

Article

Polytopic Aspects of the Mind Representation System

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Abstract: Using classical clinical observations, we first outline an elementary conceptual model for the Mind Representation System, then move to a more elaborate mathematical model that refers to discrete cofibration with enriched fibers.

Keywords: Involuntary memories, causal logic and semiological logic; unconscious; mathematical model of the mind-matter relation; idioptope; category; discrete cofibration.

1. Introduction

For about fifty years now, the question of a scientific, functionally representable and conceptualizable approach to consciousness has been studied based on neural activity which is considered as the quantifiable root of all mental processes. This approach was first largely developed through the mathematisation of visual perception. Therefore this kind of mathematical modelling may be considered as appended to the very emergence of the phenomenon of consciousness.

With larger scale psychic phenomena we usually use the term psychology. Although this level has brought many rich conceptual elaborations, it seems to escape any regular kind of mathematisation. And the very idea of such a mathematisation may even seem outrageous to some. The present article does not claim to achieve mathematical modelisation, but to offer a few paths forward. It shows that, even at an unusually large scale of human psychic behaviour, it is possible to delineate certain working models, if not exactly of consciousness, of something which we could call "mind representation system".

Our model is developed in two steps. The first, based on clinical observations, emphasizes the polytopic nature of the mind representation system and describes its operating mode, clearly discriminating daytime and night-time functioning, normal and crisis status. The second step is a mathematical development based on category theory. It is intended to be a means to grasp the conceptual mastery of the local/global relationship between the various zones of this polytopic structure as we have defined it.

2. The starting point

There are famous examples of evidence of psychic conglomerates, or psychic units, which suddenly emerge from the past wholly formed; they seem to have been present in the mind, inaccessible until a triggering incident brings them to the fore. Such situations are generally classified as involuntary memories.

A very famous example is found in the French literature, in Marcel Proust's novel *Du côté de chez Swann* (*Swann's way*): as the narrator savours a *madeleine* cake soaked in tea, he is suddenly and unexpectedly carried back into his childhood, when his aunt gave him tea and cake, and is flooded with very precise sensations and memories of that time.

In the English literature, a prominent example is *Henry Hoare's torn finger-nail*, from Samuel Butler's *Notebooks* [1]. Hoare was at Cambridge with Butler and became a close friend; it is perhaps relevant to the narrative that he was a member of a well-known banking family. The story of this torn nail is an even more interesting example, since this memory is the occasion of a retrospective re-construction. As a young man of about twenty-five, Hoare tore his finger-nail, and this reminded him that, as a child of about seven, the same thing had happened when he was idly stuffing a piece of paper into a hole of the wooden frame of his sickbed, in his aunt's house in Hertfordshire. Recollecting the circumstances of this action, he remembered that, at the same time, there was a great disturbance in this house about a missing five pound note. *Then, there flashed upon him an idea with irresistible force ... the piece of paper he had stuffed into the hole of the bedstead was the missing five-pound note ... At that time he was so young that a five-pound note was to him only a piece of paper; when he heard that five pounds were missing he had thought it was five sovereigns.* The pitch of this story is that, with his memory reactivated, he decided to return to the aunt's house and, there, with the help of a bed-key, he retrieved the piece of paper, which was revealed to be a five-pound note. It is all the more interesting that, at about the same time, Sigmund Freud was conceptualizing this type of retrospective re-elaboration under the name of *deferred action*¹ [2]: namely the emergence of a new meaning in a subsequent context enriched with further information. But he was not able to produce such an undisputable occurrence.

Proust's and Butler's episodes seem particularly simple situations where the memories in question were awoken by the reproduction of similar bodily sensations: taste of a piece of cake in the first case, pain of a torn nail in the second.

3. The idiotopic assumption

From these observations, let us introduce the following definition and assumptions.

3.1. A basic model of the Mind Representation System

Idiotope: autonomous preconscious psychic agency, constituted around memorial, experiential and axiological loads that have a degree of coherence and around certain representations that are attached to them and have a certain homogeneity. Of course, from one idiotope to another, there may be significant inconsistencies. We shall observe later on that certain idiolectal recurrences are often attached to the different idiotopes; this is what usually enables us to isolate them.

Examples: in Hoare's case, we shall use *SV* (seven years) to represent the psychic agency concerned by his stay in his aunt's house in the past the details of which come back to his mind with the torn nail, as specified in Butler's notes; we shall use *TWF* to represent the agency which was active when, at the age of twenty five, Hoare once again tore his finger-nail. It is clear from the context that the content of the idiotope *SV* was absolutely outside the idiotope *TWF* before the nail incident. It was a kind of sleeping agency which was woken up concomitantly with *TWF* by the nail incident. It would certainly have remained inaccessible without it.

Basic tenet: in any individual several idiotopes exist all the time; but, during the day, at a given moment only one idiotope is enabled: this is what allows a coherent perception of external reality and can give rise to right action. If the situation changes, another idiotope may be called upon.

¹ *Nachträglichkeit* in german; *après-coup* in french.

Of course, during sleep, the rule of uni-activation has no use, the link with the real world having been broken; thus many idiotopes can be safely activated during sleep². This would explain the chaotic feeling when we emerge from our psychic activity of dreaming: chaotic, but not dangerous since external reality was not there.

We may, of course, wonder about the insensible character of any change in idiotopic activation during the daytime; it could be compared to the lack of attention paid to the action of changing gears while driving. One of the interests of this basic tenet is evidently to give a handle on the high degree of plasticity of the mind's activity in its normal daytime functions.

In any individual, one could suppose the existence of at least two very large idiotopes: *Home* which is enabled when this individual is in the place of his highest safety and *Outh* (for Out-of-home) when he is out of this place. According to their definitions, these two idiotopes must also always be supposed in a situation of mutual exclusion. In an ordinary life with an ordinary home, the passage from *Home* to *Outh* is performed by simply crossing the doorstep.

Crisis situation: it is characterized by the fact that external circumstances accidentally require the concurrent activation of two or more idiotopes. The crisis is short and opportune when this multi-activation leads to a decision or to a psychic elaboration which terminates the multi-activation: eating the *madeleine* certainly contributed to plan the creation of what became the monument of literature that is *In Search of Lost Time*³; a torn nail led to a quick re-evaluation of a certain memory and to the decision to check it. If this multi-activation persists, it then blocks any possibility of firmly positioning oneself in front of external reality and can be, as we shall see herebelow, the origin of psychic suffering.

3.2. Causal logic vs. semiotical logic

Causal logic is totally indexed on the notion of *presence*: the presence of a given item, or entity, implies the presence of another such item. There would also be, dare we say, a *semiotical logic*, which is clearly from another angle, but no less important for the human mind. As for semiotical logic, it is ruled by certain specific modes of relationship with the notion of *absence*. We have long known this, for instance, from linguistics: the paradigmatic dimension of the language makes the meaning of a word in a given sentence mainly determined by the exclusion of the other possible words which could have taken its place. This is especially clear from the comparison between the linguistic chromatic tables in English and in Welsh as described by L. Hjelmslev in [3], see also [4]:

English	Welsh
green	gwyrd
blue	glas
gray	llwyd
brown	

² Clearly this remark is valid during hypnosis sessions as well.

³ *Du côté de chez Swann* is the first volume of *In Search of Lost Time*; there are seven, published from 1913 to 1927. The episode of the *madeleine* is so important that it reappears explicitly in the last volume *Time Regained*.

But *absence*, with its unlimited nature, is at once non-faceted and multi-faceted; in a way it is rationally intractable⁴. This is probably what gives this "semiotical logic" its (apparently disordered) way of development by means of associations. So the major interest of an idiotopic distribution for the Mind Representation System would lie, by restraining the field of its contextual awareness, in limiting the pressure of the semiotical logic for the benefit of the causal one. It is this benefit which produces the internal coherence inside each idiotope as well as its specific power of investigation and action in the *presence* of particular aspects of the real world.

In Hoare's case, we said that the multi-activation of the idiotopes *TWF* and *SV* was certainly based upon the association of the similar painful circumstances; but it is not so simple because we do not know why this association was effectively performed, in other words because, at the semiotico-associative level, we have no possibility of knowing what exactly this association was excluding. On the contrary, once the multi-activation is done, we clearly witness the causal logic in action.

4. Lucy's cure

Historically, it is well known that advances in knowledge of human anatomy and physiology came from the study of its disorders (i.e. illnesses). We are in a similar situation concerning the human mind. So it will not be so surprising that the narratives of some psychic disorders can help us to exemplify our ideas about idiotopes.

We found interesting data for that in three of Freud's reports from *Studies on Hysteria* [5], published in German in 1895, when psycho-analytic therapy was in its early stages and still called the *talking cure*. We developed them in idiotopic terms in [6]. It is noticeable that, at these early stages, all these cures and their resolutions only dealt with recent events; they did not refer to any event going back far in time and thus did not refer to any later psychoanalytic theoretical tool (such as the Oedipus complex, child sexuality, etc.). One of these reports which we shall briefly summarize here concerns *Miss Lucy R., age 30*.

The very beginning of our interest for this case emerged from Freud's observation about the last moments of the treatment during the ultimate session:

... when, two days later, Miss Lucy visited me once more, I could not help asking her what had happened to make her so happy. She was as though transfigured. She was smiling and carried her head high.

Lucy came to Freud *suffering from depression and fatigue*, so this undisputable *transfiguration* testifies that something irresistible occurred during the treatment. In physical manifestation alone, this something is not immediately and clearly understandable. Better, it could be considered as the expression of a silent, ante-linguistic message, as was the *stab at the heart* which flagged the beginning of Lucy's psychic disorder as we shall see later on.

Miss Lucy was a young English lady who came to Austria some years before as a governess in the house of the managing director of a factory in Outer Vienna to take care of his two daughters. Their mother, a distant relation⁵ of Lucy's own mother, died of an acute illness shortly after Lucy's hiring.

After many years of quietly living in the house, Lucy began to suffer from depression and fatigue and to be tormented by subjective sensations of smell (burnt pudding) which she found most distressing; she was finally referred to Freud at the end of 1892. The treatment lasted a total of nine weeks and was

⁴ *What is not there cannot be counted*, Ecclesiastes 1:15, International Standard Version.

⁵ The *eine entfernte Verwandte* of Freud seems more precise than the *distant relation* of Strachey.

successful despite one lengthy interruption as well as the fact that the sessions were very short because of her duties and the long journey to Vienna.

Here is how Freud describes the penultimate session: *When I enquired once more about the smell of burnt pudding, she informed me that it had quite disappeared but that she was being bothered by another, similar smell, resembling cigar-smoke... But this time, she didn't know where the subjective olfactory sensation came from-on what important occasion it had been an objective one. "People smoke everyday in our house" However, at Freud's insistence, a picture gradually emerged: the dining room, all sitting around the table: the Director, the servants, the children and herself. Ah, there was a guest after all, the chief accountant, an old man who is as fond of the children as though they were his own grandchildren... "We're getting up from the table; the children say their good-byes, and they go upstairs with us as usual to the second floor." - "And then?" ... "It is a special occasion after all. I recognized the scene now. As the children say good-bye, the accountant tries to kiss them. My employer flares up and actually shouts at him: "Don't kiss the children!" I feel a stab at my heart⁶; and as the gentlemen are already smoking, the cigar-smoke sticks in my memory."*

This scene happened two months before the one with the burnt pudding.

"Why this stab? The Director's reprimand wasn't aimed at you".

"It wasn't right of him to shout at an old man who was a valued friend of his and, what's more, a guest. He could have said it quietly."

Then a memory of another scene emerges, something that happened a few months earlier: *a lady who was an acquaintance of her employer's came to visit them, and on her departure kissed the two children on the mouth. Their father, who was present, managed to restrain himself from saying anything to the lady, but after she had gone his fury burst upon the head of the unlucky [Lucy]. He said that he held her responsible if any one kissed the children on the mouth... that she was guilty of a dereliction of duty if she allowed it; if it ever happened again, he would entrust his children's upbringing to other hands. This had happened at a time when she still thought he loved her⁷, and was expecting a repetition of [a] first friendly talk. This scene had crushed her hope. [Behaving like this], the Director can never have had any warm feelings for [her], or they would have taught him to treat [her] with more consideration.*

And now back again to the ultimate session: *...when, two days later, Miss Lucy visited me once more, I could not help asking her what had happened to make her so happy. She was as though transfigured, smiling and carrying her head high. I thought for a moment that after all ... the children's governess had become the Director's fiancée. She dispelled my notion. "Nothing has happened. It's just that you don't know me. You have only seen me ill and depressed. I'm always cheerful as a rule. When I woke up yesterday morning the weight was no longer on my mind and since then I have felt well."* Four months later, Freud met her by chance in a summer resort, she was in good spirits and assured him that her recovery had been maintained.

Freud did not give any other explanation of what he noticed from this ultimate session than that of the effect of the recalling of the initial trauma (the unfair reprimands of the Director after the lady's visit) which would fully justify the vanishing of the symptoms.

5. An idiotopic conglomerate underlying the case

The explanation appears to fall short: one would think that the memory of the lady's visit should have emerged right at the beginning of the cure, when Lucy had accepted Freud's interpretation of her love

⁶ Emphasis added.

⁷ At the beginning of the cure, from the first elements of the anamnesis, Freud stated that she was repressing the idea of becoming the Director's wife. Lucy totally agreed with this interpretation. But, to his great disappointment, it had absolutely no effect on the symptoms.

expectations (see footnote 7) and was questioned about its end. But she only found a poor rationalisation: the vain waiting for a second *friendly talk*. Why could this memory not appear at the right moment, why did it take such a long time, exactly nine weeks? Secondly, what specifically happened in this penultimate session to produce the spectacular final result that was Lucy's relief?

Let us compare the two reminiscences of this session:

- a) a lady visiting the family kisses the children on the mouth; in front of her, the Director restrains himself, but after her departure severely reprimands Lucy and threatens to dismiss her if this happens again;
- b) a regular guest of the house, the chief accountant, an old man who is as fond of the children as though they were his own grandchildren tries to kiss them; the Director shouts at him.

The parallelism between the two scenes is clear. What is most striking for Lucy? Among all her various ways of thinking of this man: accountant, guest, "grandfather", valued friend, she ultimately emphasized the guest: "*What's more a guest*". So, despite their similar position as guests, the lady and the accountant were meaningfully treated differently. If the Director did not restrain himself with the man, we can only conclude that:

- c) for him, this man is neither a guest, nor a friend, nor a kind of a grandfather for the children, but a mere employee;
- d) Lucy, having been treated in the same way, is for the Director a mere employee as well.

For us, this is the revelation which Lucy, more or less consciously, has access to at the end of the cure. After remembering the two scenes she discovers that, notwithstanding the family relationship with her mother and the many years of faithful service in the house, the Director does not see her as more than a mere employee. By this disclosure, Lucy is freed from an illusion: having a *special position* in the house⁸. And this release allows her to regain her cheerful mood and her independence of judgement⁹.

So, we answered our second question: what specifically happened in the penultimate session is the disclosure we just described. As for the first question, if we assume the idea of the disclosure, the obstacle is certainly hidden somewhere in Lucy's illusion of having a special position in the house; an illusion which would explain her amazing deafness of the threat to her employment associated with the lady's visit. So we must look for the reasons behind the illusion in the time before the lady's visit. In fact, we already mentioned one reason: Lucy's mother and the Director's wife were related, a fact noted, but neglected, by Freud.

Furthermore, her way of presenting the accountant as *an old man who is as fond of the children as though they were his own grandchildren* suggests a tendency to confuse fondness for the children and position inside the family. In addition, her way of describing her behaviour with the children is a bit awkward: "*While the children were having this game with me, there was suddenly a strong smell. They had forgotten the pudding they were cooking and it was getting burnt.*". These are the words of an elder sister playing with them rather than of a governess, who would not incriminate the children. Incidentally, we could also wonder whether the discussion initiated by the Director regarding *the lines along which children should be brought up* was not in reference to her own rather maladjusted behaviour. Awkward as well is her way of thinking of this discussion as an *exchange of views*¹⁰ during a *friendly talk* rather than as instructions from an employer.

⁸ Actually this analysis corroborates the feeling of the other servants who, according to Lucy, *thought that [she] was putting [her]self above [her] station*. She was unable to understand why.

⁹ The last words of Lucy quoted by Freud in his report are: "*After all, I can have thoughts and feelings to myself.*"

¹⁰ *Vertraulichem Gedankenaustausche* in German.

In any case, it seems that here lies some powerful psychic conglomerate, in other words an idiotope, delineating a figure which could be more precisely labelled under the name of *House Daughter* and which would be grounds for the illusorily special position.

So, this idiotope *House Daughter* would be rooted in the family links between the two mothers, and settled by Lucy's promise to the children's mother on her death-bed *of devoting [her]self with all [her] power to the children, ... of not leaving them ... and of taking their mother's place with them* (in the literature of the XIX Century, it is a conventional formula for an elder daughter in such a situation). It would shed light on her deafness to the threat to her employment: really a foolish idea, a *House Daughter* can't be dismissed. It would also create some suspicion about the quickness with which Lucy accepts Freud's interpretation in terms of repression of some love expectations¹¹, namely the suspicion of avoiding the perhaps more difficult confrontation with the illusion of her special position. A repressed idea is indeed not generally accepted so easily, and, even more important here, this acceptance had no effect on the symptoms.

Now we have an answer to the first question: the nature of the obstacle to remembering the Lady's visit is the presence of this deeply rooted idiotope *House Daughter*. We can even safely add that the delay of nine weeks is just the time taken by the dissolution of this idiotope. Whence a new question: would it be possible to give some hints about the way the "work" of the talking cure operated this dissolution?

6. Psychic disorder and idiotopic multi-activation

We isolated the idiotope *House Daughter*. But there is also, with no ambiguity, another one, the idiotope *Employee* which is active at the beginning of the talking cure when Lucy described her situation and which was certainly not created for this occasion. Roughly speaking, let us say that *House Daughter* is enabled inside the Director's house and *Employee* outside¹².

- 1) The reprimand that follows the lady's visit is received by the idiotope *House Daughter* which was up to now the only one enabled inside the house, and it is stored here, most likely as an unfair manifestation of an (up to now) unknown personality trait of the Director. The threat to employment is not heard.
- 2) The scene with the accountant again happens while *House Daughter* is active and stored here. But, this time, the parallelism between the two scenes and the effective condition of the dismissing [kissing the children] certainly trigger an alarm to *Employee* and activate it as well; this is manifested by *the stab at the heart*. Here is the beginning a crisis situation: *House Daughter* and *Employee* simultaneously activated inside the Director's house. This crisis is actualized by the tension with the other servants.
- 3) Now we get a more precise answer to our first question: at the beginning of the cure (outside the house), while speaking with Freud, only *Employee* is active. This explains why, when the question of the intimate relationship with the Director emerges, the memory of the lady's visit, confined in *House Daughter*, remains inaccessible.
- 4) There was an interview with the Director, at her initiative some time after the scene with the chief accountant, about the "intrigue" of the other servants. Getting no support, she gave notice; he kindly suggested that she think the matter over for a couple of weeks. This is a tipping point in the balance between the idiotopes *House Daughter* and *Employee* fighting together inside the Director's house. Before the interview, *House Daughter* prevails; after her giving notice, it is *Employee*. This new balance is definitively sustained by Lucy's decision to stay because of the children's affection for her. The depression can then

¹¹ Again, see footnote 7.

¹² We could think *Employee* as a kind of ambassador of the idiotope *House Daughter* outside the Director's house.

be understood as a diverted expression of the complete disqualification and dereliction of the still active *House Daughter*.

5) We can precisely observe the paradigmatic dimension mentioned in Section 3.2 in action when Lucy itemizes all the possible positions of the chief accountant to Freud: Director's employee, old man, grandfather, valued friend, and finally eliminates them when she finds a meaningful stop at guest. This stop determined the reminiscence of the Lady's visit.

6) The idiolectal specification of *House Daughter* is characterized by a tendency to confuse fondness for the children and position inside the family, viz. i) the accountant considered by her as a kind of *grandfather*; ii) the discussion with her employer experienced by her as an *intimate exchange* of views.

Of course a single analysis of this kind cannot serve as the basis for a conceptual framework such as the one proposed in Section 3.1. Actually we developed four other similar idiotopic analysis as well in [6] and we refer the reader to them. Two cases are again from *Studies on Hysteria*. First, the case of *Katharina* which is very similar to Lucy's and even more spectacular: only one conversation and the same kind of outcome with a transfiguration effect, this time observed on the spot by Freud himself: *At the end of these two sets of memories, she came up to a stop. She was like someone transformed. The sulky, unhappy face had grown lively, her eyes were bright, she was lightened and exalted* [5]. And then *Miss Elisabeth von R.*. Afterwards we extended the method to the much more difficult and intricate cases of *Dora* and of the *Rat Man*. In all these cases, the method used for the isolation of the different idiotopes relies upon strongly discriminating techniques coming from the semiotics of action as initiated by W. Propp [7] and then revisited by A.J. Greimas [4], see also [8]. Actually, in the work [6], the same semiotical method was already very strictly applied to Lucy's case as well; we did not develop it here and only briefly sketched it, because the semiotical method is not our point here.

7. The happy outcome of Lucy's cure

The happy outcome of the talking cure, understood as an outcome of an idiotopic crisis situation, must be interpreted as an idiotopic reorganisation. However this very quick outcome (the two days between the penultimate and ultimate sessions) leads to the supposition that this reorganisation was without extensive collateral effect and much simpler than usual in any situation of high psychic disorder. Clearly the idiotope *House Daughter* is largely loaded by Lucy's fondness and real interest for the children. We suggested that this fondness was perhaps more that an elder sister than that of a governess. This idiotopic reorganisation could then be sketched as a mere shifting of the idiotope *Employee* into the place of the idiotope *House Daughter*, embodying all its material and then creating a new idiotope *Ēmployee* active inside the Director's house: the elder daughter yields to the governess who is thus able to understand the regular nature of the Director's *lines along which the children should be brought up*, namely the guidelines of an employer. It would be this shift alone with no further effect that would explain the unusual quickness of the happy outcome.

Now let us further address the process of this shift at the end of the talking cure. Certainly the major tool of any kind of psychotherapy is a certain way of scrambling or disturbing the representation system in which the patient is too tightly enmeshed. We can think, among other things, to the paradoxical slogan of the Palo-Alto school: "Prescribe the symptom".

So, here, we have to consider the peculiar kind of scrambling used by Freud at that time. On the one hand, in avoiding the face-to-face interview, the setting of the talking cure inhibits the usual major condition wherein the individual, in order to position himself, is directly facing reality. On the other hand, many authors have noticed a sort of parallelism between the setting of a psychoanalytic cure and some aspects of dream activity:

- inhibition of any action;
 - rejection of critical faculties and full reliance on free association;
 - and even strong use of the dream material which is ordinary neglected during daytime consciousness.
 In this way, if these observations are valid, the protected setting of a psychoanalytic cure certainly promotes idiotopic multi-activation similar to the model of sleep functioning, or, to use Freud's terms, it promotes: an *extension of memory, a different state of consciousness, or an enlarging ... [of] consciousness.*

Perhaps, at this early stage of the development of the psychoanalytic cure, the instructions about the three points hereabove were not so clearly stated; what is certain from Freud's report is that Lucy *simply lay quietly ... with her eyes closed all the time.* So, it would have been a progressive adaptation to an idiotopic multi-activation during the sessions of the cure, which brought about the peaceful contact of the two memories stored in *House Daughter* with the other representations stored in *Employee*, finally making the threat to employment, which was the origin of the crisis situation, audible. This contact would in turn have produced the shifting we described, freeing Lucy from the illusion of a special position in the Director's house and offering relief. Once again, what is very interesting here is that we got only a bodily expression of this relief, without any reflexive analysis by Lucy; in other words, everything, despite its complexity, stayed at an ante-linguistic level. The table below provides a dynamic description of the case:

Chronol. of events	Idiotopic activation	Symptom
lady's visit	House Daughter	
accountant's scene	House Daughter/Employee	stab at heart
interview with Director she gives notice	tipping point	depression
her decision to stay	Employee/House Daughter	depression
first visit to Freud	Employee	depression
talking cure	Employee + House Daughter	
penultimate session	peaceful contact between the two memories	
ultimate session	re-organization and creation of \tilde{E} mployee	vanishing
four months later in a summer resort	Outh	vanishing confirmed

Our idiotopic assumption allowed us to explain Lucy's recollection of the lady's visit by the simultaneous access, thanks to the specific setting of the cure, to the two distinct idiotopes *House Daughter* and *Employee* which were normally mutually exclusive in the daytime before the crisis. But we also supposed that, after the episode concerning the accountant, both *House Daughter* and *Employee* were simultaneously active inside the house as well. So we must stress, once again, an important and clear distinction: facing the actual reality of life inside the Director's house, the two idiotopes were in competition, creating disconcerting interferences leading to the discord with the other servants, and producing a painful feeling. Inside the protected setting of the cure (protection by the kind attention of the therapist and protection from the immediate pressure of external reality), they could more quietly coexist and produce peaceful groupings of representations, opening the possibility of unprecedented interactions.

Let us go back now to Proust and Hoare. What differentiates their situations from Lucy's is that the concomitant activation of two idiotopes was very short and was followed, as we observed, by the strengthening of some decision in the first case and by a re-elaboration of a given event from the past in the second. In both cases, the representations were dealing with facts and with something from the far past with no pressing effect on the present time. In Lucy's case, however, the representations were dealing with feelings or emotions directly concerning her present life. It is most likely why the antagonistic

multi-activation lasted so long, and why she was led to psychotherapy to get treatment. In any case, while the decision, the re-elaboration and the reorganisation can be described and understood, the "logic" of the multi-activation remains enigmatic¹³.

Certainly, in Hoare's case, we could speak of an idiotopic reorganisation as well. We could roughly describe it along the following lines:

- 1) torn nail and sudden activation of the idiotope *SV* while *TWF* is enabled;
- 2) causal deduction following information coming from both idiotopes, and checking decision;
- 3) integration of the amended version of *SV* inside *TWF*, and vanishing of *SV* as an "autonomous" idiotope; again a kind of shifting as in Lucy's case.

8. Elements towards a more complex structure

It is quite natural at this stage to try to articulate our basic idiotopic model with the Freudian psychic classification: *conscious/preconscious/unconscious*.

8.1. Chasing the unconscious

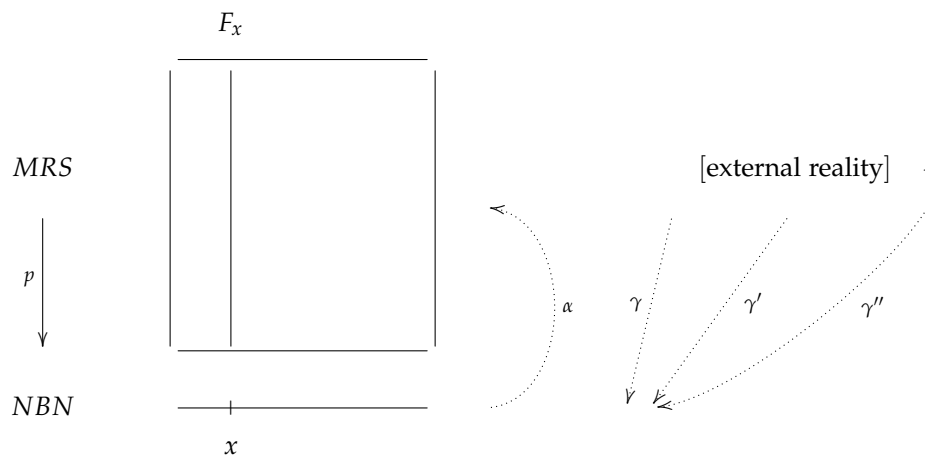
Clearly the *preconscious* would be formed by the content of all the idiotopes, while the *conscious* would consist of the content of the only enabled one. But what about the *unconscious*? Even in Freud the nature of the unconscious is far from being clear: sometimes it seems to be thought of as a certain quality of psychic activity and sometimes as the reservoir of inaccessible (repressed) representations, namely as a kind of agency in the same way as the *preconscious* and the *conscious*.

In our model, the *unconscious* refers to how the brain dispatches the various representations generated by the confrontation with external reality in the diverse idiotopes and to how it determines their activation. So the inaccessible dimension of the *unconscious* (which is its major conceptual distinction in Freud's theory) would be assumed by the dispersion of the representations inside various idiotopes, as we saw in Lucy's case with the impossibility of remembering the lady's visit, stored inside *House Daughter*, whereas in Freud's office only the idiotope *Employee* was active. Inside this frame of reference, the Freudian concept of repression could be understood as a repression from one idiotope to another.

So we immediately have to complete our model by introducing the following graph, where MRS (Mind Representation System) denotes the set of all representations, and NBN (Neural Brain Nets) the set

¹³ *Droll thing life is – that mysterious arrangement of merciless logic for a futile purpose. The most you can hope from it is some knowledge of yourself – that comes too late – a crop of inextinguishable regrets.* Heart of Darkness by J. Conrad.

of neural intrications which rule the organisation of the representations, and where $p : MRS \rightarrow NBN$ is a mapping associating a neural brain net with any given representation:



Above the neural net x , the fibre $F_x = p^{-1}(x)$ denotes the idiotope (= the set of all the representations) which is enabled by the net x . The dotted arrows γ , γ' , γ'' roughly sketch how the external reality multi-impacts NBN. Finally the dotted arrow α denotes the unconscious synthetic function of selecting and enabling the idiotope which is the best equipped to position oneself and to move oneself into action in the given circumstance where γ , γ' and γ'' intervene.

Certainly these neural brain nets (NBN) must be clearly distinguished from the so-called neural correlates of consciousness (NCC) of the neuroscientists which are isolated from very strict specific conscious percepts and thought to be in one to one correspondence with them. This distinction is a consequence of the large change of scale we are interested in. In this diagram, only the full arrow is a mapping in the mathematical sense, while the dotted arrows roughly outline some aspects of the so-called "hard problem of consciousness".

We already mentioned unprecedented groupings of representations as an effect of idiotopic multi-activation during the cure. Now, there is a figure which is correlative to grouping, namely fragmentation. Quite naturally this figure offers a way forward in cases of traumatic or only completely unprecedented confrontation with reality: a representation having no usual or immediately adaptable idiotopic reception could be fragmented along different idiotopes (even non-active), being then quasi-necessarily affected by some losses. This would explain the feeling of being confronted with some maladjusted jigsaw pieces during the free multi-activated idiotopic mode of dreams.

Coming back to Lucy, we did wonder about the exact status of "the threat to her employment" clearly expressed by the Director during the lady's visit. Lucy recorded it, but she did not seem to understand or even hear it at the moment. So where exactly did she record it? During the lady's visit, she certainly was first affected by the unknown violence of her employer's unjustified behaviour concerning *a matter ... [for which she was] not in the least responsible*. And certainly she was not able to pay much attention to the threat. We supposed in Section 6 that the whole memory of this visit was stored in *House Daughter*, active at that moment. But we could now surmise that it was fragmented: the unfair part in the active idiotope *House Daughter* which was put on special alert, while the specific threat was stored in the inactivated idiotope *Employee*, but with such a minor intensity that *Employee* was not activated. The quiet multi-activated idiotopic mode of the talking cure, as described above, would have allowed her to collect the various pieces and to completely restore the whole circumstance of the lady's visit.

8.2. A step towards the structure of discrete cofibration

We said that, in any individual, one could suppose the existence of at least two idiotopes: *Home* which is enabled when this individual is in the place of his highest safety and *Outh* when he is out of this place.

In the case of Lucy, *Home* is entrenched in the Director's house. Certainly the idiotope *House Daughter* is a specific part of this idiotope, while the idiotope *Employee* is a specific part of *Outh*. Here, there is something which is not very easy to conceptualize: certainly *House Daughter* is a "part" of *Home*, but also this part has many facets. So, the word part is not quite appropriate, but let us keep it for the moment to roughly express a certain type of oriented correlation. So the neural brain nets $p(\text{HouseDaughter})$ and $p(\text{Home})$ must be correlated as well, in some way which, at this stage, could be conceptualized by the presence of one or several arrows:

$$p(\text{HouseDaughter}) \rightarrow p(\text{Home})$$

the plurality of arrows depending on the many facets of the idiotope *House Daughter* inside the idiotope *Home*. Inversely the presence of any of these arrows in NBN should imply that any representation registered in *House Daughter* is also registered in some way in *Home*, but inside a larger context of representations and consequently with a reduced, less intensive or at least different resonance. So that any arrow in NBN would be, at the same time, a consequence and a cause of some arrow in MRS.

So, coming back to our above scheme, we could now more easily see MRS and NBN as categories and $p : \text{MRS} \rightarrow \text{NBN}$ as a functor, both notions introduced by Eilenberg and Mac Lane, [9]:

Definition 8.1. A category \mathbb{C} is a two-level structure with objects $x, y \dots$, and arrows $f: x \rightarrow y$ having a domain x and a codomain y . In the following condition of incidence, there is a composition of arrows:

$$\begin{array}{ccc} & y & \\ f \nearrow & & \searrow g \\ x & \xrightarrow{g \cdot f} & z \end{array}$$

which is associative. Moreover for any object x , there is an identity arrow $1_x : x \rightarrow x$ with a neutral effect on composition: given any arrow $f : x \rightarrow y$, we get: $1_y \cdot f = f$ and $f \cdot 1_x = f$

A functor $F : \mathbb{C} \rightarrow \mathbb{D}$ is a two-level mapping which preserves objects, arrows, domain, codomain, identity arrows and composition.

The objects of MRS are the representations, and the arrows between representations describe how a "same" representation¹⁴ is dispatched between two different idiotopes when these idiotopes are correlated. The objects of NBN are the neural brain nets while the arrows are determined by the "oriented interactions" between these nets.

Now the precise relationship between the arrows in NBN and the registration arrows in MRS, roughly sketched hereabove with the words "a consequence and a cause", could be conceptually dominated by

¹⁴ Semiotically speaking, it is not the "same" representation since it is enrolled in a distinct (here larger) context and in a different distribution of relationship with other representations which give this representation another paradigmatic dispatching power, in other words, which give it another "meaning". Again, see the chromatic tables of Section 3.2.

saying that the functor $p : MRS \rightarrow NBN$ is a *discrete cofibration*, namely a particular case of cofibration [10], see also [11]:

Definition 8.2. A functor $U : \mathbb{C} \rightarrow \mathbb{D}$ is said to be a *discrete cofibration*, when, given any object x in \mathbb{C} and any arrow $f : U(x) \rightarrow z$ in \mathbb{D} , there is a unique arrow $\phi : x \rightarrow \bar{z}$ such that $U(\phi) = f$.

So here, p is a functor such that any pair (r, f) of a representation $r \in MRS$ and of an oriented interaction $f : p(r) \rightarrow y$ in NBN determines a unique arrow $\phi : r \rightarrow r'$ in MRS with $p(\phi) = f$ and consequently $p(r') = y$; this arrow is called the *translation arrow* (along the oriented interaction f) relating the representation r belonging to the idiotope $F_{p(r)}$ to its translation r' inside the idiotope F_y .

Discrete cofibrations $\mathbb{C} \rightarrow \mathbb{D}$ are in one to one correspondence with functors $\mathbb{D} \rightarrow Set$, the category of sets, by a restriction of the so-called Grothendieck construction [10]. Here, this correspondence simply associates with the functor p the functor $F : NBN \rightarrow Set$ where F_x is nothing but the idiotope above the neural net x .

Let us try now to show how this notion of discrete cofibration allows us to compensate for what we suggested to be an inappropriate use of the word "part", when we introduced conceptualization through the use of arrows. As for Hoare, we distinguished two idiotopes SV and TWF . It is legitimate to suppose the existence of an idiotope HF (for Hoare Family) containing all the representations, in Henry Hoare's mind, concerning the Hoare Family. We noted that the Hoares were a banking family; so that we can suppose, again with little risk, the existence of an "oriented interaction" $b_T : p(TWF) \rightarrow p(HF)$, appended to banking considerations, between the two associated neural brain nets.

Butler's notes state explicitly that there are two distinct representations, "sovereign" and "banknote", in the idiotope TWF . Let us denote by $b_T^S : "sovereign" \rightarrow "money"$ the "translation arrow" associated by the discrete cofibration with the pair $("sovereign", b_F)$ and by $b_T^B : "banknote" \rightarrow "money"$ the one associated with the pair $("banknote", b_F)$. The fact that the codomains of these two distinct arrows in MRS are the same means that the translation process along the "banking arrow" b_T in NBN is not necessarily injective. This exemplifies how much the word "part" was inappropriate and also allows us to structurally measure what we meant by "reduced, less intensive or at least different resonance" hereabove, or by "other meaning" in footnote 14.

In Section 3.1, we supposed the universal existence of two idiotopes, *Home* and *Outh*. We can now specify what we meant by "mutual exclusion". The objects $p(Home)$ and $p(Outh)$ are a kind of maximal objects in the category NBN , namely there is no arrow coming out from them, there is no arrow with these objects as domains. But they are probably not the only objects of the category NBN in this position.

8.3. Causal and semiotical logics, redux

Our idiotopic assumption is a way to rule the conflict inside the human mind between "semiotical logic" and "causal logic" in restricting the areas of effective awareness (see Section 3.2). Therefore the set F_x would actually be endowed with some kind of simple structure M_x of "causal logic", roughly speaking a structure given by a finite number of propositions (=basic representations) and a finite number of deduction rules taken from the propositional calculus. The representations a and b given in Section 5 could indeed be easily broken down into conjunctions of this type of basic representations, and the representations c and d could appear as the result of a relatively simple propositional calculus from these breakdowns of a and b inside the extended multi-activated idiotopic context of the talking cure which contains the two representations a and b in question and therefore contains their breakdowns.

So, our discrete cofibration would actually come from a functor $F : NBN \rightarrow Mod$, where Mod is the category whose objects are the pairs (X, M_X) of a set X endowed with some "causal model" M_X and whose arrows are those applications $f : X \rightarrow Y$ which are compatible with the respective models M_X and M_Y . This kind of discrete cofibration would help us understand the conceptual coordination of the local/global relationship between the distinct local causal logics of the various idiotopes. Thus, ultimately, our way of modelling the "mind representation system" is in terms of discrete cofibration with fibers enriched in Mod .

The idea of such modelling is rooted in the mathematical notion of *sheaf* introduced by J. Leray. A sheaf of structure (e.g. groups or vector spaces), see [12] or [13], is defined on a topological space T whose preorder of open sets is denoted by $\mathcal{U}(T)$:

- it is a *presheaf*, namely a functor $F : \mathcal{U}(T)^{op} \rightarrow \mathbb{C}$, where $\mathcal{U}(T)^{op}$ is the opposite preorder of $\mathcal{U}(T)$ and \mathbb{C} any category of algebraic structure,
- satisfying a "gluing axiom" with a condition of existence and unicity.

But, for us, the restriction of the domain of F to a preorder is far too restrictive with respect to modelling mind activity. An interesting attempt to mix mathematical sheaf theory with the theory of games and decisions was initiated in [14].

9. Coda

Our basic model of the mind representation system is not based upon a very original idea: each human being is multiple, each human being is diversely defined or determined. What is new is its strict formalisation, clearly discriminating daytime and night-time functioning, normal and crisis status, as precisely described in Section 3.1. Since this formalisation is rather simple, it offers many opportunities to check its relevance. Among other things, honing one's sensitivity to the subtle idiolectal variations of their patients is the major art of any psychotherapist; and this sensitivity is the best means to delineate idiotopes and to make their possible antagonisms explicit.

More generally, although the distinction between the different idiolects related to the idiotopes may be generally rather low, we could assert that certainly one of the achievements of literature is to make them audible, and pertinent for psychology, whether the literature is fictional or scientific. Using this idiolectal method, it is not very difficult to delineate some idiotopic areas for several of Dostoevski's characters; Raskolnikov or Fyodor Karamazov, the father, for instance. It is also quite common for most people to witness (albeit rarely) this kind of dissonance when listening to others.

Clearly our extensive model is more conjectural. As of now, the main interest of this more elaborated functorial model in terms of discrete cofibration with enriched fibers is to open the way to a firm heuristic perspective. In particular, one of the benefits of this mathematically structured scheme, as shown by footnote 14 and by the calculus suggested in the first paragraph of Section 8.3, is certainly to open the way to some problematisation and to some precisions about the whole concept of representation. Certainly a difficult problem, but now oriented and bounded by this heuristic frame.

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	linear dichroism

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