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Property Analysis of Riccati Difference Equation for Load Frequency Controller of Time Delayed Power System using IMMKF

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Abstract

In this paper, initially a mathematical model is formulated for transient frequency of power system considering time delays which occur while transmitting the control signals in open communication infrastructure. Time delay negligence in a power system leads to improper measurement of frequency variation in power system. The study of impact of time delays on the stability of power system is performed by estimating the decay rate of frequency wave form using Kalman Filter (KF). In power system, there is a possibility of multiple time delays. This paper also focusses on developing Interacting Multiple Model (IMM) Algorithm with multiple model space using Kalman Filter (KF) as state estimator tool. The result shows that KF provides better estimate of correct model for a particular input-set. The multiple time delays in power system is considered as multiple model space. The qualitative properties of Riccati difference equation (RDE) in terms of state error covariance of IMMKF are also analyzed and presented.

Key words: Riccati Difference equations, Power System Stability, Interacting Multiple Model Kalman Filter, Load frequency controller, Time Delays.

AMS Classification:$39A10, 39A30, 39A60, 93C80$.

1 Introduction

In many industrial applications, direct measurement of the state variables is impossible which required state estimation based on the output in the course of the process of state controller model [1, 3]. Among the various filters used, Kalman filter is incorporated in several applications to evaluate the state for power system, signal processing and navigation [3, 5] systems with measurement noises. This filter maintains an optimal recursive solution for the least square state estimation problem and its properties including stability. The Riccati equation and its variation, the Matrix Riccati equation appear in numerous scientific and engineering

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applications such as optimal control and filtering problems [10, 20] which plays an important role in the stability analysis of different variations of the Kalman filter. The results are later extended to the non monotonic case [9]. The properties of the solutions of the Riccati equation convergence like monotonicity and stabilizability were discussed in [6, 7, 8, 12] by selecting appropriate initial covariance matrix through the monotonic properties [11, 20, 25]. These assumptions were then composed of detectability and stabilizability for continuous time systems [28] and for discrete-time systems [8]. Interrelated results were acknowledged [3] for time-varying detectable and stabilizable systems. The detailed relations between various Riccati equations and the closed-loop stability of linear quadratic optimal control and estimation are illustrated in [6, 7, 12].

Power system is one of the most intricate system in the eminent field of engineering theory. Enormous literature is available on the frequency measurement techniques for power system. Frequency is significantly determined by active power whereas the voltage is determined by the reactive power. The obstacles in power system control are the frequency(active power)control and the voltage(reactive power) regulation. The active power control with frequency control is also known as the Automatic Load Frequency Control (ALFC). Time delays in voltage and frequency play a vital role in evaluating amplitude, phase of voltage and current signals. Time-delays in communication between a controller and generators are one of the eminent impediments in stability. Minimal research has been performed in the field of power system stability due to time delay. In order to maintain the stability of power system, multifarious control loops are required. Proper control and maintenance of the frequency stability of a power system requires primary, secondary and tertiary frequency control loops. Generally, Primary Frequency Control (PFC)loop is important for intercepting the frequency decline before triggering the under/over frequency protection relays. The PFC is performed by the governor droop resulting in the steady state errors. The secondary frequency control (Load Frequency control (LFC)) [4, 22] is used to regulate the frequency in power systems into a desirable range and to control the interchange power between the different control areas through major tie-lines. Finally, tertiary control level is resending the generated units and secondary reserve after a distinct disturbance. An LFC model to consider the effects of frequency control loops for two-area power systems is already developed [13]. The main objective of LFC during load sharing is to restore the balance between load and generation in every control area under definite limits. The delay dependent stability of power system using frequency domain approach is analyzed in [15, 16, 17]. The delay dependent stability of the LFC scheme by using the Lyapunov-theory based delay-dependent criterion and Linear Matrix Inequalities (LMIs) techniques was earlier investigated in [19]. The wide range of communication networks in power system control causes unavoidable time delays. The impact of these delays on the stability of one-area and two-area LFC systems and an analytical method to determine delay margins, for stability is proposed in [23]. The application of Rekasius substitution into delay margin computation of LFC system plays a vital role in this work. The infinite dimensional characteristic equation with transcendental terms of the time delayed LFC system can be transformed into a finite dimensional regular polynomial by using the Rekasius substitution. By the application of Routh Hurwitz stability criterion, the critical root, the corresponding oscillation frequency and the delay margin for stability can be determined. The computation of gain and phase margin based stability delay margin using Rekasius substitution is investigated in [24]. An LFC approach based on LMI theory to design a robust controller in delayed control signals is proposed in [2] and of the equivalent seven stability criteria is established in [26]. The present estimation problem is based on the measured data to approximate the values of the unknown parameters used from the measured data.

Motivation and Contribution

Extended Kalman Filter has been applied for estimating various parameters in PFC [21], whereas there is no report in the literature on the tracking and estimation of frequency decay in Secondary Frequency Controller (SFC) with this kind of deriving mathematical model and discretization of the system. This paper focuses on the application of an IMMKF for estimating the decay rate in the transient frequency of power system by considering various time delays as different models. This helps as a guide to analyze the impact of time delays on power system.

The current research article is organized as follows. In section 2, the model of the power system incorporated with communication delay is developed and discussed and also the mathematical formulation for transient frequency variation of power system with time delay is presented. Section 3 proposes an IMMKF algorithm for estimating the state of the system and the evaluation of the properties of the Riccati Difference equation through state error covariance is also presented. A detailed discussion on the IMMKF algorithm and the Properties of the Riccati Difference equation such as monotonicity and stability are stated in Section 4. Results of estimating the measurement vector using Kalman filter and identifying the correct input set in multiple model space using IMMKF is presented in section 5. Conclusions are drawn in Section 6.

2 Dynamical model of power system

2.1 Model of Power system

An illustration of the dynamic model with the components of the ALFC loop in power system is shown in Figure 1 [19]. One of the main limitations in communication network is the time delay arising in the transmission of control signals.This consists of governor, turbine and load generator which are modeled by a transfer function of first order.

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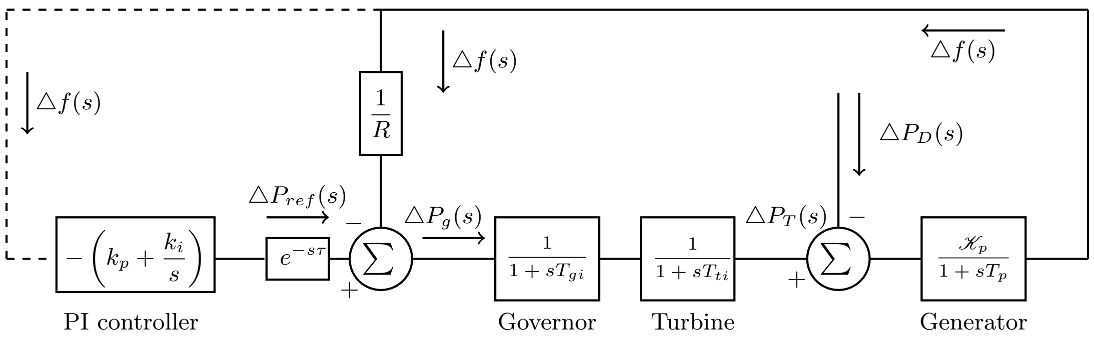

Figure 1. Block diagram of Load Frequency Control

The transfer function of governor G_H is $\dfrac{1}{1+sT_{gi}}$ where T_{gi} is a governor time constant (s), the transfer function of nonreheat turbine G_T is $\dfrac{1}{1+sT_{ti}}$ where T_{ti} is a turbine time constant (s), the Transfer function of PI controller is $-\left(k_p + \dfrac{k_i}{s}\right)$ and finally the transfer function of Generator G_P is $\dfrac{\mathscr{K}_p}{1+sT_p}$. Also, as G_H, G_T and G_P contain atleast one time constant each, the resultant $\Delta f(t)$ will be of third order, which is a complicated polynomial to solve. $(T_{gi} \ll T_{ti} \ll T_p)$ where T_p is generally 20s. $T_{gi} \approx T_{ti} \leq 1s$, and thus by assuming $T_{gi} = T_{ti} = 0$, the values of G_H and G_T become 1 each. Multiple factors contribute to the time delay in LFC system. To analyze the stability of LFC system, it is adequate to determine the variation of roots of the characteristic equation with respect to time delay. All these delays are appended as a single delay denoted by $e^{-s\tau}$, where τ is the time delay constant.

Let $\triangle f(s)$ is the frequency deviation which acts as an output signal which can be represented as

$$\triangle P_{ref}(s) = -\left(k_p + \frac{k_i}{s}\right) e^{-s\tau} \triangle f(s). \tag{2.1}$$

This is the reference set power which acts as a control input. Here, $\triangle f(s)$ is fed to an PI controller which controls the speed changer position. From the Figure 1, For a step load change in the system, $\Delta f(s)$ can be written as

$$\Delta f(s) = [\Delta P_T(s) - \Delta P_D(s)]\, G_p \tag{2.2}$$

$$= \left[\left(-\left(k_p + \frac{k_i}{s}\right) e^{-s\tau} \Delta f(s) - \frac{1}{R}\Delta f(s)\right) G_{HT} - \Delta P_D(s)\right] G_p \quad \text{where } G_{HT} = G_H \times G_T$$

$$= -\frac{G_p}{1 + \left[\left(k_p + \frac{k_i}{s}\right) e^{-s\tau}\right] G_{HT}G_p + \frac{1}{R}G_{HT}G_p}\Delta P_D(s). \tag{2.3}$$

Here, $\Delta P_D(s) = -\dfrac{M}{s}$ where M is the magnitude of step change in load.

$$\text{Then,} \quad \Delta f(s) = \frac{\dfrac{\mathscr{K}_p}{1+sT_p}}{1 + \left[\left(k_p + \frac{k_i}{s}\right) e^{-s\tau}\right] \dfrac{\mathscr{K}_p}{1+sT_p} + \dfrac{1}{R}\dfrac{\mathscr{K}_p}{1+sT_p}} \frac{M}{s}. \tag{2.4}$$

Because of the exponential term in equation(2.4), the characteristic equation becomes transcendental and has an infinite number of roots which makes the stability analysis complicated. In order to overcome this problem, using an exact substitution for the transcendental term we make use of the Rekasius substitution [15, 23, 24] $e^{-s\tau} = \dfrac{1-sh}{1+sh}, \tau \in \Re^{+}, h \in \Re$, which is defined only on the imaginary axis in the complex plane. The characteristic polynomial of the system given in equation (2.4) can be written as $\Delta(s,\tau) = \det(sI - f - \omega e^{-s\tau})$. It can be seen that the system would be asymptotically stable if and only if all the roots of the characteristic equation are on the left half of the complex plane.

Multiplying the numerator and denominator of equation(2.4) by $sR(1+sT_p)$, with the help of Rekasius substitution $e^{-s\tau} = \dfrac{1-sh}{1+sh}$ and on simplification we obtain

$$\Delta f(s) = \frac{R\mathscr{K}_p(1+sh)M}{s^3 + J_2 s^2 + J_1 s + J_0} \tag{2.5}$$

$$\text{where} \quad J_2 = \frac{Rh + RT_p - Rhk_p\beta\mathscr{K}_p + h\mathscr{K}_p}{RT_p h},$$

$$J_1 = \frac{R - Rhk_i\beta\mathscr{K}_p + Rk_p\beta\mathscr{K}_p + \mathscr{K}_p}{RT_p h},$$

$$J_0 = \frac{k_i\beta\mathscr{K}_p}{T_p h}.$$

2.2 Mathematical Formulation of Transient Frequency Deviation

The mathematical formulation for the transient frequency variation of power system is formulated by assuming the values for the parameters [16] R, T_p, M, k_p, k_i, $\mathscr{K}_p$, β with different values of h and by inverse laplace transformation in equation(2.5) as

$$f(t) = a_1 e^{-\mu_1 t} + a_2 e^{-\mu_2 t}\cos\omega t + a_3 e^{-\mu_2 t}\sin\omega t. \tag{3.1}$$

The above can be expressed as $f(t) = y_1(t) + y_2(t)$ (3.2)

where $y_1(t) = a_1 e^{-\mu_1 t}$,(exponentially decaying component)

$y_2(t) = a_2 e^{-\mu_2 t}\cos\omega t + a_3 e^{-\mu_2 t}\sin\omega t$(an oscillatory component).

$y_1(t)$ and $y_2(t)$ are the solutions of the following differential equations

$$\dot{y_1} = -\mu_1 y_1. \tag{3.3}$$

$$\ddot{y_2} = -2\mu_2\dot{y_2} - \left(\mu_2^2 + \omega^2\right) y_2. \tag{3.4}$$

Using state-space representation, we get

$$x_1 = y_1 \tag{3.5}$$

$$x_2 = y_2 \tag{3.6}$$

$$\dot{x_2} = x_3. \tag{3.7}$$

With the definitions of the equation(3.5) to (3.7), equation(3.3) and (3.4) can be rewritten as

$$\dot{x_1} = -\mu_1 x_1, \tag{3.8}$$

$$\dot{x_2} = x_3, \tag{3.9}$$
$$\dot{x_3} = -2\mu_2 x_3 - \left(\mu_2^2 + \omega^2\right) x_2. \tag{3.10}$$

Kalman filter is used to estimate the states x_1, x_2 and x_3 and the parameters a_1, a_2, μ_1, μ_2, ω. The parameters are evaluated as additional states invariant with time. Using Forward Euler discretization method, the augmented state equations are given by,

$$x_1(k+1) = (1 - x_4(k)\Delta t)\, x_1(k), \tag{3.11}$$
$$x_2(k+1) = x_2(k) + x_3(k)\Delta t, \tag{3.12}$$
$$x_3(k+1) = (1 - 2x_5(k)\Delta t)\, x_3(k) - x_6(k)x_2(k)\Delta t, \tag{3.13}$$
$$x_4(k+1) = x_4(k), \tag{3.14}$$
$$x_5(k+1) = x_5(k), \tag{3.15}$$
$$x_6(k+1) = x_6(k), \tag{3.16}$$

where $x_4 = \mu_1$, $x_5 = \mu_2$ and $x_6 = \mu_2^2 + \omega^2$.

Here, k is the sampling instance and Δt is the sampling interval.

The system of equations(3.11) to (3.16) can be written in the most general form as

$$x(k+1) = f\,(x(k), k) + \omega(k). \tag{3.17}$$

The measurement, (i.e) instantaneous frequency $z(k)$ is given by

$z(k) = x_1(k) + x_2(k)$, which can also be rewritten in general form as

$$z(k) = h\,(x(k), k) + v(k) \tag{3.18}$$

where f is the vector valued nonlinear state function and h is the vector-valued nonlinear measurement or output function, $x(k)$ is the state vector of the equivalent model, $w(k)$ is the process noise vector and $v(k)$ is the measurement noise vector. In order to handle the nonlinear function, Jacobian matrices are calculated in order to linearize the equations at each time step. The coefficient matrices F, H are the Jacobian matrices which are formulated as follows;

$$F = \frac{\partial f(x(k), k)}{\partial x(k)}\Big|_{\widehat{x}(k)}$$
$$= \begin{bmatrix} 1 - x_4(k)\Delta t & 0 & 0 & -x_1(k)\Delta t & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & -x_6(k)\Delta t & 1 - 2x_5(k)\Delta t & 0 & -2x_3(k)\Delta t & -x_2(k)\Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.19}$$

and $$H = \frac{\partial h(x(k), k)}{\partial x(k)}\Big|_{\widehat{x}(k)}$$
$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.20}$$

The system of equations (3.17) to (3.18) is structural and depends upon the parametric change. In

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the estimation of state variable of the system, the parameter time delay h plays a major role. For adaptive estimation, IMMKF is a strong approach. By assuming these different time delays as the base model set in the IMMKF, the possible system behavior patterns and the overall state estimate are derived.

3 The IMMKF algorithm

In this section, we present the estimation of state variable of the dynamic system with the help of Kalman filter is discussed and the development of interacting multiple model is presented. The IMM algorithm incorporates r interacting filters functioning simultaneously with every filter corresponding to a model of the following stochastic hybrid system [5] with the application of equations(3.19) and (3.20) by considering different values of h as model set is

$$x(\bar{k}) = F_{m(k)}x(\bar{k}-1) + w_{m(k)}(\bar{k}) \tag{4.1}$$

$$z(\bar{k}) = H_{m(k)}x(\bar{k}) + v_{m(k)}(\bar{k}) \tag{4.2}$$

where $x \in \Re^n$ is the state of every system and $z \in \Re^P$ is the measurement vector.

$$E\left\{\begin{bmatrix} w_j \\ v_j \end{bmatrix}\begin{bmatrix} {w_j}^T {v_j}^T \end{bmatrix}\right\} = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}\delta_{\hat{i}\hat{j}} \tag{4.3}$$

with Q non-negative definite ($Q \geq 0$), R positive definite ($R > 0$) and $\delta_{\hat{i}\hat{j}}$ is the Kronecker delta. By factoring Q and R we have $R = \left(R^{1/2}\right)\left(R^{1/2}\right)^T$ and $Q = LL^T$. The matrices $F_{m(k)}$ and $H_{m(k)}$ of the system corresponding to every model $m(k) \in \{1, 2 \ldots r\}$ at time $\bar{k}$: $w_{m(k)}(\bar{k})$ and $v_{m(k)}(\bar{k})$ are uncorrelated Gaussian noise vectors with white zero mean, $Q_{m(k)}$ and $R_{m(k)}$ are the respective covariance matrices. The evolvement of the model $m(k)$ is denoted by $p\left[m(k) = \hat{j}|m(k-1) = \hat{i}\right] = \pi_{\hat{i}\hat{j}}$ for $\hat{i}, \hat{j} = 1, 2 \ldots r$, where $\pi_{\hat{i}\hat{j}}$ is a constant; $p\,[.|.]$ symbolize a conditional probability. In the above system, for all $\hat{i}, \hat{j} = 1, 2 \ldots r,$ we consider that $F_{\hat{j}}$ is non-singular with $0 < \xi_1 I \leq Q_{\hat{j}} \leq \xi_2 I$, $0 < \xi_3 I \leq R_{\hat{j}} \leq \xi_4 I$ where ξ_1, ξ_2, ξ_3, ξ_4 are inverses of the maximum acceptable values by Bryson's rule and I is the identity matrix.

Let $Z^k := \left\{z(1), z(2), \cdots z(k)\right\}$ be the set of measurements up to time $\bar{k}$. The IMM algorithm calculates the relative posterior mean $\hat{x}_{\hat{j}}(\bar{k})$ and $P_j(k)$ for each Kalman filter $\hat{j}$, and the mode probability $\mu_{\hat{j}}(k) := p\left[m(k) = \hat{j}|Z^k\right]$. Let us suppose that, $\mu_{\hat{j}}(\bar{k}-1)$, $\hat{x}_{\hat{j}}(\bar{k}-1)$,and $P_{\hat{j}}(\bar{k}-1)$ for $\hat{j} = 1, 2 \ldots r$ are estimated from the last iteration at time $\bar{k}-1$, then the IMM algorithm computes the following for every $\bar{k}$:

- **Interaction/Mixing**

  Calculate the mixing probability

  $$\gamma_{\hat{j}\hat{i}}(\bar{k}-1) := p\left[m(k-1) = \hat{i}|m(k) = \hat{j}, Z^{k-1}\right] = \frac{1}{\sum_{\ell=1}^{r}\pi_{\ell\hat{j}}\mu_\ell(\bar{k}-1)}\pi_{\hat{i}\hat{j}}\alpha_{\hat{i}}(\bar{k}-1). \tag{4.4}$$

  For every Kalman filter $\hat{j}$ the initial conditions are calculated as

  $$\hat{x}_{\hat{j}0}(\bar{k}-1) = \sum_{\hat{i}=1}^{r}\gamma_{\hat{j}\hat{i}}(\bar{k}-1)\hat{x}_{\hat{i}}(\bar{k}-1), \tag{4.5}$$

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$$P_{\hat{j}0}(\bar{k}-1) = \sum_{\hat{i}=1}^{r} \left\{ P_{\hat{i}}(\bar{k}-1) + [\hat{x}_{\hat{i}}(\bar{k}-1) - \hat{x}_{\hat{j}0}(\bar{k}-1)][\hat{x}_{\hat{i}}(\bar{k}-1) - \hat{x}_{\hat{j}0}(\bar{k}-1)]^T \right\} \gamma_{\hat{j}\hat{i}}(\bar{k}-1). \tag{4.6}$$

- **Kalman Filtering**

  Compute $\hat{x}_{\hat{j}}$ and $P_{\hat{j}}(\bar{k})$using Kalman filter for each $\hat{j}$ model

  $$\hat{x}_{\hat{j}}(\bar{k}) = F_{\hat{j}}\dot{\hat{x}}_{\hat{j}0}(\bar{k}-1) + K_{\hat{j}}(\bar{k})\upsilon_{\hat{j}}(\bar{k}) \tag{4.7}$$

  $$P_{\hat{j}}(\bar{k}|\bar{k}-1) = F_{\hat{j}}P_{\hat{j}0}(\bar{k}-1)F_{\hat{j}}^T + Q_{\hat{j}} \tag{4.8}$$

  $$P_{\hat{j}}(\bar{k}) = \left[P_{\hat{j}}^{-1}(\bar{k}|\bar{k}-1) + H_{\hat{j}}^T R_{\hat{j}}^{-1} H_{\hat{j}}\right]^{-1} \tag{4.9}$$

  where $\upsilon_{\hat{j}}(\bar{k}) = z(\bar{k}) - H_{\hat{j}}F_{\hat{j}}\dot{\hat{x}}_{\hat{j}0}(\bar{k}-1)$ represents the residual, $K_{\hat{j}}(\bar{k})$ is the gain of the Kalman filter, and $P_{\hat{j}}(\bar{k}), (P_{\hat{j}}(\bar{k}|\bar{k}-1))$ is the posterior (prior) state covariance.

  By defining $E = R^{-1/2}H$, we can rewrite the Riccati equation in the normalized form as

  $$P_{\hat{j}}(\bar{k}) = F_{\bar{k}}P_{\hat{j}}(\bar{k}|\bar{k}-1)F_{\bar{k}}^T - F_{\bar{k}}P_{\hat{j}}(\bar{k}|\bar{k}-1)H_{\bar{k}}^T(H_{\bar{k}}P_{\hat{j}}(\bar{k}|\bar{k}-1)H_{\bar{k}}^T + I)^{-1}H_{\bar{k}}P_{\hat{j}}(\bar{k}|\bar{k}-1)F_{\bar{k}}^T + LL^T. \tag{4.10}$$

- **Model Probability Update**

  Herewith $\Lambda_{\hat{j}}(\bar{k}) := N_p\big(\upsilon_{\hat{j}}(\bar{k}); 0, S_{\hat{j}}(\bar{k})\big)$ is the Likelihood function with p dimension and $\upsilon_{\hat{j}}(\bar{k}); N_p(\cdot; 0, \sum)$ is a $p-$ dimensional multivariate Gaussian probability density function along with mean zero and $\sum$ covariance. The model probability is then given by $\alpha_{\hat{j}}(\bar{k}) = \dfrac{1}{\sum\limits_{\ell=1}^{r} \Lambda_\ell(\bar{k})\alpha_{\bar{\ell}}(\bar{k})} \Lambda_{\hat{j}}(\bar{k}) \left(\sum\limits_{\hat{i}=1}^{r} \pi_{\hat{i}\hat{j}}\alpha_{\hat{i}}(\bar{k}-1) \right).$ (4.11)

- **State Estimate and Covariance combiner**

  The state estimator and the covariance combiner are respectively calculated as

  $$\hat{x}(\bar{k}) = \sum_{\hat{j}=1}^{r} \alpha_{\hat{j}}(\bar{k})\hat{x}_{\hat{j}}(\bar{k}). \tag{4.12}$$

  $$P(\bar{k}) = \sum_{\hat{j}=1}^{r} \left\{ P_{\hat{j}}(\bar{k}) + [\hat{x}_{\hat{j}}(\bar{k}) - \hat{x}(\bar{k})][\hat{x}_{\hat{j}}(\bar{k}) - \hat{x}(\bar{k})]^T \right\} \alpha_{\hat{j}}(\bar{k}). \tag{4.13}$$

4 Properties of Riccati Difference Equation

This section is dedicated for analyzing the stability and monotonicity properties of solutions $\{P(\bar{k})\}$ of the Riccati difference equation (RDE) of optimal filtering [6, 7]. The associated Riccati Difference equation is of the form $P(\bar{k}+1) = FP(\bar{k})F^T - FP(\bar{k})H^T \left(HP(\bar{k})H^T + R\right)^{-1} HP(\bar{k})F^T + Q.$ (5.1)

To obtain the results of monotonicity and stability, the following fundamental theorems are needed.

4.1 Monotonicity

The following theorems characterize the monotonic property of the solutions of the RDE, which performs an important characteristics in the stability analysis of optimal filters. The existent theorem relates the solutions of two Riccati difference equations i.e the difference between two sequence solutions of the RDE is also a solution of the RDE.

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Theorem 1.[9]

Let $P_1(\bar{k})$ and $P_2(\bar{k})$ be two solutions of the RDE (5.1) with the matrices F, H and R and initial conditions $P_1(0) = P_1 \geq 0$ and $P_2(0) = P_2 \geq 0$ with $\bar{k} \geq 0$. Consider the difference between the two solutions $\bar{P}(\bar{k}) = P_2(\bar{k}) - P_1(\bar{k})$, where

$$P_1(\bar{k}+1) = FP_1(\bar{k})F^T - FP_1(\bar{k})H^T \left(HP_1(\bar{k})H^T + R\right)^{-1} HP_1(\bar{k})F^T + Q, \tag{5.2}$$

$$P_2(\bar{k}+1) = FP_2(\bar{k})F^T - FP_2(\bar{k})H^T \left(HP_2(\bar{k})H^T + R\right)^{-1} HP_2(\bar{k})F^T + Q, \tag{5.3}$$

and have the two solutions $P_1(\bar{k}) \geq 0$ and $P_2(\bar{k}) \geq 0$. Then, $\bar{P}(\bar{k})$ satisfies the following RDE

$$\bar{P}(\bar{k}+1) = \bar{F}_1(\bar{k})\bar{P}(\bar{k})\bar{F}_1(\bar{k})^T - \bar{F}_1(\bar{k})\bar{P}(\bar{k})H^T \left(H\bar{P}(\bar{k})H^T + \bar{R}(\bar{k})\right)^{-1} H\bar{P}(\bar{k})\bar{F}_1(\bar{k})^T. \tag{5.4}$$

where $\bar{F}_1(\bar{k}) = F - FP_1(\bar{k})H^T \left(HP_1(\bar{k})H^T + R\right)^{-1} H$

and $\bar{R}(\bar{k}) = HP_1(\bar{k})H^T + R$.

Theorem 2.[7]

Suppose, that the RDE (5.1) has a solution $P(\bar{k}) \leq 0$ and $\bar{k} = 0$. Then, $P(\bar{k})$ is monotonically non increasing for all subsequent time

$$P(\bar{k}) \leq P(\bar{k}+1) \text{ for all } \bar{k} \geq 0. \tag{5.5}$$

Theorem 3.[7]

Suppose that the RDE (5.1) has a solution $P(\bar{k}) \geq 0$ and $\bar{k} = 0$. Then, $P(\bar{k})$ is monotonically non decreasing for all subsequent time

$$P(\bar{k}) \geq P(\bar{k}+1) \text{ for all } \bar{k} \geq 0. \tag{5.6}$$

Let us also note that, $\{P(\bar{k})\}$ which satisfy the RDE is a monotonic non increasing sequence, then $Q(\bar{k})$ is not necessarily a monotonic sequence of non negative definite matrices defined by

$$Q(\bar{k}) = P(\bar{k}) - FP(\bar{k})F^T + FP(\bar{k})E^T \left(EP(\bar{k})E^T + I\right)^{-1} EP(\bar{k})F^T \tag{5.7}$$

satisfying $Q(\bar{k}) \geq LL^T$, $E = R^{-1/2}H$ and also $Q(\bar{k}) \geq 0$.

4.2 Stability

By factoring Q and R we have $R = \left(R^{1/2}\right)\left(R^{1/2}\right)^T$ and $Q = LL^T$ and by defining $E = R^{-1/2}H$ we can rewrite the Riccati equation in a normalized form as

$$P(\bar{k}+1) = FP(\bar{k})F^T - FP(\bar{k})E^T \left(EP(\bar{k})E^T + I\right)^{-1} EP(\bar{k})F^T + LL^T. \tag{5.8}$$

The closed-loop state transition matrix of the Kalman filter is

$$F(\bar{k}) = F - FP(\bar{k})E^T \left(EP(\bar{k})E^T + I\right)^{-1} E = F - K(\bar{k})E. \tag{5.9}$$

Theorem 4.[6]

Consider the RDE (5.8) with initial condition P_0 and solution sequence $P(\bar{k})$. Define the sequence of matrices $Q(\bar{k})$ by (5.7). If

1. $[E, F]$ is detectable,

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2. $[\digamma, L]$ is stabilizable, and

3. $P_0 \geq 0$ is such that $Q_0 \geq LL^T$,

then the solution sequence $\{P(\bar{k})\}$ of the RDE is stabilizing for all $\bar{k} \geq 0$,i.e $|\lambda_i(\digamma(\bar{k}))| < 1$ for all $\bar{k} \geq 0$ and for $i = 1, 2, \ldots n$, with $\digamma(\bar{k})$ defined by (5.9) and $\lambda_i(.)$ denoting the individual eigenvalues.

Theorem 5.[6]

Consider the RDE (5.8). Define Q_0 as in (5.7).If

1. $[E, \digamma]$ is detectable,

2. $\left[\digamma, Q_0^{1/2}\right]$ is stabilizable, and

3. $P_0 \geq 0$ is such that $Q_0 \geq LL^T$,

then the solution sequence $\{P(\bar{k})\}$ with initial condition P_0 is stabilizing for each $\bar{k}$.

To conclude that, A real symmetric non negative definite solution P of the RDE is called a *strong solution* if the corresponding state transition matrix $\bar{\digamma} \triangleq \digamma - \digamma PE^T \left(EPE^T + I\right)^{-1} E$ has all its eigen values inside or on the unit circle and hence it is stabilizing.

Let us also note that, $\{P(\bar{k})\}$ which satisfy the RDE is a monotonic non increasing sequence, then $\check{Q}(\bar{k})$ is not necessarily a monotonic sequence of non negative definite matrices defined by

$$\check{Q}(\bar{k}) = P(\bar{k}) - \check{\digamma} P(\bar{k})\check{\digamma}^T + \check{\digamma} P(\bar{k})\check{E}^T \left(\check{E}P(\bar{k})\check{E}^T + I\right)^{-1} \check{E}P(\bar{k})\check{\digamma}^T \tag{5.10}$$

satisfying $\check{Q}(\bar{k}) \geq LL^T$ and also $\check{Q}(\bar{k}) \geq 0$. By comparing the RDE (5.7) with (5.10) we have $\check{Q}(\bar{k}) = LL^T + \text{P}(\bar{k}) - \text{P}(\bar{k}+1)$, which leads to monotonicity.

5 Results and Observation

In this paper, Kalman filter is introduced as a tool to estimate the state variables of power system. The Multiple Kalman filter compares parallelly, how well each Kalman Filter matches true measurement from the estimated measurements of the system. The information flow in the proposed IMMKF algorithm is illustrated in Figure 5. By implementing the IMMKF algorithm, each Kalman filter acts as models of different time delays. Although number of Kalman Filters in IMM framework is conceptually unlimited, the computational requirement of IMM increase linearly as more models are evaluated. In this section, the mathematical formulation of IMM is established. The parameters that define IMM and the results of IMM is presented with KF as the estimator of individual models. When IMM was developed as a tool for target tracking environment, the basic formula can also be applied to the estimation of impact of time delay on the frequency waveform of power system. The set of time delayed power system models are defined as the model space in IMM. Each model is defined by the decay rate (μ_1) of exponential component and decay rate (μ_2) of oscillatory

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component which indirectly related to the study of impact of time delays on the frequency wave form of power system. In this research, model space consists of 2 models. These models are chosen in order to mean the power system is subjected to different time delays while transmitting the control signals in LFC system. The operation of KF as the estimation for IMM is considered. To demonstrate the application of IMMKF algorithm for the estimation of frequency decay in Power system, a single area LFC system is considered. The corresponding nomenclature is given in Appendix A. The parameters of the system [16] are as follows;
$R = 2$, $T_p = 20$, $M = 0.01$, $k_p = 0.1$, $k_i = 0.1$, $\mathscr{K}_p = 1$, $\beta = 0.51$ and $h = 5$

Substituting these values in equation (2.4) we obtain

$$\triangle f(s) = \frac{0.02(1+5s)}{s^3 + 0.27245s^2 + 0.01296s + 0.00051}$$

where $a_1 = -0.05908$, $a_2 = 0.05908$, $a_3 = 2.1356$, $\mu_1 = 0.22491$, $\mu_2 = 0.02377$ and $\omega = 0.04126$.

The initial values assigned for the elements of initial state vector of the equations (3.11) to (3.16), $\widehat{X} = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6]^T$ is derived from equation (2.5) and the initial values for $h = 5s$ is calculated as $[-0.05908 \;\; 0.05908 \;\; 0.08671 \;\; 0.22491 \;\; 0.02377 \;\; 0.002267]^T$.

The initial covariance matrix P_0, which is of the form of a (6×6) matrix is assumed to have all elements as 15 and the noise vector Q is given a value of 0.05 and R as 0.001. The coefficient matrix $\digamma, H$ are the Jacobian matrices are obtained as follows.

$$\digamma = \begin{bmatrix} 0.9775 & 0 & 0 & 0.0059 & 0 & 0 \\ 0 & 1.0000 & 0.1000 & 0 & 0 & 0 \\ 0 & -0.0002 & 0.9952 & 0 & -0.0172 & -0.0059 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For different h values, the initial values of the state vector are calculated and presented in table 1.

When h increases, the decay rate decreases which means that for the higher value of time delays, the system may go unstable. This is verified through simulation. Initially, Kalman filter is applied for estimating the measurement vector of the system. It is presented in the following figures 2(a)-2(c).

For $h = 5s$, estimation of the state vector x_1 which is the exponential component presented in Figure 2(a). Figure 2(b) represents the estimation of the oscillatory component x_2. Figure 2(c) shows the Frequency deviation at a particular bus, following a step load distribution in the power system. This figure is the combination of the exponential component and oscillatory component which is the measurement vector. The figure shows that the Kalman filter is more effective in tracking the frequency decay of the power system.

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Table 1: Initial values of the state vector

	h=5	h=10	h=15	h=20	h=35
x_1	-0.05908	-0.48073	-1.1227	-1.8424	-4.1422
x_2	0.05908	0.48073	1.1227	1.8424	4.1422
x_3	0.08671	0.128495	0.15516	0.1785	0.2440
x_4	0.22491	0.148758	0.12903	0.120229	0.11007
x_5	0.02377	0.0118462	0.00505	0.00111	-0.00452
x_6	0.002267	0.0016805	0.001318	0.00106	0.000662
$\dot{x}_1$	0.01329	0.0715	0.14486	0.2215	0.4559
$\dot{x}_2$	0.08671	0.128495	0.15516	0.1785	0.2244
$\dot{x}_3$	-0.004256	-0.003852	-0.003047	-0.002349	-0.0005363
μ_1	0.22491	0.14876	0.12903	0.120229	0.11007
μ_2	0.02377	0.0118462	0.00505	0.00111	-0.00452
ω	0.04126	0.039246	0.03595	0.03255	0.02533

Similarly, on increasing the value of the time delay $h = 10s$, estimation of the state vectors for exponential and oscillatory components is presented in figures 3(a) and 3(b) respectively. Also, tracking the frequency decay using Kalman Filter is shown in Figure 3(c). For different values of h, Figure 4(a) represents the exponential component and oscillatory component is presented in Figure 4(b). The work is further extended by applying Interacting Multiple Model Kalman Filter with two different time delays with $h = 5s$ and $h = 35s$ as two different models. The operation of KF as the estimation for IMM is considered. The result shows that KF provides more accurate estimate of correct model for a particular input set. Figure 4(c) shows the model estimate of IMMKF when the input data set is for $h = 5s$.

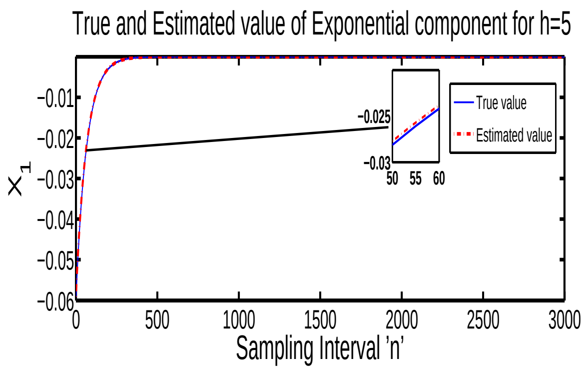

Figure 2(a)

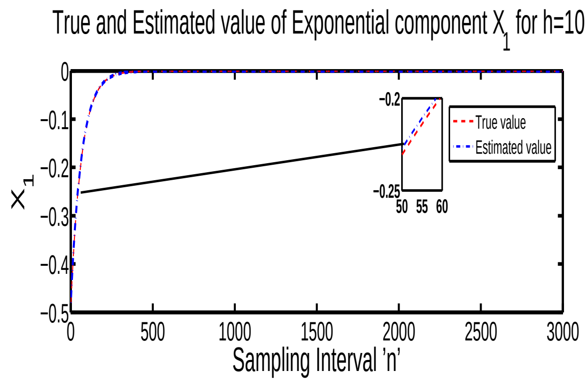

Figure 3(a)

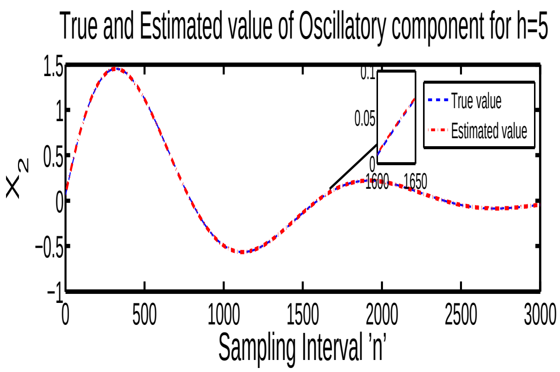

Figure 2(b)

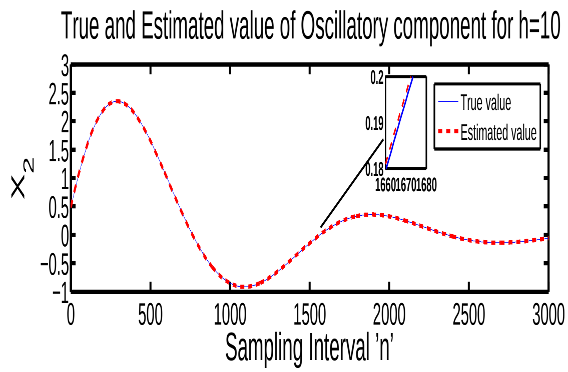

Figure 3(b)

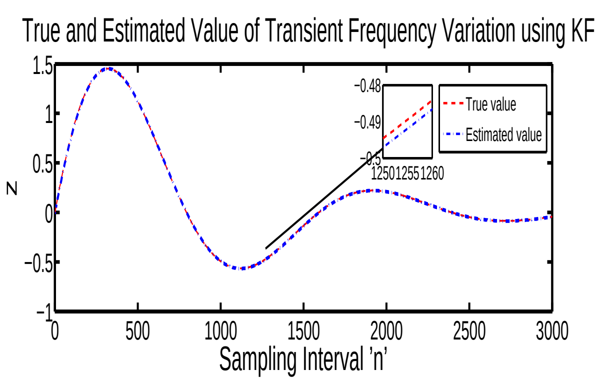

Figure 2(c)

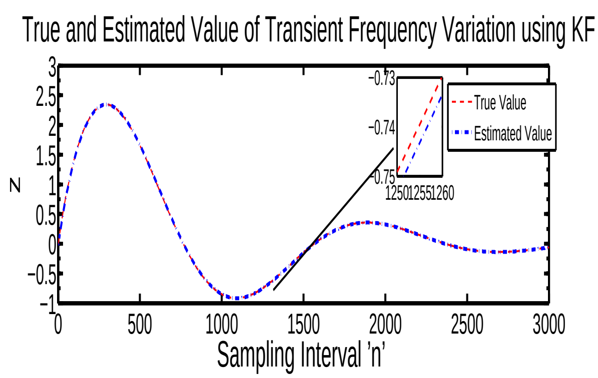

Figure 3(c)

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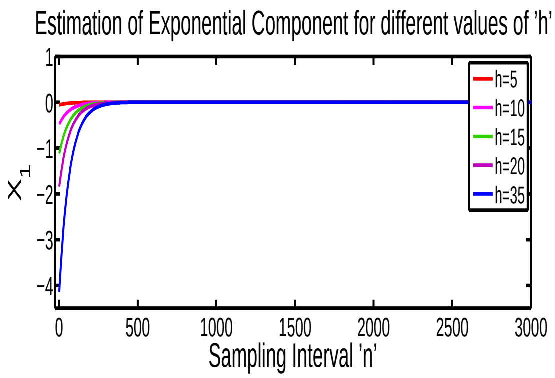

Figure 4(a)

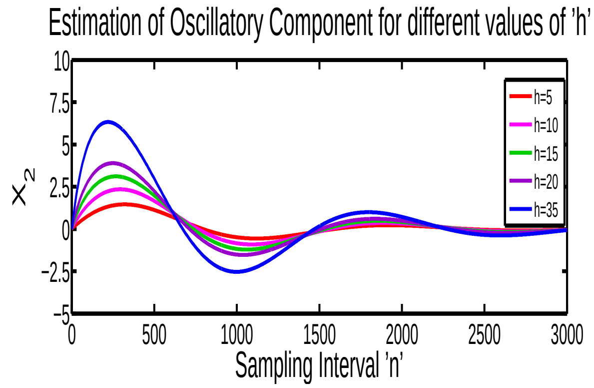

Figure 4(b).

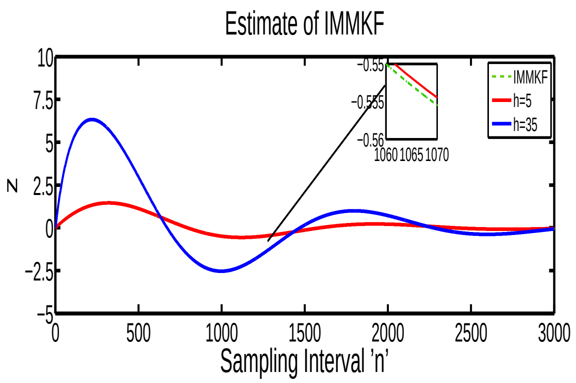

Figure 4(c).

5.1 Numerical Example.

The properties of the RDE such as monotonicity are also analyzed and verified by selecting appropriate initial covariance matrix P_0 which is an arbitrary non negative definite matrix. The results are later extended to the monotonic case. Using the equation(5.5), the solution of the RDE (5.4) is

$$P = 1.0e-014 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0888 & 0.1776 & 0.0888 & 0.0888 & 0.0888 & 0.0888 \\ 0.3553 & 0.3553 & 0.1776 & 0.3553 & 0.3553 & 0.3553 \\ 0.3553 & 0.3553 & 0 & 0.3553 & 0.3553 & 0.3553 \\ 0.3553 & 0.3553 & 0 & 0.3553 & 0.3553 & 0.3553 \\ 0.3553 & 0.3553 & 0 & 0.3553 & 0.3553 & 0.3553 \end{bmatrix}$$

Therefore, $P_0 \geq 0$ and $P_0 \geq P$.

Before extending the results, the conditions of Theorem 4. are reviewed in order. The first condition of detectability is essential for the well defined state estimation problem. It is also verified. Condition 2. states that $[F, L]$ is stabilize where F has eigen values lie inside the unit circle. The third condition is necessary for

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monotonicity of $P(\bar{k})$, that $Q_0 \geq LL^T$ implies $P_0 \geq P$. In generalizing Theorem 4, the stability condition on $[F, L]$ is replaced by $[F, Q_0^{1/2}]$, since $[F, L]$ need not be stabilizable. By using Theorem 4 and Theorem 5, we can conclude that P is a strong solution and the eigen values are 0.9775 1.0000 0.9976 0.9976 1.0000 1.0000 which lie either inside or on the unit circle. Hence, the stability of the system is verified.

In the same manner, using Theorem 1 to Theorem 3, the values of P_1 and P_2 are the solutions of the RDEs then the difference is also satisfying the RDE (5.1). And using $Q_0 = LL^T + P_0 - P_1$, with $P_0 \geq P_1$ and $P_1 \geq P2$ it is observed that the state error covariance matrix of Riccati equation is monotonic in nature.

6 Conclusion

In this paper, the transient frequency deviation in the power system following a load disturbance is mathematically formulated. The Kalman filter is introduced as a means to estimate the decay rate of the frequency waveform which indirectly relates to the study of impact of time delay on the stability of the power system. The application of the IMMKF technique for estimation of frequency decay, in one area LFC of power system with different time delays, is considered and the results of IMM when using KF as a state estimator for each model is also presented. KF provides a more accurate estimate of the correct model. The impact of time delays in the power system on the decay rate of frequency variation is also analyzed and presented. The variation of decay rate for various time delays is also shown in results. Moreover, the stability and monotonicity properties of Riccati Difference Equation is analyzed.

Appendix A

Nomenclature

$\bar{k}$ - time instant.

$x(\bar{k})$ - State Vector at time k.

$z(\bar{k})$ - Output Vector.

$\omega(\bar{k})$ - Process Noise Vector.

$v(\bar{k})$ - Measurement Noise Vector.

F - State Transition Matrix.

H - Measurement Matrix.

Q - Process Noise Covariance Matrix.

R - Measurement Noise Covariance Matrix.

P - State Error Covariance Matrix.

$K(\bar{k})$ - Kalman Filter Gain.

$p\{.\}$ - Probability.

$\triangle P_{ref}$ - Change in the reference power setting.

$\triangle P_D$ - Load disturbance (p.u.MW).

$\Delta f(s)$ - Frequency deviation (Hz).

$\mathscr{K}_p$ - Generator gain constant.

T_p - Generator time constant (s).

T_{gi} - Governor time constant (s).

T_{ti} - Turbine time constant (s).

R - Speed regulation due to governor action (Hz/p.u MW).

M - Step change in load.

k_p - Proportional gain.

k_i - Integral gain.

Appendix B

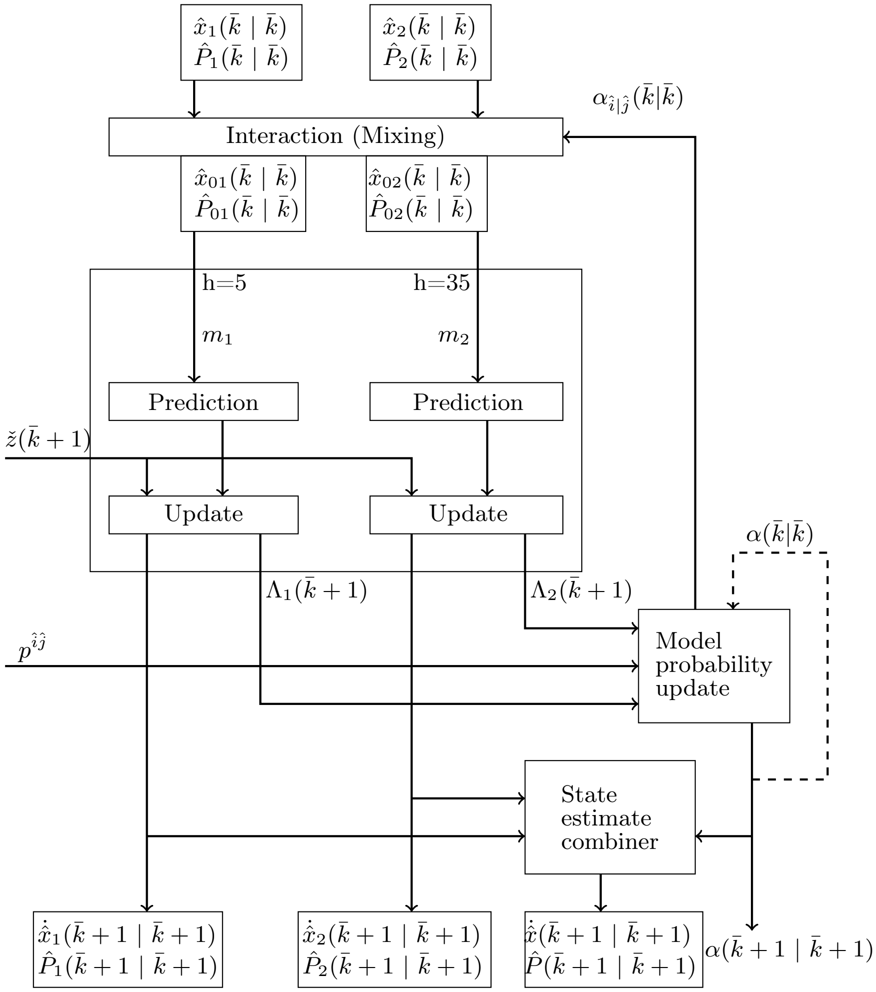

Figure 5. Flow diagram of IMM Kalman Filter

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