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Entropy, Information and Symmetry, Ordered is Symmetrical, II: System of Spins in the Magnetic Field

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Abstract: The second part of the paper develops the approach, suggested in the *Entropy* **2020**, *22*(1), 11; https://doi.org/10.3390/e22010011 , which relates ordering in physical systems to their symmetrizing. Entropy is frequently interpreted as a quantitative measure of "chaos" or "disorder". However, the notions of "chaos" and "disorder" are vague and subjective to a much extent. This leads to numerous misinterpretations of entropy. We propose to see the disorder as an absence of symmetry and to identify "ordering" with symmetrizing of a physical system; in other words, introducing the elements of symmetry into an initially disordered physical system. We explore the initially disordered system of elementary magnets exerted to the external magnetic field $\vec{H}$. Imposing symmetry restrictions diminishes the entropy of the system and decreases its temperature. The general case of the system of elementary magnets demonstrating the *j*-fold symmetry is treated. The interrelation $T_j = \frac{T}{j}$ takes place, where T and T_j are the temperatures of non-symmetrized and *j*-fold-symmetrized systems of the magnets correspondingly.

Keywords: entropy; symmetry; ordering; elementary magnets; magnetic field; *j*-fold symmetry

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1. Introduction

Entropy is a key concept in the characterization of ordering in physics [1-2], chemistry [3], biology [4-6] and engineering [6]. At the same time the notion of entropy remains one of the most abstract and least intellectually transparent quantities of physics [7-9]. The widespread illustrative interpretation of entropy is "the measure of disorder" in macroscopic systems built from a large number of particles [10]. However equating entropy with disorder was criticized recently [8]. In the first part of our manuscript we suggested that that "ordering" may be strictly related to the symmetry, and may be quantified by symmetry [11]. In turn, "chaos" or "disorder" are understood as an absence of symmetry [11]. We already illustrated this suggestion with the simplest binary 1D and 2D systems built of elementary magnets, which can point only up or down, fixed in a

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space and aligned [11]. They formed a binary, non-interacting system. We demonstrated that introducing elements of symmetry necessarily diminishes the entropy, which is true for true for 1D and 2D systems built of the elementary magnets [11]. In the present work we generalize the approach, reported in ref. 11 for the initially disordered systems of elementary magnets, embedded into the magnetic field $\vec{H}$ and symmetrized by the *j*-fold symmetrizing procedure.

2. Symmetry and entropy of binary magnetic systems embedded into the magnetic field

2.1. Symmetrizing and entropy of 1D systems exposed to the magnetic field $\vec{H}$

Consider first a binary 1D system built of non-interacting magnets (spins) $\vec{\mu}$ illustrated in **Figure 1A**. We assume that there are *N* separate and distinct sites fixed in a space and aligned as shown in **Figure 1A** [11]. Attached to each site is an elementary magnet $\vec{\mu}$, which can point only up or down. The system of magnets is embedded into the magnetic field $\vec{H} \neq 0$, leading to the orientation of spins. The potential energy of the single elementary magnet in the magnetic field is given by:

$$U_1 = -\vec{\mu} \cdot \vec{H} \tag{1}$$

Magnetic field leads to the orientation of magnets; consider the configuration of magnets demonstrating the spin excess *2m* defined by Eq. 2 (the number *N* is supposed to be even):

$$\frac{1}{2}N + m - \left(\frac{1}{2}N - m\right) = 2m, \tag{2}$$

corresponding to the configuration where $\frac{1}{2}N + m$ of magnets are oriented “up” and $\frac{1}{2}N - m$ are oriented “down”. The total potential energy of the system of magnets characterized by the spin excess *2m* is given by [12-14]:

$$U(2m) = -2m\mu H \tag{3}$$

The entropy *S* of this system is given by [12-14]:

$$S(N, m) = k_B lng(N, m), \tag{4a}$$

$$g(N, m) \cong 2^N \left(\frac{2}{\pi N}\right)^{1/2} exp\left(-\frac{2m^2}{N}\right), \tag{4b}$$

$$S(N, m) \cong k_B \left[Nln2 - \frac{1}{2} ln\frac{2}{\pi N} - \frac{2m^2}{N}\right] = S_0(N) - \frac{k_B U^2}{2N\mu^2 H^2} \tag{4c}$$

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where $g(N,m)$ is the multiplicity function, i.e. the number of states having the same value of m [12-14]; $S_0(N) = k_B\left[Nln2 - \frac{1}{2}ln\frac{2}{\pi N}\right]$. Eqs. 4b-c hold for $m \gg 1; N \gg 1; \frac{|m|}{N} \ll 1$.

Now let us restrict the possible configurations of elementary magnets by introducing the symmetry axis, shown with the dashed line in **Figure 1B**. After introducing the symmetry axis only the symmetric configurations of the elementary magnets are available, as depicted in **Figure 1B**, this implies the decrease in the number of "states" available for the symmetrized system to $g(\frac{N}{2},m)$. The multiplicity function for the symmetrized, ordered, binary, non-interacting system is given by [12-14]:

$$g(\frac{N}{2},m) \cong 2^{N/2}\left(\frac{2}{\pi(\frac{N}{2})}\right)^{1/2} exp\left(-\frac{4m^2}{N}\right), \tag{5}$$

Hence, the entropy of the symmetrized, ordered, binary, non-interacting system is given by:

$$S_2(N,m) = k_B \ln g\left(\frac{N}{2},m\right) \cong S_{02}(N) - \frac{k_B U^2}{N\mu^2H^2}\ , \tag{6}$$

where the subscript "2" indicates the presence of the axis of symmetry of the second order and $S_{02}(N) = k_B\left[\frac{N}{2}ln2 - \frac{1}{2}ln\frac{2}{\pi\frac{N}{2}}\right]$ takes place. Combining Eqs. 3-6 and trivial transformations yield:

$$S - S_2 = k_B\left[\frac{N}{2}ln2 + \frac{2m^2}{N}\right] = k_B\left[\frac{N}{2}ln2 + \frac{U^2}{2N\mu^2H^2}\right] > 0 \tag{7}$$

It is seen that introducing symmetry decreases the entropy, whatever are the spin excess $2m$, energy of the system U and the value of the magnetic field $\vec{H}$ (recall that Eq. 7 holds for $m \gg 1; N \gg 1; \frac{|m|}{N} \ll 1$). The larger spin excess $2m$ the stronger a decrease in entropy emerging from symmetrizing. Thus, the generalization of the results reported in ref. 11 is achieved.

Consider the temperatures of the original and symmetrized systems of magnets; Eqs. 4c and 6 yield {12-14]:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_N = -\frac{k_B U}{N\mu^2H^2} \tag{8a}$$

$$\frac{1}{T_2} = \left(\frac{\partial S_2}{\partial U}\right)_N = -\frac{2k_B U}{N\mu^2H^2} \tag{8b}$$

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Recall that $U < 0$ takes place. It is recognized that the interrelation $T_2 = \frac{1}{2}T$ takes place; in other words, the symmetrized system of magnets is "colder" than the non-symmetrized one, when the spin excess and the energy of the systems are the same. This result is intuitively quite expectable.

2.2. Symmetrizing and entropy of the 2D systems possessing axes of symmetry of various orders (*j*-fold symmetry)

Consider the 2D system of elementary magnets possessing the axes of symmetry of the *j*-th order (**Figure 2** depicts the sample system of spins with $j = 6$). Again the number of the available states for the j-fold-symmetrical system is given by $g(\frac{N}{j}, m)$. Indeed, keeping the *j*-fold symmetry demands simultaneous re-orientation of *j* magnets. The entropy of such a *j*-fold system of magnets is supplied, in turn, by:

$$S_j = k_B lng(\frac{N}{j}, m) \cong k_B \ln \{2^{N/j} \left(\frac{2j}{\pi N}\right)^{1/2} exp\left(-\frac{2jm^2}{N}\right)\} = S_{0j}(N, j) - \frac{2k_B jm^2}{N} =$$

$$= S_{0j}(N, j) - \frac{k_B jU^2}{2N\mu^2 H^2} \qquad (9a)$$

$$S_{0j}(N, j) = k_B \left[\frac{N}{j} ln2 + \frac{1}{2} ln\left(\frac{2j}{\pi N}\right)\right] \qquad (9b)$$

The initial entropy of the 2D non-symmetrical binary system of magnets is given by Eqs. 4 (2D location of elementary magnets does not matter). Combining Eqs. 9 and 4 yields:

$$S - S_j = k_B(j-1)\left[\frac{N}{j} ln2 + \frac{2m^2}{N}\right] = k_B(j-1)\left[\frac{N}{J} ln2 + \frac{U^2}{2N\mu^2 H^2}\right] > 0 \qquad (10)$$

Again introducing symmetry decreases the entropy, whatever are the order of the symmetry axis *j*, the spin excess *2m*, energy of the system *U* and the value of the magnetic field $\vec{H}$ (recall that Eq3. 9a-10 hold for $m \gg 1; N \gg 1; \frac{|m|}{N} \ll 1$). It is easily seen that:

$$\frac{\partial S}{\partial j} = -k_B\left(\frac{Nln2}{j^2} - \frac{1}{2j} + \frac{2m^2}{N}\right) \cong -k_B\left(\frac{Nln2}{j} + \frac{2m^2}{N}\right) < 0 \qquad (11)$$

Eq. 11 holds when the condition $\frac{N}{j} \gg 1$ takes place, and it means that increase in the order of the symmetry axis *j*, decreases the entropy of the system. It is also seen from Eq. 9a, that Eq. 12 is true:

$$\frac{1}{T_j} = \left(\frac{\partial S_2}{\partial U}\right)_{N,j} = -\frac{k_B jU}{N\mu^2 H^2}, \qquad (12)$$

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where T_j is the temperature of the system of magnets, possessing the axis of symmetry of the order of j, i.e. the j-fold symmetry. Comparing Eqs. 12 and 8a supplies:

$$T_j = \frac{T}{j} \tag{13}$$

Again symmetrizing of the system of magnets "cools" it; moreover, the larger is the value of j the cooler is the system. The presented results support the idea that ordering (understood as symmetrizing) necessarily decreases the multiplicity of the system and consequently decreases the entropy.

Conclusions

We conclude that introducing of elements of symmetry orders the system of elementary magnets exposed to the external magnetic field and consequently diminishes its multiplicity, entropy and temperature. The idea is illustrated with the binary system built from elementary magnets $\vec{\mu}$ embedded into magnetic field $\vec{H}$. Symmetrizing of the initially disordered system of N magnets diminishes the multiplicity function $g(N,m)$, where $2m$ is the spin excess, and consequently the entropy $S(N,m)$. The simplest 1D exemplification of the binary systems is treated. Introducing two-fold symmetry decreases the entropy, whatever are the spin excess $2m$, energy of the system U and the value of the magnetic field $\vec{H}$. The general case of the system of elementary magnets demonstrating the j-fold symmetry and exposed to the magnetic field $\vec{H}$ is addressed. Symmetrizing decreases the multiplicity and the entropy of the system whatever is the value of j, and $\frac{\partial S(j)}{\partial j} < 0$ is true. The interrelation $T_j = \frac{T}{j}$ takes place, where T and T_j are the temperatures of non-symmetrized and j-fold-symmetrized systems of the magnets correspondingly. Thus, symmetrizing necessarily "cools" the discussed system.

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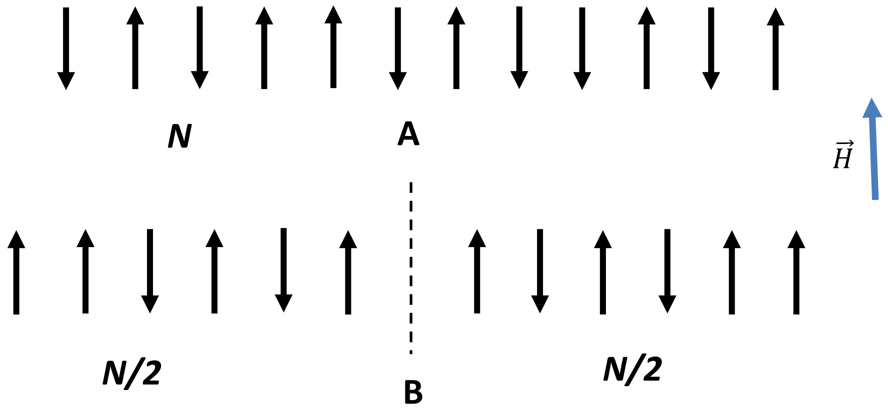

Figure 1. A. The binary 1D system of N non-interacting elementary magnets is shown, exposed to the external magnetic field $\vec{H} \neq 0$. The spin excess of the system is given by $2m = \frac{1}{2}N + m - \left(\frac{1}{2}N - m\right)$. **B.** The axis of symmetry shown with the dashed line restricts the number of available configurations of magnets.

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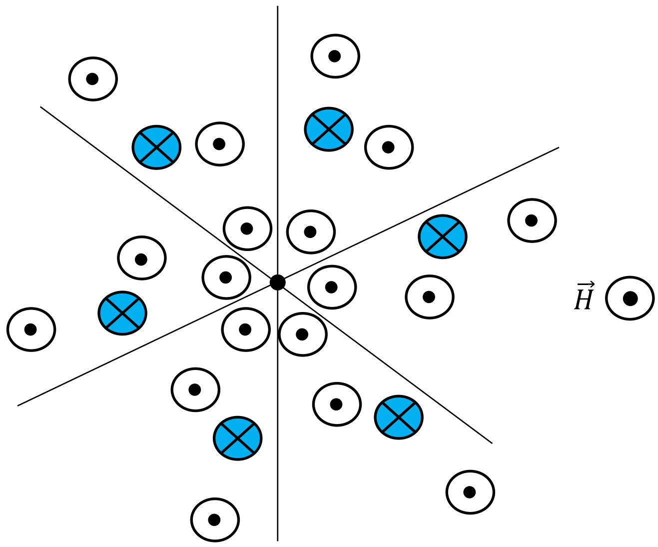

Figure 2. System of the elementary magnets possessing the axis of symmetry of the order of six embedded into magnetic field $\vec{H}$ is shown. Magnetic moments and the magnetic field $\vec{H}$ are normal to the image plane. Keeping the 6-fold symmetry demands simultaneous re-orientation of six magnets (for example re-orientation of the magnets, marked in the Figure with a blue color).