A resolution of the Trans-Planckian problem in the $R_{\rm h} = ct$ universe

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Abstract –The recent measurement of a cutoff $k_{\rm min}$ in the fluctuation power spectrum $P(k)$ of the cosmic microwave background may vitiate the possibility that slow-roll inflation can simultaneously solve the horizon problem and account for the formation of structure via the growth of quantum fluctuations in the inflaton field. Instead, we show that $k_{\rm min}$ may be interpreted more successfully in the $R_{\rm h} = ct$ cosmology, as the first mode exiting from the Planck scale into the semi-classical Universe shortly after the Big Bang. In so doing, we demonstrate that such a scenario completely avoids the well-known trans-Planckian problem plaguing standard inflationary cosmology, with some additional clues that may motivate further development of Kaluza-Klein cosmologies, string theories and supergravity.

Introduction. – The Friedmann-Lemaître-Robertson-Walker (FLRW) metric, based on the cosmological principle and its assumption of isotropy and homogeneity on large scales, is the backbone of modern cosmology. All the available observational evidence appears to support its essential spacetime basis, so any conceptual or foundational hurdles arising with the expansion of the Universe are attributed to other factors—notably an incomplete understanding of the physics underlying the evolution of its contents.

Over the past four decades, several crucial amendments and additions have been introduced to the basic picture in order to address some of these difficulties, chief among them the well-known horizon problem associated with the uniformity of the cosmic microwave background (CMB) temperature, $T_{\rm cmb}$. In the context of ΛCDM, CMB photons emitted near the surface of last scattering (LSS) at redshift $z \sim 1100$ from opposite sides of the sky would be causally disconnected without an anomalous accelerated expansion in the early Universe [1]. Yet $T_{\rm cmb}$ has the same value in all directions, save for $\sim 10^{-5}$ variations associated with fluctuations seeded at, or shortly after, the Big Bang.

A very elegant solution to this problem was introduced in the early 1980's [2], based on an expected phase transition in grand unified theories (GUTs), when the strong and electroweak forces may have separated at an energy scale $\sim 10^{16}$ GeV, or $\sim 10^{-35}$ seconds after the Big Bang. As long as the scalar field, ϕ, associated with this spontaneous symmetry breaking had the 'right' potential, $V(\phi)$, one could envisage an evolution at almost constant energy density, ρ_ϕ, producing a transient near-de Sitter cosmic expansion [3]. Such an inflationary phase would have exponentially stretched all observable features well beyond the Hubble radius, $R_{\rm h} = c/H$, where $H(z)$ is the redshift-dependent Hubble parameter, causally connecting the spacetime throughout the visible Universe today.

Perhaps even more importantly, this event is believed to have also produced the large-scale structure via the seeding of quantum fluctuations in ϕ and their subsequent growth to classically relevant scales during the inflated expansion [4]. A near scale-free spectrum $P(k)$ would have been generated as modes with comoving wavenumber k successively crossed $R_{\rm h}$ and classicalized, freezing their amplitude at a mode-dependent crossing time t_k. Thus, inflation appears to have simultaneously solved the $T_{\rm cmb}$ horizon problem and provided an explanation for the origin of $P(k)$.

In spite of this initial success, however, the inflationary paradigm is nonetheless conceptually incomplete for several reasons. For example, the recent discovery of the Higgs particle [5] has reminded us that ΛCDM is subject to several horizon problems—at several different epochs—not just one, so the GUT transition at $\sim 10^{-35}$ seconds is looking more like an overly customized solution focusing solely on $T_{\rm cmb}$, rather than providing an over-arching paradigm to account for our entire causally-connected Universe. A second well-motivated phase transition should have occurred when the electric and weak forces separated at a critical temperature $T_{\rm Higgs} \sim 159.5 \pm 1.5$ GeV, i.e., $t \sim 10^{-11}$ seconds after the Big Bang [6–10], too far beyond the GUT scale to have been affected by the hypothesized first transition [11]. This second spontaneous symmetry breaking would have inevitably led to its own horizon problem, having to do with the 'turning on' of the Higgs field and its vacuum expectation value, which today appears to be universal, even on scales exceeding the regions that were causally connected at the time of the electroweak phase transition.

This is not so much an argument against inflation per se, though it does weaken the claim that a GUT phase transition could account for all of the major features we see today; it apparently does not. A more serious problem with the slow-roll inflationary paradigm has been uncovered by another recent study of the angular correlation function measured in the CMB by *Planck* [12]. The solution to the $T_{\rm cmb}$ horizon problem, and a generation of a near scale-free fluctuation spectrum $P(k)$, are intimately connected via the initiation and extent of the inflationary phase. But the CMB angular-correlation function now provides compelling evidence—at a confidence level *exceeding* 8σ—that $P(k)$ has a non-zero cutoff $k_{\rm min} = (4.34 \pm 0.50)/r_{\rm cmb}$, where $r_{\rm cmb}$ is the comoving distance to the LSS [13]. Since $k_{\rm min}$ would have been the first mode crossing $R_{\rm h}$ during inflation, it would signal the precise time, as a function of $V(\phi)$, at which the de Sitter expansion started. Unfortunately, its measured value shows that none of the slow-roll potentials proposed thus far can simultaneously account for the uniformity of $T_{\rm cmb}$ across the sky and the observed $P(k)$ in the CMB [14]. The conclusion from this is that, if slow-roll inflation is to work, it must function in a more complicated way than has been imagined thus far.

As we shall see in this *Letter*, the measurement of $k_{\rm min}$ in the angular correlation function of the CMB not only constrains the time when inflation could have started, but apparently provides direct evidence of quantum fluctuations at the Planck scale. This topic broaches one of the most serious fundamental problems with inflation, one that has eluded satisfactory resolution for over three decades. It is generally understood that to solve the horizon problem in ΛCDM, a minimum of 60 e-folds of inflationary expansion must have occurred, even more in many variants of the basic model. Thus, cosmological scales of observational relevance today must have expanded from sub-Planckian wavelengths at the start of inflation [15–18]. But the physics we have today cannot adequately handle such processes, a situation known as the 'trans-Planckian problem' (TP) [20]. This signals a potentially fatal incompleteness of inflationary theory at a fundamental physics level.

Phenomenological Approach to TP Physics in ΛCDM. – One can easily understand why this constitutes a potentially insurmountable problem, given that the Planck mass is defined by the equality of its Compton wavelength and Schwarzschild radius. Since the former increases as the latter shrinks towards the Big Bang, it is simply not possible to characterize the behaviour of modes below the Planck scale using quantum mechanics and general relativity separately. The semi-classical physics we use to describe the evolution of quantum fluctuations as the Universe expands does not apply for mode scales shorter than their Compton wavelength.

This problem manifests itself in several ways, particularly via the mode normalization that one must use to calculate $P(k)$ for a comparison with the CMB data. The amplitude of the modes is typically inferred by minimizing the expectation value of the Hamiltonian, but with a time-dependent spacetime curvature at the Planck scale, the frequencies themselves depend on time and non-inertial effects. Early attempts at addressing this issue extended the birth of fluctuation modes into the very distant conformal past, well below the Planck scale, arguing that the simple harmonic oscillator is recovered there, allowing one to impose a Minkowski vacuum—called the Bunch-Davies vacuum in this context [19]—as the background for the fluctuations. But given that the physics below the Planck scale is unknown, we have a conceptual problem understanding whether or not the Bunch-Davies vacuum is even the correct choice for sub-Planckian modes.

The consensus today is that Planck-scale physics probably should have created an imprint on the CMB, but with no established theory of quantum gravity, no one knows how to predict such features with any confidence. Instead, the approaches taken over the past two decades have been based on phenomenological treatments, including (1) modifications to the dispersion relation for quantum modes on short scales [15, 16, 20–23]; (2) the use of string-inspired changes to the Heisenberg uncertainty relation [17, 18, 24]; and (3) noncommutative geometry [25–27].

All of these are really probes of the CMB to suggest how basic theory ought to be modified rather than robust attempts at using a well-justified model of physics at short distances to predict a trans-Planckian signature. To understand the scale we are considering here, we define the Planck length $\lambda_{\rm P}$ to be the Compton wavelength $\lambda_{\rm C} \equiv 2\pi/m_{\rm P}$ of a (Planck) mass $m_{\rm P}$ for which $\lambda_{\rm C}$ equals its Schwarzschild radius $R_{\rm h} \equiv 2Gm_{\rm P}$. The Planck energy is therefore $m_{\rm P} \approx 1.22 \times 10^{19}$ GeV. Estimates of how big trans-Planckian corrections might be, based on the above phenomenological approaches, range from $(\lambda_{\rm P}/R_h[t_{\rm inf}])^2$ (see, e.g., refs. [28, 29]) to as large as $\lambda_{\rm P}/R_h(t_{\rm inf})$ [18, 30–32]. In these expressions, $R_{\rm h}(t_{\rm inf})$ is the Hubble radius during inflation (which is more or less constant in the slow-roll approximation).

Thus, if inflation is associated with a GUT phase transition at $\sim 10^{16}$ GeV, these phenomenologically motivated corrections fall in the range 10^{-6} to 10^{-3}. Additional support for such a claim—especially towards the high-end of this range—is provided by arguments [31, 32] that curvature effects at the Planck scale probably produce deviations of the ϕ quantum state from the local vacuum state on the order of $\lambda_P/R_h(t_{\rm inf})$, but no one really knows for sure. If reasonable, this range includes effects potentially large enough to affect the primordial power spectrum $P(k)$ in measurable ways (see, e.g., [30]). Of course, a final resolution of whether or not trans-Planckian effects manifest themselves observationally must await the formulation of a well-motivated quantum gravity theory. We may find that such corrections are larger than expected and inconsistent with the data, thereby eliminating any possibility that the quantum fluctuations were seeded below the Planck scale.

Certainly, the measurement of $k_{\rm min}$ already seems to argue against the premise that inflation might have started early enough to solve the temperature horizon problem, while simultaneously explaining the origin of $P(k)$. As shown in ref. [14], the interpretation of this cutoff as the first mode to cross $R_{\rm h}$ once slow-roll inflation began is inconsistent with the accelerated expansion required to provide us with a causally-connected Universe today. In the next section, we present an alternative interpretation of $k_{\rm min}$ that avoids these conceptual problems and, at the same time, completely eliminates the trans-Planckian inconsistency.

A Resolution of the TP Problem in $R_{\rm h} = ct$. – The FLRW cosmology known as the $R_{\rm h} = ct$ universe [33–37] has had a sustained level of success accounting for over 25 different kinds of cosmological data at least as well as the standard model. A recent summary of these comparative tests may be found in Table 2 of ref. [38]. In this model, the cosmic fluid obeys the zero active mass condition from general relativity, in which the total density and pressure are related via the constraint $\rho + 3p = 0$. A notable feature of the expansion implied by this scenario is that it lacks any horizon problem, eliminating the need for an inflated expansion of the early Universe. Thus, if the zero active mass condition was evident at the earliest times, t, it is straightforward to show [39] that an incipient (though non-inflationary) scalar field ϕ would have had the well-defined potential

$$V(\phi) = V_0 \exp\left\{-\frac{2\sqrt{4\pi}}{m_P}\,\phi\right\} . \tag{1}$$

ϕ is therefore a special member of the class of minimally coupled fields explored in the 1980's, that produced power-law inflation [40–43] except that, with the zero active mass equation-of-state, this ϕ produced a constant expansion rate $a(t) = t/t_0$ and did not inflate.

In $R_{\rm h} = ct$, quantum fluctuations in ϕ with a wavelength $\lambda_k < 2\pi R_{\rm h}$, where k is the comoving wavenumber and $R_{\rm h}$ is the Hubble radius, oscillate, while those with $\lambda_k > 2\pi R_{\rm h}$ do not [39]. Thus, mode k oscillated in the semi-classical Universe once it emerged across the Planck scale. But the critical question is "When did it emerge?" From the expression $k = 2\pi a(t)/\lambda_k(t)$, it is clear that the observed value of $k_{\rm min}$ indicates the time $t_{\rm min}$ when the first mode appeared. Therefore,

$$t_{\rm min} = \frac{4.34\, t_P}{\ln(1 + z_{\rm cmb})} , \tag{2}$$

in terms of the redshift, $z_{\rm cmb}$, at the surface of last scattering.

In the concordance ΛCDM model, $z_{\rm cmb} \sim 1080$, for which $t_{\rm min} \sim 0.6 t_P$. With the expansion scenario implied by $R_{\rm h} = ct$, this redshift could be quite different, but the dependence of $t_{\rm min}$ on the location of the last scattering surface is so weak, that even a redshift $z_{\rm cmb} \sim 50$ would result in an initial emergence time of $t_{\rm min} \sim 1.1 t_P$. Therefore, it appears that $k_{\rm min}$ in $R_{\rm h} = ct$ represents the first mode exiting the Planck region at about the Planck time, a compelling indication that the cutoff $k_{\rm min}$ corresponds to the first mode that could have physically emerged into the semi-classical Universe after the Big Bang.

Unlike the situation with an inflaton field, in which these modes were seeded in the Bunch-Davies vacuum and oscillated across the trans-Planckian region, the quantum fluctuations associated with a non-inflationary scalar field in the $R_{\rm h} = ct$ cosmology could well have been formed at the Planck scale and then evolved according to standard physical principles in the semi-classical Universe. Such an idea—that modes could have been created at a particular (perhaps even fixed) spatial scale—is not new. It has been proposed and discussed by several other workers, principally Hollands and Wald [44], but also in refs. [24, 27], among others.

Assuming that all subsequent modes continued to emerge across the Planck scale with a wavelength $\lambda_k = 2\pi\lambda_P$, though at progressively later times $t_k \equiv k\lambda_P t_0$, it is trivial to show [39] that the resultant power spectrum is almost scale free, with an index n_s slightly less than one, consistent with the value measured by *Planck* [12]. Thus, a non-inflationary scalar field in the $R_{\rm h} = ct$ universe can account for both the measured cutoff $k_{\rm min}$ and for the observed distribution of fluctuations in the CMB. Most importantly for the main theme of this paper, the first reliable measurement of a minimum cutoff in the power spectrum $P(k)$ signals a direct link between the CMB anisotropies—and the subsequent formation of structure in the Universe—and quantum fluctuations at the Planck scale. In so doing, this interpretation eliminates one of the principal inconsistencies with the basic slow-roll inflationary model, i.e., the well-known trans-Planckian problem.

Conclusion. – In this paper, we have discussed the implications of the fact that, in addition to the well-studied power spectral index n_s and amplitude of the CMB fluctuations, we now have a robust measurement of a third parameter characterizing the primordial perturbation spectrum, i.e., the wavenumber cutoff $k_{\rm min}$, which differs from zero at a confidence level exceeding 8σ. This cutoff appears to invalidate basic slow-roll inflationary models attempting to simultaneously account for the 60 e-folds of exponential expansion at the GUT scale and the generation of anisotropies in the CMB from quantum fluctuations in the inflaton field. An additional well-known inconsistency with this scenario is the trans-Planckian problem, referring to the required transition of modes from below the Planck scale into the semi-classical Universe, a process that cannot adequately be described by quantum mechanics and general relativity separately.

Contrasting with this deficiency in the standard model, we have also demonstrated that the interpretation of $k_{\rm min}$ in the $R_{\rm h} = ct$ cosmology suggests it corresponds to the first quantum fluctuation that could have physically emerged from the Planck scale shortly after the Big Bang. This scenario thus avoids the trans-Planckian problem if one invokes the idea that all fluctuations in the incipient scalar (though non-inflationary) field were seeded at a fixed spatial scale—in this case, the Planck scale—though at progressively later times depending on the wavenumber k of the mode. This interpretation is fully consistent with the quantum mechanical meaning of the Planck length, representing the shortest physical size of any causally connected region in the early Universe.

Looking to the future, this interpretation of $k_{\rm min}$ may offer clues concerning how to extend our current semi-classical description of the early Universe to scales below the Planck length, thereby heralding the initiation of an observationally-motivated quantum gravity theory. In concert with such ideas, we point out that, if the $R_{\rm h} = ct$ cosmology is the correct description of nature, the potential of the (non-inflationary) scalar field present just after the Big Bang is precisely known (Eq. 1). Interestingly, such exponential forms are well motivated in string theories, Kaluza-Klein cosmologies, and supergravity. This work may therefore offer a deeper, more meaningful development in these areas.

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REFERENCES

[1] Melia, F., *Astron. & Astrophys.*, **553** (2013) id. A76.
[2] Guth, A. H., *Phys. Rev. D*, **23** (1981) 347.
[3] Linde, A., *Phys.Lett. B*, **108** (1982) 389.
[4] Mukhanov, V. F. et al., *Phys. Rep.*, **215** (1992) 203.
[5] Aad, G. et al. (ATLAS Collaboration), *Phys. Lett. B*, **716** (2012) 1.
[6] Fileviez, P. P. & Patel, H. H., *Phys. Rev. D*, **79** (2009) 055024.
[7] Weir, D. J., *Phil. Trans. R. Soc. A*, **376** (2018) 20170126.
[8] Noble, A. & Perelstein, M., *Phys. Rev. D*, **78** (2008) 063518.
[9] Dolan, M. J. et al., *High-energy Physics*, **1210** (2012) 112.
[10] Barr, A. J. et al., *High-energy Physics*, **1502** (2015) 016.
[11] Melia, F., *European Phys. J. C Lett.*, **78** (2018) 739.
[12] Planck Collaboration, *Astron. & Astrophys.*, **in press, eprint arXiv:1807.06209** (2018) .
[13] Melia, F. & López-Corredoira, M, *Astron. & Astrophys.*, **618** (2018) A87.
[14] Liu, J. & Melia, F., *European Phys. J. C*, **submitted** (2019) .
[15] Brandenberger, R. & Martin, J., *Mod. Phys. Lett.*, **A16** (2001) 999.
[16] Niemeyer, J., *Phys. Rev. D*, **63** (2001) 123502.
[17] Kempf, A., *Phys. Rev. D*, **63** (2001) 083514.
[18] Easther, R. et al., *Phys. Rev. D*, **64** (2001) 103502.
[19] Bunch, T. S. & Davies, P.C.W., *Proc. R. Soc. A*, **360** (1978) 117.
[20] Martin, J. & Brandenberger, R. H., *Phys. Rev. D*, **63** (2001) 123501.
[21] Joras, S.E. & Marozzi, G., *Phys. Rev. D*, **79** (2009) 023514.
[22] Ashoorioon, A. et al., *Journal Cosm. and Astropart. Phys.*, **06** (2011) 034.
[23] Zhu, T. et al., *Phys. Rev. D*, **93** (2016) 123525.
[24] Hassan, S. F. & Sloth, M. S., *Nuc. Phys. B*, **674** (2003) 434.
[25] Chu, C.-S. et al., *Mod. Phys. Lett. A*, **16** (2001) 2231.
[26] Lizzi, F. et al., *J. High-energy Phys.*, **06** (2002) id.049.
[27] Brandenberger, R. & Ho, P.-M., *Phys. Rev. D*, **66** (2002) 023517.
[28] Kempf, A. & Niemeyer, J. C., *Phys. Rev. D*, **64** (2001) 103501.
[29] Kaloper, N. et al., *Phys. Rev. D*, **66** (2002) 123510.
[30] Easther, R. et al., *Phys. Rev. D*, **66** (2001) 023518.
[31] Brandenberger, R. & Martin, J., *Int. J. Mod. Phys. A*, **17** (2002) 3663.
[32] Danielsson, U. H., *Phys. Rev. D*, **66** (2002) 023511.
[33] Melia, F., *Mon. Not. R. Astronom. Soc.*, **382** (2007) 1917.
[34] Melia, F. & Abdelqader, M., *Int. J. Mod. Phys. D*, **18** (2009) 1889.
[35] Melia, F. & Shevchuk, A.S.H., *Mon. Not. R. Astronom. Soc.*, **419** (2012) 2579.
[36] Melia, F., *Frontiers of Phys.*, **11** (2016) 119801.
[37] Melia, F., *Frontiers of Phys.*, **12** (2017) 129802.
[38] Melia, F., *Mon. Not. R. Astronom. Soc.*, **481** (2018) 4855.
[39] Melia, F., *Class. Quant. Grav.*, **34** (2017) 015011.
[40] Abbott, L. F. & Wise, M. B., *Nucl. Phys.*, **244** (1984) 541.
[41] Lucchin, F. & Materrese, S., *Phys. Rev. D*, **32** (1985) 1316.
[42] Barrow, J., *Phys. Lett. B*, **187** (1987) 12.
[43] Liddle, A. R., *Phys. Lett. B*, **220** (1989) 502.
[44] Hollands, S. & Wald, R. M., *Gen. Rel. Grav.*, **34** (2002) 2043.