Dark Energy Stars with Quadratic Equation of State

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Abstract: Recent astronomical observations with respect to measurements in distant supernovas, cosmic microwave background and weak gravitational lensing confirm that the Universe is undergoing a phase of accelerated expansion and it has been proposed that this cosmological behavior is caused by a hypothetical dark energy which has a strong negative pressure that allows explain the expanding universe. Several theoretical ideas and models related dark the energy includes the cosmological constant, quintessence, Chaplygin gas, braneworld and tachyonic scalar fields. In this paper, we have obtained new relativistic stellar configurations considering an anisotropic fluid distribution with a charge distribution which could represents a potential model of a dark energy star. In order to investigate the effect of a quadratic equation of state in this anisotropic model we specify particular forms for the gravitational potential that allow solving the Einstein-Maxwell field equations. For these new solutions we checked that the radial pressure, metric coefficients, energy density, anisotropy factor, charge density , mass function are well defined and are regular in the interior of the star. The solutions found can be used in the development of dark energy stars models satisfying all physical acceptability conditions but the causality condition and strong energy condition are violated. We expect that these models have multiple applications in astrophysics and cosmology.

Keywords: dark energy; stellar configurations; anisotropic fluid distribution; quadratic equation of state; einstein-maxwell field equations; metric coefficients

1. Introduction

The nature of the energy content in the Universe is a fundamental issue in cosmology and currently there is sufficient observational evidence as measurements of supernovas of type Ia and microwave background radiation that favors an accelerate expansion [1]. The explanation for this cosmological behavior in the framework of general relativity requires assuming that a considerable part of the Universe consists of a hypothetical dark energy with a negative pressure component [2]. Several authors have suggested that this dark energy is a cosmic fluid parameterized by an equation of state $\omega = p/\rho < -1/3$ where p is the spatially homogeneous pressure and ρ the dark energy density [1-5]. The range for which $\omega < -1$ has been denoted phantom energy and possesses peculiar properties, such as

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negative temperatures and the energy density increases to infinity in a finite time, resulting in a big rip [2-4]. It also provides a natural scenario for the existence of exotic geometries such as wormholes [5-7].

The structure of relativistic stars and the research on gravitational collapse is of fundamental importance in astrophysics since the formulation of general theory of relativity. In the construction of theoretical models of relativistic stars we can refer to the pioneering works of Schwarzschild [8], Tolman [9], Oppenheimer and Volkoff [10]. Schwarzschild [8] found analytical solutions that allowed describing a star with uniform density, Tolman [9] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [10] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention Chandrasekhar's contributions [11] in the model production of white dwarfs in presence of relativistic effects and the work of Baade and Zwicky [12], they propose the concept of neutron stars and identify a astronomic dense objects known as supernovas.

The notion of dark energy is that of a homogeneously distributed cosmic fluid and when extended to inhomogeneous spherically symmetric spacetimes, the pressure appearing in the equation of state is now a negative radial pressure, and the tangential pressure is then determined via the field equations [2,3]. Lobo [3] explored several configurations, by imposing specific choices for the mass function and studied the dynamical stability of these models by applying the general stability formalism developed by Lobo and Crawford [13]. Chan et al. [14] propose that the mass function is a natural consequence of the Einstein´s field equations and considered a core with a homogeneous energy density, described by the Lobo´s first solution [3]. Malaver and Esculpi [15] presented a new model of dark energy star by imposing specific choice for the mass function which correspond an increase in energy density inside of the star. Bibi et al.[4] obtained a new class of solutions of the Einstein-Maxwell field equations which represents a model for dark energy stars with the equation of state $p_r=-\rho$. Malaver et al.[16] found a new family of solutions to the Einstein-Maxwell system considering a particular form of the gravitational potential $Z(x)$ and the electric field intensity with a linear equation of state that represents a model of dark energy star. Ananda and Bruni [17] studied the dynamics of some homogenous cosmological models containing a fluid with a quadratic equation of state. Parsaei and Rastgoo [18] propose several phantom wormhole solutions with the non-linear equation of state $p=\omega\rho+\omega_1\rho^2$. According Chan et al.[14] the denomination dark energy is applied to fluids which violate only the strong energy condition given by $\rho+p_r+2p_t \geq 0$ where ρ is the energy density, p_r and p_t are the radial pressure and tangential pressure, respectively.

Stellar models consisting of spherically symmetric distribution of matter with presence of anisotropy in the pressure have been widely considered in the frame of general relativity [19-31]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [32] or another physical phenomenon by the presence of an electrical field [33]. Many researchers and scientists have used a vast and great variety of mathematical techniques to try and test in order to obtain solutions of the Einstein-Maxwell field equations for anisotropic relativistic stars since it has been demonstrated by Komathiraj and Maharaj [34], Thirukkanesh and Maharaj [35], Maharaj et al.[36], Thirukkanesh and Ragel [37,38], Feroze and Siddiqui [39,40], Sunzu et al.[41], Pant et al. [42] and Malaver [43-47]. These analyses indicate that the

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system of Einstein-Maxwell equations is very important in the description of ultracompacts objects.

The aim of this paper is to generate new class of solutions of the field equations which represents a potential model of dark energy stars with a quadratic equation of state using a specify forms for the gravitational potential and the electric field intensity. We assume that the denomination dark energy is applied to fluids which violate the strong energy condition [14]. This article is organized as follows, in Section 2, we present Einstein´s field equations. In Section 3, we make a particular choice of gravitational potential $Z(x)$ that allows solving the field equations and we have obtained new models for dark energy stars consistent alone of dark matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. Einstein-Maxwell Field Equations

We consider a spherically symmetric, static and homogeneous spacetime. In Schwarzschild coordinates the metric is given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2 \tag{2}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2 \tag{3}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2 \tag{4}$$

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$$\sigma = \frac{1}{r^2} e^{-\lambda} (r^2 E)' \tag{5}$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity,

p_t is the tangential pressure and primes denote differentiations with respect to r. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2 y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji [48], the Einstein field equations can be written as

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \tag{6}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \tag{7}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \tag{8}$$

$$p_t = p_r + \Delta \tag{9}$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \tag{10}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \tag{11}$$

σ is the charge density, $\Delta = p_t - p_r$ is the anisotropy factor and dots denote differentiation with respect to x. With the transformations of [48], the mass within a radius r of the sphere takes the form

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\left(\rho^* + E^2\right)dx \tag{12}$$

where $$\rho^* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)c$$

The interior metric (1) with the charged matter distribution should match the exterior spacetime described by the Reissner-Nordstrom metric:

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{13}$$

where the total mass and the total charge of the star are denoted by M and Q^2, respectively. The junction conditions at the stellar surface are obtained by matching the first and the second fundamental forms for the interior metric (1) and the exterior metric (13).

In this paper, we assume the following equation of state proposed by Parsaei and Rastgoo [18]

$$p_r = \omega\rho + \omega_1\rho^2 \tag{14}$$

ω is the dark energy parameter and $\omega_1 = -(2\omega + 4)$. As $-1 < \omega < -1/3$ is related to dark energy regime, the range of values $-10/3 < \omega_1 < -2$ is acceptable in this regime.

3. Specific Models of Dark Energy Stars

In this research, we take the form of the gravitational potential $Z(x)$ as $Z(x) = (1 - ax)^n$ proposed for Thirukanesh and Ragel [38] and Malaver [43] where a is a real constant and n is an adjustable parameter. This potential is regular at the origin and well behaved in the interior of the sphere. Following Ngubelanga et al.[49] for the electric field we make the particular choice

$$\frac{E^2}{2c} = x(a + bx) \tag{15}$$

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This electric field is finite at the center of the star and remains continuous in the interior. We have considered the cases for n=1,2.

For the case n=1, using $Z(x)$ and eq.(15) in eq.(6), we obtain

$$\rho = c(3a - ax - bx^2) \tag{16}$$

Substituting (16) in eq.(14) the radial pressure can be written in the form

$$p_r = \omega c(3a - ax - bx^2) + \omega_1 c^2 (3a - ax - bx^2)^2 \tag{17}$$

Using (16) in (12), the expression of the mass function is

$$M(x) = \frac{\left(35a - 7ax - 5bx^2\right)x^{3/2}}{70\sqrt{c}} \tag{18}$$

With (15) and *Z(x)* in (11) , the charge density is

$$\sigma^2 = \frac{2c^2(1-ax)(3a+4bx)^2}{(a+bx)} \tag{19}$$

Replacing (14), (15) and *Z(x)* in eq.(7) we have

$$\frac{\dot{y}}{y} - \frac{a}{4(1-ax)} = \frac{\omega(3a - ax - bx^2)}{4(1-ax)} + \frac{\omega_1(3a - ax - bx^2)^2}{4(1-ax)} - \frac{x(a+bx)}{4(1-ax)} \tag{20}$$

Integrating (20) we obtain

$$y(x) = c_1(ax-1)^{A*} e^{Bx^4 + Cx^3 + Dx^2 + Ex} \tag{21}$$

Where for convenience we have let

$$A* = -\frac{\left(9a^6 - 6a^5 + a^4 - 6a^3 b + 2a^2 b + b^2\right)c\omega_1 + \left(3a^5 - a^4 - a^2 b\right)\omega + a^5 - a^4 - a^2 b}{4a^5} \tag{22}$$

$$B = -\frac{3a^3 b^2 c\omega_1}{48a^4} \tag{23}$$

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$$C=-\frac{\left(8a^4b+4a^2b^2\right)c\omega_1}{48a^4} \tag{24}$$

$$D=-\frac{\left(6a^5-36a^4b+12a^3b+6ab^2\right)c\omega_1-6a^3b\omega-6a^3b}{48a^4} \tag{25}$$

$$E=-\frac{\left(-72a^5+12a^4-72a^3b+24a^2b+12b^2\right)c\omega_1-\left(12a^4+12a^2b\right)\omega-12a^4-12a^2b}{48a^4} \tag{26}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda}=\frac{1}{1-ax} \tag{27}$$

$$e^{2\nu}=A^2c_1{}^2(ax-1)^{2A*}e^{2\left(Bx^4+Cx^3+Dx^2+Ex\right)} \tag{28}$$

and the anisotropy factor Δ is given by for

$$\Delta=4xc(1-ax)\left[\frac{\left(A*^2-A*\right)a^2}{(ax-1)^2}+\frac{2aA*\left(4Bx^3+3Cx^2+2Dx+E\right)}{ax-1}+12Bx^2+6Cx+2D+\left(4Bx^3+3Cx^2+2Dx+E\right)^2\right]-2axc\left(\frac{aA*}{ax-1}+4Bx^3+3Cx^2+2Dx+E\right)-2xc(a+bx) \tag{29}$$

The metric for this model is

$$ds^2=-A^2c_1^2\left(acr^2-1\right)^{2A*}e^{2\left(Bc^4r^8+Cc^3r^6+Dc^2r^4+Ecr^2\right)}dt^2+\frac{dr^2}{\left(1-acr^2\right)}+r^2(d\theta^2+\sin^2\theta d\varphi^2)$$

(30)

The figures 1, 2, 3 , 4, 5, 6 represent the graphs of ρ, p_r , σ^2 , $M(x)$, anisotropy Δ and strong energy condition respectively for n=1, ω= -0.4 and ω_1= -3.2 with a=0. 5, b= 0.2 , c=1 and a stellar radius of r = 1.33 Km.

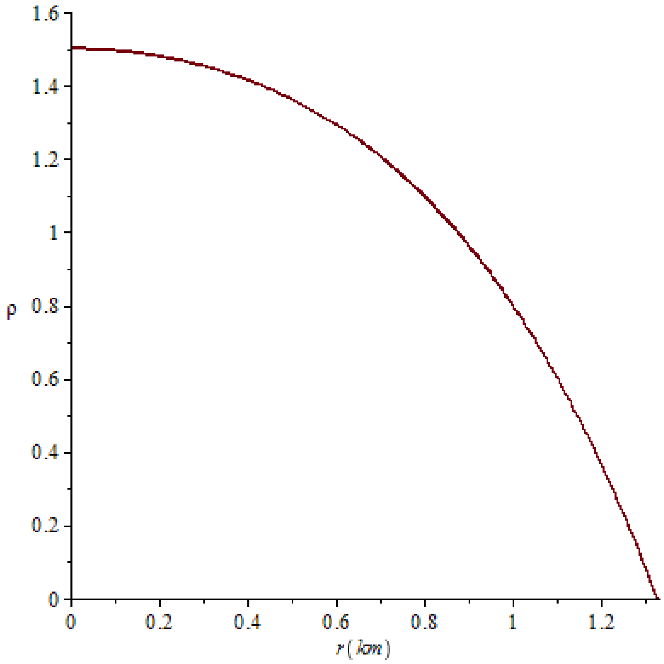

Figure 1. *Energy density vs radial coordinate with n=1, a=0.5, b =0.2, c=1*

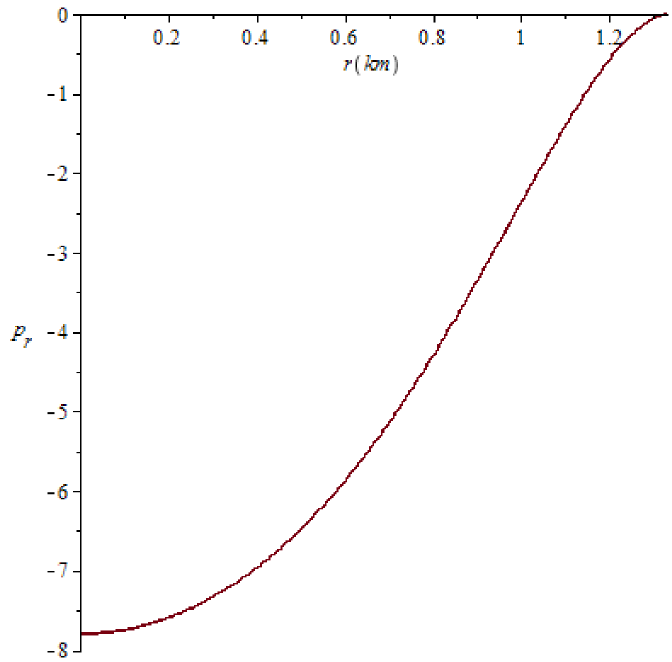

Figure 2. *Radial pressure vs radial coordinate with n=1, ω= -0.4, ω₁= -3.2 , a=0.5, b=0.2, c=1 .*

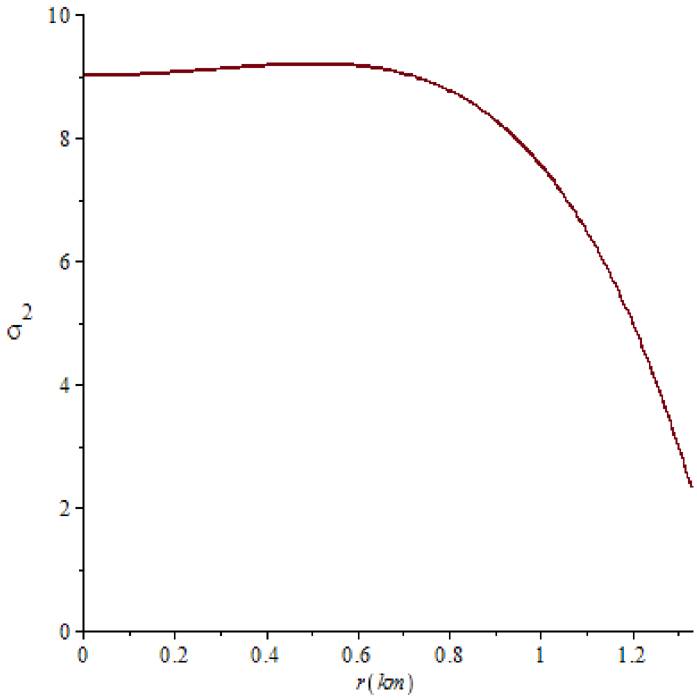

Figure 3. *Charge density vs radial coordinate with n=1, ω= -0.4 , ω_1= -3.2 , a=0.5, b=0.2 and c=1*

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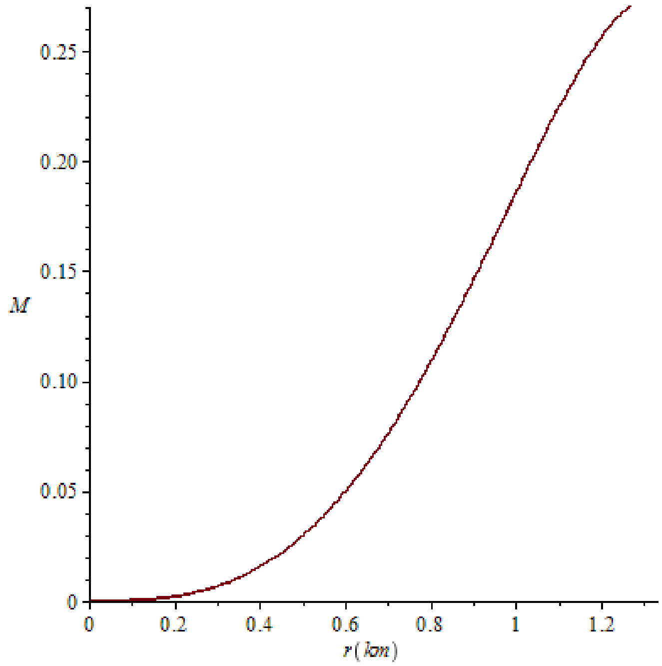

Figure 4. *Mass function vs radial coordinate with n=1, a=0.5, b=0.2 and c=1*

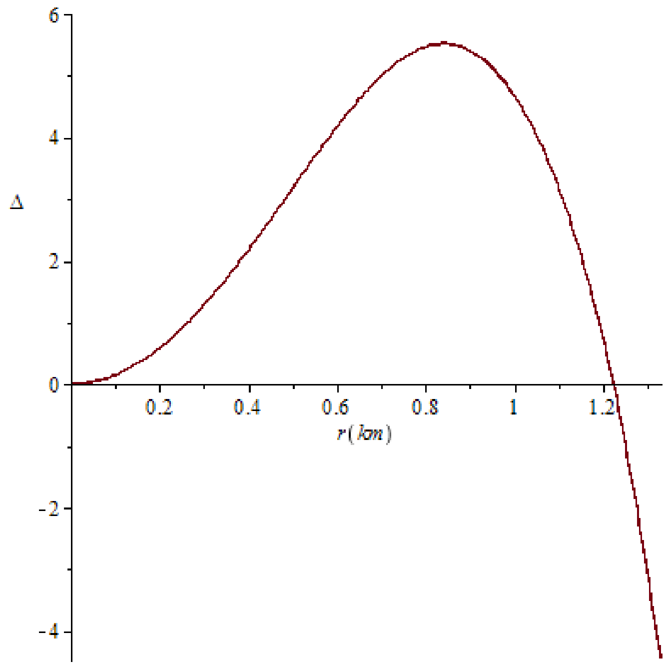

Figure 5. *Anisotropy vs radial coordinate with n=1, ω= -0.4, ω_1= -3.2, a=0.5, b=0.2 and c=1*

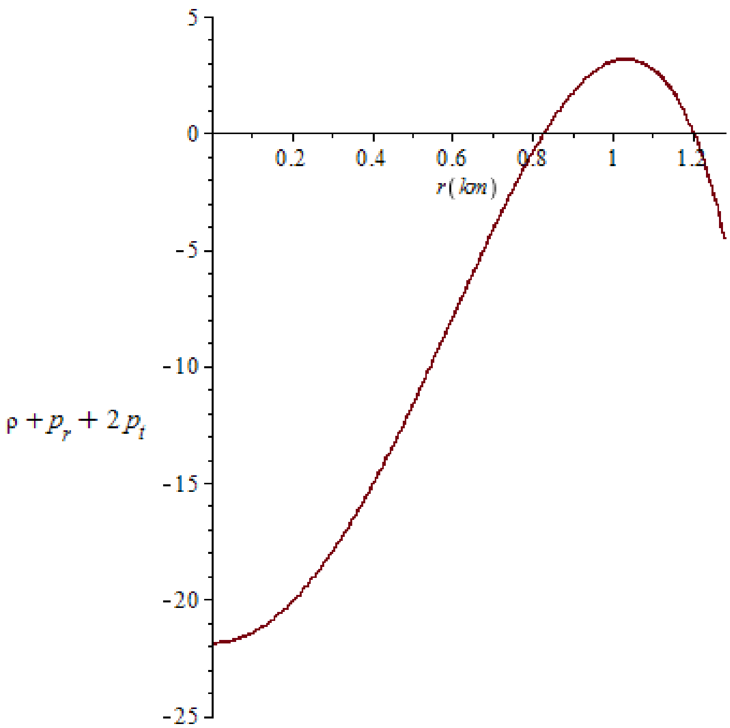

Figure 6. *Strong energy condition vs radial coordinate with n=1, ω= -0.4, ω₁= -3.2, a=0.5, b=0.2 and c=1*

With n=2, the expression for the energy density is

$$\rho = c\left[6a - \left(5a^2 + a\right)x - bx^2\right] \tag{31}$$

Replacing (31) in (14) we have for the radial pressure

$$p_r = \omega c\left[6a - \left(5a^2 + a\right)x - bx^2\right] + \omega_1 c^2\left[6a - \left(5a^2 + a\right)x - bx^2\right]^2 \tag{32}$$

and the mass function is

$$M(x) = \frac{\left[70a - \left(35a^2 + 7a\right)x - 5bx^2\right]x^{3/2}}{70\sqrt{c}} \tag{33}$$

Substituting eq. (15) and *Z(x)* in eq. (11) we obtain for the charge density

$$\sigma^2 = \frac{2c^2(1 - ax)^2(3a + 4bx)^2}{(a + bx)} \tag{34}$$

With the eq. (15), (32) and *Z(x)* the eq.(7) becomes

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$$\frac{\dot{y}}{y}-\frac{(2a-a^2x)}{4(1-ax)^2}=\frac{\omega\left[6a-\left(5a^2+a\right)x-bx^2\right]}{4(1-ax)^2}+\frac{\omega_1 c\left[6a-\left(5a^2+a\right)x-bx^2\right]^2}{4(1-ax)^2}-\frac{x(a+bx)}{4(1-ax)^2}$$

(35)

Integrating (35) we obtain

$$y(x)=c_2\left(ax-1\right)^F e^{\frac{Gx^4+Hx^3+Ix^2+Jx+K}{12a^5(ax-1)}} \quad (36)$$

Again for convenience we have let

$$F=-\frac{\left(10a^6-8a^5-2a^4-6a^3b-6a^2b-4b^2\right)c\omega_1+\left(5a^5+a^4+2a^2b\right)\omega+a^5+a^4+2a^2b}{4a^5}$$

(37)

$$G=a^4b^2c\omega_1 \quad (38)$$

$$H=\left(15a^6b+3a^5b+2a^3b^2\right)\omega_1 \quad (39)$$

$$I=\left(75a^8+30a^7+3a^6+9a^5b+9a^4b+6a^2b^2\right)c\omega_1-3a^4b\omega-3a^4b \quad (40)$$

$$J=\left(-75a^7-30a^6-3a^5-24a^4b-12a^3b-9ab^2\right)c\omega_1-12a^3b\omega-3a^3b$$

(41)

$$K=\left(-3a^6+6a^5-3a^4+6a^3b-6a^2b-3b^2\right)c\omega_1+\left(3a^4-3a^5+3a^2b\right)\omega-3a^5+3a^4+3a^2b$$

(42)

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda}=\frac{1}{(1.-ax)^2} \quad (43)$$

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$$e^{2\nu} = A^2 c_2^2 (ax-1)^{2F} e^{\frac{Gx^4+Hx^3+Ix^2+Jx+K}{6a^5(ax-1)}} \tag{44}$$

and for the anisotropy factor Δ we have

$$\Delta = 4xc(1-ax)^2 \left[\frac{(F^2-F)a^2}{(ax-1)^2} + \frac{2aF}{(ax-1)}\left(\frac{4Gx^3+3Hx^2+2Ix+J}{12a^5(ax-1)} - \frac{Gx^4+Hx^3+Ix^2+Jx+K}{12a^4(ax-1)^2} \right) + \frac{12Gx^2+6Hx+2I}{12a^5(ax-1)} - \frac{4Gx^3+3Hx^2+2Ix+J}{6a^4(ax-1)^2} + \frac{Gx^4+Hx^3+Ix^2+Jx+K}{6a^3(ax-1)^3} + \left(\frac{3aGx^4+2(aH-2G)x^3+(aI-3H)x^2-2Ix-(aK+J)}{12a^5(ax-1)^2} \right)^2 \right]$$
$$- 2ac(1-ax)\left[1+2x\left(\frac{aF}{ax-1} + \frac{4Gx^3+3Hx^2+2Ix+J}{12a^5(ax-1)} - \frac{Gx^4+Hx^3+Ix^2+Jx+K}{12a^4(ax-1)^2} \right) \right]$$
$$2ac - a^2cx - 2xc(a+bx) \tag{45}$$

The metric for this model is

$$ds^2 = -A^2 c_2^2 (acr^2-1)^{2F} e^{\frac{Gx^4+Hx^3+Ix^2+Jx+K}{6a^5(ax-1)}} dt^2 + \frac{dr^2}{(1-acr^2)^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{46}$$

The figures 7, 8, 9 , 10, 11, 12 represent the graphs of ρ, p_r , σ^2 , $M(x)$, anisotropy Δ and strong energy condition respectively for n=2, ω= -0.95 and ω_1= -2.1 with a=0. 5, b= 0.01 , ,c=1 and a stellar radius of r = 1.33 Km.

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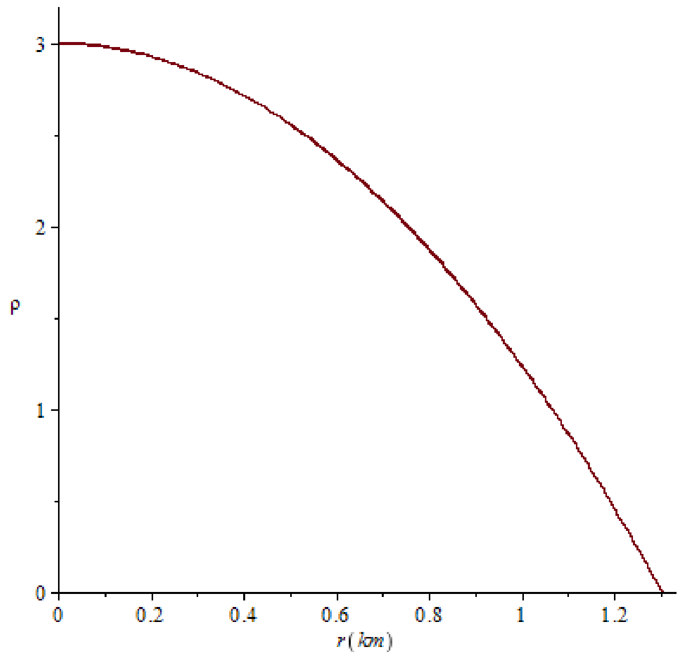

Figure 7. *Energy density vs radial coordinate with n=2, a=0.5, b =0.01, c=1*

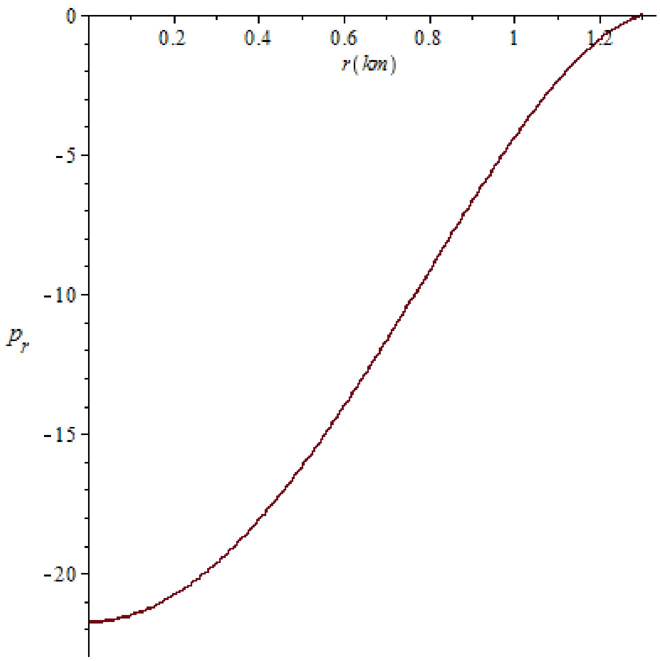

Figure 8. *Radial pressure vs radial coordinate with* $n=2$, $\omega= -0.95$, $\omega_1= -2.1$, $a=0.5$, $b=0.01$, $c=1$.

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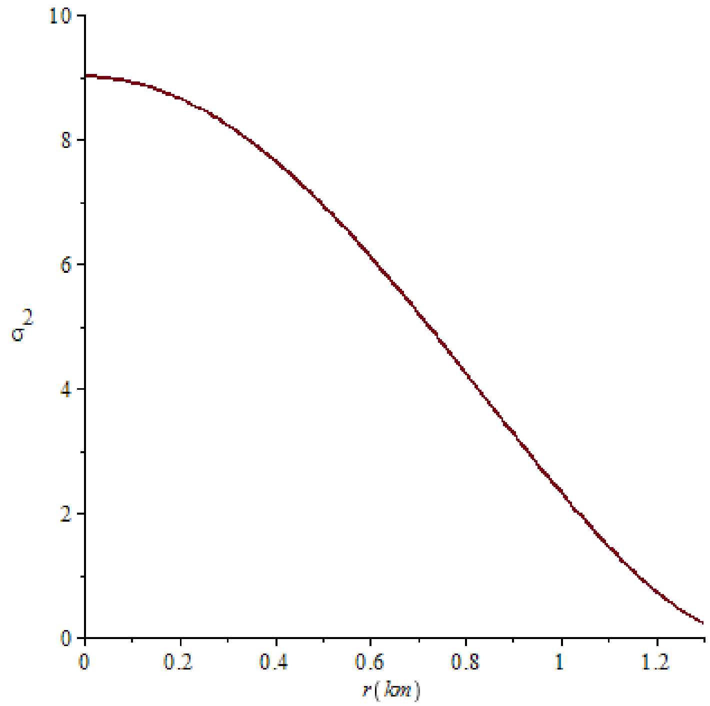

Figure 9. *Charge density vs radial coordinate with n=2,* ω*= -0.95,* ω_1*= -2.1, a=0.5, b=0.01 and c=1*

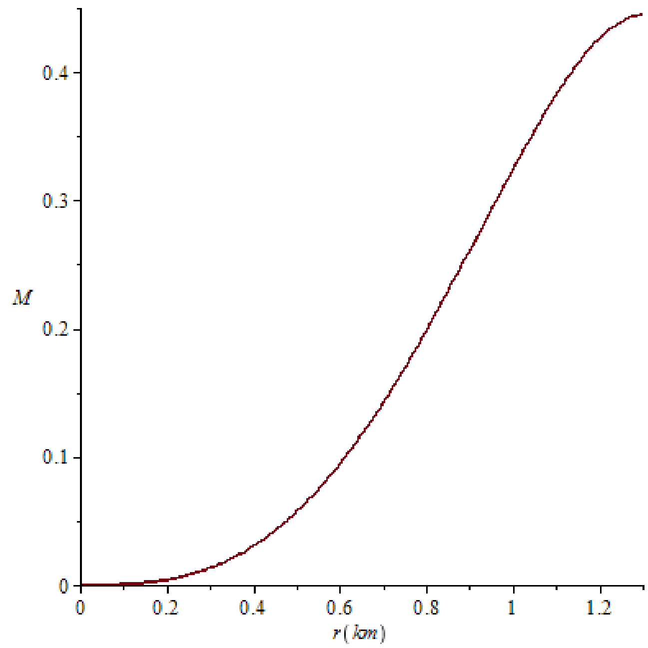

Figure 10. *Mass function vs radial coordinate with n=2, a=0.5, b=0.01 and c=1*

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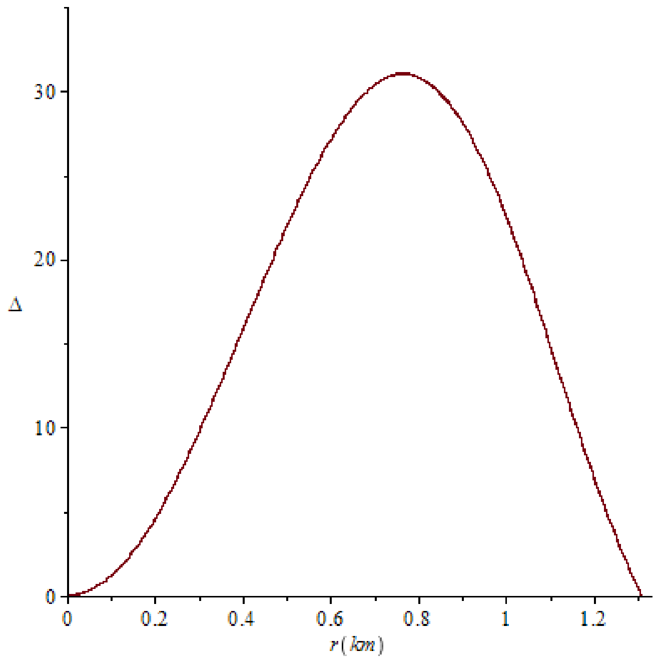
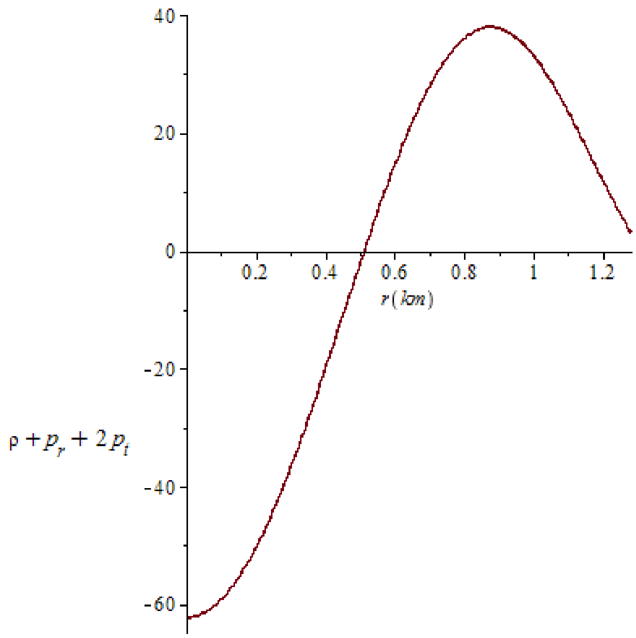


Figure 11. *Anisotropy vs radial coordinate with n=2, ω= -0.95, ω_1= -2.1, a=0.5, b=0.01 and c=1*



Figu re 12. *Strong energy condition vs radial coordinate with n=2, ω= -0.95, ω_1= -2.1, a=0.5, b=0.01 and c=1*

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4. Physical Analysis of the New Models

In order for a solution to be physically acceptable and viable must satisfy the following physical properties [4,14,38]:

(i) The energy density must be well defined, must be positive and a decreasing function of the radial pressure
(ii) The radial pressure must decrease as the radius increases and it must vanish at the surface of the sphere but for negative pressure this condition is not satisfied.
(iii) Regularity of the gravitational potentials in the origin.
(iv) For anisotropic solutions, the radial and the tangential pressure are equal to zero at the centre $r=0$
(v) The consideration of dark energy is applicable only to fluids that violate the strong energy condition.
(vi) Electric field intensity E should be well defined throughout of the sphere.

The models obtained are physically acceptable and constitute another new family of solutions for anisotropic charged matter with quadratic equation of state. For the case n=1 with ω=-0.4 and ω_1=-3.2 the gravitational potentials are regular at the origin since $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_1^2(-1)^{2A^*}$ and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$. In the center $r=0$ $\rho(0)=3ac$ and $p_r(0)=3\omega ac+9\omega_1 a^2c^2$, therefore the energy density will be non-negative at the center and $p_r(0) < 0$. For the case n=2 with ω=-0.95 and ω_1=-2.1 $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_2^2(-1)^{2F}e^{-\frac{K}{6a^5}}$ and in the origin $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$. Again the gravitational potentials are regular in $r=0$ and $\rho(0)=6ac$, $p_r(0)=6\omega ac+36\omega_1 a^2c^2$. From equations (17) and (32) we can deduce that the radial pressure is negative and not decreasing function of the radial coordinate. In these new models the mass is increasing function, continuous, finite and the charge density not present physical singularity at the center and behaves well in the interior of the star.

In figures 1 and 7, it is observed that the energy density is finite and decreasing from the center to the surface of the star in the two studied cases. In figures 2 and 8, the radial pressure is continuous, also is finite and monotonically increasing function. In figures 3 and 9, the charge density is non-singular at the origin, non-negative and decreases. In figures 4 and 10, the mass function is strictly increasing, continuous and finite. The variation of the measure of anisotropy in two cases is shown in figures 5 and 11. In both cases, $\Delta(0)=0$ at the center because the radial and tangential pressures should be equal in $r=0$. The degree of anisotropy reaches a maximum value near at 0.8 km and then remains finite and decreasing throughout the interior of the star. In figures 6 and 12 it is observed as the strong energy condition is violated in the two cases what it means that the consideration of dark energy is valid in these new models.

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5. Conclusions

Considering a particular form of gravitational potential and electric field, we have generated a physically valid category of exact solutions to the Einstein-Maxwell system of equations with a quadratic equation of state that represents a model of dark energy star where $\omega_1 = -(2\omega + 4)$ and the range of values $-1 < \omega < -1/3$, $-10/3 < \omega_1 < -2$. The radial pressure, energy density, anisotropy, mass function, charge density and all the coefficients of the metric behaves well inside the stellar interior and are free of singularities. For these new models the consideration of dark energy is applied only to the cases where the values of ω and ω_1 not satisfied the strong energy condition. The solutions obtained fulfilled all the requirements for a compact negative energy stellar object and can be used in the description and modeling of different relativistic configurations.

The possible existence of dark energy, responsible for the current accelerated expansion of the Universe has opened up new research topics in theoretical physics. Evidence of this expansion has been found from measurements of supernovae of type Ia and in cosmic microwave background radiation [2]. In this work the dark energy consist of a cosmic fluid governed by the quadratic equation of state $p_r = \omega\rho + \omega_1\rho^2$ where the strong energy condition is violated. According Lobo [3] and Parsaei and Rastgoo [18] the limits for the parameter ω are within the range $-1 < \omega < -1/3$. The dark energy star may have their origin in a density fluctuation in the cosmological background where the inclusion of Hubble parameter dependent term in the equation of state could result in the nucleation of a dark energy star through a density perturbation [3].

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