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# Enhanced Flexible Algorithm for the Optimization of Slot Filling Factors in Electrical Machines

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**Abstract:** The continuous development in the field of industrial automation and electric mobility has led to the need for more efficient electrical machines with high power density. The improvement of electrical machines slot filling factors is one of the measures to satisfy these requirements. In recent years, this topic has aroused greater interest in the industrial sector, since the evolution of the winding technological manufacturing processes allows an economically sustainable realization of ordered winding arrangements, rather than random ones. Moreover, the manufacture of electrical machines windings must be preceded by an accurate design phase in which it is possible to evaluate the maximum slot filling factor obtainable for a given wire shape and for its dimensions. For this purpose, this paper presents an algorithmic approach for the evaluation of maximum slot filling factors in electrical machines under ideal geometric premise. In particular, this algorithm has a greater degree of flexibility with respect to the algorithm approaches found in the literature, since the study has been extended to round, rectangular and hexagonal wire sections. Furthermore, the slot filling factor calculation was carried out both for standard and non-standard slots. The algorithmic approach proposed can be considered as an additional useful tool for the fast design of electrical machine windings.

**Keywords:** Electrical motors, Slot Filling factor, Optimization Algorithm, Windings, Magnetic Wire, Filling Factor Optimization.

## 1. Introduction

The development of more and more efficient electrical machines has become a topic of interest for various industrial sectors such as automation or electric traction. In particular, high efficiency, high power density and cost-effective manufacturing are required in the automotive industry [1-3]. One possible solution to meet these requirements is to optimize the copper filling factor of stator winding [4-11]. In particular, a high copper filling factor involves a more rational and efficient use of copper with economic benefits and improved energy savings. Therefore, the optimization of slot filling factor is a key point in winding technology. The improvement of the slot filling factor depends mainly by the winding pattern schemes and the adopted winding manufacturing process. The simplest type of winding pattern is the so-called “random winding”. In this case, the random winding process is sustainable for mass production and it is characterized by low manufacturing requirements. The main advantage is represented by the high production speed while the disadvantage consists generically of lower filling factors. The highest possible filling factor is achieved by the “orthocyclic winding pattern” for round wires. This winding pattern presents a high packing density of wire but the winding process is more complex and so more costly than random

winding process. In particular, the orthocyclic winding process requires very high manufacturing requirements to obtain an ordered wire positioning within each slot. Another possible winding pattern is the “layer winding”, where the wires are uniformly arranged in layers [12]. This winding pattern allows to obtain higher filling factors than those of the random winding pattern. The choice of typology winding pattern depends on the functions and the design requirements of the electrical machine. Therefore, orthocyclic or ordered winding structures are appropriate for high power density applications.

In the past, the realization of commercial solutions that allow economically sustainable manufacturing of distributed windings with ordered structures was very difficult due to the high economic burdens. The automated winding process technologies available for making distributed windings are: the insertion winding technology, the flyer winding technology and needle winding technology [13]. In recent years, these winding process technologies have undergone a great technological evolution that has allowed to reduce manufacturing costs and windings process time with an ordered structure [13-14]. In [12], a new and innovative needle winding method that allows shifted layer winding structures for distributed round wire applications is described, thus increasing significantly the copper filling factor. In this regard, it is of considerable interest to accurately evaluate the slot filling factor obtainable during the design phase of electrical machines. In the previous century, this task was performed by means of manual graphic analysis or by testing the stator of the electrical machine during the pre-production phase. In order to carry out this process, the resources in terms of money, time and technical staff are not indifferent [15-17]. Therefore, a preliminary analysis of the maximum possible value of filling factor is an important step forward for the design of electrical machines windings for a given slot, wire shape and their dimensions.

In the scientific literature, there are several definitions of slot filling factors and generally, an electrical slot filling factor  $f_{cu}$  and a mechanical slot filling factor  $f_{me}$  are defined. In this study, the electrical slot filling factor  $f_{cu}$  is given by the ratio between the total wire copper cross-section ( $N_w A_{cu}$ ) and the total slot cross-section  $A_{slot}$ . Instead, the mechanical slot filling factor  $f_{me}$  is defined as the ratio between the total wire cross-section ( $N_w A_w$ ) and the effective slot area  $A_{eff}$ , which is defined as the difference between the total slot cross-section  $A_{slot}$  and the area occupied by slot insulation. The two filling factors can be described by the following mathematical equations:

$$f_{cu} = \frac{N_w A_{cu}}{A_{slot}} \quad (1)$$

$$f_{me} = \frac{N_w A_w}{A_{eff}} \quad (2)$$

where  $N_w$  is the number of wires contained within a slot,  $A_{cu}$  is the maximum copper cross-section of a single wire (useful cross-section) and  $A_w$  is the maximum cross-section of a single wire (including its insulation layer). For the purpose of this work, the electrical slot filling factor values are considered and discussed. This paper proposes an algorithmic approach for the preliminary determination of the slot filling factors. The objective is an extension of work done in [18] with reference to various types of wires and slots. The algorithm, described here, allows determining the maximum slot filling factor for a given slot and wire shape. More in detail, with a given cross-section geometry and given wire dimensions, the algorithm allows calculating the maximum number of wires that can be placed inside the slot. In this work, both wire and slot insulation are considered, because together they cause a reduction of the useful slot area. In detail, three different wire geometries, namely round, rectangular and hexagonal, are taken into consideration and results are discussed.

This paper is structured as follow: Section II describes the state of art of optimization algorithms of the slot filling factors, Section III describes the algorithmic approach proposed and Section IV illustrates several cases of study that have been carried out and the relevant obtained results.

## 2. State of Art of Slot Filling Factors Optimization Algorithms

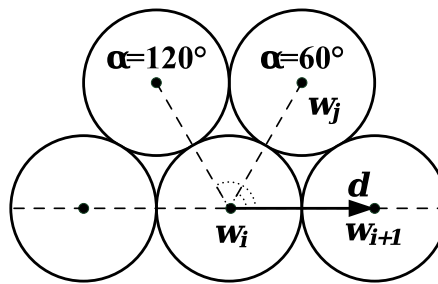
At present time, the scientific literature does not include many works regarding the optimization of the slot filling factors with algorithmic approaches. Furthermore, the algorithmic approaches found in the literature refer to the case where the wire has a round cross-section. The optimal winding pattern is invariant respect to the axis perpendicular to the stator cross-section. Therefore, the determination of the optimal winding pattern can be carried out with a bi-dimensional approach. A family of widespread algorithms is that of the orthocyclic windings algorithms (OWA). In [19] the results of this kind of algorithm are discussed. A study was carried out on the variation of the value of the mechanical and electrical slot filling factors when the radius of the circular conductor varies. In the first phase the algorithm sets the coordinates of the center ( $x_i$ ;  $y_i$ ) of the first wire:

$$w_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \quad (3)$$

Then the wires of the same layer are plotted and the distance between the centers of two adjacent wires is equal to the diameter of the wire  $d$ . Once the first layer is finished, the wires coordinates of the upper layer are determined through the following relation:

$$w_j = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \cdot d + w_i \quad (4)$$

with  $\alpha$  equal to  $60^\circ$  or  $120^\circ$  (Figure 1).



**Figure 1** Round wires with orthocyclic distribution.

The algorithm ends when it is not possible to find a new wire position whose area intercepts the slot contour or its insulation contour: then the slot is full.

Another algorithm approach, named Filling Factor Estimation Algorithm (FFEA), is described in [15]. Goal of this algorithm is to reduce the design time of the winding by estimating the filling factors. In this case, the first wire is placed centrally at the slot bottom. The next wire is placed by checking all valid positions along a circle around the first wire with a radius equal to the distance  $d_w + t_w$ , where  $d_w$  and  $t_w$  are the diameter of the wire and the safety distance or the minimum gap between two wires, respectively. For each new wire positioned, the algorithm checks whether there are overlaps with the contour of the slot taking into account the presence of the slot insulation. If the new position is valid, the algorithm places the new neighbour wire and continues going around until all positions have been tried. The procedure ends when it is not possible to position wires that do not overlap the slot profile. The result of this algorithm is a random positioning of the wires forming the winding. Since the value of the determined filling factor depends on the initial position of the first wire, the procedure is repeated for several different initial positions.

Another interesting approach is described in [19]. This algorithm is based on the basic rules of the two algorithms described above. This algorithm is called the Needle Winding Simulation Algorithm (NWSA) because the objective is to optimize the positioning of each wire in order to simulate the process of needle winding. In this case, for the positioning of wire, a cost function is taken into consideration accounting for the position and the downward force acting on the wires. In the algorithm, there are constraints to avoid overlapping between wires and between the wire and the slot

profile. Optimal positioning is achieved by identifying the global minimum of the cost function. Authors use a genetic algorithm to search the global minimum in order to reduce convergence times.

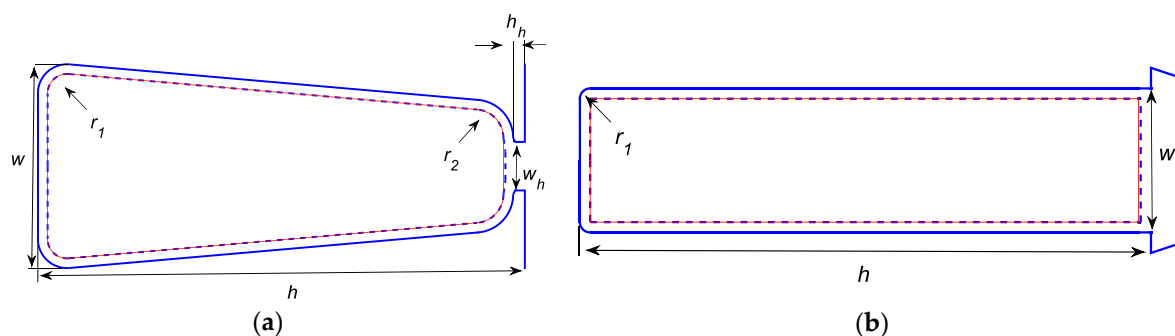
Beyond the approaches described above, there are the dense packing algorithms also called the wire inflation algorithms (WIA). The goal of these algorithms is to find the maximum radius of spheres within a given boundary and their corresponding coordinates when the number of spheres is fixed. In [20] an algorithm is described that simulates a system of billiard spheres, within a limited space, whose radius is made to grow until it reaches a state of the blockade of the system. This algorithm approach is used for dense packing of spheres in circle, triangles and hexagons space [21-23]. This algorithm performs an event based on physical simulation of a billiard system where the coordinates, the number and the speed of the spheres are set in advance as input data. Events are represented by the collisions of each sphere with other spheres or with the borders of the system. Depending on the event type (either sphere-obstacle or sphere-sphere interaction) an elastic impact is performed and the new velocities are calculated. At the increasing radii of the spheres, an eventual jamming occurs resulting in a dense packing. For the purposes of maximizing the slot filling factor, the objective is to maximize the number of magnetic wires, with given radius, within the slot. In this sense in [19], the Authors, in order to make a comparison with the NWSA, have modified the described approach. In particular, the wires are understood as charged particles exerting a Coulomb force on each other, leading to movement of the spheres during the simulation. The results of the comparison are widely discussed.

### 3. Proposed Enhanced Algorithmic Approach

The basic rules of the algorithm proposed in this paper are mainly inspired to the approach proposed in [15] and treating in detail the winding patterns with ordered structures [18]. The algorithm is based on a general approach that is valid for different stator slot and for different wire shapes. In detail, for round and hexagonal wires, the algorithm focuses on the windings with orthocyclic arrangement of the wires within the slot. Moreover, the algorithm has been designed so as to be able to define some critical aspects such as the coordinates of the first wire, the possible presence of slot insulation, the positioning of the wire parallel to the flank or to the bottom of the slot and any safety distances between the wires forming the winding. In this way, the algorithm presents a high degree of flexibility that allows investigating the slot filling factors obtainable in many cases of study. The algorithm has been implemented in the Matlab environment and it is described in detail below.

#### 3.1. Slot geometrical features definition

Compared to the works found in the literature, where simplified models of slot have been taken into account, in this work both a standard slot (STSL) model and the non-standard (NSTSL) ones are used. The slot profiles are defined with the presence of insulation sheet and, therefore, its thickness  $d_{ins}$  is taken into account. In general, the slot insulation sheet is pre-folded and does not substantially modify the slot cross-section profile available for the positioning of the wires. The geometries and the related contour data of a standard slot, typically used for wires, and a non-standard one are shown in Figure 2 (a) and in Figure 2 (b) [16], respectively.



**Figure 2** Cross-section of the standard slot (a) and non-standard slot with contour parameters.

Generally, the slot profiles are supplied with CAD drawings by the manufacturers and, therefore, the geometrical parameters are easy to be determined. The dimensions of each slot are known and are reported in Table 1 and Table 2.

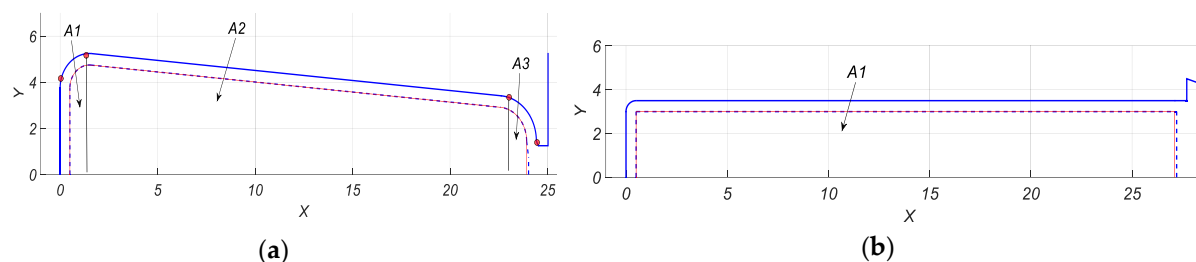
The slot profiles are defined in the  $xy$  reference frame by means of characteristic points connected through lines and arcs. The implementation of the geometric model of the slot profile is carried out as described in [18]. The cross-sections of slots are calculated in a numeric way by the trapezium rule in the same way of [15]. In particular, the slots areas are divided into different sections whose characteristic points are known (Figure 3 (a-b)). The calculated cross-sections show a maximum deviation of less than one percent from the cross-section given by specifications.

**Table 1** Standard slot geometric parameters.

Parameter	Value [mm]+
$w$	10.360
$h$	24.930
$w_h$	2.500
$h_h$	0.500
$r_1$	1.500
$r_2$	1.971
$d_{ins}$	0.500

**Table 2** Non-standard slot geometric parameters.

Parameter	Value [mm]+
$w$	10.360
$h$	24.930
$r_1$	1.500
$d_{ins}$	0.500



**Figure 3** Half cross-section of the standard slot (a) and non-standard slot (b) divided into sections.

### 3.2. Magnet wire data

In this study, investigations on round, rectangular and hexagonal wires has been carried out. The geometrical data of the round and rectangular wires used in this paper are obtained from [24-25]. In these standards, the nameplate data, the insulation degree, tolerances and maximum dimensions allowed for each wire shape are reported. For rectangular shape, the standard defines the shape of the wire with rounding corners. Instead, there is not an international standard for hexagonal wires. The use of the latter could be an innovative idea, as they can substantially improve the filling factors. Improvements must be compared with the production costs which, compared to the past, have been reduced thanks to the technological evolution of the wires manufacturing processes [13], [17]. Therefore, the geometric data of the hexagonal wires will be hypothesized. The geometrical reference quantities for each wire shape are defined below for:

#### 1. Round wire geometrical data:

- $d_{cu}$  diameter wire without insulation;
  - $d_{max}$  maximum diameter of wire with insulation;
  - $x_c$  and  $y_c$  coordinates of wire center.
2. Rectangular wire geometrical data:
- $L_{1cu}$  width of the rectangular wire without insulation;
  - $L_{2cu}$  height of the rectangular wire without insulation;
  - $L_{1max}$  maximum width of the rectangular wire with insulation;
  - $L_{2max}$  maximum height of the rectangular wire with insulation;
  - $r_{corner}$  corner radius;
  - $x_c$  and  $y_c$  coordinates of the wire center.
3. Hexagonal wire geometrical data:
- $r_{cu}$  radius of circumference circumscribed to the hexagon without insulation;
  - $r_{max}$  maximum radius of circumference circumscribed to the hexagon with insulation;
  - $x_c$  and  $y_c$  coordinates of the wire center.
  - $phi$  ( $\phi$ ) rotation angle of the hexagon.

Figure 4 shows the wire cross-sections with the related contour data.

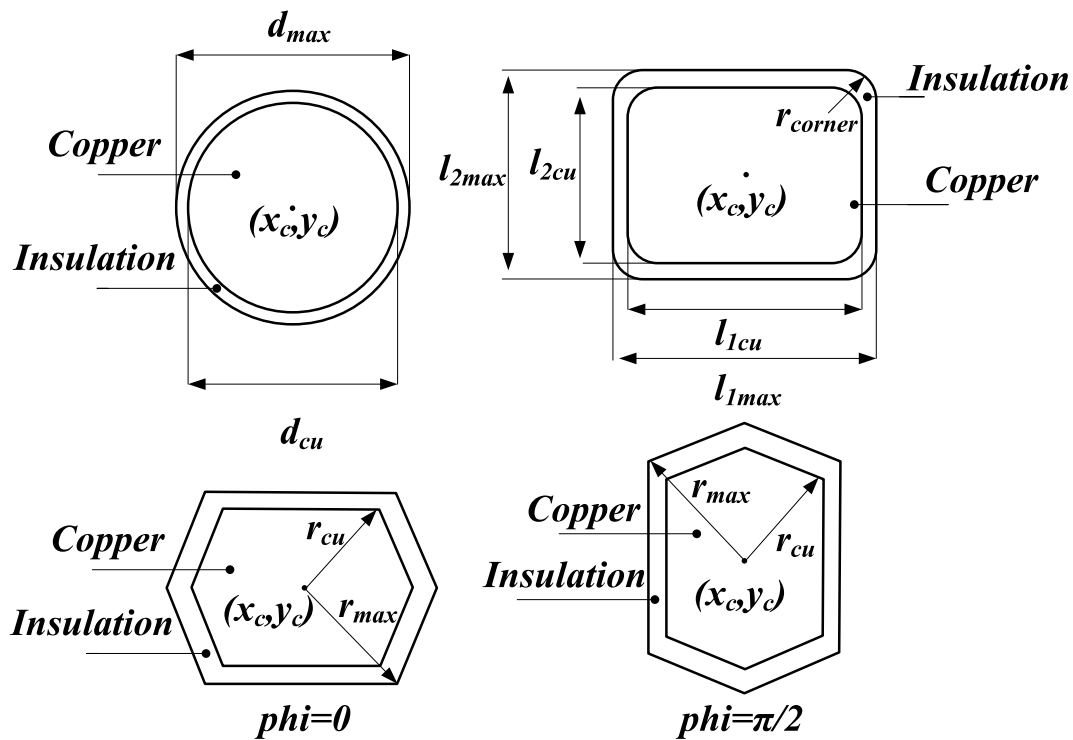


Figure 4 Wires cross-sections with contour data.

### 3.3. Constraints for wires distribution and placement

In the algorithm, the slot models have been implemented in a  $xy$  plane with the bottom or the ground of each slot parallel to the  $y$ -axis. Each slot profile has been divided into several reference sections Figure 3 (a-b). These sections are referred to a one-half slot. The distribution and placement conditions of wires are defined for each section and applied in a specular way for each half of the slot. In order to avoid overlapping between the wire and the slot profile and between adjacent wires, several constraints are taken into account. Therefore, for each wire that must be positioned within the slot profile, the following gaps are considered and investigated:

- distance between the wire and a part (arc or line) of the slot insulation sheet profile;
- distance between the new wire and an existing wire.

In Figure 5 the distances between the slot profile and the round and rectangular wires are shown. In each region of the slot profile, the minimum of the various distances is evaluated. In particular, for round wires, the distances are defined with respect to the center of the wire and it is imposed that

$$\min\{|d_1|, |d_2|, \dots, |d_n|\} \geq r_{max} \quad (5)$$

whereas in the case of rectangular and hexagonal wires, these distances are evaluated with respect to the vertices of each wire shape (Figure 6) and it is imposed that

$$\min\{|d_1|, |d_2|, \dots, |d_n|\} \geq 0 \quad (6)$$

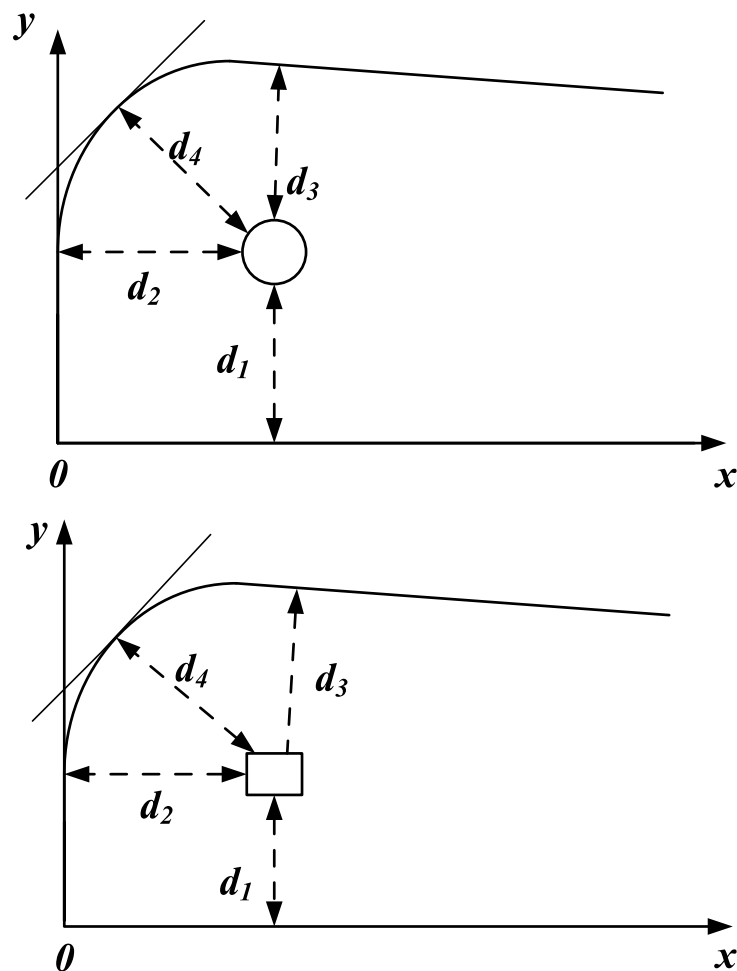


Figure 5 Distances between the wires and the slot profile.

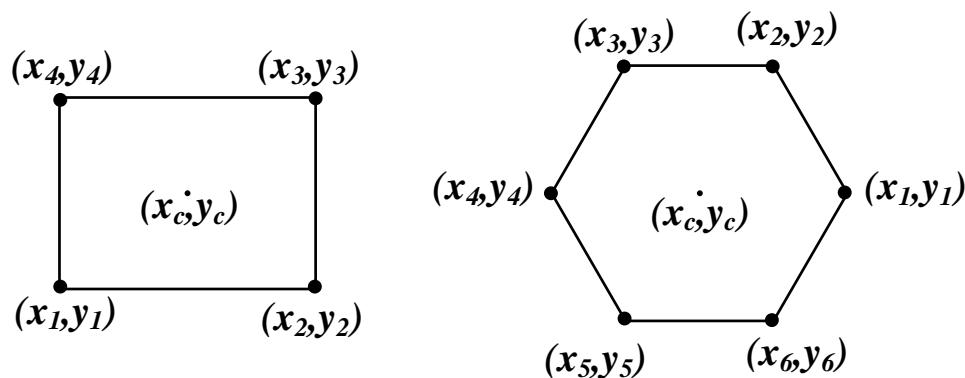


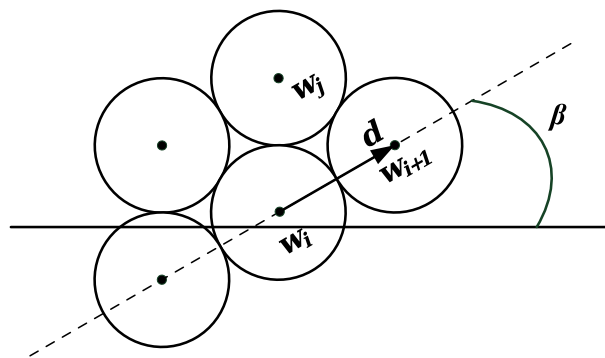
Figure 6 Characteristic points of the rectangular and hexagonal wires.



Regarding the distance between wires, it is possible to add an additional safety distance  $t_w$ . The winding positioning pattern adopted in this work is the orthocyclic one that can be obtained in the case of round and hexagonal wires. Instead, an ordered arrangement was taken into consideration for rectangular shaped wires. Moreover, it is possible to vary the angle formed between the line joining the centers of the wires and the horizontal one of the  $xy$  plane (Figure 7). In this case, the new arrangement is obtained by applying the following coordinate transformation to the center of wires, for round wire, and also to the vertices for rectangular and hexagonal wires:

$$\begin{pmatrix} x_{inew} \\ y_{inew} \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} \quad (7)$$

In this sense, it is possible to evaluate the value of the slot filling factors in the case where the wires are arranged parallel to the flank or the bottom of the slot. This study has been performed with particular attention on round and rectangular wires, since in the hexagonal wire case, as described above, it is possible to set the rotation angle.



**Figure 7** Rotation of the orthocyclic distribution.

### 3.3. The algorithm procedure

As described above, the algorithm has been designed in such a way as to arbitrarily define the coordinates of the center of the first wire. In this way, the algorithm can be executed several times in order to evaluate the filling factors when the coordinates of the center of the first wire vary. In this procedure, the coordinates of the first wire center  $(x_{w1}, y_{w1})$  are selected in such a way that it is positioned at the lower wedge (i.e. the lower rounded corner) of the slot:

$$x_{w1} = x_{init} + d_{ins} \quad (8)$$

$$y_{w1} = y_{init} - \frac{w}{2} \quad (9)$$

where  $d_{ins}$ ,  $x_{init}$ ,  $y_{init}$  are the slot insulation foil thickness and the initial wire coordinates, respectively, which are chosen in such a way as to avoid overlaps with the slot profile. In order to place the next wires, the cross-section of the slot is divided into a grid formed by  $i$  columns and  $j$  rows. The maximum number of columns  $i_{max}$  and rows  $j_{max}$  is calculated according to the slot and wire dimensions. For round, rectangular and hexagonal wires these values are calculated by means the following relationships, respectively:

$$i_{max} = \text{round} \left( \frac{h}{\frac{\sqrt{3}}{2} d_{max}} \right) \quad (10)$$

$$j_{max} = \text{round} \left( \frac{w}{d_{max}} \right) \quad (11)$$

$$i_{max} = \text{ceil} \left( \frac{h}{L_{1max}} \right) \quad (12)$$



$$j_{max} = \text{ceil}\left(\frac{w}{\frac{\sqrt{3}}{2}L_{2max}}\right) \quad (13)$$

$$i_{max} = \text{ceil}\left(\frac{h}{\frac{\sqrt{3}}{2}r_{max}}\right) \quad (14)$$

$$j_{max} = \text{ceil}\left(\frac{w}{\frac{3}{2}r_{max}}\right) \quad (15)$$

where  $\text{round}(x)$  and  $\text{ceil}(x)$  are functions that round each element of  $x$  to the nearest integer and to the nearest integer greater or equal to that element  $x$  itself, respectively. The distribution of the wires occurs through two for loops that change the indices  $i, j$  in order to position the wires along with the whole slot profile. In the case of round and hexagonal wire, in order to obtain an ortocyclic winding pattern, the coordinates of the successive wire are calculated as previously described in Section II. The rectangular case is widely described in [13]. The positioning of the wires is considered valid only if it meets the overlapping conditions described above. In short, the algorithm works with the following steps:

1. The algorithm asks as input data the wire shape, the type of positioning and the slot profile to be used;
2. The geometric dimensions of the wire and the slot, the initial coordinates of the first wire and the value of the safety distance are defined in an input file;
3. The algorithm proceeds by plotting the slot profile;
4. Subsequently, the algorithm proceeds to position the wires and for each of them it checks the overlapping conditions;
5. The algorithm calculates the profile of the slot containing the wires, the number of wires positioned and the value of the electrical and mechanical filling factors.

#### 4. Cases of study

In order to validate the proposed algorithm, several cases of study have been performed. This study has been carried out both for the STSL and for the NSTSL and for each wire shape. From international standards [24-25], it is possible to deduce the nominal dimensions, the insulation grades, the tolerances and the outer maximum dimensions for each typology of wire. In this work, the largest allowed outer dimensions are adopted to consider the worst case for the evaluation of slot electrical filling factor. An insulation grade<sup>1</sup> 3 and a grade 2 for the round-shaped wire and for the rectangular-shaped one have been taken into consideration, respectively. The dimensions of the hexagonal wire have been deduced assuming the cross-section equal to that of the round wire. In this way, it is possible to compare the slot filling factors and evaluate the benefits deriving from the use of hexagonal wires. In this study, six different dimensions have been chosen for each wire shape, respectively. For each dimension, three additional interspaces  $t_w$ , respectively equal to 0 mm, 0.05 mm and 0.1 mm, were taken into consideration. Furthermore, as regards the standard slot, a study was carried out on the possibility of positioning the wires parallel to the ground or the bottom (PG) and to the flank (PF) of the slot. The dimensions of the wires, used for this study and expressed in mm, are reported in

<sup>1</sup> According to [24-25] the grade is defined as “the range of thickness of the insulation wire”.

Table 3 and Table 4.

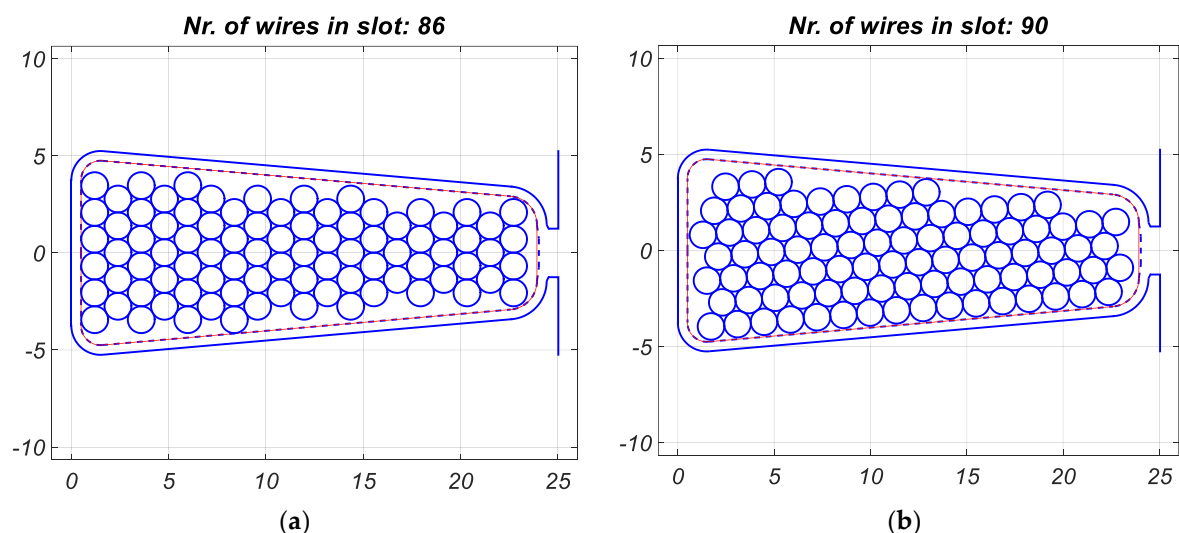
**Table 3** Geometrical features of round and hexagonal wires.

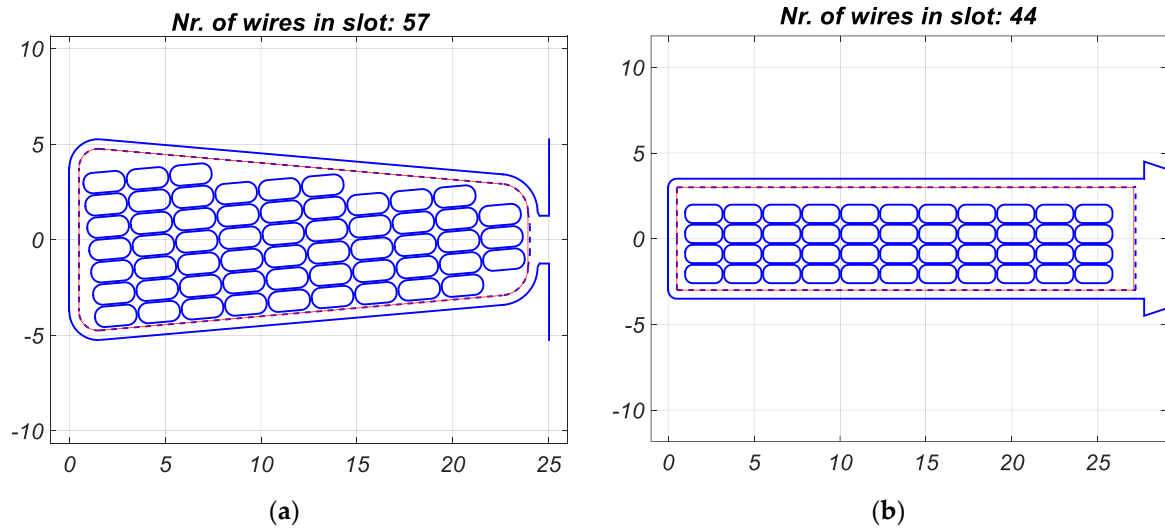
$s_{cu}$ [mm <sup>2</sup> ]	$d_{cu}$ [mm]	$d_{max}$ [mm]	$r_{cu}$ [mm]	$r_{max}$ [mm]
0.636	0.90	1.018	0.495	0.560
0.785	1.00	1.124	0.550	0.618
0.985	1.12	1.248	0.616	0.686
1.227	1.25	1.381	0.687	0.759
1.539	1.40	1.535	0.770	0.854
2.010	1.60	1.740	0.880	0.957

**Table 4** Geometrical features of rectangular wires.

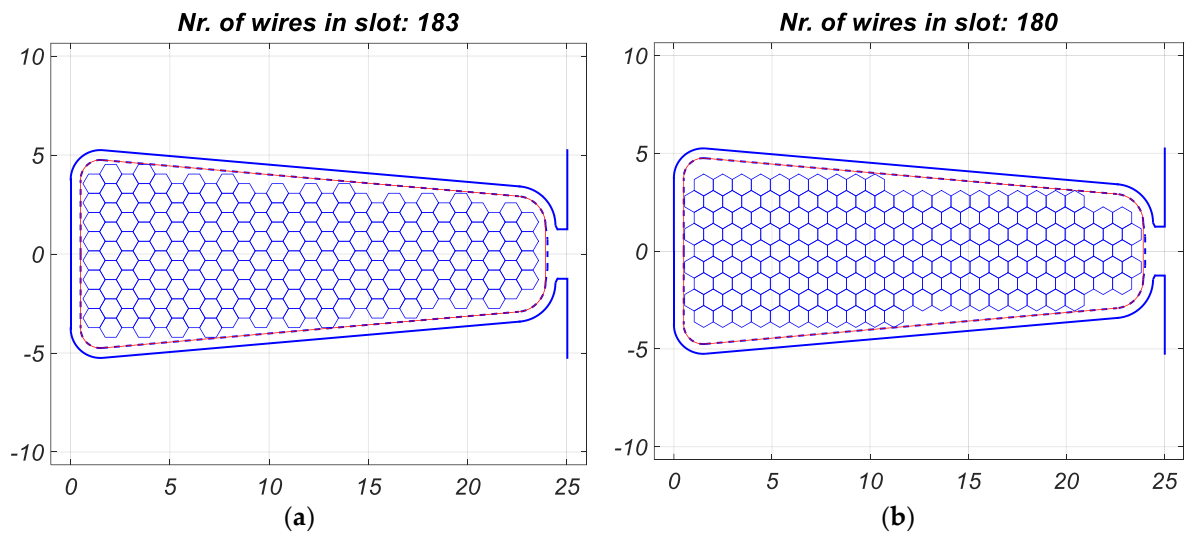
$s_{cu}$ [mm <sup>2</sup> ]	$L_{1cu}$ [mm]	$L_{2cu}$ [mm]	$L_{1max}$ [mm]	$L_{2max}$ [mm]	$r_{corner}$ [mm]
1.626	2.00	0.90	2.17	1.07	0.45
2.025	2.24	1.00	2.41	1.17	0.50
2.920	2.50	1.25	2.67	1.42	0.50
3.705	2.80	1.40	2.97	1.57	0.50
4.825	3.15	1.60	3.32	1.77	0.50
5.465	3.55	1.60	3.72	1.77	0.50

From Figure 8 to Figure 11 it is possible to notice how the algorithm returns the slot profile with the desired distribution of the wires and also shows the number of inserted wires. In particular, in the case of round wires, the winding pattern presents an orthocyclic structure where the positioning is parallel to the bottom in one case and parallel to the flank of the slot in the other case, respectively. In the case of rectangular wire, the winding pattern is ordered. In the case of hexagonal wire, an orthocyclic winding pattern is obtained both with  $\phi$  equal to zero and with  $\phi$  equal to  $\pi/2$ . Furthermore, it is possible to notice how the structure remains ordered also with the use of an additional interspace  $t_w$ .

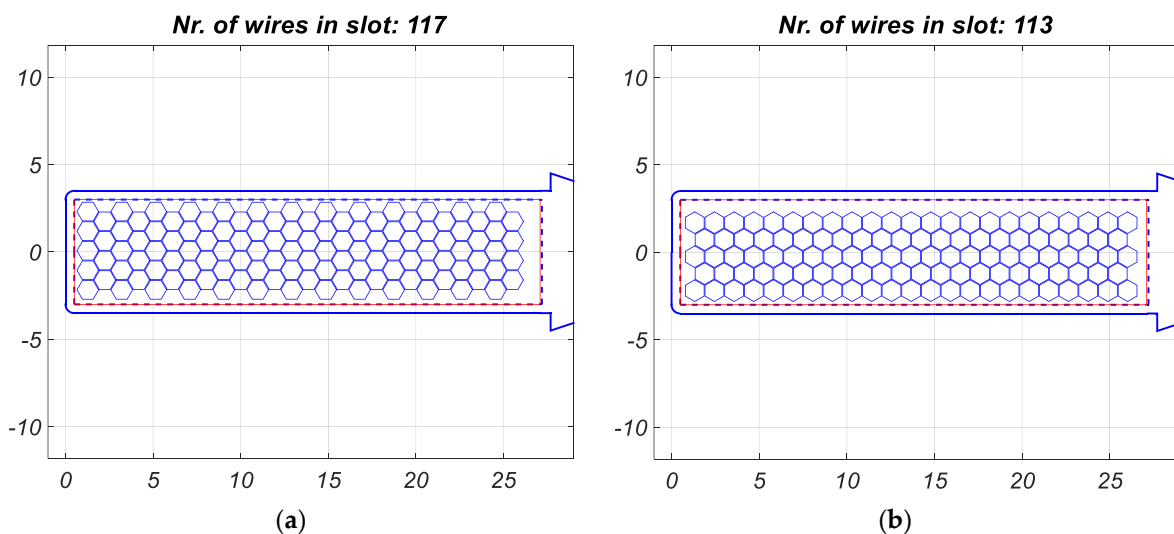
**Figure 8** Orthocyclic distribution of the round wires with the parallel ground disposition (a) and with the parallel flank disposition in the standard slot (b) ( $t_w=0\text{ mm}$ ,  $d_{max}=1.381\text{ mm}$ ).



**Figure 9** Ordered distribution of the rectangular wires with parallel disposition to the flank in standard slot (a) and in non-standard slot (b) ( $t_w=0.1\text{ mm}$ ,  $L_{1\max}=2.17\text{ mm}$ ,  $L_{1\max}=1.07\text{ mm}$ ).

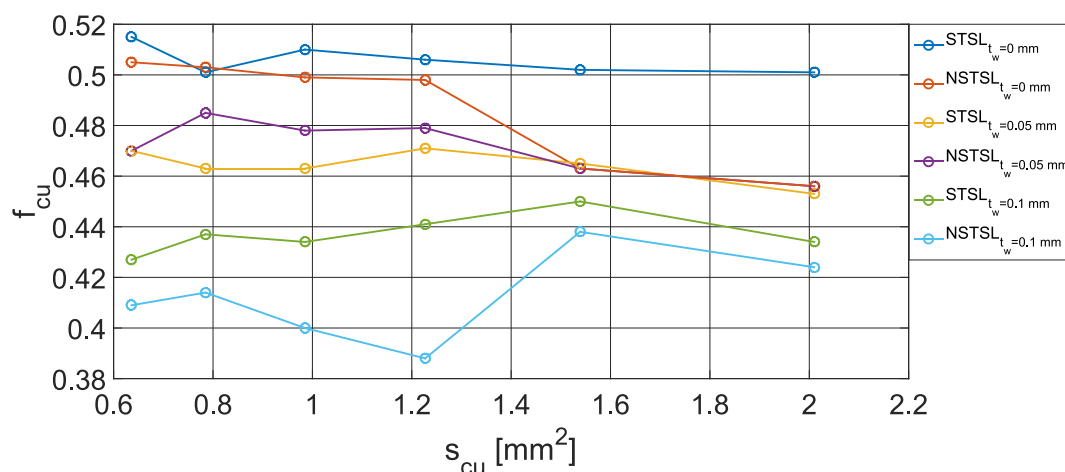


**Figure 10** Orthocyclic distribution of the hexagonal wires in standard slot ( $\phi=0$ ,  $t_w=0\text{ mm}$ ,  $r_{\max}=0.313\text{ mm}$ ) (a) and ( $\phi=\pi/2$ ,  $t_w=0\text{ mm}$ ,  $r_{\max}=0.313\text{ mm}$ ) (b).

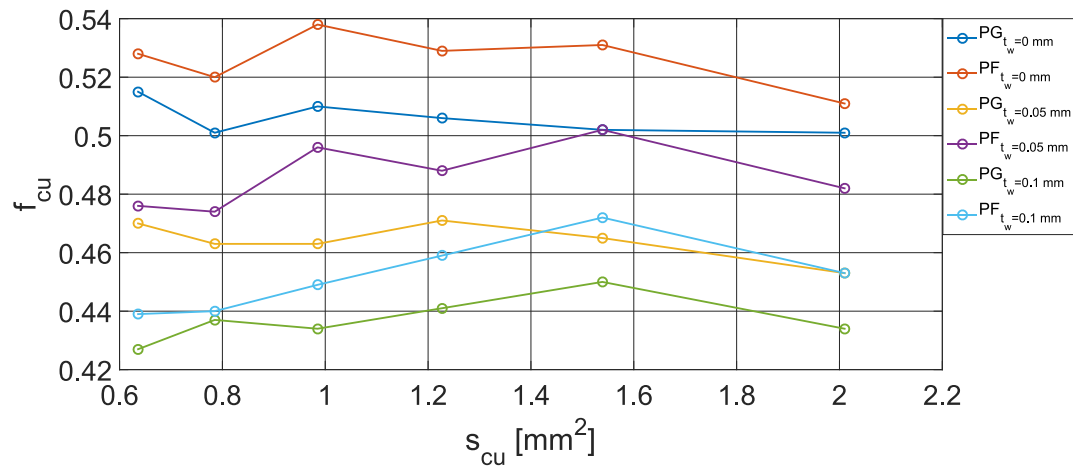


**Figure 11** Orthocyclic distribution of the hexagonal wires in non-standard slot ( $\phi=0$ ,  $t_w=0\text{ mm}$ ,  $r_{\max}=0.618\text{ mm}$ ) (a) and ( $\phi=\pi/2$ ,  $t_w=0.05\text{ mm}$ ,  $r_{\max}=0.618\text{ mm}$ ) (b).

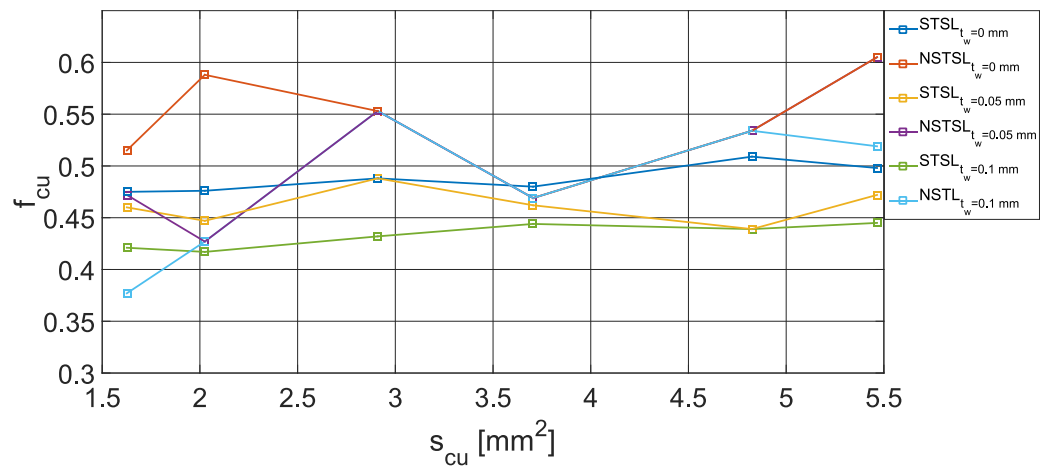
In Figure 12, the value of slot electrical filling factors obtained for the case of round wire, with a parallel bottom disposition, both for STSL and for the NSTSL, are reported. A comparison, between the slot electrical filling factors obtained with a parallel bottom disposition and that one obtained with the parallel flank disposition of standard slot, are reported in Figure 13. This comparison shows that the slot electrical filling factor is higher for a parallel flank disposition for each additional interspace  $t_w$ . A similar study was performed for rectangular wires. Figure 14 shows the trend of slot electrical filling factors as a function of the cross-section area. On average, the filling factors obtained in the rectangular case are higher than those of the round case for the dimensions chosen. Also, in the rectangular case, the slot electrical filling factor is higher in a parallel flank disposition (Figure 15). In this work, a comparison between the slot electrical filling factors obtained in the round and rectangular wire cases, with equal cross-section, has not been reported because it has been widely discussed in [18] and in [26]. Furthermore, the rectangular wires are used for medium and high-power applications whereas the round ones are mostly used for low power applications. Figure 16 to Figure 17 show the comparison between the value of electrical slot filling factors obtained for hexagonal wires with  $\phi$  equal respectively to 0 and  $\pi/2$ . This study is performed both for the standard slot and for the non-standard slot, for each value of the additional interspace  $t_w$ . In the standard slot, the slot electrical filling factors are higher when  $\phi=0$ , whereas, in the non-standard slot, in some cases, higher electrical slot filling factors are obtained with  $\phi=\pi/2$ . Particularly interesting is the comparison between the slot filling factors obtained in the case of round wire and in the hexagonal one, with the same cross-section. In Figure 18, the comparison between the slot electrical filling factors of round wires with the parallel flank disposition and the slot electrical filling factors of hexagonal wires with  $\phi=0$ , is reported. From this comparison, it can be seen that the filling factors obtained in the case of hexagonal wires are always higher than that obtained in the case of round wires. This difference decreases as the additional interspace increases. Therefore, the use of hexagonal wires can provide innovation in the field of electrical machine windings. Obviously, this improvement must be contextualized with any additional costs and the current state of the art of the specially shaped wires manufacturing process. Therefore, the case studies shown demonstrate how the algorithm allows to accurately estimate the value of the filling factor and it is a useful tool for the design of electrical machines windings. The results, like electrical filling factor, are calculated under the assumption of the ideal geometric shape of each wire and slot filling paper. For practical considerations tolerances of shape and diameter must be taken into account and also the winding process itself gives not an ideal orthocyclic winding distribution in the slot. This deviations from ideal geometry and winding process can be considered by additional geometric tolerances. With the given algorithm the influence of non-ideal conditions on filling factor and maximal turns can be calculated within seconds and motor design engineers and process engineers can make decisions based on a reliable calculation concept.



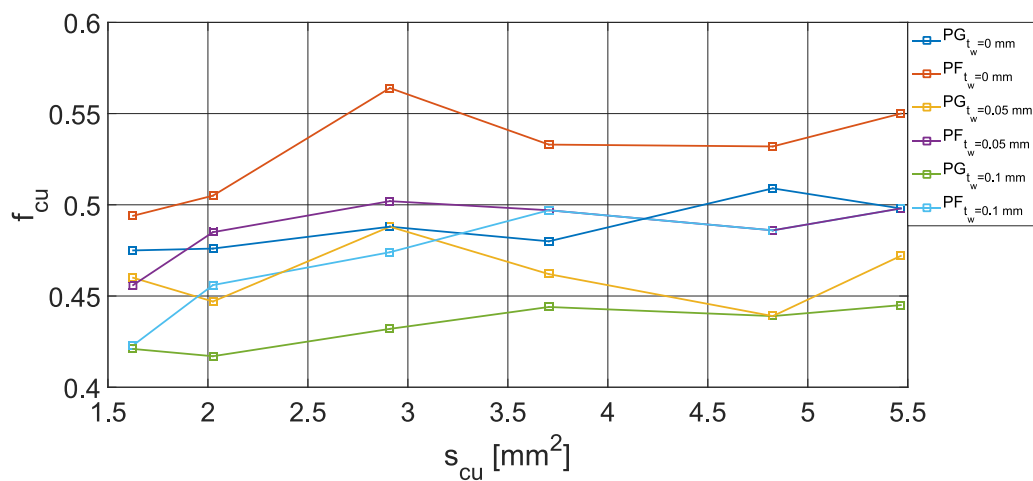
**Figure 12** Slot electrical filling factor as a function of wire copper cross-section area for round wires.



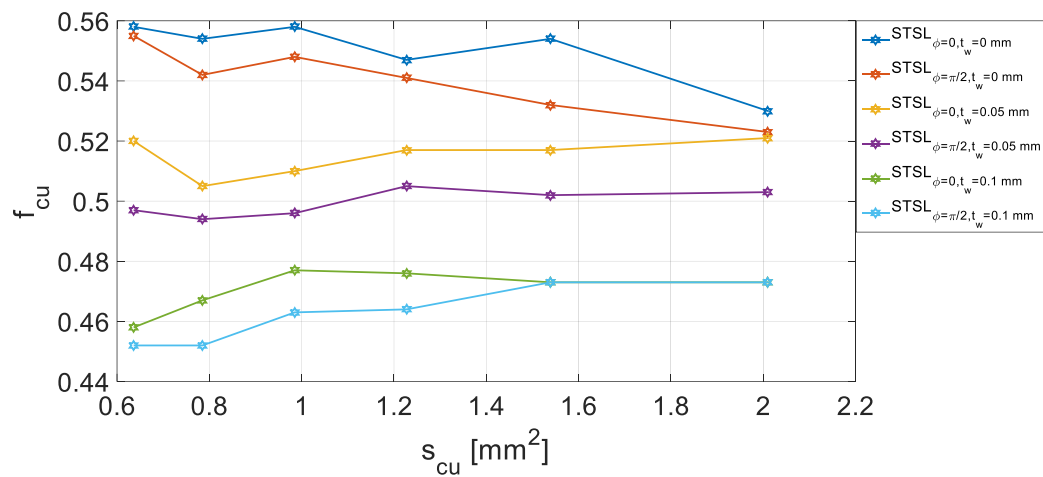
**Figure 13** Comparison between the slot electrical filling factors obtained with round wires distribution parallel to the ground/bottom (PG) and to the flank (PF) of the standard slot.



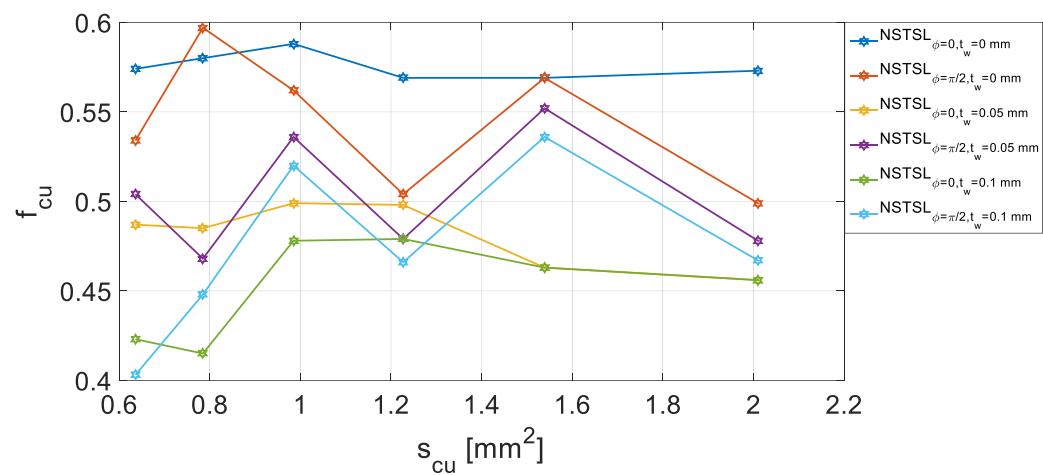
**Figure 14** Slot electrical filling factor as a function of wire copper cross-section area for rectangular wires.



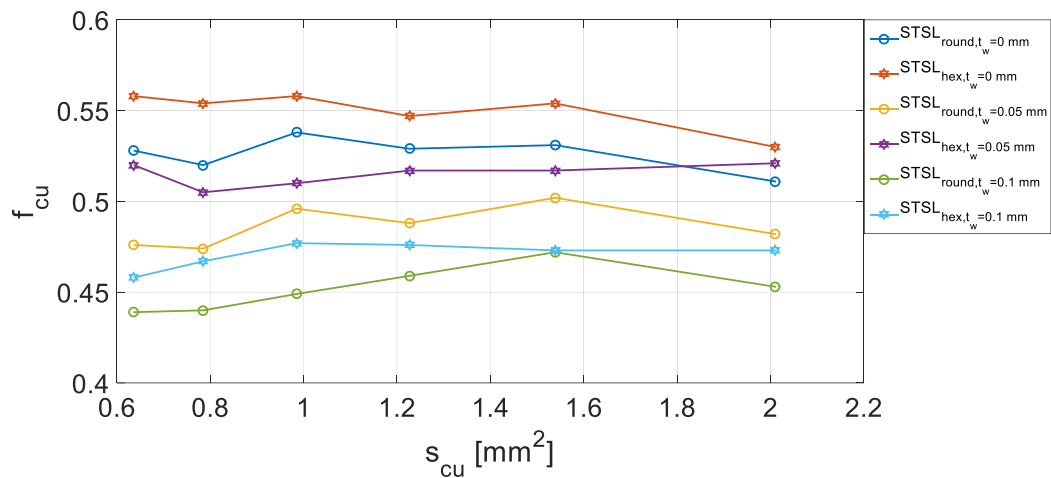
**Figure 15** Comparison between the slot electrical filling factors obtained with rectangular wires distribution parallel to the ground/bottom (PG) and to the flank (PF) of the standard slot.



**Figure 16** Slot electrical filling factor as a function of wire copper cross-section area for hexagonal wires in a standard slot (STSL).



**Figure 17** Slot electrical filling factor as a function of wire copper cross-section for hexagonal wires in a non-standard slot (NSTSL).



**Figure 18** Comparison between the slot electrical filling factors obtained with round wires distributed parallel to the flank of the slot and that one obtained with hexagonal wires as a function of wire copper cross-section area.



## 5. Conclusions

High filling factors make it possible to improve the electrical machines performances, meeting the design requirements of several application fields. This improvement requires an optimal arrangement of the wires inside the slots and a careful evaluation of the maximum slot filling factor obtainable in the design phase. In this paper, an algorithm approach is proposed to perform the calculation of slot filling factors in electrical machines. The algorithm requires as input data the geometrical data of slot, the insulation thickness, the shape of the wire, the dimensions of wire and the type of disposition inside the slot. From this data, the algorithm determinates the maximum slot filling factors, the number of wires positioned and a graphic distribution of wires inside the slot. The algorithm has a high degree of flexibility and requires very little computation time (about 2-3 seconds). The conducted study proves that the algorithm is very simple and can give useful results in the designing processes of winding layouts. Furthermore, the algorithm can be used as investigation tool because it allows to compare the electrical filling factor values when different wire shapes with the same cross-section are employed. In detail, from the investigations here presented, it has been shown that the use of hexagonal wires provides higher filling factors than those obtained with the use of round wires. Although the comparison between the slot filling factors obtainable with rectangular wires and hexagonal wires with the same wire cross-section is not presented in this paper, the use of hexagonal wires allows obtaining higher filling factors than those obtained with the use of rectangular wires. This result is due to the orthocyclic arrangement of hexagonal wires that optimally occupies the slot area unlike the ordered arrangement of rectangular wires. Non-ideal geometry of the magnetic wires and tolerances out of the winding process can be easily considered by additional geometric factors and practical problems can be addressed. These results can be of considerable interest for the optimization of electrical machine windings field.

**Author Contributions:** Authors contributed equally to this work. Authors of this manuscript jointly conceived theoretical developments, revised the state of the art, provided suggestions to obtain a flexible algorithm for the optimization of slot filling factors in the design phase of electrical machines.

**Acknowledgements:** This work was financially supported by MIUR - Ministero dell'Istruzione dell'Università e della Ricerca (Italian Ministry of Education, University and Research) and by SDESLab—Sustainable Development and Energy Saving Laboratory of the University of Palermo. In this section, you can acknowledge any support given which is not covered by the author contribution or funding sections. This may include administrative and technical support, or donations in kind (e.g., materials used for experiments).

**Conflicts of Interest:** The authors declare no conflict of interest.

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