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General Quantum Theory No Axiom Presumption: II

----Measuring & Identical Theorems, Origins & Classifications of Entanglements and Solution to Crisis of Wave Collapse

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Abstract

This paper derives measurement and identical principles, then makes the two principles into measurement and identical theorems of quantum mechanics, plus the three theorems derived earlier, we deduce the axiom system of current quantum mechanics, the general quantum theory no axiom presumptions not only solves the crisis to understand in current quantum mechanics, but also obtains new discoveries. We deduce the general Schrödinger equation of any n particles from two aspects, and the wave function not only has particle properties of the complex square root state vector of the classical probability density of any n particles, but also has the plane wave properties of any n particles. Thus, the current crisis of the dispute about the origin of wave-particle duality of any n microscopic particles is solved. This paper displays the classical locality and quantum non-locality for any n particle system, shows entanglement origins, and discovers not only any n-particle wave function system has the original, superposition and across entanglements, but also the entanglements are of interactions preserving conservation or correlation, the three kinds of entanglements directly gives lots of entanglement sources. This paper discovers, one of two pillars of modern physics, quantum mechanics is a generalization (mechanics) theory of the complex square root (of real density function) of classical statistical mechanics. Thus, all current studies on various entanglements and their uses to quantum computer, quantum communications and so on must be further updated and classified by the three kinds of entanglements. Finally, this paper and our previous paper together solve the crisises of basses of quantum mechanics, e.g., wave-particle duality & the first quantization origins, quantum nonlocality, entanglement origins & classifications, wave collapse and so on.

Key words: quantum mechanics, operator, basic presumptions, wave-particle duality, principle of measurement, identical principle, superposition principle of states, entanglement origin, quantum communication, wave collapse, classical statistical mechanics, classical mechanics

I.Introduction

Quantum mechanics is the science relevant to small matters, e.g., explaining the behaviors of matters and their interactions on the scales of atoms, subatomic particles and so on. Classical mechanics explains matter and energy on scales familiar to human experience, and is still used in modern science [1].

Statistical mechanics is one key pillar for modern physics with a large number of degrees of freedom, and the theory is constructed on statistical methods and microscopic physical laws [2].

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Statistical mechanics can be used to explain the thermodynamic behaviors of large systems, and shows how the concepts from macroscopic observations, e.g., temperature & pressure, are relevant to microscopic states of fluctuating around an average state. It links thermodynamic quantities, e.g., heat capacity, to microscopic behaviors [3].

Resolving inconsistencies between the observed phenomena and classical theory resulted in two major physics revolutions: relativity theory and quantum mechanics [4]. Physicists discovered the limitations of classical physics, and roughly developed the main concepts of the quantum theory in the early decades of the 20th century [4].

Some aspects of quantum mechanics are usually counter intuitive and can be paradoxical [5], because their microscopic behaviors are quite different from larger scales. The famous quantum physicist Richard Feynman said: quantum mechanics deals with "nature as She is – absurd " [6]. The disputes of Albert Einstein and Niels Bohr were a series of public debates on quantum mechanics. Their disputes had been remembered because of their importance to scientific development. The debates with different opinions regarding quantum mechanics (but Bohr and Einstein had a mutual admiration lasting the rest of their lives) were recorded in refs. [7-10].

The disputes showed the highest viewpoints of scientific research in the first half of the twentieth century because the disputes called attention to some key elements of quantum theory, e.g., quantum non-locality, which is just the central to the modern understanding of the physical laws [11]. The debates and the lasting deeper debates have been continuing up to now [12,13], the fundamental probabilistic character of quantum measurement is definitively established [14].

Following the pioneers' steps [15-18], up to now, professional physicists have done various efforts in quantum theory and have met a series of some crisises of interpretations of basses of quantum mechanics [12,13,19-27], e.g., wave-particle duality origin and the first quantization interpretation, quantum nonlocality, quantum entanglement origins, wave collapse from quantum measurement and so on, ref.[28] and this paper want to solve these crisises by generalizing classical statistical mechanics to quantum mechanics and using relevant mathematical theories so that we not only can cancel all axiom presumptions of quantum theory, but also get real pictures of our physics world.

Especially, it is well-known that quantum mechanics is established on the five well-known basic axiom presumptions: wave function, operator, measurement, evolution and identical axiom presumptions, utilizing the five axiom presumptions, people can deduce quantum mechanics [29,30], ref.[28] has shown that the wave function axiom presumption, operator axiom presumption and evolution axiom presumption can be reduced as the wave function theorem, operator theorem and evolution theorem, respectively, this paper further proves that measurement axiom presumption and identical axiom presumption can be reduced as measurement theorem and identical theorem, respectively, so that we can both deduce general quantum theory cancelling all the axiom presumptions and solve these crisises and so on.

Especially, ref.[31] reviewed development of quantum Physics, ref.[32] discussed probability, information and statistical physics, and ref.[33] systematically studied fundamental principles of theoretical physics, concepts of quasiaverages, quantum protectorate and emergence.

Arrangements of this paper: Sect. 2 deduces the measuring theorem; Sect. 3 shows identical theorem; Sect.4 displays superposition theorem of states, entanglement origins and three kinds of entanglements; Sect. 5 gives discussion and application; Sect. 6 is summary and conclusions.

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2.Measuring theorem

It is generally believed that the classical limit of quantum mechanics is classical mechanics, which is a direct extension of classical mechanics to quantum mechanics. However, classical mechanics does not have the problem of measuring collapse, everything is certain, while quantum mechanics does, so there is no way to understand the problem of measuring collapse in quantum mechanics. Thus, we consider that the classical limit of quantum mechanics first is classical statistical mechanics, and then macroscopic limit of classical statistical mechanics is classical mechanics, instead of going straight to classical mechanics. Otherwise, it will lead to many problems that are difficult to understand, for example, it will also lead to the difficult crisis of the current interpretation of quantum mechanics measurement collapse and so on [12,13].

Collapse effect in classical statistical mechanics [2,3]: A density state means that a particle is in a density state of multiple superposition possibilities, but once an observer has observed it, it can only fall into a density state of specific possibility. Although each measurement must collapse to some possible density state, a large number of observations reveal a probability density distribution, which just shows that the matter is described by probability density.

In the current quantum mechanics [29,30], the fourth axiom presumption is the measurement axiom presumption: Quantum systems are generally in the superposition state of various eigenstates of wave functions (i.e., eigenwave functions). When measured, it will lead to the collapse of the wave function of the quantum system and make the wave function of the quantum system jump to a certain eigenstate with a certain probability.

We now reduce the fourth axiom presumption (the measurement axiom presumption) to the measurement theorem, which is to prove the measurement theorem from classical statistical mechanics.

When operators $F_i(\vec{r},t)$ for all i are one in the average value expression $\bar{F}(t)=\int_V \sum_{i=1}^{n}\psi_i^*(\vec{r},t)F_i(\vec{r},t)\psi_i(\vec{r},t)d\vec{r}$, e.g., eq.(2.3) in ref.[28], in classical statistical mechanics, the expression is reduced as density integration $\bar{F}(t)=\int_V \sum_{i=1}^{n}\psi_i^*(\vec{r},t)\psi_i(\vec{r},t)d\vec{r}=1$, thus similar to research of ref.[28], we can use Lagrange multipliers λ_i ($i=1,2,3,\ldots$) to build up a variational system

$$\delta\bar{\hat{A}}'=\delta\left(\int\phi^+\hat{A}\phi d\tau-\int\phi^+\lambda\phi d\tau\right) \quad , \tag{2.1}$$

so that we can consider the deviation $\bar{\hat{A}}'$ between $\int\phi^+\hat{A}\phi d\tau$ and $\int\phi^+\lambda\phi d\tau$, where λ is a Lagrange multiplier diagonal matrix, $d\tau$ may be different integration measures, e.g., coordinate space, momentum space and so on. Therefore, using variational theory to deduce

$$\delta\bar{\hat{A}}'=\int\delta\phi^+(\hat{A}-\lambda)\phi d\tau+\int[(\hat{A}-\lambda)\phi]^+\delta\phi d\tau+\int\phi^+(\delta\hat{A}-\delta\lambda)\phi d\tau=0 \quad . \tag{2.2}$$

Using eq.(2.2) and independence of complex functions $\delta\phi^+$ and $\delta\phi$, it follows that

$$(\hat{A}-\lambda)\phi=0, \quad i.e., \quad \hat{A}\phi=\lambda\phi \quad . \tag{2.3}$$

They also indicate that: the eigenvalues of all operators (i.e., Lagrange multipliers) are the eigenvalues corresponding to the extreme values of the variational system in the transition from classical statistical mechanics to quantum mechanics [28].

Ref.[28] uses last term to deduce general Hellmann-Feynman theorem in transition from classical statistical mechanics to quantum mechanics. This shows that ordinary Hermitian operators and eigenvalues also satisfy this important theorem.

In particular, the Hellmann-Feynman theorem is established under the condition that not only the eigenvalue equations of all operators are corresponding to the extreme values of their variational systems, but also under the condition that the variational system is taken as the extreme values.

Therefore, the whole variational system takes the extreme value among all possibly taking values, and the equation derived when taking this extreme value is the eigenvalue equation in the transition from classical statistical mechanics to quantum mechanics.

Similar to deduction of eq.(2.1) for generalizing to any n particle system, we can extensively use the below variation

$$\delta\bar{\hat{\mathrm{H}}}'=\delta\int_{-\infty}^{\infty}\phi^{\dagger}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\hat{\mathrm{H}}(T_{total},V_{total})\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}d\vec{p}_j$$

$$-\delta\int_{-\infty}^{\infty}\phi^{\dagger}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)E\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}d\vec{p}_j$$

$$=\int_{-\infty}^{\infty}\delta\phi^{\dagger}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)[\hat{\mathrm{H}}(T_{total},V_{total})-E]\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}d\vec{p}_j$$

$$+\int_{-\infty}^{\infty}\phi^{\dagger}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)[\hat{\mathrm{H}}(T_{total},V_{total})-E]\delta\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}d\vec{p}_j+$$

$$\int_{-\infty}^{\infty}\phi^{\dagger}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)[\delta\hat{\mathrm{H}}(T_{total},V_{total})-\delta E]\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}d\vec{p}_j \tag{2.4}$$

where $E=E_1+E_2+...+E_n$ is the total energy eigenvalue**,** according to eq.(2.4) and using the independence of complex functions $\delta\phi^{+}$ and $\delta\phi$, i.e., the deduced eigen-equation (2.3) can be extensively analogously written as

$$\begin{aligned}&\hat{\mathrm{H}}(T_{total},V_{total})\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)=\hat{\mathrm{H}}(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,\vec{r}_1,\vec{r}_2,...,\vec{r}_n)\phi(\vec{p}_1,\\&\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)=E\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\end{aligned} \tag{2.5}$$

where $\hat{H}$ is the total Hamiltonian operator in **c**lassical statistical mechanics, that is, in eq.(2.2), $d\tau$ can be taken as the measure of the momentum representation. They also indicate that the eigenvalue equation of all operators is the eigenvalue equation corresponding to the extreme value of their variational system.

Similar to ref.[28], then the general Hellmann-Feynman theorem for any n variables in the transition from classical statistical mechanics to quantum mechanics can be deduced from the last term in eq.(2.4), which will be studied in another paper due to limit of paper's length.

This relation implies that ordinary Hermitian operators and their eigenvalues also satisfy this important relation. In particular, the Hellmann-Feynman theorem for any n particles is established under the condition that not only the eigenvalue equations of all operators are the eigenvalue equations corresponding to the extreme values of their variational system, but also it's the result of taking the extreme value of the variational system for the system of any n particles.

Using $e^{\sum_{j=1}^{n} i(\vec{p}_j\cdot\vec{r}_j - tE_j)/\hbar}/(2\pi\hbar)^{2n}$ to multiply eq.(2.5) and making integration of $\Pi_{j=1}^{n} d\vec{p}_j dE_j$ from minus infinity to positive infinity, namely, the Fourier integral transformation of the momentum and energy of the plane wave of any n particles, we deduce

$$\frac{1}{(2\pi\hbar)^{2n}}\int_{-\infty}^{\infty} E\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n} e^{i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}d\vec{p}_j dE_j$$
$$=\frac{1}{(2\pi\hbar)^{2n}}\int_{-\infty}^{\infty}[T(\vec{p}_1,\vec{p}_2,...,\vec{p}_n)+\sum_{j<k}V(\vec{r}_j-\vec{r}_k)]\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n} e^{i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}d\vec{p}_j dE_j$$
$$=\int_{-\infty}^{\infty}[T(-i\hbar\nabla_1,-i\hbar\nabla_2,...,-i\hbar\nabla_n)+\sum_{j<k}V(\vec{r}_j-\vec{r}_k)]\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n}\frac{e^{i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}d\vec{p}_j dE_j}{(2\pi\hbar)^{3n/2}}$$
$$=i\hbar\frac{\partial}{\partial t}\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\hat{\mathrm{H}}(\hat{\vec{p}}_1,\hat{\vec{p}}_2,...,\hat{\vec{p}}_n,\vec{r}_1,\vec{r}_2,...,\vec{r}_n)\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t) \quad , \quad (2.6)$$

where we have deduced $\hat{\mathrm{H}}(\hat{\vec{p}}_1,\hat{\vec{p}}_2,...,\hat{\vec{p}}_n,\vec{r}_1,\vec{r}_2,...,\vec{r}_n)=i\hbar\frac{\partial}{\partial t}$, and we have considered that $T(\vec{p}_1,\vec{p}_2,...,\vec{p}_n)$ may be generally expanded as sum of series of powers about $(\vec{p}_1,\vec{p}_2,...,\vec{p}_n)$, e.g., $\sum_{j_1,j_2,...,j_n=1}^{n} a_{j_1 j_2\cdots j_n}\vec{p}^{j_1}\vec{p}^{j_2}...\vec{p}^{j_n}$, and

$$\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\frac{1}{(2\pi\hbar)^{2n}}\int_{-\infty}^{\infty}\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)\prod_{j=1}^{n} e^{i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}d\vec{p}_j dE_j$$
$$=\frac{1}{(2\pi\hbar)^{3n/2}}\int_{-\infty}^{\infty}\varphi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,t)\prod_{j=1}^{n} e^{i\vec{p}_j\cdot\vec{r}_j/\hbar}d\vec{p}_j . \quad (2.7)$$

Eq.(2.7) for any n particles is $\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)$ about Fourier transformation of plane wave energy $E_1,E_2,...,E_n$ and momentum $\vec{p}_1,\vec{p}_2,...,\vec{p}_n$ for any n particles, i.e., probabilistic wave $\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ is deduced via that general complex square root state vector $\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)$ of the probability density of any n particles in classical statistical mechanics is projected to the plane wave basic vector $e^{\sum_{j=1}^{n} i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}$ of any n particles, and the superposition integral from minus infinity to positive infinity is needed.

This makes $\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n, t)$ not only have the characteristic of the state vector of the complex square root of the probability density of any n particles, but also have the characteristic of the plane wave of any n particles, that is, make $\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n, t)$ have the characteristics of the state vector of wave-particle duality of any n particles at the same time.

From the above derivation, it can be seen that eq.(2.5) is the eigenvalue equation related to the complex square root function of the classical probability density function of any n particles in the classical statistical mechanics, the state function $\phi(\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n)$ is derived from the complex square root of the probability density function of any n particles in the complex domain. These $\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n$ are all the classical physics quantities of any n particles in classical statistical mechanics, and they reflect the particle properties of any n particles.

When, in four dimensional spacetime, taking Fourier transformation (2.6) of eq.(2.5) for any n particles, that is, to project eq.(2.5) onto the four-dimensional spacetime of the state vector $e^{\sum_{j=1}^{n} i(\vec{p}_j \cdot \vec{r}_j - tE_j)/\hbar}$ of the plane wave function of any n particles and to integrate them, we get Schrӧdinger equation (2.6) for any n particles.

Schrӧdinger equation (2.6) of any n particles not only reflects the particle property of any n particles, but also displays the wave property of any n particles, in other words, eq.(2.5) only reflects the particle properties of any n particles, eq.(2.6) of any n particles is transformed into Schrӧdinger equation in quantum mechanics which reflects particle-wave duality of any n particles.

The complex square root function $\phi(\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n)$ of the classical probability density of any n particles in the complex number field is transformed into the complex square root function $\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n, t)$ of the probability density of any n particles in quantum mechanics by eq.(2.7), it not only has the particle characteristic of the state vector of the complex square root of the probability density of any n particles, but also has the wave characteristic of any n particles. In other words, state vector $\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n, t)$ of any n particles at the same time has the characteristics of wave-particle duality.

Thus, the crisis of the dispute over the origin of the wave-particle duality of any n microscopic particles, e.g., a crisis of a fundamental interpretation of quantum mechanics has been debated for nearly a century, is solved.

Consequently, eq.(2.5) and eq.(2.6), respectively, show the classical locality and quantum non-locality for any n particle system, $\phi(\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n)$ in eq.(2.7) (from the complex square root of the classical probability density of any n particles in the complex domain) and eq.(2.7) directly, respectively, shows the classical locality and quantum non-locality for any n particle system.

According to classical statistical mechanics [2,3], when a system is not measured, it evolves according to its own classical statistical mechanics. When measured in the sample space of classical statistical mechanics, any physical quantity measurements are collapsing at a certain probability to observe the state, because we have already taken the micro-particle system state described by a general function of complex square root of real density function (It can be called the complex square root of the probability density function, which is projected onto the plane wave and doing their Fourier integration), so we have entered a special system different from the classical statistical mechanics description of the system, we call it a quantum mechanics system. If the quantum system is in the eigenstate of the probability wave function (i.e. the eigenwave function), the result of measuring the quantum mechanics quantity is the eigenvalue of the quantum system.

If the quantum system is not in the eigenstate of wave function, the measurement will lead to the wave function collapse of quantum systems (because the measurement must act to the system through the measurement instrument (this effect is ignored in classical statistical physical measurements), which causes the system to change to a certain state), namely the measurement for extracting the information of the quantum system must cause some effects on the quantum isolated system, the measurement makes the wave function of the quantum system jump to some eigenstate with certain probability, and this probability can be calculated strictly according to quantum mechanics.

These measurement processes are nonlocal, decoherent, stochastic and irreversible even from the classical statistical point of view, because classical measures cause the taking eigenvalue corresponding to the variational system's choosing extreme value in process transforming classical statistical mechanics to quantum mechanics and in the quantum mechanics so that people can measure the eigenvalues. On the bases of the nonlocal, decoherent, randomness and irreversibility of these classical statistical mechanics (which have been understood no problem in classical statistical mechanics [3]), the nonlocal, decoherent, randomness and irreversibility of plane waves of any n particles are superimposed. Therefore, these properties of nonlocal, decoherent, random and irreversible measurement collapse caused by quantum mechanics measurement collapse are thoroughly proved and understood in the systems.

Therefore, we derive the fourth axiom presumption of quantum mechanics, namely the measurement axiom presumption, from the classical statistical mechanics, that is, it should be reduced to the measurement theorem, i.e., we have proved the measurement theorem from classical statistical mechanics, and the quantum theory is just the current quantum mechanics.

3. Identical Theorem

In classical statistical mechanics, when the identical particles of a system have indiscernible property, the physical state of the system composed of identical particles will not be changed due to the exchange of identical particles, and the particles with all intrinsic properties such as the same mass, charge, spin and isospin can be called identical particles [2,3].

In classical mechanics, in general, it is always possible to distinguish different particles from different orbits of particle movement. When it is impossible to distinguish the characteristics of different particles from different orbits of particle movement, and when the state of the microscopic particle system is described by the general complex function of the complex square root of real density function, the wave function $\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ is obtained (in sect. 2) by

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integrating the weight $e^{\sum_{j=1}^{n} i(\vec{p}_j\cdot\vec{r}_j - tE_j)/\hbar}$ of the plane wave of any n particles from minus infinity to positive infinity of momentum and energy $(\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n)$ with the function $\phi(\vec{p}_1, \vec{p}_2, ..., \vec{p}_n, E_1, E_2, ..., E_n)$ of the complex square root of the classical probability density, then, in the above studies, we have proved that the system begins to enter a special description system that is different from the classical statistical mechanics system, namely, the system is called as quantum mechanics. Therefore, in the present quantum mechanics, each particle corresponds to a probability wave, which is the probability state vector of the general complex function obtained by taking the complex square root of the classical density function and projecting it onto the plane wave for integral. It's well known that waves always overlap as they travel, and that plus the identity make it impossible tell which probabilistic wave belongs to which particle, namely, the identical particle is indistinguishable in quantum mechanics.

In fact, for the indistinguishability of identical particles, it's actually classical statistical mechanics where people can (or cannot) distinguish different particles from different orbitals of their motions. Since all waves will overlap in the process of propagation, and the identical property makes it impossible distinguish which probability wave belongs to which particle. For the indistinguishability of identical particles in quantum mechanics, because at the same time the state of the microscopic particle system is described by the general complex function of complex square root of the real density function, and which is projected to the plane probability wave with integral. And the system composed of identical particles has symmetry: the Hamiltonian operator of the identical particle system has the invariance of commutative symmetry (due to the indistinguishability of identical particles), and then the indistinguishability of identical particles in quantum mechanics is derived directly from classical statistical mechanics. So now we should not continuously call it an identical axiom presumption, but an identical theorem.

For the multi-particle system, we have obtained eq.(2.6) which is Schrödinger equation of the multi-particle system in quantum mechanics, in which we may have

$$\hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2, ..., \hat{\vec{p}}_m; \vec{r}_1, \vec{r}_2, ..., \vec{r}_m) = \sum_{i=1}^{m} T_i(\hat{\vec{p}}_i) + \frac{1}{2}\sum_{i,j=1, i\neq j}^{m} V(\vec{r}_i - \vec{r}_j) \quad . \tag{3.1}$$

According to the identical theorem, when making the exchange $\hat{P}_{ji}\hat{P}_{ij}$ of any two particles in Schrödinger equation (2.6) of a multi-particle system in quantum mechanics, Schrödinger equation (2.6) remains unchanged, we deduce $P_{ji}P_{ij} = 1$, where we have used

$$\hat{P}_{ij}\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_i, ..., \vec{r}_j, ...,, \vec{r}_m, t) = P_{ij}\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_j, ..., \vec{r}_i, ...,, \vec{r}_m, t) \tag{3.2}$$

and the identical theorem requires Hamiltonian invariance

$$\hat{P}_{ij}\hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2, ..., \hat{\vec{p}}_i, ..., \hat{\vec{p}}_j, ..., \hat{\vec{p}}_m; \vec{r}_1, \vec{r}_2, ..., \vec{r}_i, ..., \vec{r}_j, ..., \vec{r}_m) = \hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2, ..., \hat{\vec{p}}_j, ..., \hat{\vec{p}}_i, ..., \hat{\vec{p}}_m; \vec{r}_1, \vec{r}_2, ..., \vec{r}_j, ..., \vec{r}_i, ..., \vec{r}_m)$$
$$= \hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2, ..., \hat{\vec{p}}_i, ..., \hat{\vec{p}}_j, ..., \hat{\vec{p}}_m; \vec{r}_1, \vec{r}_2, ..., \vec{r}_i, ..., \vec{r}_j, ..., \vec{r}_m) \;, \tag{3.3}$$

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The Hamiltonian with identical particles is not changed by identical particle exchange.

Since there is a general solution $P_{ji}P_{ij} = \left|P_{ij}\right|^2 = 1$ (i.e., $P_{ji} = P_{ij}*$), thus the general solution is

$$\hat{P}_{ij}\psi(\vec{r}_1,\vec{r}_2,...,i,...,j,...,\vec{r}_m,t) = e^{i\alpha_{ij}}\psi(\vec{r}_1,\vec{r}_2,...,j,...,i,...,\vec{r}_m,t) \quad , \tag{3.4}$$

where $\alpha_{ij} = \nu\Delta\varphi_{ij} = -\alpha_{ji}$, $\Delta\varphi_{ij}$ is the azimuth of particles i and j around the middle point in the line connecting particles i and j. Since it is the exchange of the two particles, this azimuth can only be $\pm\pi$, $\nu = \eta + 2n$ are real parameters, $0 \le \eta \le \eta_{\max}$ ($\eta_{\max}$ is the possible maximum spin), and n is an integer.

Using operator $\hat{P}_{ij}$ to act to both sides of Schrὂdinger equation (2.6) for a multi-particle system and by using eqs.(3.2) and (3.3), we can obtain that both sides of Eq.(3.4) are multiplied by a constant $e^{i\alpha_{ij}}$. One can remove this constant, then we deduce that using operator $\hat{P}_{ij}$ to act on Schrὂdinger equation (2.6) of a multi-particle system is invariant. That is, the general multi-particle system remains unchanged under the exchange of any two particles, but its probability wave function can have a variation of eq.(3.4).

There are two extremes: (i) when $\eta = 0$, exchanging any two particles in the wave function is symmetric (for example, identical particles with an integer number of spins are called bosons, such as photons etc); (ii): when $\eta = 1$, exchanging any two particles in the wave function is antisymmetric (for example, identical particles with half an odd number of spins are called fermions, such as electrons, protons, neutrons and so on). However, their Hamiltonian operator, namely Schrὂdinger equation (2.6) of their multi-particle system, has the invariance of exchange symmetry. In general, $0 \le \eta \le \eta_{\max}$, that is, one can generically derive that the exchange of any two particles in the wave function of the many particle system so that the wave function is of general fractional symmetry. And their Hamiltonian operator, i.e., their system has the invariance of the exchange symmetry. Bose and Fermi statistical symmetries are special examples of general fractional statistics. Therefore, η is a parameter reflecting the characteristics of the particle system, namely, spin, it can be fractional spin, such as fractional spin may have the quantum Hall effect. In fact, the research in this section also extends the research in literature [28].

Thus, we derive the fifth axiom presumption of quantum mechanics from classical statistical mechanics, the identical axiom presumption is reduced as identical theorem, i.e., we proved the identical theorem from classical statistical mechanics.

4. State Superposition Theorem, All Entanglement Origins and Three Kinds of Entanglements

Using the final line of eq.(2.6), we have

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\hat{\mathrm{H}}(\hat{\vec{p}}_1,\hat{\vec{p}}_2,...,\hat{\vec{p}}_n,\vec{r}_1,\vec{r}_2,...,\vec{r}_n)\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)\,, \tag{4.1}$$

$$\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\sum_a c_a\psi_a(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)\quad , \tag{4.2}$$

where $\psi_a(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ for all a satisfy just expression of the final line of eq.(2.6), c_a are general complex constants and satisfy the normalization condition $\sum_a |c_a|^2=1$. Therefore, we naturally deduce expressions (4.1) and (4.2) of the principle of superposition of states. Because the whole process is naturally deduced out and is not based on the fundamental presumption, the principle of superposition of states is reduced as the theorem of superposition of states.

Because $\psi_a(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ for all a satisfy just expression of the final line of eq.(2.6) and have the original entanglement stratifying Schrӧdinger equation (i.e., dynamic system) of any n particle, namely, $\psi_a(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ is obtained in eq.(2.7), i.e., from the superposition of the classical complex square root state vector $\phi(\vec{p}_1,\vec{p}_2,...,\vec{p}_n,E_1,E_2,...,E_n)$ of the probability density of any n particles, which are, in the same time, projected to the plane wave basic vector $e^{\sum_{j=1}^n i(\vec{p}_j\cdot\vec{r}_j-tE_j)/\hbar}$ of any n particles and then doing the integral from minus infinity to positive infinity.

This makes $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ have the properties of the state vector with the complex square root of the classical probability density of any n particles, and have the properties of the plane wave of any n particles, in other words, it has the properties of wave-particle duality of any n particles at the same time.

Especially, $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ may compositely have both the original entanglement and different entanglements, that is, when $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ cannot be expressed as

$$\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\Pi_{j=1}^n\psi_j(\vec{r}_j)\quad , \tag{4.3}$$

then the many particle quantum system has both original entanglement and different entanglements, the original entanglement is coming from the interactions between the probability waves obtained in eq.(2.7).

For the composite entanglements, further considering theorem of superposition of states, when $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ cannot be written as

$$\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)=\sum_a c_a b_{aj}\Pi_{j=1}^n\psi_j(\vec{r}_j)=(\sum_a c_a)\Pi_{j=1}^n\psi_j(\vec{r}_j), b_{aj}=1\,, \tag{4.4}$$

then the many particle quantum system has the superposition entanglement that is coming from the theorem of superposition of states, i.e., from the interactions of the superposition waves.

The superposition entanglement more directly gives a lot of entanglement sources (e.g., more directly by c_a in eq.(4.2)) , which thus give lots of chances for developing quantum communications, quantum computer, parallel quantum works (e.g., network [31-33]) and so on.

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Further, the many particle quantum system has the cross entanglement between the original and the superposition entanglements, which are coming from the interactions of the waves of the original and the superposition entanglements.

In fact, in all current quantum communications, quantum computer and so on, entanglement is the key source of all the theories, if no entanglement, there must not be all the current quantum communication theories and so on. Because we have generally deduced general quantum theory, we generally give the realistic entanglement origins. Therefore, all the studies on various entanglements and their uses must be further studied and classified by the three kinds of entanglements, otherwise, all studies on various entanglements are not perfect and exact.

5. Discussions and applications

From another aspect, substituting the deduced operators $\hat{\vec{p}}_j = -i\hbar\nabla_j \, (j=1,2,...,n)$ (in eq.(2.6)) into the deduced Hamiltonian operator $\hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2,..., \hat{\vec{p}}_n, \vec{r}_1, \vec{r}_2,..., \vec{r}_n)=i\hbar\partial/\partial t$ (see below line of eq.(2.6)), we achieve $\hat{\mathrm{H}}(-i\hbar\nabla_1, -i\hbar\nabla_2,..., -i\hbar\nabla_n, \vec{r}_1, \vec{r}_2,..., \vec{r}_n) = i\hbar\partial/\partial t$, utilizing the deduced Hamiltonian operator $\hat{\mathrm{H}}(-i\hbar\nabla_1, -i\hbar\nabla_2,..., -i\hbar\nabla_n, \vec{r}_1, \vec{r}_2,..., \vec{r}_n) = i\hbar\partial/\partial t$ to act on a general wave function $\psi(\vec{r}_1, \vec{r}_2,..., \vec{r}_n, t)$ of any n particles, we derive a general Schrödinger equation of any n particle systems

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r}_1, \vec{r}_2,..., \vec{r}_n, t) = \hat{\mathrm{H}}(-i\hbar\nabla_1, -i\hbar\nabla_2,..., -i\hbar\nabla_n, \vec{r}_1, \vec{r}_2,..., \vec{r}_n)\psi(\vec{r}_1, \vec{r}_2,..., \vec{r}_n, t) \quad , \quad (5.1)$$

which show that our investigations are consistent with all the relevant studies for this article.

Considering the collapse of classical statistical mechanics by rolling coins and regular hexahedral dices, since, in classical mechanics, the motion states of any n coins and regular hexahedral dices are determined by the solutions of Newton's equations and the initial and boundary conditions. If its initial and boundary conditions are completely given, we get a completely deterministic description with no collapse. If the initial and boundary conditions are given in the form of probability, the solutions must also appear in the form of probability, so the resulting form of motion is also in the form of probability.

Since the thrower (observing with free will) usually cannot make himself exact decisions in automatically and instantaneously selecting a set of initial conditions and boundary conditions to roll the coin or dice, in other words, because of the instantaneous collapse into the conditions and state of being the involuntary instantaneous selection of a probability in a space where all the possibilities add up to one, then this system instantaneously collapses to a certain motion state, i.e., automatically obtains a specific state.

For simplicity, we first consider flipping a coin. The probability space formed by flipping a coin is the analogous to the probability space formed by neutral π^0 meson decaying into positive and negative electrons. In other words, in flipping a coin, when one side is up, the other side must be down. In the similar way, when one measures a system of positive and negative electrons, because of entanglement, when one measures the spin of one electron is up, the spin of the other must be down. The positive and negative electron systems are separable, and the two sides of a coin are tied together, which is the stronger entanglement and is independent of the separation of coordinates, and people can see this be the extreme case of quantum entanglement.

Therefore, it is an independent space coordinate system with overall symmetry. This system can be expressed as a function system of momentum representations independent of spacetime coordinates. If we define the system flipping a coin: heads up is spin 1/2 and heads down is negative spin 1/2, then the two systems have a conservation of spin.

Considering when the coin is small enough that the Planck constant effect cannot be ignored, and further considering when the two sides of the coin can be separated in such a small case, in this time, we need to introduce spacetime coordinates to represent the separate states. And when the mass of this coin is further reduced to have the property of a plane wave (from wave-particle duality) with spacetime coordinates and superimposed for integration on the process, as eq.(2.7), the object described by classical statistical mechanics is transformed into the object described by quantum mechanics. The effect of flipping the quantum coin is mathematically identical with the effect measuring the collapse effect of the system of positive and negative electrons. So we get the conservation of spin angular momentum in a quantum coin toss. That is, no matter how far apart they are, they have to preserve the conservation of the quantum spin angular momentum through this entanglement of the quantum coin toss, e.g., for the conserved quantum angular momentum system of two particles, the angular momentum state of the quantum system is in a superposition of all possible states. When the angular momentum of one particle is measured, the quantum state of the other particle will collapse to the other certain state in a way that instant exceeds the speed of light, so as to maintain the conservation of spin angular momentum of the whole system.

Especially, the moment the coin is flipped, a set of initial conditions and boundary conditions are selected from the set of the initial conditions and boundary conditions corresponding to heads up or down, the system collapses. The operation motion variation of the later coin is the entanglement motion satisfying Newtonian mechanics with the initial and boundary conditions.

In the quantum case, there is entanglement before measurement, and the collapse during the measurement is the instantaneous collapse that the entanglement is maintained. In the classical case it could be separated, that is, collapse and then entanglement to maintain the conservation or correlation of the system. For quantum mechanics, because quantum mechanics wants to go back to the limits of classical statistical physics, by the transition from classical statistical physics to quantum mechanics, we find that the instantaneous collapse measured in quantum mechanics is actually the instantaneous hypervelocity entanglement to maintain the conservation or correlation of the system. So it's actually made up of the two processes.

By comparing the classical with the quantum, we can understand the essence of both classical and quantum collapses and entanglements.

Thus, we conclude that the velocities of such quantum collapse and entanglement are instantaneous and infinite big, rather than just the super light velocity of collapsing and entangling to maintain the corresponding conservation or correlation.

Consequently, we discover that entanglements are of interactions (among all wave function state vectors) that preserves conservation and correlation. The two examples above illustrate two extreme entanglements in order to preserve conservation and correlation. The entangled interaction in order to maintain some kind of conservation or correlation is a new quantum phenomenon, not a known quantum phenomenon. We call this new quantum state interaction as the interaction of entangled quantum states to maintain some kind of conservation or correlation in the system, for short, entanglement interaction of quantum states.

Therefore, collapse phenomena in quantum mechanics should also include those in classical statistical mechanics, because the classical limit of quantum mechanics is classical statistical mechanics.

6. Summary and conclusions

Following ref.[28], this paper continues to generalize the density function in classical statistical mechanics to a product of a general complex function for any n particles and its complex Hermitian conjugate function, naturally derives the last two axiom presumptions in the five axiom presumptions of quantum mechanics in literature [29,30]: the measurement principle and the identical principle, and naturally makes the two axiom presumptions into the measurement theorem and the identical theorem of quantum mechanics. The two deduced basic theorems not only solve the crisis that has been very difficult to understand in current quantum mechanics, but also obtain important new physics and new discoveries.

Therefore, this paper and ref.[28] together not only naturally deduce the axiom system of quantum mechanics [29,30], but also build up general quantum theory no axiom presumption.

This paper uses Lagrange multipliers λ_i ($i=1,2,3,...$) to build up a general variational system, the whole variational system takes the extreme value among all possible values, and the equations derived when taking this extreme value are the eigenvalue equations in the transition from classical statistical mechanics to quantum mechanics.

Furthermore, we deduce the general Schrὂdinger equation of any n particle systems from two aspects, from which we see that the some keys of deducing the general Schrὂdinger equation of any n particle systems are the deductions of the momentum operators $\hat{\vec{p}}_j = -i\hbar\nabla_j$ ($j = 1, 2, ..., n$) and the Hamiltonian operator $\hat{\mathrm{H}}(\hat{\vec{p}}_1, \hat{\vec{p}}_2, ..., \hat{\vec{p}}_n, \vec{r}_1, \vec{r}_2, ..., \vec{r}_n) = i\hbar\partial/\partial t$.

The basic interpretation of quantum mechanics has been debated for nearly a century, but the crisis of the dispute about the origin of wave-particle duality of the microscopic particles of any n particles is solved in this paper.

The properties of quantum mechanical measurements of resulting in nonlocal, decoherent, random and irreversible measurement collapse are thoroughly proved and understood in this paper. Namely, the fourth axiom presumption for measurement can be given both in the same way in classical statistical mechanics and by superimposing the nonlocal, decoherent, random and irreversible collapse effects of **plane waves** of any n particles. Therefore, people can intrinsically show and understand the fourth axiom presumption for measurement.

Therefore, we derive the fourth axiom presumption of quantum mechanics from classical statistical mechanics, i.e., the measurement axiom presumption, that is, it should be reduced to the measuring theorem. Namely, we prove the measuring theorem from classical statistical mechanics.

Because all waves always overlap in the process of propagation, and the homogeneity makes it impossible to distinguish which probability wave belongs to which identical particle. Therefore, there is the indistinguishability of identical particles in quantum mechanics. This is because the state of the micro-particle system is simultaneously described by the deduced general complex function $\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n, t)$ (2.7) with wave-particle duality and, in the same time, the system of identical particles has symmetry: the Hamiltonian operator of an identical particle system has the invariance of commutative symmetry (due to the indistinguishability of identical particles), which can be derived directly from classical statistical mechanics. So now we should not call it an identical axiom presumption, but an identical theorem.

Therefore, we derive the fifth axiom presumption of quantum mechanics from classical statistical mechanics, namely, the universal identical axiom presumption is reduced to the universal identical theorem. Namely, we prove the identical theorem from classical statistical mechanics.

This paper shows states' superposition theorem and entanglement origins. Namely, we naturally deduce expressions (4.1) and (4.2) of the principle of superposition of states. Because the whole process is naturally deduced out and is not based on the fundamental presumption, the principle of superposition of states is reduced as the theorem of superposition of states.

Especially, this paper discovers that any n-particle wave function $\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ (2.7) may have the original entanglement satisfying Schrödinger equation (i.e., dynamical system of any n particles), that is, when $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ cannot be expressed as eq.(4.3), then the many particle quantum system has the original entanglement and the other entanglements, the original entanglement is coming from the interactions between the inner components of probability waves (2.7).

Further considering theorem of superposition of states, when $\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_n,t)$ cannot be written as eq.(4.4), then the many particle quantum system has the general superposition entanglement that is coming from the theorem of superposition of states.

The many particle quantum system further has the cross entanglement from the interactions of the waves of the original entanglement and the superposition entanglement.

Therefore, for a general quantum system of any n (>1) particles, this paper, for the first time, discovers three kinds of entanglements: original, superposition and across entanglements. The three kinds of entanglements directly give lots of entanglement sources. Thus, all the studies on various entanglements and applications need to be further studied and classified by the three kinds of entanglements. Otherwise, all the current studies on various entanglements aren't perfect or exact.

Analogous to taking the square root of Klain-Gooden equation, we get Dirac equation Fermi system, in terms of the studies in this paper and ref.[28], we discover that one of two pillars of modern physics, **quantum mechanics actually is** a generalization mechanic theory of the complex square root of real density function of classical statistical mechanics, for short, quantum mechanics actually is just a generalization theory of the complex square root of classical statistical mechanics, which is both key new physics and a revolutionary discovery.

Finally, ref.[28] and this paper solve a series of crisises of basses of quantum mechanics, e.g., wave-particle duality origin and the first quantization origin, quantum nonlocality, quantum entanglement origins, wave collapse from quantum measurement and so on, following the new deduced general quantum theory, a lot of works related to quantum communications, quantum computer and so on can be further supplied, classified and updated.

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