Article

Identical Quantum Particles, Entanglement and Individuality

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Abstract: Particles in classical physics are distinguishable objects, which can be picked out individually on the basis of their unique physical properties. By contrast, in quantum mechanics the standard view is that particles of the same kind ("identical particles") are completely indistinguishable from each other. This standard view is problematic: Particle indistinguishability is irreconcilable not only with the very meaning of "particle" in ordinary language and in classical physical theory, but also with how this term is used in the practice of present-day physics. Moreover, the indistinguishability doctrine prevents a smooth transition from quantum particles to what we normally understand by "particles" in the classical limit of quantum mechanics. Elaborating on earlier work, we here discuss an alternative to the standard view that avoids these and similar problems. As it turns out, this alternative approach connects to recent discussions in quantum information theory concerning the question of when identical particles can be considered to be entangled.

Keywords: Identical quantum particles; indistinguishability; the concept of a particle; emergence; entanglement

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0. Introduction

Our everyday environment is filled with distinguishable objects, which differ from each other by one or more physical characteristics. Classical physics has extended this picture into the domain of the not directly observable by the introduction of the concept of a *particle*: a classical particle is an entity characterized by an individuating set of values of physical quantities (mass, electric charge, position, momentum, etc.). It is true that two or more classical particles may have a number of such physical properties in common, but they will differ at least in their spatial positions since the particles of classical theory are assumed to be impenetrable.

It is therefore possible to assign a physically meaningful identity to each classical particle, expressed by a unique name or label. Because classical particle labels have this physical content (in practice usually because they refer to particle positions and trajectories), there is no need to invoke an additional non-physical ("metaphysical") notion of particle identity ("*haecceity*" or "primitive thisness"). This is as we expect it to be: it would be bad scientific methodology to introduce notions into physics that cannot be related at least in *some* way to physical quantities used in the relevant physical theories.

In particular, we expect a physically meaningful concept of particle identity to play a role in the dynamics specified by the theory. This is needed to guarantee that particles are individually addressable by empirical means—that we can pick them out by using physical interactions. In the case of classical physics this requirement is always fulfilled in principle: by using detection techniques that are sensitive to spatial position, we can in principle identify and follow classical particles.

In the usual $6N$-dimensional phase space representation of classical many-particle states the use of these physically meaningful individual labels is taken for granted. The particle with label i has its own x_i (position) and p_i (momentum) axes of the total phase space associated with it, and can in

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principle be addresses individually via a measurement interaction that makes contact with only its part of phase space. It is true that in the case of a collection of classical particles *of the same kind*, with the same intrinsic properties like rest mass and electric charge, any measuring device must couple in the same way with each particle, so that the interaction Hamiltonian has to be symmetric in the particle labels. Nevertheless, it is possible to interact with only one specific particle by using a position dependent interaction potential such that only the particle in question affects the measuring device (because all other particles are too far away). Classical particle labels are thus measurable quantities.

The standard way in which quantum mechanics models systems of particles of the same kind follows the classical example. The particles are assumed to be labeled, $1, 2, ..., N$ (in a collection of N particles), and each particle is allotted its own part of the total state space. In quantum mechanics, state spaces are Hilbert spaces and we are led to a tensor product structure of the total state space: $\mathcal{H}^N = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \otimes ... \otimes \mathcal{H}_N$, where the factor space $\mathcal{H}_i$ is the one-particle Hilbert space belonging to the particle labeled i. This factor space of the total tensor product space is the analogue of the "i-particle part" of the classical phase space, i.e. the part associated with the axes x_i and p_i. As in the classical case, the Hamiltonian representing interactions with a collection of quantum particles of the same kind ("identical particles") must be symmetric in the particle labels, since different labels are not associated with different intrinsic particle properties, so that the interaction has the same form for all particles [1].

There is nevertheless an essential difference between the quantum and classical cases. In classical physics each single particle label is associated with a unique one-particle state that differs from the states of all other particles. This is not so in quantum mechanics, as a consequence of the *symmetrization postulate*: quantum mechanics ordains that the states of collections of identical particles must be symmetric (bosons) or anti-symmetric (fermions) under permutations of the particle labels.

For the simplest situation, namely a system consisting of two identical particles described in $\mathcal{H}_1 \otimes \mathcal{H}_2$, only states of the following form are accordingly allowed:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\phi\rangle_1|\psi\rangle_2 \pm |\psi\rangle_1|\phi\rangle_2\}, \tag{1}$$

with the plus sign holding for the bosons and the minus sign for fermions.

States of the product form $|\Psi\rangle = |\phi\rangle_1 \otimes |\psi\rangle_2$ are consequently forbidden in the quantum mechanics of identical particles. If this product form *were* allowed, we could hope to complete the analogy with classical mechanics by associating the labels 1 and 2 with the one-particle states $|\phi\rangle$ and $|\psi\rangle$, respectively; and we could in this case imagine that the labels would be empirically accessible through measurements distinguishing these two states (which would certainly be possible in the case of orthogonal states). The symmetrization postulate, however, has the consequence that both labels are *symmetrically* associated with $|\phi\rangle$ and $|\psi\rangle$. In fact, the one-particle states derivable from (1) by partial tracing are the same in $\mathcal{H}_1$ and $\mathcal{H}_2$, namely $1/2\{|\phi\rangle\langle\phi| + |\psi\rangle\langle\psi|\}$. So there can be no way to identify the labels 1 and 2 experimentally, on the basis of results of measurements—the particle states associated with 1 and 2 predict identical expectation values for all observables. The "particles" labeled by factor space indices can therefore not be individually addressed by experimental methods.

The standard doctrine that the labels of the factor spaces in the tensor product Hilbert space of an identical particles system refer to single particles[1] has therefore the undesirable consequence that the thus defined particles become metaphysical entities, in the sense that it is impossible to make contact with them individually by experimental techniques. It should be noted that this impossibility is not due to imperfections in our technical capabilities, which could be overcome at a later point in time. Rather, the lack of physical individuality of "factor space particles" is a point of principle that derives

1 This doctrine has been dubbed "factorism" [2].

directly from the symmetrization postulate, which puts all "factorist" particles in exactly the same physical state.

However, in circumstances in which classical physics provides an adequate description of (part of) the world, particles—in the ordinary sense of the word—do possess their own individuality based on identifying physical properties. It follows that the factorist particles represented in the factor spaces of the total Hilbert space, and labeled by the indices of these factor spaces, cannot correspond to what we ordinarily call particles: in the classical limit of quantum mechanics the symmetrization postulate remains untouched, so that even in this limit factorist particles will all be in the same state. Therefore, we need another definition of what a quantum particle is and how it is to be represented in the formalism if we wish to achieve a gradual transition from quantum mechanics to the classical world picture, and if we want to avoid empirical inaccessibility of individual quantum particles.

In earlier work [3–6] such an alternative definition was proposed. The core idea of that proposal is to associate particles with *one-particle states* occurring in symmetrized many-particle states like (1). So instead of associating particles with the *labels* 1 and 2 in (1), as the standard approach does, the idea is to look for *states* like $|\phi\rangle$ and $|\psi\rangle$ and to interpret these states as representing individual particles. This non-standard point of view, which we shall work out further in the sections to come, holds the promise of leading to the correct classical limit: if $|\phi\rangle$ and $|\psi\rangle$ are orthogonal, the associated systems are fully distinguishable and thus can be assigned their own empirically accessible identities.

This alternative view of how quantum particles are to be represented is at variance with the standard quantum mechanical doctrine, and leads to new questions. One of the most significant of these concerns the notion of entanglement. According to standard definitions a state of the form (1) represents two particles, identified by the labels 1 and 2, that are entangled with each other, because their total state is not a product of two one-particle states. However, on the view that (1) represents one particle characterized by the state $|\phi\rangle$ and one particle represented by $|\psi\rangle$, one may well wonder whether these particles should really be considered entangled with each other—intuition tells us that in the case of orthogonal states, for example of particles prepared at a large distance from each other, such particles may well be completely uncorrelated and therefore not entangled. This connects to a two decades old discussion in the physics literature going back to work by Ghirardi at al. [8], about the definition of entanglement for identical quantum particles. This discussion has recently attracted considerable attention because of its relevance for quantum information theory. As we shall argue, central issues in this discussion become conceptually clearer in light of the alternative particle concept explained and defended here.

1. The standard approach and its problems

Consider a situation of two identical particles that only have a non-vanishing detection probability in two non-overlapping spatial volumes, to the Left and to the Right, respectively. For the sake of simplicity we assume in this example that the particles do not possess internal degrees of freedom, and that their total state has the following symmetrical form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2\}. \tag{2}$$

Here, the kets $|L\rangle$ and $|R\rangle$ correspond to wave functions with non-overlapping domains to the left and right, respectively. The one-particle states $|L\rangle$ and $|R\rangle$ occur in exactly the same way in the two factor spaces labeled by 1 and 2. If these labels are taken to refer to the two particles, so that the state of particle 1 is represented in factor space 1 and that of particle 2 in factor space 2, we have to conclude that our two particles are in precisely the same state. More generally, in the case of an (anti-)symmetrical total state of an N-particle system all factor spaces contain the same one-particle states in exactly the same way so that there can be no measurable differences between the particles represented in these factor spaces.

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The same conclusion can be reached more formally by determining single-particle states via the procedure of taking "partial traces": tracing out, in state (2), over the parts labeled by 2 we obtain the mixed state $W = 1/2\{|L\rangle\langle L| + |R\rangle\langle R|\}$; and exactly the same state by tracing out over 1. According to the standard approach these two identical mixed states are the quantum states of the two particles labeled by 1 and 2. This conclusion generalizes to (anti-)symmetric N-particle states, with $N > 2$: partial tracing leads to identical results in each factor Hilbert space.

So if identical quantum particles are taken to correspond to factor space labels, and if the quantum mechanical description is taken to be complete, all particles in a collection of particles of the same sort must be in exactly the same one-particle state and must possess exactly the same physical properties. This is an extremely strange conclusion that conflicts with the very idea of a particle. For example, because the symmetrization postulate applies to all particles of any given kind in the whole universe, e.g. all existing electrons, the factorist must hold that each single electron is equally present at all positions in the universe at which there is "electron presence". So it would not make sense to speak about the specific electrons fired by an electron gun: all electrons in the universe equally partake in being fired by this gun.

This is in conflict not only with how the notion of a particle is defined in classical physics but also with how it is used in the actual practice of physics. This is relevant because the motivation for speaking about particles in quantum physics at all comes from analogies with classical physics and from the use of the particle concept in experimental practice.

The Einstein-Podolsky-Rosen-Bohm state, much discussed in the context of Bell inequalities, furnishes a concrete illustration of the dilemma. This state has the form

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2\} \otimes \{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\}, \tag{3}$$

where $|L\rangle$ and $|R\rangle$ as before are states localized in non-overlapping regions to the left and right.

The standard approach, consistently applied, tells us that in the situation described by (3) there are two particles, labeled 1 and 2, and each in exactly the same spatial state, namely $1/2(|L\rangle\langle L| + |R\rangle\langle R|)$, so that each of the particles would be "evenly spread out" over left and right. This means that the way the EPR case is commonly described both in the fundamental physics literature and in experimental practice, namely as a situation in which there are two distinct and localized systems at a large distance from each other, is at odds with the official standard account in which the indices 1 and 2 are particle labels.[2]

That collections of particles of the same kind are to be represented by (anti-)symmetrized states is a general principle of quantum mechanics, which is not invalidated by taking the classical limit—whatever the details of the limiting procedure may turn out to be. The sameness of partial traces in all factor spaces is therefore a generic and robust feature of quantum mechanics that survives the classical limit. This means that even after taking this limit, all particles that were defined in the quantum formalism through reference to factor space labels still possess exactly the same properties. But particles in the sense of the classical theory are of course individual entities that are physically different from each other. Therefore, the particles that we know from classical physics cannot correspond to their quantum namesakes if the latter are defined according to the standard factorist approach. For example, the electrons in the classical Maxwell theory of the electromagnetic field cannot be approximated by the electrons in the quantum mechanical version of this theory. It seems obvious that this is an undesirable situation. The very introduction of the particle concept in modern physics is motivated by our classically describable experience and we should require that the quantum description approximates

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[2] As we shall discuss later, the rejection of factorism does make it possible to interpret the state of Eq.(3) as a representation of two localized systems. However, as we shall also see, the particular form of the superposition in (3) prevents a full localized particle interpretation. This will turn out to be the background of the non-locality manifested by violations of Bell inequalities in states like (3).

the classical one when quantum effects become negligible. We should therefore reconsider the way in which single particles are represented in the quantum mechanics of identical particle systems.

2. Quantum particles

An alternative to the standard way of using the particle concept in quantum mechanics was proposed in [2,5,6]; its original idea goes back to [3,4]. The central idea is to associate particles not with labels in the tensor space formalism, but instead with *one-particle states* that occur in the total N-particle state.

To make the basic motivation clear, we may consider the anti-symmetric state

$$\frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2 - |R\rangle_1|L\rangle_2\}, \tag{4}$$

in which $|L\rangle$ and $|R\rangle$ stand for two non-overlapping wave packets at a distance from each other, to the left and to the right, respectively. According to the standard view, factorism, this state represents two particles that are both in the mixed state $W = 1/2\{|L\rangle\langle L| + |R\rangle\langle R|\}$, so both equally "smeared out" over left and right. However, the nature of this state, with its two widely separated and narrow spatial regions in which something can be detected at all, suggests something else, namely that we are dealing with a situation in which there is one particle to the left and one to the right; the results of position measurements certainly agree with this interpretation. Actually, as already noted before, this interpretation is adopted without question in the actual practice of physics (for example in discussions of the EPR set-up). In order to flesh out this alternative view of the meaning of state (4), individual particles should be associated with the states $|L\rangle$ and $|R\rangle$, respectively, even though each of these states occurs in *both* factor spaces.

A theoretical framework employed by Ghirardi, Marinotti and Weber [8] turns out to be useful to make some of the ideas mathematically precise. This scheme starts from the observation that, independently of the symmetrization postulate, all observables of systems of identical particles must be symmetric in the factor labels (because measurement interactions cannot distinguish between different labels, as already pointed out in the previous section). Observables corresponding to measurable single particle properties must accordingly be represented by symmetric projection operators (telling us whether or not the property in question is instantiated, via the eigenvalues 1 and 0, respectively). Therefore we should not consider operators of the form $P_1 \otimes I_2$ (with I_2 the unity operator in factor Hilbert space 2) if we want to represent empirically accessible particle properties, but rather projection operators of the form

$$P_1 \otimes I_2 + I_1 \otimes P_2 - P_1 \otimes P_2, \tag{5}$$

with P standing for the projection operator to be used for the relevant property in the case of a one-particle system (in which there is only one factor space). The expectation value of the operator in (5) in an (anti-)symmetric state is the probability of finding at least once the result 1 in a double P measurement. The final term in (5) can be omitted in the case of fermions.[3]

The consistent use of symmetric projection operators makes it sometimes possible to associate definite one-particle properties with a system of identical particles, even though the total state is symmetric or anti-symmetric. This follows because (anti-)symmetric many-particle states may be eigenstates with eigenvalue 1 of symmetric projection operators like (5), in which case it is certain that the corresponding property will be found in a measurement. For example, the state (4) is an eigenstate with eigenvalue 1 of the symmetric projector $|L\rangle\langle L| \otimes I_2 + I_1 \otimes |L\rangle\langle L|$.

[3] The last term of (5) is added to allow for the possibility that the same one-particle state occurs twice in the total state, which may happen with bosons. In this case the probability would become greater than 1 without the correction term. In the case of fermions one-particle states cannot occur more than once, so the term is not needed.

For such cases Ghirardi, Marinotto and Weber [8] argue that the many-particle system can be thought of as built up from components possessing the one-particle properties that have probability 1. Their suggestion is that we thus obtain a description of an N-particle state in terms of N one-particle states, but that as a point of principle we cannot say *which* particle occupies each of these one-particle states. For example, in the case of state (4), we are entitled to say that the system consists of one particle to the left and one to the right, but it is impossible to know whether it is particle 1 or particle 2 that is located at these respective positions.

Our own proposal is similar in spirit, but deviates in a subtle but conceptually significant way. As explained before, we propose to *define* and identify particles through their physical properties and to reject any notion of particle identity that is not empirically accessible. On this view, the factor space labels in the tensor product formalism cannot be taken to refer to single particles at all, as they are not measurable quantities. It is then inconsistent to associate particles with definite and measurable physical properties, and at the same time to think that they are referred to by factor indices like 1 and 2. So the statement that there is one particle to the left and one to the right, but that we cannot know which one is particle 1 and which one particle 2, does not make sense according to our proposal. In our view, there is one particle to the left, characterized by $|L\rangle$, and one to the right, represented by $|R\rangle$; the labels 1 and 2 merely refer to factor spaces in the total product Hilbert space and are not particle names. In other words, in our view the factor space indices are mathematical quantities that play an important role in the definition of the total state space, but do not possess the physical significance of particle labels.

This interpretation of the factor labels as mathematical rather than physical quantities was recently called into question by Goyal, who writes [11, pp. 8–9]:

> However, such a claim leaves the challenge of formulating an alternative understanding of these indices which, for instance, is capable of rendering intelligible the usual procedures for interpreting measurement operators. For example, if one applies the symmetrization procedure to the electrons in a helium atom, the measurement operator $(x_1 - x_2)^2$ is ordinarily interpreted as representing a measurement of the squared-distance between the two electrons; but it is unclear how one would justify such an interpretation if the indices 1 and 2 have a "merely formal significance".

Consider, in order to respond to this challenge, the application of the observable $(x_1 \otimes I_2 - I_1 \otimes x_2)^2$ to the state (4):

$$(x_1 \otimes I_2 - I_1 \otimes x_2)^2 \frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2 - |R\rangle_1|L\rangle_2\} = (l-r)^2 \frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2 - |R\rangle_1|L\rangle_2\}. \tag{6}$$

State (4) is therefore an eigenstate of $(x_1 \otimes I_2 - I_1 \otimes x_2)^2$ with eigenvalue $(l-r)^2$, with l denoting the coordinate at which $|L\rangle$ is centered and r the position of $|R\rangle$, so that $(l-r)^2$ is the squared distance between our two particles defined and labeled by L and R, respectively. Therefore, $(x_1 \otimes I_2 - I_1 \otimes x_2)^2$ indeed represents the squared distance between the particles, but the particles in question are our two physical particles labeled by L and R. This mutual distance is expressed by $(x_1 \otimes I_2 - I_1 \otimes x_2)^2$ without invoking any other meaning of the indices 1 and 2 than the mathematical meaning of indices of the two factor spaces in the tensor product space—there is no need to think of these indices as particle labels. Is it intelligible why the mutual distance can be expressed this way? Yes, this is because $|L\rangle$ and $|R\rangle$ in state (4) always occur in *different* factor spaces, either in the order $1-2$ or $2-1$. That *both* these orders appear in the total state illustrates again that 1 or 2 itself cannot refer to one definite particle.

The formalism and main line of reasoning of Ghirardi et al. [8] are independent of these conceptual issues, however. Their argument demonstrates how it is possible, and why it is reasonable, to associate N pure one-particle states with an (anti-)symmetric N-particle state if this total (anti-)symmetric state can be obtained from symmetrizing or anti-symmetrizing an N-fold product state. Such (anti-)symmetrized product states are always eigenvectors of symmetric projection operators of the form (5) or its generalizations to N-fold products. In the case of (anti-)symmetrized product states we can therefore always find N one-particle states that define one-particle subsystems.

In the case of anti-symmetrized product states (fermions) we can always find *mutually orthogonal* single-particle states with the help of the above recipe. These states are fully distinguishable by empirical means. In the case of bosons this mutual orthogonality of states is not guaranteed. Accordingly, fermionic states that are anti-symmetrized product states can always be interpreted in terms of fully distinguishable particles.

Bosonic symmetrized product states do not always allow such a distinguishable particle description, however, because the one-particle states occurring in them may overlap and even coincide. In such situations bosons are better described as assemblies of field quanta (in a Fock space occupation number representation—see [5–7]) than as individual particles. In such a field picture the number N should not be seen as the result of counting N individual entities, but rather as a mass noun referring to a total quantity.[4]

This proposal for using the notion of a particle fits the actual practice of physics. Again, in the two-fermion state (4) we may conclude that there is one particle characterized by $|L\rangle$ and one particle characterized by $|R\rangle$. In contrast to the factorist approach, this interpretation does not face the objection that it is not able to reproduce what we expect and require from the transition to the classical limit, namely classical particles that emerge from their quantum counterparts as localized entities, characterized by relatively narrow wave packets following approximately classical trajectories.[5]

Summing up, what we propose is the definition and identification of quantum particles with the help of distinct physical properties, represented by one-particle projection operators and their mutually orthogonal eigenstates.[6] The thus defined quantum particles can be labeled on the basis of their individuating physical characteristics. These physical labels do clearly not coincide with the factor indices occurring in the total quantum state—the latter remain uniformly distributed over all one-particle states, even in the classical limit, and do not refer to individual particles.

3. Particles without factor labels

The irrelevance of factor space indices for the identification of individual particles in a collection of identical particles raises the question of whether it is possible to develop a theoretical treatment in which particles are represented without appealing to factor indices at all. A formalism in which this is achieved was introduced by Lo Franco et al. [12]; see also [13–15].

The core idea is to introduce a new many-particle Hilbert space (corresponding to the (anti-)symmetrical sector of the usual tensor product space) spanned by vectors of the form $|\alpha, \beta\rangle$, which represent one particle characterized by the one-particle state $|\alpha\rangle$ and one in the state $|\beta\rangle$. The

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[4] Think of the analogy with a total quantity of N liters of a liquid, which does not consist of N well-defined individual liter-entities [7].

[5] Very narrow quantum wave packets disperse very quickly, and will therefore only move according to approximately classical trajectories for very short time spans. For the correct classical limit, conditions must therefore be fulfilled that counteract the effects of dispersion. Decoherence, which can be thought of as the effect of very frequent position measurements by the environment, must be assumed to play a vital role here, in addition to the usual conditions of Ehrenfest's theorem.

[6] We should mention the important point that the decomposition in terms of such states as given in (1) is not unique. The equality of the coefficients appearing in front of the terms in the (anti-)symmetric superposition is responsible for a degeneracy that allows infinitely many alternative decompositions, in addition to the one in terms of $|L\rangle$ and $|R\rangle$. So the set of properties that distinguish the quantum particles is underdetermined by the procedure as we have outlined it. To make the definition of the particles unique some additional ingredient is needed, which picks out a privileged particle-properties basis. It is plausible, and in accordance with the classical picture of particles as localized entities, to take the position basis as privileged [5]. This ties in with the argument, to be discussed in section 4, that the possibility of interpreting a total state as a representation of several individual particles does not only depend on the form of the state, but also depends on the form of the interactions with the environment. Measurement interactions used to verify the presence of particles have a local character, and this endows "position" with a privileged status (see [9] for a further tentative exploration of this idea).

state of Eq. (4) would in this formalism be written as $|L, R\rangle$, with the interpretation that there is one particle on the left-hand side (L), the other on the right-hand side (R).[7]

To fix the properties of the new two-particle Hilbert space we have to equip it with an inner product. This can be done in the following way [12,13]:

$$\langle \alpha', \beta' | \alpha, \beta \rangle := \langle \alpha' | \alpha \rangle \langle \beta' | \beta \rangle + \eta \langle \alpha' | \beta \rangle \langle \beta' | \alpha \rangle, \tag{7}$$

where $\eta = \pm 1$ and where $\langle \phi | \psi \rangle$ is the usual one-particle inner product of the standard formalism.

The two possible values of η, -1 and $+1$, correspond to the difference between fermions and bosons. It follows from (7) that the order of α and β in the state $|\alpha, \beta\rangle$ is not without significance: $|\alpha, \beta\rangle = \eta |\beta, \alpha\rangle$, so that exchanging one-particle states in the total state leads to a global phase shift in the case of fermions.

All two-particle state vectors of the form $|\alpha, \beta\rangle$ span a two-particle Hilbert space that is isomorphic to the (anti-)symmetric subspace of the tensor product space $\mathcal{H} \otimes \mathcal{H}$, but does not possess a tensor product structure itself. This suggests that, in general, states like $|\alpha, \beta\rangle$, even when they can be interpreted as instantaneously representing one individual particle in state $|\alpha\rangle$ and one in state $|\beta\rangle$, do not represent a system of two objects each having its own dynamical evolution in its own state space. Indeed, the form of the inner product of Eq. (7) shows that two one-particle states at an instant t' may contain contributions from both one-particle states at an earlier instant t, $t' > t$. The classical notion of a particle that retains its identity over time so that its evolution may be described in its own part of phase space need therefore not be applicable in quantum mechanics, even though we may have initially well-defined and empirically discernible particle states.[8] However, it is also visible from (7) that there can be limiting situations in which identity over time does become an applicable concept. This is the case when the second term on the right hand side of (7) vanishes because $\langle \alpha' | \beta \rangle$ and $\langle \beta' | \alpha \rangle$ are both zero (we here think of $|\alpha', \beta'\rangle$ as the time-evolved version of $|\alpha, \beta\rangle$). In this case the ordinary tensor product form of the inner product is recovered, so that we are entitled to represent $|\alpha\rangle$ and $|\beta\rangle$ as vectors in two different factor spaces in a tensor product space. This shows once again, at least in principle, that the classical particle concept can emerge from the quantum description of the world if we define particles through their states.

4. Particles and entanglement

Classical particles possess their properties in an autonomous way: each particle has its own properties, which could stay in place—logically speaking—even if all other matter would suddenly disappear. Of course, particles interact with each other and will be subject to forces, and as a result their properties will generally *change* over time. But this dynamical picture only makes sense on the very assumption that each particle can be assigned its own properties in the first place.

In quantum mechanics the situation is fundamentally different. Many-particle systems may be in entangled states, and in this case there are correlations between measurement results that cannot be explained by correlations between preexisting particle properties. In such situations of entanglement the usual notion of a particle as an autonomous individual becomes problematic: an

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7 As in the previous section, we here deviate slightly but from a conceptual viewpoint significantly from the literature [12,14,15] in that we *define* our particles by means of the characteristics L and R. As a result, these particles are physically identifiable entities; their "labels" L and R are measurable quantities. By contrast, Lo Franco et al. [12] introduce their formalism by stating: "Indistinguishability requires that the identical particles cannot be individually addressed", which is still based on the notion that a referential function can be attributed to the factor labels. This perpetuates the confusing notion that we have a set of distinguishable one-particle states, but cannot know *which* particle is in any given state.

8 It is interesting to compare our argument here, which takes its starting point in the formalism of quantum mechanics, with the analysis in [11], which starts from an operational point of view and proposes that interpretations of empirical data with and without particles should be seen as complementary. We reserve a detailed comparison with that viewpoint for another occasion.

entangled "N-particle system" cannot be thought of as being composed of N independent particles whose properties determine measurement outcomes in accordance with local interaction laws.[9]

It is therefore to be expected that the notion of individual component particles possessing their own properties only becomes applicable in identical particle systems whose total state is not entangled. This raises the question of whether states of the form $|\alpha, \beta\rangle$, in the formalism of the previous section, are entangled. The way we have discussed these states, in terms of one particle fully characterized by $|\alpha\rangle$ and one by $|\beta\rangle$, suggests the absence of entanglement. However, in the standard formalism the corresponding two-particle state is $\frac{1}{\sqrt{2}}\{|\alpha\rangle_1|\beta\rangle_2 \pm |\beta\rangle_1|\alpha\rangle_2\}$, which is not a product state and therefore rather suggests the presence of entanglement.

This issue was addressed by Ghirardi, Marinotto and Weber in their seminal paper [8]. Their conclusion (which is in line with our discussion in sections 1 and 2) is that (anti-)symmetrized product states of identical particles, like $\frac{1}{\sqrt{2}}\{|\alpha\rangle_1|\beta\rangle_2 \pm |\beta\rangle_1|\alpha\rangle_2\}$, should be considered as *not* entangled.

The important premise in the argument by Ghirardi et al. for this conclusion is that the observables that are measured on the system probe genuine one-particle properties and therefore do not mix the states $|\alpha\rangle$ and $|\beta\rangle$. That is, the measured observables are assumed to be such that they have non-vanishing expectation values either in $|\alpha\rangle$ or in $|\beta\rangle$, but not in both. This assumption is valid for the observables normally considered in EPR-Bohm type of experiments: these are of the form $P_L.\sigma_n \otimes P_R.\sigma_{n'} + P_R.\sigma_{n'} \otimes P_L.\sigma_n$, with $P_L = |L\rangle\langle L|$ and $P_R = |R\rangle\langle R|$ the projector operators on the non-overlapping wave packets to the left and right, respectively. When we now consider an anti-symmetrized product state of the form

$$\frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2 - |R\rangle_1|L\rangle_2|\downarrow\rangle_1|\uparrow\rangle_2\}, \tag{8}$$

it is easy to see that all results of measurements in this state are identical to the results of measuring $P_L.\sigma_n \otimes P_R.\sigma_{n'}$ in the product state $|L\rangle_1|R\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2$. This shows that Bell inequalities will not be violated: under the mentioned measurement conditions (8) is effectively a product state. By contrast, measuring the same symmetrical observables in the EPR-Bohm state (3) will result in the appearance of cross terms in the expectation values, which is responsible for the fact that in this state Bell inequalities *are* violated.

So the result that (anti-)symmetrized products of states are effectively equivalent to product states depends on a restriction on the class of observables that we are allowed to measure: it is assumed that we will only be interested in observables whose expectation values will not show interference between the states of the two particles (defined by the states $|\alpha\rangle$ and $|\beta\rangle$ in $|\alpha, \beta\rangle$).

When this restriction on permissible measurements is taken into account, we have to qualify our earlier statements suggesting that (anti-)symmetrized product states like (8) represent a collection of autonomous single particles *tout court*: this interpretation can only be maintained with the proviso that observables that mix one-particle states will not be considered.

In classical physics individual particles differ at least in their positions, and this motivates the use of the notion of individual quantum particles especially in cases in which the relevant one-particle states have (approximately) non-overlapping spatial parts—in particular, this is appropriate when situations close to the classical limit are considered. In such situations we have to require, in order to take the just-mentioned restriction on measurements into account, that only *local* one-particle observables are measured, because these keep one-particle states with non-overlapping spatial parts separated.

If this condition of only local measurements is not fulfilled, the results of measurements on (anti-)symmetrized products of localized one-particle states like (8) may violate Bell inequalities. At

[9] If one assumes the existence of instantaneous many-body interactions, as in the Bohm theory, a different analysis applies. Our argument here is meant to be understood within standard quantum mechanics.

first sight such non-local measurements might appear difficult to realize because interactions are typically governed by local field theories with potentials that are position dependent. However, the effects of non-local measurements can be reproduced by combinations of local interactions, as shown by the well-known Ou-Mandel experiment [16,17]. For two electrons the core idea of this experiment translates into the following.

Suppose that we have two electron guns, one to the Left and one to the Right, and suppose that each of these devices fires exactly one electron—one with spin up in the z-direction, the other with spin down. This is exactly the situation we discussed before: two individual particles, each with its definite spin property. Since the two electrons are identical fermions, we have to anti-symmetrize the total wave function, so that the total state has the form (8). Now, suppose further that the two electron wave packets are each split by beam splitters, in local processes, in such a way that both original packets are distributed over two new locations, L' and R', respectively.

In more detail, suppose that the time evolution can be represented as follows:

$$|L\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|L'\rangle + |R'\rangle\right) \tag{9}$$

$$|R\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|L'\rangle - |R'\rangle\right), \tag{10}$$

where $|L'\rangle$ and $|R'\rangle$ correspond to wave packets localized at L' and R', respectively.

After the evolution, the total state is still an anti-symmetrized product state: if we call the states into which $|L\rangle$ and $|R\rangle$ have evolved ϕ and ψ, respectively, the final total state can be written as

$$\frac{1}{\sqrt{2}}\{|\phi\rangle_1|\psi\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2 - |\psi\rangle_1|\phi\rangle_2|\downarrow\rangle_1|\uparrow\rangle_2\}. \tag{11}$$

So according to the criterion proposed by Ghirardi, Marinotto and Weber ([8], see also [18]), and our discussion in section 1 and 2, this state still represents two independent particles, and we would not expect violations of Bell inequalities. This interpretation would be confirmed if we were to detect and identify the particles by performing measurements of $|\phi\rangle\langle\phi|$ and $|\psi\rangle\langle\psi|$.

However, if we perform *local* measurements, by using electron detectors positioned at L' and R', an interpretation of the results in terms of independent particles becomes problematic, in spite of the fact that both initial one-particle states were localized far apart from each other and evolved independently and unitarily. This is so because each of the two initial particles can be found both at L' and at R'. This circumstance is responsible for the appearance of correlations between measurement results that can only be explained by entanglement [16].

In detail, when in a series of repetitions of the experiment we post-select the measurements in which one electron is found at L' and one at R', correlations between the particle spins found at the two different locations can be computed from the component of the state (11) that has the spatial parts $|L'\rangle_1|R'\rangle_2$ or $|R'\rangle_1|L'\rangle_2$. We thus find the effective state

$$|\Phi\rangle = \frac{1}{2}\{|L'\rangle_1|R'\rangle_2 - |R'\rangle_1|L'\rangle_2\} \otimes \{|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\}, \tag{12}$$

which is not an anti-symmetrized product: it is the superposition of two anti-symmetrized product states (a superposition of two "Slater determinants"), and this qualifies it as an *entangled* two-electron state. Therefore, if we condition on experiments in which both detectors fire, we find correlations between outcomes of spin measurements that cannot be explained locally on the basis of preexisting values. These correlations are able to violate Bell inequalities.[10] The existence of entanglement

[10] The spin part of (12) is of course different from the EPR-Bohm singlet state, and it is not rotation invariant. The correlation between spin measurements predicted by (12) in the directions α and β (with α and β the angles with respect to the

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of this kind ("measurement-induced entanglement") has been established as an important tool in experimental practice—it has become a significant resource in quantum information research (see, e.g., [19–21]).

5. Classical particles as *emergent*

In earlier work, as in the beginning sections of this article, we have explained and defended the interpretation of states that are (anti-)symmetrized products of one-particle states as the representation of individual particles, each characterized by distinctive physical properties that correspond to the one-particle states occurring in the total state. But the argument from the previous section shows that even quantum particles defined in this way will not automatically give rise to phenomena that naturally fit into what we expect from a particle picture. In the classical limiting situation we should not only have particles that possess their own characteristic properties, but we should also make sure that these particles are (approximately) localized and that "measurement-induced entanglement" is avoided. The classical particle picture will therefore only become (approximately) applicable under special circumstances. The world describable by classical particle physics can therefore only be a fringe area of the quantum world.

To start with, general so-called "many-particle quantum states" will usually not admit a particle interpretation at all, since most of them are not (anti-)symmetrized product states. The EPR-Bohm state (3) already illustrates this: although its spin part $\{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\}$ is an anti-symmetrized product, the complete state is entangled even according to the criterion that anti-symmetrized products do not count as such. This entanglement is responsible for the non-factorizability of joint probabilities of measurement outcomes on the two wings of a Bell experiment, and consequently for violations of Bell inequalities. In order to have factorizable probabilities for measurements on the two wings of the experiment, and in order to be consistent with the picture of two independent particles with complete sets of definite one-particle properties, we should rather have a state like (8), which *is* an anti-symmetrized product. In this state there will be no violations of Bell inequalities and therefore no no-go results for local models. In the usual Alice-Bob scenario state (8) can be taken to represent a situation in which there are two particles, one to the left and one to the right, possessing spin up and spin down, respectively. The important difference with the EPR-Bohm case is that in (8) the locations L and R are correlated with definite spin eigenstates, whereas in (3) the spin part is independent of location and thus represents a global—non-particulate—property of the system.

States like (8) therefore come close to representing a classical situation, in which the particles can be labeled by their properties: $L, \uparrow$ and $R, \downarrow$, respectively (note that this labeling is "non-factorist", since the particle labels differ from the factor space labels 1 and 2).

Even so, results of measurements will only accord with a classical particle picture when the measurements are local and do not mix the two particle states; otherwise measurement interactions could introduce entanglement after all.

The possibility of a physically meaningful particle interpretation of states of "many identical particles" is therefore not at all given *a priori*: most of such states allowed by quantum mechanics will *not* allow a particle interpretation. In addition, even states that do qualify as the representation of distinct entities characterized by one-particle states will only display particle-like behavior when subjected to measurements if the measurement interactions themselves do no not re-introduce entanglement.

Speaking in all generality, quantum states manifest features of holism and non-locality, and it is only in special situations that the notions of individual and local physical components of systems

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polarization direction z) is $\sin\alpha.\sin\beta - \cos\alpha.\cos\beta$. It is simple to find directions such that this correlation function produces violations of the CHSH inequality.

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become applicable. In this sense the classical world with its classical particles *emerges* from quantum mechanics.[11]

6. Discussion

In the world of everyday experience individual objects constitute an important part of "what there is". Classical physics has generalized this everyday conception of the world by introducing the concept of a particle as a basic category. These classical particles are individuated by their properties: each single particle has a unique set of physical characteristics. At first sight, this picture seems completely irreconcilable with the quantum mechanics of systems of "identical particles": because of the symmetrization rules all identical quantum particles appear to be in the same physical state and therefore they seem to lack all individuality. But as we have noted, this conclusion depends on the adoption of "factorism", the doctrine that the labels of the factor spaces in the tensor product Hilbert space refer to single particles. However, in spite of the fact that factorism is part of standard wisdom, it is physically unreasonable. Moreover, there is an alternative, namely the identification of particles with the help of one-particle states that occur in the total many-particle state.

This alternative holds the prospect of leading to the right classical limit, in which quantum particles mimic the behavior of classical particles. It is also in accordance with how the particle concept is used in the experimental practice of physics, even in present-day high-energy physics.

The alternative approach also connects to recent work in quantum information theory. It is essential for quantum information theory to have a robust notion of entanglement, since entangled systems constitute the instrument with which to achieve non-classical information transfer. The usual way of defining entanglement is to say that any state that is not a product of one-particle states is entangled. Because of the symmetrization rules, this definition has the consequence that all many-particle states of particles of the same kind (identical particles) are entangled. However, this conclusion is challenged by the fact that systems described by such states can behave as consisting of independent and individual particles, for which standard quantum information protocols do not work. The identification of particles with one-particle states instead of factor labels makes it possible to solve this problem, by making a distinction between physically spurious entanglement between "*factor particles*" and genuine entanglement between *physical* particles. Even so, it must be taken into account that the qualification as independent particles should be seen as relative to a restricted class of interactions with which the system is to be probed.

In general, the many-particle states of identical particles allowed by quantum mechanics are not (anti-)symmetrized products. Such states are genuinely entangled. They correspond to situations in which it is not natural to interpret the total system as built up from individual entities each characterized by its own one-particle state. Such general, entangled, states are exactly the states that are responsible for the typical quantum features of non-local correlations and violations of Bell inequalities. This illustrates the trade-off between the applicability of the notion of a particle on the one hand and the occurrence of typical quantum effects on the other. It is precisely when holism, non-locality and entanglement disappear from sight that the classical world and its particles emerge. By contrast, on the fundamental level of quantum mechanics there is overwhelming evidence that the concept of global systems as being composed of autonomous individuals is not theoretically fruitful.

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[11] Evidently, the details of the physical mechanisms that are responsible for washing out entanglement and making the classical particle concept applicable need a separate and much more precise discussion. In anticipation of a full discussion of the classical limit, it seems clear however that the fact that ordinary measurement interactions have a local character is important here: such local measurements will not mix localized states unless they are combined in special setups like those discussed in section 4. Moreover, in most everyday situations frequent local interactions with the environment will lead to quick and efficient decoherence, which will make the effects of entanglement between different positions invisible. As a consequence, an effective description in terms of localized wave packets will become applicable. Since our usual interactions with the quantum world possess this local character, the particle properties that we measure are correlated with positions, so that measurements will usually lead to effective states of the form of (8).

Awareness of how particles should be represented in quantum mechanics, of why the quantum world in its basic features is non-particulate, and of how in principle the classical world emerges from the quantum world, provides new conceptual resources for the study of quantum processes. Phenomena like violations of Bell inequalities and non-classical information transfer become better understandable when we realize that the systems involved do not consist of particles in the usual sense; that, for example, we should not speak and think of Alice's particle with "its" spin, and similarly for Bob's particle. Typically, in these situations there are (approximately) localized states that are *not* correlated with definite spin states, so that the full applicability of the notion of a particle defined by a complete one-particle state fails. Adjusting our explanations better to these features of the quantum formalism may enable a better understanding of what is going on in the quantum world (see for some attempts [9,10]).

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