.13	0.18	0.01	0.01	0.00	0.01	0.01	0.52
10	0.15	0.17	0.17	0.01	0.01	0.01	0.01	0.01	0.54
11	0.22	0.20	0.20	0.01	0.01	0.01	0.01	0.02	0.68
12	0.19	0.16	0.17	0.01	0.01	0.01	0.01	0.02	0.59
13	0.17	0.23	0.19	0.02	0.01	0.02	0.01	0.02	0.66
14	0.21	0.19	0.18	0.01	0.01	0.01	0.01	0.02	0.64
15	0.17	0.22	0.19	0.01	0.01	0.01	0.01	0.02	0.65
16	0.17	0.15	0.16	0.01	0.01	0.01	0.01	0.01	0.52
17	0.17	0.22	0.20	0.01	0.01	0.01	0.01	0.02	0.66
18	0.23	0.27	0.24	0.01	0.01	0.01	0.01	0.02	0.82
19	0.22	0.22	0.19	0.01	0.01	0.01	0.01	0.02	0.69
20	0.16	0.18	0.20	0.01	0.00	0.01	0.01	0.01	0.58
21	0.21	0.29	0.22	0.02	0.00	0.02	0.01	0.02	0.78
22	0.28	0.35	0.32	0.02	0.00	0.02	0.01	0.02	1.01
23	0.30	0.28	0.32	0.01	0.01	0.01	0.01	0.02	0.96
24	0.16	0.18	0.16	0.01	0.00	0.01	0.01	0.01	0.54
Total	4.09	4.34	4.18	0.26	0.15	0.20	0.19	0.36	13.76

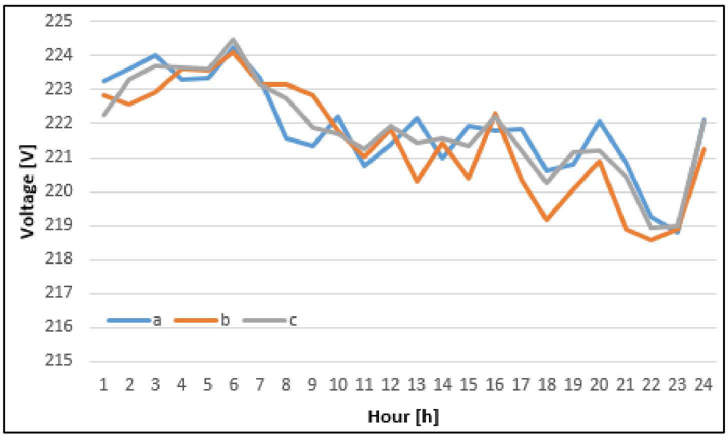

Figure 11. Improvement of voltage quality at the farthest pillar (P95), after applying the proposed algorithm

The results were compared with other algorithms to emphasize the accuracy of the smart meter data-based proposed algorithm (SMD): from heuristic (the minimum count of loads adjustment (MCLA) algorithm [32]) and metaheuristic (Particle Swarm Optimization (PSO) algorithm [28] and Genetic Algorithm (AG) [17]) categories.

Regarding the *UC* coefficient, the obtained value with the proposed algorithm is identical with AG (1.0017) at the SP level, being smaller than in the case of MCLA and PSO, as shown in Figure 12.

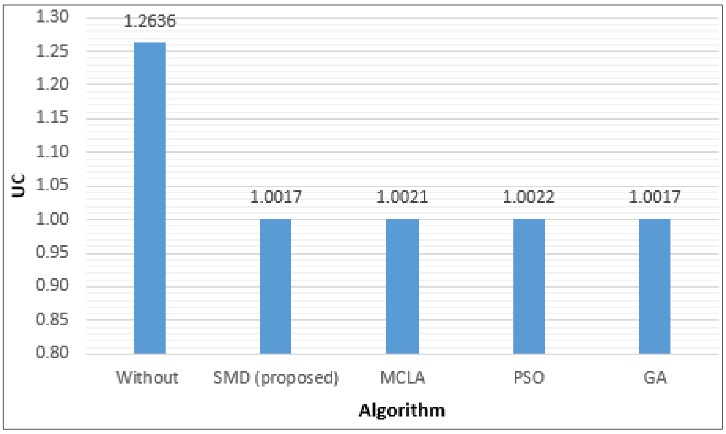

Figure 12. Comparison between the average values of *UC* at the SP level, calculated with different algorithms

This result does not guarantee that effects will be identical on the decrease of the current in the neutral conductor (and implicit on the energy losses) or on improving the voltage quality at the level of each pillar. To highlight these effects, the steady-state regimes were calculated, having as input data the load matrices obtained with each algorithm. The average value of the current in the neutral conductor, on the first branch (SP – Pillar), is shown in Figure 13, for each algorithm. It can be observed that the smallest value was obtained applying the proposed algorithm (2.07 A), with 22.7 % better than GA.

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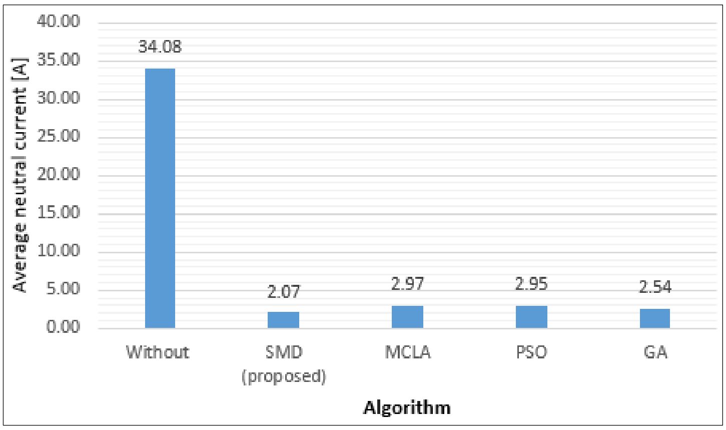

Figure 13. Comparison between the average values of neutral current in the first branch, SP-Pillar 8, calculated with different algorithms

Regarding the energy losses, Table 10 presents the values calculated on the phase and neutral conductors on the branching and main conductors. The analysis of the results indicates smaller energy losses in the case of the proposed algorithm compared to the other algorithms, as shown in Table 10 and Figure 14. The energy losses decreased by 0.20 %, more than in the case of AG. The difference from the MCLA algorithm is higher, with 19.01 %.

The errors of the energy losses (δΔW), given in percent, are indicated in Table 10. The calculation relation is the following:

$$\delta \Delta W = \left| \frac{\Delta W_{without} - \Delta W_{algorithm}}{\Delta W_{without}} \right| \cdot 100, \, [\%] \quad (17)$$

where *algorithm* is SMD, MCLA, PSO, and GA.

Table 10. Comparison between the energy losses calculated with different algorithms, [kWh]

Algorithm	Main Conductors				Branching Conductors				Total	δΔW [%]
	a	b	c	Neutral	a	b	c	Neutral		
Without	1.13	16.93	3.48	13.34	0.13	0.41	0.03	0.37	35.81	-
SMD (proposed)	4.09	4.34	4.18	0.26	0.15	0.20	0.19	0.36	13.76	61.57
MCLA	4.14	6.23	4.98	4.32	0.33	0.05	0.16	0.36	20.57	42.56
PSO	4.44	4.43	3.77	0.32	0.23	0.17	0.15	0.36	13.86	61.30
GA	3.66	4.62	4.50	0.51	0.14	0.19	0.21	0.36	14.19	60.37

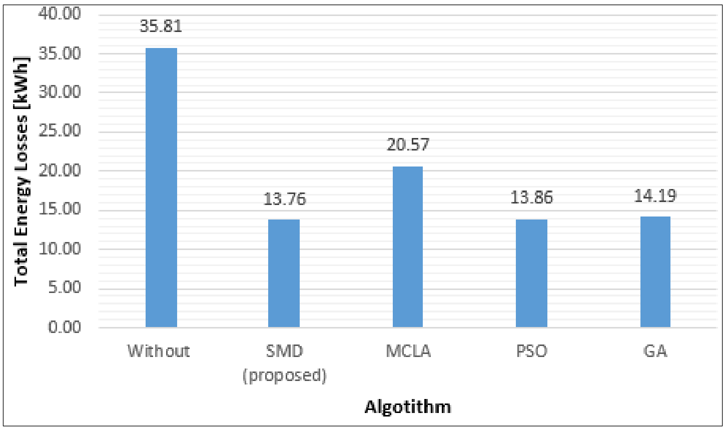

Figure 14. Comparison between the total energy losses, calculated with different algorithms

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The voltage quality was evaluated based on the minimum phase voltage at the level of the farthest pillar (P95). The results are presented in Table 11, where the minimum values are highlighted with bold. An analysis of results highlighted that the phase voltages are inside of admissible limits (rated voltage ± 10 %, where rated voltage is 230 V). The approximately equal phase voltages were obtained in the case of the proposed algorithm, with differences in the range [0.13V, 0.36 V], having an improvement of value on phase b by 14.58 V (7.15 %). The biggest differences, in the range [5.59 V, 12.9 V], were obtained in the case of the MCLA algorithm.

Table 11. The minimum value of the phase voltages at the level of the farthest pillar (P95)

Algorithm	Phase		
	a	b	c
Without	224.33	**204.00**	226.71
SMD (proposed)	218.81	**218.58**	218.94
MCLA	218.90	**211.59**	224.49
PSO	**218.19**	219.03	218.55
GA	219.41	**217.28**	219.07

The detailed results for each algorithm are presented in Table B1, B2, B3, and B4 from Appendix B.

4. Conclusions

At the worldwide level, most DNOs have incorporated automation systems in own EDNs, which use common standards to communicate with the data concentrators from the SPs or/and with the DMCL. In this context, the absorption and integration of the smart technologies which to make an easy transition towards the smart grids can be technically achievable. One of these technologies concerns to the real-time PLB based on smart systems, integrated into the SMS, containing two devices: a smart meter and a PLBD. In this situation, the general structure of the control system must be modified to implement and integrate new algorithms to identify the optimal PLB solutions for each EDN.

In the paper, a PLB algorithm, with the following advantages, was proposed: it can be implemented in the EDNs with hybrid structures of consumption points (switchable and non-switchable consumers); it can work in both operation modes (on-line and off-line), uploading information from different databases of the DNO which contain the consumers' characteristics, real loads of the consumers integrated in the SMS, and loads from the TLPs for the consumers non-integrated in the SMS; the convergence is rapid due to the fast recognition of EDN topology with the help of a structure vectors based-algorithm.

The testing of the algorithm was made in a real rural EDN from the northeastern region of Romania, having a hybrid structure of the consumption points (only 114 1-P consumers (70.8 %) are integrated into the SMS with the possibility to have PLBS, the others consumers having standard meter). The obtained results were analyzed and compared with other algorithms from the heuristic category (Minimum Count of Loads Adjustment (MCLA) algorithm) and the metaheuristic category (Particle Swarm Optimization (PSO) and Genetic Algorithm (AG)).

The best performances were recorded for the proposed algorithm, obtaining the smallest value of the unbalance coefficient (1.0017), in comparison with MCLA (1.0022) and PSO (1.0021) algorithms. The same value (1.0017) was also obtained in the case of AG. But, after performing the steady-state calculations, the obtained results highlighted that technical effects were not identical to the decrease of the current in the neutral conductor (and implicit on the energy losses) or to improve the voltage quality at the level of each pillar. The average value of the current in the neutral conductor decreased with 94% from the average value by 34.08 A (initial case) at 2.07 A. This value is smaller with 22.70 % than AG, 42.51 % than PSO, and 43.47 % than MCLA. The similar results were obtained

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for the energy losses and phase voltages in the farthest pillar (P95). The energy losses decreased with 61.75 % compared to the initial case using the data obtained with the proposed algorithm. This value is smaller with 0.20 % than AG, 0.27 % than PSO, and 19.01 % than MCLA. The minimum value of the phase voltage at the farthest pillar (P95) was improved with 7.14 % (14.48 V) compared to the case. This value is higher with 0.45 % than AG, 0.17 % than PSO, and 3.19 % than MCLA.

Currently, the technical solution is developed by the producers to be on-line implemented by the DNO in the EDNs. In the on-line operation mode, the control system installed in the data concentrator sends the command to PLBD as soon as the optimum phase for each switchable consumer has been determined. The solution can be introduced by the DNOs to ensure the transition toward the smart grids, but only on the base of a feasibility analysis which to justify the investment. This analysis follows the costs in each of the next stages: the identification of feasible technology, the planning of assembly at consumers, the commissioning of the system, integration in the SMS, testing the communication with data concentrator from the supply point, and not least the maintenance plan. Of course, the steps of the transition process should be implemented based on analyses of the DNOs in the "hot" areas where there are EDNs with high values of unbalance degree and loads.

The authors work now at an improved variant of the proposed algorithm which considers the weight of each switchable consumer at the unbalance degree so that to find the optimal number of PLBD which minimizes the unbalance coefficient and the investment costs.

Author Contributions: G.G. proposed the implementation methodology, mathematical modeling, validation and writing—original draft preparation B.C.N. has implemented the software, data curation, and validation; C.B. and I.T. improved the methodology, performed simulations, and writing; M.G. performed simulations, and reviewed the manuscript. All authors discussed the results and have agreed with the structure of the paper.

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Conflicts of Interest: The authors declare no conflict of

Nomenclature:

0	Neutral conductor
1-P	Single-phase consumer
3-P	Three-phase consumer
EDN	Electric distribution network
LV	Low voltage
TLP	Typical load profile
DNO	Distribution Network Operator
SMS	Smart Metering System
SMD	Smart Meter Data
PLB	Phase load balancing
PLC	Power-line communication
SCADA	Supervisory Control and Data Acquisition
APLBD	Automatic phase load balancing device
DMCL	Decision-Making Central Level
PSO	Particle Swarm Optimization
AG	Genetic Algorithm
MCLA	Minimum Count of Loads Adjustment
H	The analysed time period, [hours]
B_i	Vector of the input nodes of branches
B_j	Vector of the end nodes of branches
a, b, c	The phases of the EDN
abc	3-P consumer in the input data files
{ph}	The set of phases {a, b, c}
TV1	Topology vector containing the number of branches from each vicinity level

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TV2	Topology vector containing the branches placed in the order of the vicinity levels
SP	Supply Point
N_C	The total number of consumers from the EDN
CP	Vector of the connected pillars, size ($N_C \times 1$)
PB	Vector of the branching phase, size ($N_C \times 1$)
CS	Vector of the consumption sector of the consumers, size ($N_C \times 1$)
CC	Vector of the consumption class of the consumers from a certain consumption sector, size ($N_C \times 1$)
INT	Vector of the integration mode in the SMS, size ($N_C \times 1$)
BS	Vector of the PLBD status, size ($N_C \times 1$)
IC	Vector of the hourly loads for all consumers, size ($N_C \times H$)
SN	Vector of the serial numbers corresponding the smart meters, size ($N_C \times 1$)
r_0	Specific resistance, [Ω/km]
x_0	Specific reactance, [Ω/km]
UC	The unbalance coefficient
I_a, I_b, I_c	The currents on the phases a, b, and c
$I_{average}$	The average value of the phase currents
h	The current hour (h = 1,…, H)
N_p	The number of pillars from the EDN
p	The analysed current pillar (p = 1, …, N_p)
d	Pillar located downstream by pillar p
$UC^{(p),h}$	The unbalance coefficient calculated at the pillar p and hour h
index	Vector of the indices corresponding to pillar p in vector CP
$I_a^{(p),h}$	The current on the phase a, at the pillar p and hour h, [A]
$I_b^{(p),h}$	The current on the phase b, at the pillar p and hour h, [A]
$I_c^{(p),h}$	The current on the phase c, at the pillar p and hour h, [A]
$I_{a,ns}^{(p),h}$	The total current of the non-switchable consumers on the phase a, pillar p and hour h, [A]
$I_{b,ns}^{(p),h}$	The total current of the non-switchable consumers on the phase b, pillar p and hour h, [A]
$I_{c,ns}^{(p),h}$	The total current of the non-switchable consumers on the phase c, pillar p and hour h, [A]
$I_{a,s}^{(p),h}$	The total current of the switchable consumers on the phase a, pillar p and hour h, [A]
$I_{b,s}^{(p),h}$	The total current of the switchable consumers on the phase b, pillar p and hour h, [A]
$I_{c,s}^{(p),h}$	The total current of the switchable consumers on the phase c, pillar p and hour h, [A]
$I_a^{(d),h}$	The currents on the phase a, pillar d, and hour h, [A]
$I_{b,s}^{(d),h}$	The currents on the phase b, pillar d, and hour h, [A]
$I_{c,s}^{(d),h}$	The currents on the phase c, pillar d, and hour h, [A]
j	Index of the non-switchable consumer connected on the phase a, pillar p, and hour h
k	Index of the non-switchable consumer connected on the phase b, pillar p, and hour h
l	Index of the non-switchable consumer connected on the phase c, pillar p, and hour h
m	Index of the switchable consumer connected on the phase a, pillar p, and hour h
n	Index of the switchable consumer connected on the phase b, pillar p, and hour h
o	Index of the switchable consumer connected on the phase c, pillar p, and hour h
$N_{a,ns}^{(p),h}$	The number of the non-switchable consumers connected on the phase a, pillar p, and hour h
$N_{b,ns}^{(p),h}$	The number of the non-switchable consumers connected on the phase b, pillar p, and hour h
$N_{c,ns}^{(p),h}$	The number of the non-switchable consumers connected on the phase c, pillar p, and hour h
$N_{a,s}^{(p),h}$	The number of the switchable consumers connected on the phase a, pillar p, and hour h
$N_{b,s}^{(p),h}$	The number of the switchable consumers connected on the phase b, pillar p, and hour h
$N_{c,s}^{(p),h}$	The number of the switchable consumers connected on the phases c, pillar p, and hour h
$N_{C,ns}^{(p),h}$	The total number of the non-switchable consumers connected at the pillar p, and hour h
$N_{C,s}^{(p),h}$	The total number of the switchable consumers connected at the pillar p, and hour h

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$N_C^{(p),h}$	The total number of the consumers connected at the pillar p, and hour h
$I_{a,ns,j}^{(p),h}$	The current of the non-switchable consumer j (j = 1,…, $N_{a,ns}^{(p),h}$), [A]
$I_{b,ns,k}^{(p),h}$	The current of the non-switchable consumer k (k = 1, …, $N_{b,ns}^{(p),h}$), [A]
$I_{c,ns,l}^{(p),h}$	The current of the non-switchable consumer l (l = 1, …, $N_{c,ns}^{(p),h}$), [A]
$I_{a,s,m}^{(p),h}$	The current of the switchable consumer m (m = 1, …, $N_{a,s}^{(p),h}$), [A]
$I_{a,s,n}^{(p),h}$	The current of the switchable consumer n (n = 1, …, $N_{b,s}^{(p),h}$), [A]
$I_{a,s,o}^{(p),h}$	The current of the switchable consumer o (o = 1, …, $N_{c,s}^{(p),h}$), [A]
$\delta\Delta W$	The percentage error, [%]

Appendix A

Table A1. The allocation on pillar, phase, and the consumption sector

Pillar	Consumer' type		Branching Phase			Consumption Sector			Pillar	Consumer' type		Branching Phase			Consumption Sector		
	1-P	3-P	a	b	c	1	2	3		1-P	3-P	a	b	c	1	2	3
8	2	-	-	2	-	1	-	-	51	2	-	-	1	1	1	-	-
9	2	-	-	2	-	1	-	-	52	3	-	-	3	-	1	-	-
10	3	-	2	1	-	1	-	-	53	1	-	-	1	-	-	2	-
11	1	-	-	1	-	1	-	-	54	6	-	-	-	6	1	-	-
12	2	-	-	2	-	1	-	-	55	2	-	1	1	-	1	-	-
13	1	-	-	1	-	1	-	-	56	2	-	-	2	-	1	-	-
14	2	-	-	-	2	1	-	-	57	1	-	-	1	-	1	-	-
15	2	-	-	1	1	1	-	-	58	1	-	1	-	-	1	-	-
17	1	1	1	1	1	1	-	-	59	2	-	-	2	-	1	-	-
18	2	-	-	-	2	1	-	-	60	2	-	1	1	-	1	-	-
19	2	-	2	-	-	1	-	-	61	1	-	-	1	-	1	-	-
20	2	-	2	-	-	1	-	-	62	1	-	-	-	1	1	-	-
21	1	-	1	-	-	1	-	-	63	2	-	2	-	-	1	-	-
22	2	-	1	1	-	1	-	-	65	1	-	-	1	-	1	-	-
23	2	-	2	-	-	1	-	-	66	4	-	1	3	-	1	-	-
24	1	-	-	-	1	1	-	-	67	2	-	-	2	-	1	-	-
26	2	-	-	-	2	1	-	-	68	2	-	-	2	-	1	-	-
27	3	-	1	-	2	1	-	-	69	2	-	1	1	-	1	-	-
28	2	-	-	1	1	1	-	-	70	1	-	-	1	-	1	-	-
29	4	-	-	1	3	1	-	-	71	1	-	-	1	-	1	-	-
30	2	-	-	-	2	1	-	-	72	1	-	-	1	-	1	-	-
31	2	-	-	-	2	1	-	-	75	2	-	-	2	-	1	-	-
32	1	-	-	-	1	1	-	-	76	2	-	-	2	-	1	-	-
33	4	-	-	-	4	1	-	-	77	2	-	1	1	-	1	-	-
34	5	-	-	-	5	1	-	-	78	4	-	1	3	-	1	-	-
35	4	-	1	1	2	1	-	-	79	1	1	1	2	1	1	-	-
36	1	-	-	1	-	1	-	-	80	2	-		2	-	1	-	-
37	3	-	-	-	3	1	-	-	82	2	-	-	2	-	1	-	-
38	1	-	-	-	1	1	-	-	83	1	-	1	-	-	1	-	-
39	4	-	-	1	3	1	-	-	84	2	-	-	2	-	1	-	-
40	3	-	-	-	3	1	-	-	86	1	-	-	1	-	1	-	-
41	1	-	-	-	1	1	-	-	87	2	-	-	2	-	1	-	-
42	1	-	-	-	1	1	-	-	88	1	-	-	1	-	1	-	-

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43	2	-	-	-	2	1	-	-	89	2	-	-	2	-	1	-	-
44	2	-	-	1	1	1	-	-	90	1	-	-	1	-	1	-	-
45	4	-	-	-	4	1	-	-	91	2	-	-	2	-	1	-	-
46	2	-	-	-	2	1	-	-	92	1	-	-	1	-	1	-	-
47	3	-	1	2	-	1	-	-	93	2	-	-	2	-	1	-	-
48	3	-	1	2	-	1	2	-	94	1	-	1	-	-	1	-	-
49	2	-	-	2	-	1	-	-	95	1	-	-	1	-	1	-	-
50	1	-	-	-	1	1	-	-									

Appendix B

Table B1. Comparison between the hourly UC calculated with different algorithms at the SP level

Hour	Without	SMD (proposed)	MCLA	PSO	GA
1	1.2949	1.0000	1.0001	1.0017	1.0010
2	1.2965	1.0000	1.0005	1.0023	1.0009
3	1.2923	1.0000	1.0007	1.0024	1.0007
4	1.3016	1.0000	1.0012	1.0026	1.0011
5	1.2837	1.0000	1.0010	1.0029	1.0007
6	1.2265	1.0006	1.0005	1.0023	1.0003
7	1.1840	1.0042	1.0017	1.0010	1.0027
8	1.1700	1.0070	1.0042	1.0021	1.0046
9	1.2036	1.0050	1.0040	1.0004	1.0017
10	1.2630	1.0003	1.0022	1.0007	1.0000
11	1.3041	1.0000	1.0039	1.0018	1.0007
12	1.3339	1.0002	1.0031	1.0029	1.0019
13	1.3485	1.0003	1.0026	1.0040	1.0028
14	1.3209	1.0001	1.0028	1.0028	1.0016
15	1.3313	1.0001	1.0027	1.0031	1.0023
16	1.3078	1.0001	1.0012	1.0030	1.0013
17	1.3198	1.0001	1.0025	1.0030	1.0021
18	1.2881	1.0001	1.0018	1.0010	1.0006
19	1.2344	1.0025	1.0011	1.0001	1.0003
20	1.1843	1.0049	1.0029	1.0025	1.0032
21	1.1691	1.0070	1.0040	1.0053	1.0058
22	1.1867	1.0051	1.0031	1.0028	1.0032
23	1.2241	1.0024	1.0021	1.0007	1.0008
24	1.2562	1.0004	1.0005	1.0008	1.0001

Table B2. Comparison between the hourly neutral currents calculated with different algorithms, the first branch (SP-Pillar 8)

Hour	Without	SMD (proposed)	MCLA	PSO	GA
1	31.84	0.30	0.56	2.42	1.87
2	30.49	0.24	1.23	2.68	1.72
3	28.58	0.19	1.40	2.60	1.42

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4	29.20	0.36	1.81	2.71	1.74
5	28.43	0.21	1.67	2.85	1.39
6	22.15	1.14	1.06	2.21	0.87
7	23.59	3.58	2.27	1.75	2.85
8	24.97	5.06	3.90	2.79	4.10
9	29.18	4.59	4.07	1.34	2.66
10	33.83	1.10	3.11	1.70	0.30
11	40.52	0.22	4.57	3.07	1.98
12	39.28	0.91	3.78	3.67	2.92
13	42.20	1.26	3.67	4.49	3.80
14	40.18	0.77	3.73	3.76	2.85
15	41.18	0.76	3.68	3.98	3.39
16	35.84	0.68	2.19	3.52	2.34
17	40.77	0.64	3.59	3.96	3.33
18	43.34	0.63	3.39	2.58	1.89
19	36.19	3.72	2.49	0.74	1.37
20	29.41	4.79	3.68	3.39	3.90
21	32.43	6.61	4.97	5.71	6.03
22	39.04	6.46	5.02	4.75	5.12
23	41.72	4.29	3.99	2.30	2.44
24	33.45	1.24	1.53	1.90	0.79

Table B3. Comparison between the hourly power losses calculated with different algorithms, [kWh]

Hour	**SMD (proposed)**			**MCLA**			**PSO**			**GA**		
	a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
1	0.40	0.03	0.43	0.60	0.03	0.63	0.40	0.03	0.43	0.43	1.68	2.11
2	0.36	0.03	0.39	0.54	0.03	0.57	0.37	0.03	0.39	0.39	1.52	1.91
3	0.32	0.02	0.35	0.48	0.02	0.50	0.33	0.02	0.35	0.35	1.35	1.70
4	0.33	0.02	0.35	0.48	0.02	0.50	0.33	0.02	0.36	0.35	1.36	1.71
5	0.33	0.02	0.35	0.49	0.02	0.51	0.33	0.02	0.36	0.35	1.38	1.73
6	0.25	0.01	0.26	0.40	0.01	0.41	0.25	0.01	0.26	0.26	1.08	1.35
7	0.35	0.02	0.37	0.55	0.02	0.57	0.35	0.02	0.37	0.37	1.51	1.88
8	0.43	0.03	0.45	0.67	0.03	0.70	0.42	0.03	0.45	0.45	1.85	2.30
9	0.48	0.03	0.52	0.74	0.03	0.77	0.48	0.03	0.51	0.52	2.06	2.58
10	0.50	0.04	0.54	0.75	0.04	0.78	0.51	0.04	0.54	0.54	2.11	2.64
11	0.63	0.05	0.68	0.91	0.05	0.96	0.64	0.05	0.69	0.68	2.59	3.27
12	0.54	0.05	0.59	0.83	0.05	0.88	0.55	0.05	0.60	0.59	2.35	2.94
13	0.60	0.06	0.66	0.92	0.06	0.97	0.61	0.06	0.67	0.66	2.60	3.25
14	0.59	0.05	0.64	0.84	0.05	0.89	0.60	0.05	0.64	0.64	2.42	3.05
15	0.60	0.05	0.65	0.86	0.05	0.91	0.61	0.05	0.66	0.65	2.46	3.11
16	0.49	0.04	0.52	0.71	0.04	0.75	0.49	0.04	0.53	0.52	2.02	2.55
17	0.61	0.05	0.66	0.87	0.05	0.92	0.62	0.05	0.67	0.66	2.50	3.16
18	0.76	0.06	0.82	1.12	0.06	1.18	0.77	0.06	0.82	0.82	3.18	3.99
19	0.65	0.04	0.69	1.01	0.04	1.05	0.65	0.04	0.69	0.69	2.79	3.48
20	0.55	0.03	0.58	0.89	0.03	0.92	0.55	0.03	0.58	0.58	2.42	3.00
21	0.73	0.04	0.78	1.15	0.04	1.19	0.73	0.04	0.77	0.78	3.16	3.94
22	0.96	0.05	1.01	1.58	0.05	1.64	0.96	0.05	1.01	1.01	4.29	5.31

23	0.91	0.05	0.96	1.48	0.05	1.53	0.91	0.05	0.96	0.96	4.02	4.98
24	0.51	0.03	0.54	0.80	0.03	0.83	0.51	0.03	0.54	0.54	2.20	2.73

Table B4. Comparison between the hourly phase voltages calculated with different algorithms, at the level of the farthest pillar P95, [V]

Hour	**SMD (proposed)**			**MCLA**			**PSO**			**GA**		
	a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
1	223.25	222.85	222.25	223.28	219.05	225.96	222.50	222.46	223.38	223.81	221.64	222.90
2	223.62	222.55	223.27	223.55	219.75	226.08	222.87	222.78	223.78	224.12	222.13	223.18
3	224.01	222.94	223.69	223.91	220.40	226.29	223.28	223.20	224.17	224.45	222.65	223.54
4	223.29	223.60	223.67	223.79	220.47	226.25	223.20	223.17	224.18	224.47	222.60	223.47
5	223.33	223.55	223.59	223.82	220.35	226.26	223.30	223.07	224.10	224.35	222.57	223.55
6	224.24	224.12	224.45	224.80	221.03	226.94	224.38	223.89	224.55	224.75	223.43	224.63
7	223.33	223.16	223.14	223.26	219.58	226.72	223.09	223.16	223.38	223.64	222.57	223.42
8	221.59	223.17	222.75	222.20	218.58	226.65	222.23	222.67	222.60	222.95	221.91	222.65
9	221.36	222.83	221.88	221.84	218.08	226.07	221.69	222.03	222.35	222.69	221.29	222.08
10	222.22	221.81	221.69	221.95	218.13	225.57	221.54	221.62	222.55	222.90	220.91	221.90
11	220.73	221.02	221.25	220.94	217.16	224.83	220.43	220.62	221.95	222.37	219.82	220.81
12	221.37	221.86	221.91	223.84	216.11	225.09	221.04	221.36	222.74	223.15	220.59	221.39
13	222.15	220.28	221.45	223.49	215.53	224.75	220.57	220.86	222.45	222.89	220.05	220.93
14	220.97	221.41	221.57	221.44	217.53	224.91	220.80	220.86	222.29	222.72	220.05	221.18
15	221.95	220.37	221.35	221.37	217.39	224.85	220.71	220.76	222.20	222.70	219.83	221.14
16	221.78	222.27	222.22	222.47	218.35	225.39	221.77	221.59	222.89	223.26	220.89	222.11
17	221.85	220.34	221.23	221.31	217.20	224.85	220.73	220.63	222.06	222.56	219.66	221.19
18	220.60	219.18	220.24	220.17	215.27	**224.49**	219.58	219.62	220.82	221.41	218.47	220.14
19	220.78	220.07	221.17	220.96	215.62	225.34	220.40	220.60	221.03	221.63	219.37	221.01
20	222.07	220.89	221.23	221.49	216.34	226.24	221.15	221.66	221.38	221.99	220.40	221.80
21	220.83	218.92	220.42	221.29	214.22	224.53	219.60	220.66	219.91	220.66	219.12	220.39
22	219.25	**218.58**	**218.94**	**218.90**	**211.59**	225.06	**218.19**	**219.03**	**218.55**	**219.41**	**217.28**	**219.07**
23	**218.81**	218.91	218.98	219.75	212.05	224.71	218.55	219.04	219.11	219.98	217.33	219.38
24	222.11	221.24	222.09	222.58	217.10	225.67	221.60	221.57	222.27	222.74	220.64	222.06

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