A theoretical calculation of the cosmological constant based on a mechanical model of vacuum

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We suppose that vacuum is filled with a kind of continuously distributed matter, which may be called the $\Omega(1)$ substratum, or the electromagnetic aether. Lord Kelvin believes that the electromagnetic aether must also generate gravity. We also suppose that vacuum is filled with another kind of continuously distributed substance, which may be called the $\Omega(2)$ substratum. Based on a theorem of V. Fock on the mass tensor of a fluid, the contravariant energy-momentum tensors of the $\Omega(1)$ and $\Omega(2)$ substratums are established. Quasi-static solutions of the gravitational field equations in vacuum are obtained. Based on an assumption, relationships between the contravariant energy-momentum tensor of the $\Omega(1)$ and $\Omega(2)$ substratums and the contravariant metric tensor are obtained. Thus, the cosmological constant is calculated theoretically. The $\Omega(1)$ and $\Omega(2)$ substratums may be a possible candidate of the dark energy. The zero-point energy of electromagnetic fields will not appear as a source term in the Einstein's equations. The cosmological constant problem is one of the puzzles in physics. Some people believed that all kinds of energies should appear as source terms in the Einstein's equations. It may be this belief that leads to the cosmological constant problem. The mass density of the $\Omega(1)$ and $\Omega(2)$ substratums is equivalent to that 31.33195 protons contained in a box with a volume of $1.0m^3$.

Keywords: cosmological constant; dark energy; general relativity; electromagnetic aether; vacuum mechanics.

I. INTRODUCTION

It is known that the cosmological constant problem is one of the puzzles in physics today [1, 2]. Theoretical interpretation of the small value of the cosmological constant is still open [3].

In 1917, A. Einstein thought that his equations of gravitational fields should be revised to be ([4], p. 410)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu}^m, \quad (1)$$

where $g_{\mu\nu}$ is the metric tensor of a Riemannian space-time, $R_{\mu\nu}$ is the Ricci tensor, $R \equiv g^{\mu\nu}R_{\mu\nu}$ is the scalar curvature, $g^{\mu\nu}$ is the contravariant metric tensor, κ is Einstein's gravitational constant, c is the speed of light in vacuum, $T_{\mu\nu}^m$ is the energy-momentum tensor of a matter system, Λ is the cosmological constant.

However, it seems that the cosmological constant Λ is unnecessary when Hubble discovered the expansion of the universe. Thus, Einstein abandoned the term $\Lambda g_{\mu\nu}$ in Eqs. (1) and returned to his original equations ([4], p. 410).

The value of the cosmological constant Λ is also related to the energy-momentum tensor of vacuum ([4], p. 411). The existences and characters of dark matter and dark energy [2] are still controversy, refers to, for instance, [5–9].

The purpose of this manuscript is to propose an interpretation of the cosmological constant and establish a mechanical model of dark energy based on the theory of vacuum mechanics [10–14].

II. A BRIEF INTRODUCTION OF THE THEORY OF VACUUM MECHANICS

Descartes interpreted the celestial motions of celestial bodies based on the hypothesis that the universe is filled by a fluidic vortex aether [15]. Following Descartes and Euler [16], we suppose that vacuum is filled by an extremely thin medium which may be called the $\Omega(0)$ substratum, or the gravitational aether [11]. Thus, two sinks in the $\Omega(0)$ substratum are found to attract with each other according to the inverse-square law of gravitation [11]. A feature of this theory is that the gravitational constant depends on time and the location in space.

Thomson's analogies between electrical phenomena and elasticity helped J. C. Maxwell to establish a mechanical model of electrical phenomena [15]. Following J. C. Maxwell, we suppose that vacuum is filled with a kind of continuously distributed matter which may be called the $\Omega(1)$ substratum, or the electromagnetic aether [10]. The idea that all microscopic particles are sink flows in a fluidic substratum has been proposed by many researchers in the history, for instance, J. C. Maxwell ([15], p. 243), B. Riemann ([17], p. 507), H. Poincaré ([18], p. 171), J. C. Taylor ([19], p. 431-436). Thus, we suppose that electric charges are sources or sinks of the $\Omega(1)$ substratum [10]. Maxwell's equations in vacuum are derived by methods of continuum mechanics based on a mechanical model of vacuum [10].

We suppose that there exist a kind of basic sinks of the $\Omega(0)$ substratum, which may be called monads after Leibniz [11]. The particles that constitute the $\Omega(1)$ substratum may be called the $\Omega(1)$ particles. Lord Kelvin believes that the electromagnetic aether must also generate gravity [20]. Following Lord Kelvin [20], we suppose that the $\Omega(1)$ particles and elementary particles are

formed of monads [13, 14]. Since monads, the $\Omega(1)$ particles, elementary particles are sinks of the $\Omega(0)$ substratum, they attract with each other according to Newton's law of gravitation [11].

There exist an universal drag force exerted on each sink of the $\Omega(0)$ substratum [11]. Therefore, each monad, each $\Omega(1)$ particle and each elementary particle, as sinks of the $\Omega(0)$ substratum, will experience a universal drag force. On the other hand, all the monads, $\Omega(1)$ particles and elementary particles are undertaking stochastic movements. Based on this universal damping force and some assumptions, microscopic particles are found to obey a generalized non-relativistic Schrödinger equation [12].

For convenience, we may call these theories [10–14] as the theory of vacuum mechanics.

III. POSSIBLE ORIGIN OF THE COSMOLOGICAL CONSTANT

Lord Kelvin once said [20]: "That there must be a medium forming a continuous material communication throughout space to the remotest visible body is a fundamental assumption in the undulatory theory of light." Lord Kelvin believes that the electromagnetic aether must also generate gravity [20, 21].

Following Lord Kelvin, we propose an assumption that the particles that constitute the $\Omega(1)$ substratum, are sinks in the $\Omega(0)$ substratum [13, 22]. Thus, according to a previous theory of gravity [11], the $\Omega(1)$ substratum will also generate gravity. Further, generalized Einstein's equations in a special class of non-inertial reference frames are derived based on the theory of vacuum mechanics [22]. If the non-inertial reference frame is quasi-inertial and the gravitational field is weak, then the generalized Einstein's equations reduce to [13, 14, 22]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\kappa}{g} \left(T_{\mu\nu}^m + T_{\mu\nu}^{\Omega(1)} \right), \quad (2)$$

where $T_{\mu\nu}^m$ and $T_{\mu\nu}^{\Omega(1)}$ are the energy-momentum tensors of matter system and the $\Omega(1)$ substratum respectively, $g \equiv \text{Det } g_{\mu\nu}$.

We speculate that the so called dark energy [2] is also a kind of matter filled in the universe. Thus, we introduce the following assumption.

Assumption 1 *Vacuum is filled with a kind of continuously distributed matter which may be called the $\Omega(2)$ substratum.*

According to Assumption 1, the energy-momentum tensor $T_{\mu\nu}^{\Omega(2)}$ of the $\Omega(2)$ substratum will appear in Eqs. (2). Thus, Eqs. (2) are revised to be

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\kappa}{g} \left(T_{\mu\nu}^m + T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)} \right). \quad (3)$$

Comparing Eqs. (3) and Eqs. (1), it is natural for us to speculate that the cosmological term $\Lambda g_{\mu\nu}$ in Eqs. (1) may stem from the term $(\kappa/g) \left(T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)} \right)$ in Eqs. (3). We will investigate this clue in the following sections.

IV. GRAVITATIONAL FIELD EQUATIONS IN VACUUM

The movements of continuously distributed matters and fields should also be studied based on special relativity [23–25]. Maxwell's equations are valid in the frame of reference that attached to the $\Omega(1)$ substratum [10]. In order to compare our theory with Newtonian theory of gravitation, which is non-relativistic, it is better to single out the speed of light c as a large parameter. Thus, we introduce the following notations

$$x^0 = t, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad (4)$$

where $\{0, x, y, z\}$ is a Cartesian coordinate system for a three-dimensional Euclidean space that attached to the $\Omega(1)$ substratum, $\{0, t\}$ is a one-dimensional time coordinate. We denote this reference frame as $S_{\Omega(1)}$. An inertial reference frame is defined to be a coordinates system which is static or moving with a constant velocity relative to the reference frame $S_{\Omega(1)}$ [13].

We will use Greek indices α, β, μ, ν , etc., for the range 0, 1, 2, 3 and use Latin indices i, j, k , etc., for the range 1, 2, 3. According to a theorem of V. Fock, the characteristics of the generalized wave equation, or the d'Alembert equation

$$\frac{1}{\sqrt{-\gamma}} \frac{\partial}{\partial x^\alpha} \left(\sqrt{-\gamma} \gamma^{\alpha\beta} \frac{\partial \psi}{\partial x^\beta} \right) = 0, \quad (5)$$

is ([26], p. 432)

$$\gamma^{\mu\nu} \frac{\partial \omega}{\partial x^\mu} \frac{\partial \omega}{\partial x^\nu} = 0, \quad (6)$$

where ψ is a field variable, $\gamma_{\mu\nu}$ is a metric tensor, $\gamma = \text{Det } \gamma_{\mu\nu}$, $\omega(x^0, x^1, x^2, x^3)$ is the characteristics.

Based on the Maxwell's equations, the law of propagation of an electromagnetic wave front in the reference frame $S_{\Omega(1)}$ can be derived and can be written as ([26], p13)

$$\frac{1}{c^2} \left(\frac{\partial \omega}{\partial x^0} \right)^2 - \left(\frac{\partial \omega}{\partial x^1} \right)^2 - \left(\frac{\partial \omega}{\partial x^2} \right)^2 - \left(\frac{\partial \omega}{\partial x^3} \right)^2 = 0, \quad (7)$$

where $\omega(x^0, x^1, x^2, x^3)$ is an electromagnetic wave front, c is the velocity of light in the reference frame $S_{\Omega(1)}$.

Comparing Eq. (7) and Eq. (6), the contravariant metric tensor $g^{\mu\nu}$ must have the following limit values at infinity ([26], p. 196)

$$(g^{00})_\infty = \frac{1}{c^2}, \quad (8)$$

$$(g^{0i})_\infty = 0, \quad (9)$$

$$(g^{ij})_\infty = -\delta_{ij}, \quad (10)$$

where δ_{ij} is the Kronecker symbol.

Eqs. (8-10) can be written as

$$(g^{\mu\nu})_{\infty} = \begin{pmatrix} \frac{1}{c^2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (11)$$

The corresponding limit values of the metric tensor $g_{\mu\nu}$ will be ([26], p. 196)

$$(g_{00})_{\infty} = c^2, \quad (12)$$

$$(g_{0i})_{\infty} = 0, \quad (13)$$

$$(g_{ij})_{\infty} = -\delta_{ij}. \quad (14)$$

Eqs. (12-14) can be written as

$$(g_{\mu\nu})_{\infty} = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (15)$$

Using Eq. (15), we have $(g_0)_{\infty} \equiv \text{Det } (g_{\mu\nu})_{\infty} = -c^2$. Thus, for weak fields, we have $g_0 \approx (g_0)_{\infty} = -c^2$, where g_0 is defined by $g_0 \equiv \text{Det } g_{\mu\nu}$ in the coordinate system $S_0 \equiv \{t, x, y, z\}$.

Since the Einstein's gravitational constant κ in Eqs. (3) is not a dimensionless constant, κ depends on the choice of coordinate system. In order to determine the Einstein's gravitational constant κ in the coordinate system $S_0 \equiv \{t, x, y, z\}$, we study a weak gravitational field of a matter system. The relationship between the contravariant energy-momentum tensor $T_m^{\mu\nu}$ and the contravariant mass tensor $M_m^{\mu\nu}$ of the matter system is

$$T_m^{\mu\nu} = M_m^{\mu\nu} c^2. \quad (16)$$

Neglecting the terms $T_{\mu\nu}^{\Omega(1)}$ and $T_{\mu\nu}^{\Omega(2)}$ in Eqs. (3) and using $g_0 \approx (g_0)_{\infty} = -c^2$ and Eqs. (16), Eqs. (3) can be written as

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\kappa_0 M_m^{\mu\nu}, \quad (17)$$

where κ_0 denotes the Einstein's gravitational constant κ in the coordinate system $S_0 \equiv \{t, x, y, z\}$, $R^{\mu\nu}$ is the contravariant Ricci tensor.

Applying the same method of V. Fock ([26], p. 198-199), we have

$$\kappa_0 = \frac{8\pi G}{c^2}. \quad (18)$$

Since there is no atoms in vacuum, the field equations (3) in vacuum reduce to

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{\kappa_0}{g_0} (T_{\Omega(1)}^{\mu\nu} + T_{\Omega(2)}^{\mu\nu}). \quad (19)$$

V. ENERGY-MOMENTUM TENSOR OF THE $\Omega(1)$ SUBSTRATUM

It is known that Maxwell's equations are valid in the frame of reference $S_{\Omega(1)}$ [10]. Let T_0 be the characteristic time of a macroscopic observer of the $\Omega(1)$ substratum. We may suppose that the observer's time scale T_0 is very large comparing to the Maxwellian relaxation time τ of the $\Omega(1)$ substratum. Therefore, the macroscopic observer concludes that the $\Omega(1)$ substratum behaves like a Newtonian-fluid [10]. Thus, the contravariant mass tensor $M_{\Omega(1)S_1}^{\mu\nu}$ of the $\Omega(1)$ substratum in the coordinate system $S_1 \equiv \{ct, x, y, z\}$ can be written approximatively as ([26], p. 105)

$$M_{\Omega(1)S_1}^{00} = \rho_1 + \frac{1}{c^2} \left(\frac{1}{2}\rho_1 v^2 + \rho_1 \Pi_1 \right), \quad (20)$$

$$M_{\Omega(1)S_1}^{0i} = \frac{\rho_1 v_i}{c} + \frac{v_i}{c^3} \left(\frac{\rho_1 v^2}{2} + \rho_1 \Pi_1 + p_1 \right), \quad (21)$$

$$M_{\Omega(1)S_1}^{ij} = \frac{1}{c^2} (\rho_1 v_i v_j + p_1 \delta_{ij}), \quad (22)$$

where ρ_1 is the rest mass density of a volume element of the $\Omega(1)$ substratum in a laboratory frame relative to which the volume element moving with velocity $\mathbf{v}$, $v = \sqrt{\mathbf{v} \cdot \mathbf{v}}$, p_1 is the pressure of the $\Omega(1)$ substratum,

$$\Pi_1 = -\frac{p_1}{\rho_1^*} + \int_0^{p_1} \frac{1}{\rho_1^*} dp_1, \quad (23)$$

$$\rho_1^* = \rho_1 \sqrt{1 - \frac{v^2}{c^2}}. \quad (24)$$

The relationships between the contravariant mass tensor $M_S^{\mu\nu}$ in the coordinate system $S_1 \equiv \{ct, x, y, z\}$ and the contravariant mass tensor $M_{\Omega(1)}^{\mu\nu}$ in the present coordinate system $S_0 \equiv \{t, x, y, z\}$ are ([26], p. 198)

$$M_{\Omega(1)}^{00} = \frac{1}{c^2} M_{\Omega(1)S_1}^{00}, \quad (25)$$

$$M_{\Omega(1)}^{0i} = \frac{1}{c} M_{\Omega(1)S_1}^{0i}, \quad (26)$$

$$M_{\Omega(1)}^{ij} = M_{\Omega(1)S_1}^{ij}. \quad (27)$$

Thus, the contravariant mass tensor $M_{\Omega(1)}^{\mu\nu}$ of the $\Omega(1)$ substratum in the coordinate system $S_0 \equiv \{t, x, y, z\}$ can be written as

$$M_{\Omega(1)}^{00} = \frac{\rho_1}{c^2} + \frac{1}{c^4} \left(\frac{1}{2}\rho_1 v^2 + \rho_1 \Pi_1 \right), \quad (28)$$

$$M_{\Omega(1)}^{0i} = \frac{\rho_1 v_i}{c^2} + \frac{v_i}{c^4} \left(\frac{\rho_1 v^2}{2} + \rho_1 \Pi_1 + p_1 \right), \quad (29)$$

$$M_{\Omega(1)}^{ij} = \frac{1}{c^2} (\rho_1 v_i v_j + p_1 \delta_{ij}). \quad (30)$$

Then, for the present coordinate system $S_0 \equiv \{t, x, y, z\}$, the contravariant energy-momentum tensor

$T_{\Omega(1)}^{\mu\nu} = M_{\Omega(1)}^{\mu\nu} c^2$ can be written as

$$T_{\Omega(1)}^{00} = \rho_1 + \frac{1}{c^2} \left(\frac{1}{2} \rho_1 v^2 + \rho_1 \Pi_1 \right), \quad (31)$$

$$T_{\Omega(1)}^{0i} = \rho_1 v_i + \frac{v_i}{c^2} \left(\frac{\rho_1 v^2}{2} + \rho_1 \Pi_1 + p_1 \right), \quad (32)$$

$$T_{\Omega(1)}^{ij} = \rho_1 v_i v_j + p_1 \delta_{ij}. \quad (33)$$

Assumption 2 We have the following relationships

$$\langle \rho_1 v_i v_j + p_1 \delta_{ij} \rangle = \alpha_0 \rho_{1e} \delta_{ij}, \quad (34)$$

where $\langle \rangle$ denotes the averaged macroscopic value in the sense of statistical physics from the view point of a macroscopic observer, ρ_{1e} is the energy density of a volume element of the $\Omega(1)$ substratum, α_0 is a parameter to be determined.

Einstein's mass energy formula for a particle can be written as

$$E = mc^2, \quad (35)$$

where E is the energy of the particle, m is the mass of the particle, c is the velocity of light in vacuum.

Applying Einstein's formula (35) to a volume element of the $\Omega(1)$ substratum, we have

$$\rho_{1e} = \frac{mc^2}{V} = \frac{m_r c^2}{V \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\rho_1 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (36)$$

where V is the volume of a volume element, m_r is the rest mass of a volume element, v is the velocity of the volume element.

Using Eq. (33), Eq. (34) and Eq. (36), we have

$$\langle T_{\Omega(1)}^{ij} \rangle = \frac{\alpha_0 \rho_1 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \delta_{ij}, i, j = 1, 2, 3. \quad (37)$$

We suppose that $v^2 \ll c^2$. Thus, $v^2/c^2 \approx 0$. Neglecting those terms of order of $1/c^2$ in Eq. (31), Eqs. (32) and Eqs. (37) and using $\langle T_{\Omega(1)}^{ij} \rangle \approx T_{\Omega(1)}^{ij}$ in Eqs. (37), a macroscopic observer will obtain

$$T_{\Omega(1)}^{00} \approx \rho_1, \quad (38)$$

$$T_{\Omega(1)}^{0i} \approx \rho_1 v_i, \quad (39)$$

$$T_{\Omega(1)}^{ij} \approx \alpha_0 \rho_1 c^2 \delta_{ij}. \quad (40)$$

Using Eq. (38-40) and Eqs. (12-14), we have

$$T_{\Omega(1)} = g_{\alpha\beta} T_{\Omega(1)}^{\alpha\beta} \approx (1 - 3\alpha_0) \rho_1 c^2. \quad (41)$$

VI. ENERGY-MOMENTUM TENSOR OF THE $\Omega(2)$ SUBSTRATUM

Let T_0 be the characteristic time of a macroscopic observer of the $\Omega(2)$ substratum. We may suppose that the observer's time scale T_0 is very large comparing to the Maxwellian relaxation time τ of the $\Omega(2)$ substratum. Therefore, the macroscopic observer concludes that the $\Omega(2)$ substratum behaves like a fluid. Similar to the case of the $\Omega(1)$ substratum, the contravariant energy-momentum tensor $T_{\Omega(2)}^{\mu\nu}$ of the $\Omega(2)$ substratum in the present coordinate system $S_0 \equiv \{t, x, y, z\}$ can be written as

$$T_{\Omega(2)}^{00} = \rho_2 + \frac{1}{c^2} \left(\frac{1}{2} \rho_2 v^2 + \rho_2 \Pi_2 \right), \quad (42)$$

$$T_{\Omega(2)}^{0i} = \rho_2 v_i + \frac{v_i}{c^2} \left(\frac{\rho_2 v^2}{2} + \rho_2 \Pi_2 + p_2 \right), \quad (43)$$

$$T_{\Omega(2)}^{ij} = \rho_2 v_i v_j + p_2 \delta_{ij}, \quad (44)$$

where ρ_2 is the rest mass density of a volume element of the $\Omega(2)$ substratum in a laboratory frame relative to which the volume element moving with velocity $\mathbf{v}$, $v = \sqrt{\mathbf{v} \cdot \mathbf{v}}$, p_2 is the pressure of the $\Omega(2)$ substratum,

$$\Pi_2 = -\frac{p_2}{\rho_2^*} + \int_0^{p_2} \frac{1}{\rho_2^*} dp_2, \quad (45)$$

$$\rho_2^* = \rho_2 \sqrt{1 - \frac{v^2}{c^2}}. \quad (46)$$

Assumption 3 We have the following relationships

$$\langle \rho_2 v_i v_j + p_2 \delta_{ij} \rangle = \alpha_0 \rho_{2e} \delta_{ij}, \quad (47)$$

where ρ_{2e} is the energy density of a volume element of the $\Omega(2)$ substratum, α_0 is a parameter to be determined.

Similar to the case of the $\Omega(1)$ substratum, we have

$$\langle T_{\Omega(2)}^{ij} \rangle = \frac{\alpha_0 \rho_2 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \delta_{ij}, i, j = 1, 2, 3. \quad (48)$$

Neglecting those terms of order of $1/c^2$ in Eq. (42), Eqs. (43) and Eqs. (48) and using $\langle T_{\Omega(2)}^{ij} \rangle \approx T_{\Omega(2)}^{ij}$ in Eqs. (48), a macroscopic observer will obtain

$$T_{\Omega(2)}^{00} \approx \rho_2, \quad (49)$$

$$T_{\Omega(2)}^{0i} \approx \rho_2 v_i, \quad (50)$$

$$T_{\Omega(2)}^{ij} \approx \alpha_0 \rho_2 c^2 \delta_{ij}. \quad (51)$$

Using Eq. (49-51) and Eqs. (12-14), we have

$$T_{\Omega(2)} = g_{\alpha\beta} T_{\Omega(2)}^{\alpha\beta} \approx (1 - 3\alpha_0) \rho_2 c^2. \quad (52)$$

VII. QUASI-STATIC SOLUTIONS OF THE GRAVITATIONAL FIELD EQUATIONS IN VACUUM

Since the field is weak, we have $g_0 \approx -c^2$. Thus, the field equations (19) can be written as ([26], p. 198)

$$R^{\mu\nu} = -\frac{\kappa_0}{c^2} \left[T_{\Omega(1)}^{\mu\nu} + T_{\Omega(2)}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} (T_{\Omega(1)} + T_{\Omega(2)}) \right]. \quad (53)$$

Using Eqs. (8-10), Eqs.(38-41) and Eqs.(49-52), we have

$$\begin{aligned} T_{\Omega(1)}^{00} + T_{\Omega(2)}^{00} &- \frac{1}{2} g^{00} (T_{\Omega(1)} + T_{\Omega(2)}) \\ &\approx \frac{(1 + 3\alpha_0)(\rho_1 + \rho_2)}{2}, \end{aligned} \quad (54)$$

$$\begin{aligned} T_{\Omega(1)}^{0i} + T_{\Omega(2)}^{0i} &- \frac{1}{2} g^{0i} (T_{\Omega(1)} + T_{\Omega(2)}) \\ &\approx (\rho_1 + \rho_2) v_i, \end{aligned} \quad (55)$$

$$\begin{aligned} T_{\Omega(1)}^{ij} + T_{\Omega(2)}^{ij} &- \frac{1}{2} g^{ij} (T_{\Omega(1)} + T_{\Omega(2)}) \\ &\approx \frac{(1 - \alpha_0)(\rho_1 + \rho_2)c^2}{2} \delta_{ij}. \end{aligned} \quad (56)$$

Inertial reference frames are harmonic. For harmonic coordinates, we have approximately ([26], p. 198)

$$R^{\mu\nu} = -\frac{1}{2c^2} \frac{\partial^2 g^{\mu\nu}}{\partial t^2} + \frac{1}{2} \nabla^2 g^{\mu\nu}, \quad (57)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplace operator, i.e.,

$$\nabla^2 g^{\mu\nu} = \frac{\partial^2 g^{\mu\nu}}{\partial (x^1)^2} + \frac{\partial^2 g^{\mu\nu}}{\partial (x^2)^2} + \frac{\partial^2 g^{\mu\nu}}{\partial (x^3)^2}. \quad (58)$$

Since we are interested in the quasi-static behavior of $g^{\mu\nu}$, we suppose that ([26], p. 198)

$$\frac{\partial^2 g^{\mu\nu}}{c^2 \partial t^2} \approx 0. \quad (59)$$

Putting Eqs.(54-57) into Eqs.(53) and using Eqs. (59), we obtain

$$\nabla^2 g^{00} \approx -\frac{\kappa_0}{c^2} (1 + 3\alpha_0)(\rho_1 + \rho_2), \quad (60)$$

$$\nabla^2 g^{0i} \approx -\frac{2\kappa_0}{c^2} (\rho_1 + \rho_2) v_i, \quad (61)$$

$$\nabla^2 g^{ij} \approx -\kappa_0 (1 - \alpha_0)(\rho_1 + \rho_2) \delta_{ij}. \quad (62)$$

Newton's gravitational potential U_0 satisfies the following equation ([26], p. 199)

$$\nabla^2 U_0 = 4\pi G(\rho_1 + \rho_2), \quad (63)$$

where G is Newton's gravitational constant.

Following V. Fock ([26], p199), we introduce the following gravitational vector potentials U_i which satisfy

$$\nabla^2 U_i = 4\pi G(\rho_1 + \rho_2) v_i, i = 1, 2, 3. \quad (64)$$

Proposition 4 Suppose that the following conditions are valid: (1) $U_0 \approx 0$ and $U_i \approx 0$ at infinity; (2) in the spherical coordinates (r, θ, ψ) , U_0 and $g^{\mu\nu}$ are axially symmetric, i.e., they do not vary with the azimuthal angle ψ ; (3) the contravariant metric tensor $g^{\mu\nu}$ depends on r and θ as the Newton potential U_0 ; (4) the law of propagation of an electromagnetic wave front, i.e., Eq. (7), is valid in the reference frame $S_{\Omega(1)}$ at infinity. Then, the solutions of Eq. (60-62) are

$$g^{00} \approx \frac{1}{c^2} - \frac{(2 + 6\alpha_0)U_0}{c^4}, \quad (65)$$

$$g^{0i} \approx -\frac{4U_i}{c^4}, \quad (66)$$

$$g^{ij} \approx \left[-1 + \frac{(2 - 2\alpha_0)U_0}{c^2} \right] \delta_{ij}. \quad (67)$$

Proof of Proposition 4. From Eq. (63), we have

$$\rho_1 = \frac{1}{4\pi G} \nabla^2 U_0. \quad (68)$$

Putting Eq. (68) into Eq. (60), we have

$$\nabla^2 (g^{00} + \beta_0 U_0) \approx 0, \quad (69)$$

where

$$\beta_0 = \frac{(1 + 3\alpha_0)\kappa}{4\pi G c^2}. \quad (70)$$

Using Eq. (18), Eq. (70) can be written as

$$\beta_0 = \frac{2 + 6\alpha_0}{c^4}. \quad (71)$$

Eq. (69) is a Laplace's equation. Since U_0 and g^{00} are axially symmetric, a general form of solution of Eq. (69) can be obtained by using the spherical coordinates (r, θ, φ) ([27], p. 132)

$$g^{00}(r, \theta) + \beta_0 U_0(r, \theta) \approx \sum_n \left(A_n r^n + \frac{B_n}{r^{n+1}} \right) P_n(\cos \theta), \quad (72)$$

where A_n and B_n are arbitrary constants, n are integers, $P_n(x)$ are Legendre's functions of the first kind which is defined by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n. \quad (73)$$

We have assumed that the contravariant metric tensor $g^{\mu\nu}$ depends on r and θ as the Newton potential U_0 . Thus, we set $n = 0$ and $B_n = 0$ in Eq. (72) and obtain

$$g^{00} + \beta_0 U_0 \approx A_0, \quad (74)$$

where A_0 is a constant to be determined.

Using Eq. (64) and Eq. (18), Eqs. (61) can be written as

$$\nabla^2 \left(g^{0i} + \frac{4}{c^4} U_i \right) \approx 0. \quad (75)$$

Using Eq. (63) and Eq. (18), Eqs. (62) can be written as

$$\nabla^2 \left(g^{ij} + \frac{2-2\alpha_0}{c^2} U_0 \delta_{ij} \right) \approx 0. \quad (76)$$

Applying similar method in solving Eq. (69) to solve Eqs. (75) and Eqs. (76), we have

$$g^{0i} + \beta_1 U_i \approx A_1, \quad (77)$$

$$g^{ij} + \beta_2 U_0 \delta_{ij} \approx A_2 \delta_{ij}, \quad (78)$$

where A_1 and A_2 are constants to be determined,

$$\beta_1 = \frac{4}{c^4}, \quad \beta_2 = \frac{2-2\alpha_0}{c^2}. \quad (79)$$

Noticing $U_0 \approx U_i \approx 0$ at infinity and using Eqs. (8-10), we have

$$A_0 = \frac{1}{c^2}, \quad A_1 = 0, \quad A_2 = -1. \quad (80)$$

Thus, the solutions of Eqs. (60-62) are Eq. (74), Eqs. (77) and Eqs. (78), which can be written as Eqs. (65-67). $\square$

The variant metric tensor $g_{\mu\nu}$ can be derived from the contravariant metric tensor $g^{\mu\nu}$ and the results are ([26], p. 201).

$$g_{00} \approx c^2 - (2 + 6\alpha_0)U_0, \quad (81)$$

$$g_{0i} \approx -\frac{4U_i}{c^4}, \quad (82)$$

$$g_{ij} \approx \left[-1 - \frac{(2-2\alpha_0)U_0}{c^2} \right] \delta_{ij}. \quad (83)$$

VIII. A THEORETICAL DERIVATION OF THE COSMOLOGICAL CONSTANT

We speculate that the $\Omega(1)$ and $\Omega(2)$ substratums may be responsible for the so called dark energy. Following this idea, we set out to explore the possible value of the parameter α_0 in Eqs. (34) and Eqs. (47).

We have supposed that the velocity v of the volume element of the $\Omega(1)$ substratum, or the $\Omega(2)$ substratum, is small, i.e., $v^2 \ll c^2$. Thus, the terms $\rho_1 v_i$ in Eqs. (39) and Eqs. (50) may be neglected. Then according to Eqs. (38-40) and Eqs. (49-51), we have

$$T_{\Omega(1)}^{\mu\nu} + T_{\Omega(2)}^{\mu\nu} \approx (\rho_1 + \rho_2)c^2 \begin{pmatrix} \frac{1}{c^2} & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 \\ 0 & 0 & 0 & \alpha_0 \end{pmatrix}. \quad (84)$$

Since v , ρ_1 and ρ_2 are small enough, Newton's gravitational potential U_0 in Eq. (63) and the gravitational vector potentials U_i in Eqs. (64) are small relative to c^2 . Thus, the terms involving U_0 or U_i in Eq. (65-67) may

be neglected. Then, the contravariant metric tensor $g^{\mu\nu}$ of the $\Omega(1)$ substratum can be written as

$$g^{\mu\nu} \approx \begin{pmatrix} \frac{1}{c^2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (85)$$

In section III, we speculate that the cosmological term $\Lambda g_{\mu\nu}$ in Eqs. (1) may stem from the term $(\kappa/g_0) (T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)})$ in Eqs. (3). Now comparing Eq. (84) and Eq. (85), it is natural for us to speculate that $g^{\mu\nu}$ may depends on $T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)}$ approximately linearly. Therefore, we introduce the following assumption.

Assumption 5 $\alpha_0 \approx -1$.

Comparing Eq. (84) and Eq. (85) and applying Assumption 5, we obtain the following relationships

$$T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)} \approx (\rho_1 + \rho_2)c^2 g_{\mu\nu}. \quad (86)$$

Now we study the gravitational field of a system of matter, the $\Omega(1)$ substratum and the $\Omega(2)$ substratum. We suppose that the gravitational field is weak, i.e., Eq. (85) is valid. Thus, we have $g_0 \approx -c^2$ and Eqs. (86) are valid. Putting Eqs. (86) into Eqs. (3) and using $g_0 \approx -c^2$, we have

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \approx -\frac{\kappa_0}{c^2}T_{\mu\nu}^m - \kappa_0(\rho_1 + \rho_2)g_{\mu\nu}. \quad (87)$$

Comparing Eqs. (87) and Eqs. (1), we obtain the following relationship

$$\Lambda = \kappa_0(\rho_1 + \rho_2). \quad (88)$$

Using Eq. (18), Eq. (88) can be written as

$$\Lambda = \frac{8\pi G(\rho_1 + \rho_2)}{c^2}. \quad (89)$$

IX. A THEORETICAL CALCULATION OF THE MASS DENSITY OF THE $\Omega(1)$ AND $\Omega(2)$ SUBSTRATUMS

The mass density of the electromagnetic aether is an open problem since eighteenth century [16, 28, 29]. From Eq. (89), we have

$$\rho_1 + \rho_2 = \frac{\Lambda c^2}{8\pi G}. \quad (90)$$

Thus, the energy density $\rho_{1e} + \rho_{2e}$ of the $\Omega(1)$ and $\Omega(2)$ substratums is

$$\rho_{1e} + \rho_{2e} = (\rho_1 + \rho_2)c^2 = \frac{\Lambda c^4}{8\pi G}. \quad (91)$$

We have the following data [30–32]

$$\Lambda = 1.088(30) \times 10^{-52} \text{m}^{-2}, \quad (92)$$

$$c = 2.99792458 \times 10^8 \text{m} \cdot \text{s}^{-1}, \quad (93)$$

$$G = 6.67430(15) \times 10^{-11} \text{m}^{-2} \text{kg}^{-1} \text{s}^{-2}. \quad (94)$$

Using these data, we obtain

$$\rho_1 + \rho_2 = 5.24065 \times 10^{-26} \text{kg} \cdot \text{m}^{-3}, \quad (95)$$

$$\rho_{1e} + \rho_{2e} = 4.41006 \times 10^{-9} \text{J} \cdot \text{m}^{-3}. \quad (96)$$

The mass m_p of a proton is [30–32]

$$m_p = 1.67262192369(51) \times 10^{-27} \text{kg}. \quad (97)$$

Comparing Eq. (97) and Eq. (95), the mass density of the $\Omega(1)$ and $\Omega(2)$ substratums is equivalent to that 31.33195 protons contained in a box with a volume of $1.0m^3$.

X. A POSSIBLE CANDIDATE OF THE DARK ENERGY

According to Assumption 5, Eqs. (87) and Eqs. (86), the origin of the cosmological term $\Lambda g_{\mu\nu}$ in Eqs. (1) is the energy-momentum tensors $T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)}$ of the $\Omega(1)$ and $\Omega(2)$ substratums. Therefore, the $\Omega(1)$ and $\Omega(2)$ substratums may be a candidate of the so called concept of dark energy [2].

Applying Assumption 5, Eq. (36), Eqs. (34) and Eqs. (47), we have

$$\langle (\rho_1 + \rho_2)v_i v_j + (p_1 + p_2)\delta_{ij} \rangle = -\frac{(\rho_1 + \rho_2)c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \delta_{ij}. \quad (98)$$

The physical meaning of $\langle (\rho_1 + \rho_2)v_i v_j \rangle$ is the macroscopic dynamical pressure in the $\Omega(1)$ and $\Omega(2)$ substratums. $\langle (p_1 + p_2)\delta_{ij} \rangle$ is the macroscopic static pressure. Thus, the physical meaning of $\langle (\rho_1 + \rho_2)v_i v_j + (p_1 + p_2)\delta_{ij} \rangle$ is the macroscopic total pressure in the $\Omega(1)$ and $\Omega(2)$ substratums. Eqs. (98) show that the macroscopic total pressure in the $\Omega(1)$ and $\Omega(2)$ substratums is negative. This negativeness of the pressure of the $\Omega(1)$ and $\Omega(2)$ substratums coincides with the negativeness of the pressure of dark energy. Thus, the $\Omega(1)$ and $\Omega(2)$ substratums may be a possible candidate of the dark energy.

XI. AN OPINION ON THE COSMOLOGICAL CONSTANT PROBLEM

Various solutions have been proposed to solve the cosmological constant problem [33]. The first class of solutions is to modify the theory of gravitation. The second class of solutions is to revise the standard model of particle physics.

According to the theory of vacuum mechanics [11, 13, 22], two sinks in the $\Omega(0)$ substratum will gravitate with each other. The particles that constitute the $\Omega(1)$ and $\Omega(2)$ substratums are sinks in the $\Omega(0)$ substratum. Thus, the particles of the $\Omega(1)$ and $\Omega(2)$ substratums will also generate gravity [11]. Therefore, only three kinds of energy-momentum tensor, i.e., $T_{\mu\nu}^m$, $T_{\mu\nu}^{\Omega(1)}$ and $T_{\mu\nu}^{\Omega(2)}$, will appear as source terms in the generalized Einstein's equations (3). Not all kinds of energies will appear as source terms in the generalized Einstein's equations (3). The zero-point energy of electromagnetic fields will not appear as a source term in the generalized Einstein's equations (3).

From Eqs. (87) and Eqs. (86), we notice that the origin of the cosmological term $\Lambda g_{\mu\nu}$ in Eqs. (1) is the energy-momentum tensors $T_{\mu\nu}^{\Omega(1)} + T_{\mu\nu}^{\Omega(2)}$ of the $\Omega(1)$ and $\Omega(2)$ substratums. Thus, the cosmological term $\Lambda g_{\mu\nu}$ may not result from the zero-point energy of electromagnetic fields.

However, some people believed that all kinds of energies should appear as source terms in the Einstein's equations. It may be this belief that leads to the so called cosmological constant problem.

XII. CONCLUSION

Thomson's analogies between electrical phenomena and elasticity helped J. C. Maxwell to establish a mechanical model of electrical phenomena. Following J. C. Maxwell, we suppose that vacuum is filled with a kind of continuously distributed matter which may be called the $\Omega(1)$ substratum, or the electromagnetic aether. Lord Kelvin believes that the electromagnetic aether must also gravitate with each other and matter. We also suppose that vacuum is filled with a kind of continuously distributed matter, which may be called the $\Omega(2)$ substratum. According to a theorem of V. Fock on the mass tensor of a fluid, the contravariant energy-momentum tensors of the $\Omega(1)$ and $\Omega(2)$ substratums are established. Quasi-static solutions of the gravitational field equations in vacuum are obtained. Based on an assumption, relationships between the contravariant energy-momentum tensor of the $\Omega(1)$ and $\Omega(2)$ substratums and the contravariant metric tensor are obtained. Thus, the cosmological constant is calculated theoretically. Therefore, the $\Omega(1)$ and $\Omega(2)$ substratums may be a possible candidate of the dark energy. As a byproduct, the mass density of the $\Omega(1)$ and $\Omega(2)$ substratums is calculated theoretically and is equivalent to that 31.33195 protons contained in a box with a volume of $1.0m^3$. The zero-point energy of electromagnetic fields will not appear as a source term in the Einstein's equations. The cosmological constant problem is one of the puzzles in physics. Some people believed that all kinds of energies should appear as source terms in the Einstein's equations. It may be this belief that leads to the cosmological constant problem.

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