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Application of Statistical Mirroring in ANOVA-Like Tests

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Abstract

Statistical inferences is regarded as the general criteria for statistical conclusion and drawing generalizations. However, most of the inferential statistical tools are based on strong assumptions which create a strict limitations on their use and application. Analysis of variance (ANOVA) is one of such statistics. In this article, eight (8) new ANOVA-like methodologies were proposed, in alternative to one-way ANOVA, based on the assumptions of statistical mirroring. Methods validation in comparison with one-way ANOVA was designed to assess the suitability and statistical power of the new proposals as an alternative methods, using different sets of logically generated multivariate datasets with different problems and statistical complications. The results of comparisons validate that the eight (8) proposed ANOVA-like methodologies were suitable alternatives to ANOVA, in the sense that they require no normality assumption to be meet, used different ways to compare the data with different statistical elements rather than depending on only variance, efficient with negative values, and results interpretation is easier.

Keywords: statistical mirroring; meanic mirror; comparative optinalysis; descriptive components; inferences.

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1. Introduction

Statistics plays a vital role in all fields of research and studies ranging from culture to sciences. It helps present data accurately and draws rational, empirical and meaningful conclusions, and also makes a generalization. While presenting and analyzing data, one should be careful of using a suitable and appropriate statistical measure (Barde and Barde, 2012). One of the basically used parametric statistics and its non-parametric alternative are the analysis of variance (ANOVA) and Kruskal-Wallis test respectively, which are limited by some strict assumptions. One general limitation of ANOVA is the assumption of normality that a datasets must to pass. In case where the dataset display evidence of violating assumption(s), the researcher can perform data transformation (Ferketich & Verran, 1994). Thus, to inferentially define or explain a data, one needs to understand the extent of variability within and between multivariate entities which is expressed by the parametric tests or their alternative non-parametric tests. One-way and two-way ANOVA are the two components of ANOVA test.

The main vital role and statistical power of all statistical tools is to present the reader about the level, amount and strength of variations within or between datasets in a clear and precise interpretation that allows a researcher to make an empirical and rational conclusion. A large number of published research articles (especially in biomedical studies) have at least one kind statistical errors either in presentation (Cooper *et al.*, 2002; García-Berthou and Alcaraz, 2004) or analysis of data (Krousel-Wood *et. al.*, 2006). Enormous efforts have been made to address these statistical errors and improve quality of statistical applications (Goodman et al., 1998; Gore *et al.*, 1992; Altman *et al.*, 1983). Despite these efforts, errors are still present in published articles. One such common error is inappropriate choice of data transformation method (Rasmussen, 1989), inappropriate results presentation (Glantz, 1980; Barde and Barde, 2012), and possibly running a parametric test without checking the normality state of the data. Therefore, proper understanding and use of fundamental statistics and their application will improve reliability, interpretation, and communication of data and results to readers (Barde and Barde, 2012). Recently, the breakthrough through statistical mirroring and comparative optinalysis would likely to be one of the current leading concept and solution in data science with diverse applications (2019a; 2019b).

In this article, the application of statistical mirroring and comparative optinalysis methodologies as an alternative approach to ANOVA have identified eight (8) suitable methods that requires no normality assumption to be meet, used different ways to compare the data with different statistical elements rather than depending on variance, efficient with negative values, and results interpretation is easier.

2. Preliminaries: Basic Assumptions on Statistical Mirroring

Abdullahi (2019b) proposed the concept of statistical mirroring and the basic assumptions the govern it. These assumptions says:

i. In the statistical inferences of comparative optinalysis between two or more set of sequences, there can exist a reflector sequence (the object or source sequence) and reflecter sequence (the image sequence) such that the reflector sequence momentarily reflects in an optinalytic and intermetric manner on the reflecter sequence for inferential comparisons about their degree or level of similarity or dissimilarity.

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ii. The reflector or source sequence is the sequence in question and reflecter sequence is the *statistical (optinalytic) mirror*.

iii. Statistical (optinalytic) mirror refers to a designed sequence image on which a sequence of set of sequences can optinalytically and intermetrically reflects to give an inferential information about their comparisons (similarity and dissimilarity). Comparative optinalysis that involve a statistical mirror sequence and its object or source sequence is called *statistical mirroring.*

iv. Statistical mirror can therefore be structurally defined as a uniform amplification, in sequence, of a defined magnitude, called the *principal element or value* (e.g mean, median, and mode, minimal or maximal point of a sequence or other statistical elements) of a sequence through a defined length. The defined length of a statistical mirror must at least be equal to the length of any lengthiest object (reflector) sequence of the dataset.

v. Statistical mirror can be designed by the component of the central tendency (mean, median, and mode) of the distribution of sequence elements other statistical elements of the sequence itself.

vi. In statistics, statistical mirrors can be seen as meanic, medianic, modalic, maximalic, minimalic mirrors.

   a. Meanic mirror: designed to expresses and inferentially quantify the deviation or proximity of each sequence elements from its mean value. It measure how far or how close each element of a dataset is from the mean value. Probability level of similarity and dissimilarity expresses the proximity and deviation respectively.
   b. Medianic mirror: expresses and inferentially quantify the deviation or proximity of each sequence elements from its median value. It measure how distant or how close each element of a dataset is from the median value.
   c. Modalic mirror: expresses and inferentially quantify the deviation or proximity of each sequence elements from its modal value. It measure how far or how close each element of a dataset is from the modal value.
   d. Maximalic mirror: designed to expresses and inferentially quantify the deviation or proximity of each sequence elements from its maximal value. It measure how far or how close each element of a dataset is from the maximal value.
   e. Minimalic mirror: designed to express and inferentially quantify the deviation or proximity of each sequence elements from its minimal value. It measure how far or how close each element of a dataset is from the minimal value.

vii. In term of sequence order, statistical mirror has no define region of head and tail sequence, and a denotation headtail (HT) or tailhead (TH) can be used to describe the sequence ends.

3. Methodology

3.1 Datasets

Different types of multivariate data sequence were logically generated to present different statistical variations and complications. These generated sequences were used to validate the proposed methodologies (ANOVA-Like Tests).

3.2 ANOVA-like Methodologies

3.2.1 Specific Assumption in ANOVA-Like Tests

The proposal of Abdullahi (2019b) listed some of the statistical elements (mean, median, mode, maximum, minimum values and other statistical descriptions of a dataset) used as a principal element for the design of statistical mirrors. But he did nor clearly explained what these other statistical descriptions are, and how to employ them. In this article, other important statistical mirrors were provided, and were used in the ANOVA-Like tests.

In a multivariate or multi-clustered datasets with a certain number of replications in each variable or cluster or group. The statistical descriptions (e.g variance, standard deviation meanic deviation, geometric mean and etc) within each variable or group are considered and sequenced appropriately as a reflector (object) sequence. However, the mean between all the variables or groups are considered as a principal value of the design of a statistical mirrors. The notation below defines the argument of the assumption.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{c(p)\,\substack{(meanic\ mirror\ of\ all\ groups \\ statistical\ description)}}^{\substack{(sequence\ of\ each\ group \\ statistical\ description)}} = x(y)$$

3.2.2 List of the Proposed Methods

In general, the statistical mirroring of datasets in an ANOVA-like tests consider the following algorithmic parameters of optinalysis :

i. The sequence order of the replicate measurements of the datasets in each group was considered in an ascending. Head and tail of the sequence were denoted on the sequenced datasets left to the right respectively.

ii. Let each of the sequenced dataset optinalytically reflects head-to-headtail (H-HT) with each of its own meanic mirror sequence, by a normalization of a zero unit, such that each sequenced dataset is intermetrically similar to its own meanic mirror sequence with a resultant Kabirian coefficient of x and thus y confidence level of similarity.

Method I: Analysis of variance -II (ANOVA-II): The principal element is the average of all the groups' variance.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{c(p)\,\substack{(meanic\ mirror\ of\ all \\ groups\ variance)}}^{\substack{(sequence\ of\ each \\ group\ variance)}} = x(y)$$

Method II: Analysis of standard deviation (ANOSDE): The principal element is the average of all the groups' standard deviation.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{c(p)\,\substack{(meanic\ mirror\ of\ all \\ groups\ std.\ dev.)}}^{\substack{(sequence\ of\ each \\ group\ Std.\ dev.)}} = x(y)$$

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Method III: Analysis of standard error of mean (ANOSEM): The principal element is the average of all the groups' standard error of mean.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of\ all \\ groups\ mean\ SEM)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ SEM)}} = x(y)$$

Method IV: Analysis of similarity or meanic proximity (ANOSIM): The principal element is the average of all the groups' similarity score of meanic proxixmity.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of \\ all\ groups\ MP)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ MP)}} = x(y)$$

Method V: Analysis of dissimilarity or meanic deviation (ANODSIM): The principal element is the average of all the groups' similarity scores of meanic deviation.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of \\ all\ groups\ MD)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ MD)}} = x(y)$$

Method VI: Analysis of geometric mean (ANOGEM): The principal element is the average of all the groups' geometric mean.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of \\ all\ groups\ GEM)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ GEM)}} = x(y)$$

Method VII: Analysis of harmonic mean (ANOHAM): The principal element is the average of all the groups' harmonic mean.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of \\ all\ groups\ HAM)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ HAM)}} = x(y)$$

Method VIII: Analysis of quadratic mean (ANOQUM): The principal element is the average of all the groups' quadratic mean.

$$\bigwedge_{B}^{(\pm N=0)} : \int_{\substack{(meanic\ mirror\ of \\ all\ groups\ QUM)}\, c(p)}^{\substack{(sequence\ of\ each \\ group\ QAM)}} = x(y)$$

4.1 Data Analysis

Table 1 presented a multivariate raw data of five (5) replicate readings of five (5) treatments in six (6) different groups, logically generated to express some of the statistical problems and complications.

Graphad Prism Statistical Software (Version 8.2.1) was used to analyze the data for mean, standard deviation, and standard error of mean, geometric mean, harmonic mean, and normality test of each data sets. One-away ANOVA and Kruska-Wallis test were appropriately analyzed. Microsoft Excel statistical functions and cells were used to analyze the variance. Furthermore, meanic proximity and deviation of each dataset was analyzed by statistical mirroring (Abdullahi, 2019a and b) using an Excel programed sheets (Seen in the supplementary files).

Statistical mirroring between the sequenced datasets and their designed statistical mirrors was performed using the method of Abdullahi (2019a and 2019b). Kabirian coefficient of similarity and dissimilarity, and their translated probabilities and percentages were computed. Different algorithmic parameters and argument for each application were established to suitably and appropriately analyze the data.

The details of the data analysis and the results of comparative optinalysis were presented in Appendix A-B and Excel sheets of the supplementary materials.

4.2 Suitability Assessment Criteria

In this case, three (3) criteria were used to compare the statistical power, fitness and suitability of the proposed methodologies with ANOVA or it non-parametric alternatives. These criteria are (a) normality independence (b) efficiency with negative values (c) results interpretation and (d) results consistency across all the methods.

4.3 Results presentation

All results were presented in Tables.

5. Results and Discussion

From Table 1, the following explanations are obtained:

Normality independence: In contrast to ANOVA test, all the methodologies proposed requires no any condition of normality to be meet. They all work suitable with an interesting outcomes (results). In all the groups I to VI, normality test was only passed in group I, II and VI, and failed for the others. Due to the fact that ANOVA test completely relies on variance estimation, and variance have been shown to be sensitive to outliers (Krousel-Wood *et al.*, 2006)). Thus, the outliers are more pronounced and deviate the distribution away from the normal distribution. Statistical mirroring is therefore very resistant to the effect of large magnitude of outliers. Despite the presence of a very wide and visible variation within and between the treatment, the non-parametric alternative, Kruskal-Wallis test have however failed to detect any significant difference, but most of the proposed methodologies have able to provide a significant differences.

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Efficiency with negative values: Except for analysis of geometric mean (ANOGEM) and analysis of harmonic mean (ANOHAM), all the methods including ANOVA are very effective with negative numbers. The statistical inefficiency of these methods is not due to application of statistical mirroring, but it originate from the principal value and its reflector (geometric and harmonic mean). Literatures have well established that geometric and harmonic means cannot be estimated with negative or zero value, and therefore this created limitation on the use of ANOGEM and ANOHAM.

Results interpretation: Choosing a confidence level provide an easiest way to give an inferential interpretation and draw a general conclusion. Fortunately, all the methods have a have a deduced confidence interval. Another simplistic nature of the proposed methodologies, they all requires no any certain distribution table to trace out the significance level, the result is directly the significance level, which is not the simple case with ANOVA test.

Results consistency across all the methods: The same and consistent results cannot be assumed, because, each method has its own different subject considered. It rest with the researcher to understand what statistical element is he/she has considered to arrive at a correct, valid and consistent results.

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Table 1: Raw data of the five (5) replicate readings of the five (5) treatments in six (6) different groups

		Independent Groups					
Treatments	**Replications**	**Group I**	**Group II**	**Group III**	**Group IV**	**Group V**	**Group VI**
Treatment A	1	5.63	2.33	67.34	66.24	-45	-35
	2	5.24	3.12	23.45	26.34	45	-56
	3	6.34	3.44	12.78	12.00	-36	-46
	4	7.87	4.67	121.23	353.23	-67	-46
	5	6.98	2.45	34.56	123.45	-99	-23
Treatment B	1	8.21	5.65	76.45	23.45	-467	-55
	2	7.45	7.87	11.40	23.45	4567	-35
	3	5.98	6.78	45.21	23.66	-65	-56
	4	6.46	7.98	23.54	87.34	-76	-25
	5	7.45	7.23	89.45	23.45	-765	-45
Treatment C	1	7.36	9.45	256.34	4.00	67	-24
	2	7.45	8.89	45.32	7.00	-35	-13
	3	7.43	10.56	783.34	99.00	-65	-32
	4	6.45	9.99	38.46	9.00	89	-23
	5	5.55	8.89	134.32	8.00	23	-11
Treatment D	1	8.56	16.46	12.34	0.456	345	-234
	2	7.78	16.34	45.56	45.67	22	-433
	3	6.45	15.45	89.98	21.45	465	-456
	4	8.45	15.78	156.56	98.56	45	-654
	5	5.89	16.11	643.23	0.54	245	-312
Treatment E	1	4.78	27.56	45.65	0.56	67	-24
	2	5.98	26.99	8.65	10.35	345	-24
	3	6.89	28.02	34.77	0.43	-46	-34
	4	7.77	27.76	78.24	0.98	675	-42
	5	8.34	28.51	6897.36	56.23	-65	-15

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Table 2: Mean, standard deviation, and the significance level in each of the five (5) treatments in six (6) different groups using ANOVA and the eight (8) methodologies proposed and described

		Independent Groups					
	Treatments	**Group I**	**Group II**	**Group III**	**Group IV**	**Group V**	**Group V1**
	Treatment A	6.41±1.05	3.2±0.94	51.87±43.83	116.25±139.34	-40.4±53.56	-41.2±12.60
	Treatment B	7.11±0.89	7.1±0.95	49.21±33.41	36.27±28.55	638.8±2215.31	-43.2±13.27
	Treatment C	6.85±0.84	9.56±0.72	251.56±310.04	25.4±41.19	15.8±65.46	-20.6±8.62
	Treatment D	7.43±1.20	16.03±0.41	189.53±259.31	33.34±40.94	224.4±191.05	-417.8±160.04
	Treatment E	6.75±1.42	27.77±0.56	1412.93±3065.99	13.71±24.14	195.2±314.23	-27.8±10.40
Methods	**Results**	**Significance**					
ANOVA-I	P-value	0.6657^{ns}	<0.0001*	0.4127^{ns}	0.1000^{ns}	0.1007^{ns}	<0.0001*
ANOVA-II	K_c-value	0.956735	0.943128	0.858687	0.882755	0.859527	0.859764
	$P_{Sim.}$-value	0.8341	0.7848	0.5047	0.5802	0.5073	0.5080
	$P_{Dsim.}$-value	0.1659*	0.2152*	0.4953*	0.4198*	0.4927*	0.4920*
ANOSDE	K_c-value	0.977673	0.967532	0.875569	0.931205	0.881694	0.889431
	$P_{Sim.}$-value	0.9126	0.8742	0.5574	0.7425	0.5768	0.6018
	$P_{Dsim.}$-value	0.0874*	0.1258*	0.4426*	0.2575*	0.4232*	0.3982*
ANOSEM	K_c-value	0.977685	0.967545	0.875573	0.931198	0.881693	0.889433
	$P_{Sim.}$-value	0.9127	0.8743	0.5574	0.7425	0.5768	0.6018
	$P_{Dsim.}$-value	0.0873*	0.1257*	0.4426*	0.2575*	0.4232*	0.3982*
ANODSIM	K_c-value	0.975142	0.911944	0.973222	0.976446	0.956208	0.986948
	$P_{Sim.}$-value	0.9030	0.6763	0.8957	0.9080	0.8322	0.9485
	$P_{Dsim.}$-value	0.097*	0.3237*	0.1043*	0.0920*	0.1678*	0.0515^{ns}
ANOSIM	K_c-value	0.99834	0.995689	0.984941	0.986347	0.942348	0.997926
	$P_{Sim.}$-value	0.9934	0.9828	0.9407	0.9461	0.7820	0.9917
	$P_{Dsim.}$-value	0.0066^{ns}	0.0172^{ns}	0.0593*	0.0539^{ns}	0.2180*	0.0083^{ns}
ANOGEM	K_c-value	0.994082	0.928898	0.949564	0.906319	NES	NES
	$P_{Sim.}$-value	0.9765	0.7345	0.8079	0.6574	NES	NES
	$P_{Dsim.}$-value	0.0235^{ns}	0.2655*	0.1921*	0.3426*	NES	NES
ANOHAM	K_c-value	0.993881	0.928444	0.953638	0.902833	NES	NES
	$P_{Sim.}$-value	0.9757	0.7329	0.8228	0.6458	NES	NES
	$P_{Dsim.}$-value	0.0243^{ns}	0.2671*	0.1772*	0.3542*	NES	NES
ANOQUM	K_c-value	0.994472	0.92985	0.879789	0.931597	0.887081	0.890704
	$P_{Sim.}$-value	0.9780	0.7378	0.5708	0.7439	0.5942	0.6059
	$P_{Dsim.}$-value	0.0220^{ns}	0.2622*	0.4292*	0.2561*	0.4058*	0.3941*

* = significance at 0.05; ^{ns} = not significance difference or deviation at 0.05; NES = Not efficient to solve.
Yellow highlights indicate the dataset analyzed by an alternative non-parametric Kruskal-Wallis test.

6. Conclusion

Considering the application and results of comparison here studied, it is concluded that statistical mirroring is a suitable ANOVA-like alternative approaches with different choice of parameters. The applied method (statistical mirroring) distinguishes itself over some well-known and adopted method of analysis of variance and its non-parametric alternative (Kruskal-Wallis test) in the sense that they are independent on the assumption of normality, efficient with negative values, and results interpretation is easier.

7. Recommendations

This study recommend further application of statistical mirroring with some other datasets from real-world examples. Furthermore, a comparison with other parametric and non-parametric test should be conducted to further re-validate the proposed methodologies.

Conflict of interest

The author declares no conflict of interest.

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Appendix A

Table A1: Sequenced (in ascending order) of similarity and dissimilarity values of each group computed by comparative optinalysis prior for further comparative optinalysis

	Groups					
	G-I	**G-II**	**G-III**	**G-IV**	**G-V**	**G-VI**
	Sequenced similarity values					
Reflectors	0.9157	0.8885	0.5082	0.5442	0.0724	0.842
Sequence	0.9336	0.9474	0.611	0.5975	0.2679	0.8544
	0.9362	0.9695	0.6322	0.6112	0.5157	0.8575
	0.9502	0.9893	0.7129	0.6346	0.5833	0.881
	0.9545	0.9916	0.752	0.7895	0.7004	0.8827
	0	0	0	0	0	0
Meanic mirror	0.938	0.9573	0.6433	0.6354	0.4279	0.8635
Sequence	0.938	0.9573	0.6433	0.6354	0.4279	0.8635
	0.938	0.9573	0.6433	0.6354	0.4279	0.8635
	0.938	0.9573	0.6433	0.6354	0.4279	0.8635
	0.938	0.9573	0.6433	0.6354	0.4279	0.8635
	Sequenced dissimilarity values					
Reflectors	0.0455	0.0084	0.248	0.2105	0.2996	0.1173
Sequence	0.0498	0.0107	0.2871	0.3654	0.4167	0.119
	0.0638	0.0305	0.3678	0.3888	0.4843	0.1425
	0.0664	0.0526	0.389	0.4025	0.7321	0.1456
	0.0843	0.1115	0.4918	0.4558	0.9276	0.158
	0	0	0	0	0	0
Meanic mirror	0.062	0.0427	0.3567	0.3646	0.5721	0.1365
Sequence	0.062	0.0427	0.3567	0.3646	0.5721	0.1365
	0.062	0.0427	0.3567	0.3646	0.5721	0.1365
	0.062	0.0427	0.3567	0.3646	0.5721	0.1365
	0.062	0.0427	0.3567	0.3646	0.5721	0.1365

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Appendix B

Table B1: Sequenced (in ascending order) of different statistical elements of each group prior for comparative optinalysis

	Groups					
	G-I	**G-II**	**G-III**	**G-IV**	**G-V**	**G-VI**
			Variance			
Reflectors	0.7016	0.1716	1116.4551	582.6435	2868.8	74.3
Sequence	0.7852	0.3157	1921.3728	815.0536	4285.2	108.2
	1.1097	0.5237	67241.1426	1676.0617	36501.8	158.7
	1.4429	0.8861	96126.1455	1696.3	98741.2	176.2
	2.0155	0.8973	9400282.711	19416.0578	4907619.2	25614.2
	0	0	0	0	0	0
Meanic mirror	1.211	0.5589	1913337.565	4837.2233	1010003.24	5226.32
Sequence	1.211	0.5589	1913337.565	4837.2233	1010003.24	5226.32
	1.211	0.5589	1913337.565	4837.2233	1010003.24	5226.32
	1.211	0.5589	1913337.565	4837.2233	1010003.24	5226.32
	1.211	0.5589	1913337.565	4837.2233	1010003.24	5226.32
			Standard deviation			
Reflectors	0.8376	0.4142	33.4134	24.138	53.5612	8.6197
Sequence	0.8861	0.5618	43.8335	28.5491	65.4614	10.4019
	1.0534	0.7237	259.309	40.9397	191.0544	12.5976
	1.2012	0.9413	310.0422	41.1862	314.2311	13.274
	1.4197	0.9472	3065.988	139.3415	2215.3147	160.0444
	0	0	0	0	0	0
Meanic mirror	1.0796	0.7176	742.5172	54.8309	567.9246	40.9875
Sequence	1.0796	0.7176	742.5172	54.8309	567.9246	40.9875
	1.0796	0.7176	742.5172	54.8309	567.9246	40.9875
	1.0796	0.7176	742.5172	54.8309	567.9246	40.9875
	1.0796	0.7176	742.5172	54.8309	567.9246	40.9875
			Standard error of mean			
Reflectors	0.3746	0.1852	14.94	10.79	23.95	3.855
Sequence	0.3963	0.2513	19.6	12.77	29.28	4.652
	0.4711	0.3236	116	18.31	85.44	5.634
	0.5372	0.421	138.7	18.42	140.5	5.936
	0.6349	0.4236	1371	62.32	990.7	71.57
	0	0	0	0	0	0
Meanic mirror	0.4828	0.3209	332.048	24.522	253.974	18.3294
Sequence	0.4828	0.3209	332.048	24.522	253.974	18.3294
	0.4828	0.3209	332.048	24.522	253.974	18.3294
	0.4828	0.3209	332.048	24.522	253.974	18.3294
	0.4828	0.3209	332.048	24.522	253.974	18.3294

Table B1 (Cont…): Sequenced (in ascending order) of different statistical elements of each group prior for comparative optinalysis

	Groups					
	G-I	**G-II**	**G-III**	**G-IV**	**G-V**	**G-VI**
			Geometric mean			
Reflectors	6.344	3.1	38.35	2.676	#DIV/0!	#DIV/0!
Sequence	6.625	7.048	38.5	7.503	#DIV/0!	#DIV/0!
	6.804	9.534	87.38	11.48	#DIV/0!	#DIV/0!
	7.065	16.02	94.18	30.56	#DIV/0!	#DIV/0!
	7.345	27.76	136.3	61.96	#DIV/0!	#DIV/0!
	0	0	0	0	0	0
Meanic mirror	6.8366	12.6924	78.942	22.8358	#DIV/0!	#DIV/0!
Sequence	6.8366	12.6924	78.942	22.8358	#DIV/0!	#DIV/0!
	6.8366	12.6924	78.942	22.8358	#DIV/0!	#DIV/0!
	6.8366	12.6924	78.942	22.8358	#DIV/0!	#DIV/0!
	6.8366	12.6924	78.942	22.8358	#DIV/0!	#DIV/0!
			Harmonic mean			
Reflectors	6.277	3.007	27.9	0.9531	#DIV/0!	#DIV/0!
Sequence	6.492	6.991	28.32	1.213	#DIV/0!	#DIV/0!
	6.758	9.513	28.91	7.824	#DIV/0!	#DIV/0!
	7.02	16.02	40.97	27.53	#DIV/0!	#DIV/0!
	7.262	27.76	82.39	33.94	#DIV/0!	#DIV/0!
	0	0	0	0	0	0
Meanic mirror	6.7618	12.6582	41.698	14.292	#DIV/0!	#DIV/0!
Sequence	6.7618	12.6582	41.698	14.292	#DIV/0!	#DIV/0!
	6.7618	12.6582	41.698	14.292	#DIV/0!	#DIV/0!
	6.7618	12.6582	41.698	14.292	#DIV/0!	#DIV/0!
	6.7618	12.6582	41.698	14.292	#DIV/0!	#DIV/0!
			Quadratic mean			
Reflectors	6.481	3.311	57.57	25.57	60.64	22
Sequence	6.87	7.152	65.02	44.36	62.67	29.32
	6.889	9.578	299.5	44.75	282.1	42.71
	7.154	16.03	374.4	49.52	342.2	44.8
	7.503	27.77	3085	170.4	2082	441.6
	0	0	0	0	0	0
Meanic mirror	6.9794	12.7682	776.298	66.92	565.922	116.086
Sequence	6.9794	12.7682	776.298	66.92	565.922	116.086
	6.9794	12.7682	776.298	66.92	565.922	116.086
	6.9794	12.7682	776.298	66.92	565.922	116.086
	6.9794	12.7682	776.298	66.92	565.922	116.086