

Article

QED Response of the Vacuum

Gerd Leuchs^{1,2,*} , Margaret Hawton³ , and Luis L. Sánchez-Soto^{1,4} 

¹ Max-Planck-Institut für die Physik des Lichts, Staudtstraße 2, 91058 Erlangen, Germany

² Institute for Applied Physics, Russian Academy of Sciences, 630950 Nizhny Novgorod, Russia

³ Department of Physics, Lakehead University, Thunder Bay, ON P7B 5E1 Ontario, Canada; mhawton@lakeheadu.ca

⁴ Departamento de Óptica, Facultad de Física; Universidad Complutense, 28040 Madrid, Spain; lsanchez@fis.ucm.es

* Correspondence: gerd.leuchs@mpl.mpg.de

Abstract: We present a new perspective of the link between QED and Maxwell's equations. We demonstrate that the interpretation of $\mathbf{D} = \epsilon_0 \mathbf{E}$ as vacuum polarization is consistent with QED. A free electromagnetic field polarizes the vacuum, but the polarization and magnetization currents cancel giving zero source current. The speed of light is a universal constant, while the fine structure constant, which couples the electromagnetic field to matter, runs as it should.

Keywords: Quantum vacuum; Linear response; QED

1. Introduction

In quantum electrodynamics (QED) the vacuum is a dynamical entity, in the sense that there is a rich variety of virtual processes that can take place in it [1–3]. There are several observable effects that manifest themselves when the vacuum is perturbed in specific ways: vacuum fluctuations lead to shifts in the energy level of atoms (Lamb shift) [4], changes in the boundary conditions produce particles (dynamical Casimir effect) [5], and accelerated motion and gravitation can create thermal radiation (Unruh [6] and Hawking [7] effects).

QED conceives vacuum fluctuations as particle-antiparticle pairs that appear spontaneously, violating the conservation of energy according to the Heisenberg uncertainty principle. These pairs play a role in the value of the permittivity of the vacuum ϵ_0 : a photon will feel the presence of those pairs much the same as in a dielectric. This idea can be traced back to the time-honored works of Furry and Oppenheimer [8], Weisskopf and Pauli [9,10] and Dicke [11], who contemplated the prospect of treating the vacuum as a medium with electric and magnetic polarizability.

This approach has been recently adopted [12–14] to calculate *ab initio* ϵ_0^{-1} by using the same technique employed to determine the permittivity in a dielectric. This has prompted significant research interest in adopting an oscillator model for the virtual pairs in order to assess some properties of the vacuum [15–22]. Interestingly, the possibility that a charged pair can form an atomic bound state (which can thus be well approximated by an oscillator) was discussed by Ruark [23] and further elaborated by Wheeler [24].

In the standard relation $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, the first term on the right-hand side is often referred to as the polarization of the bare vacuum. In QED, however, the polarization of the virtual pairs is added as part of \mathbf{P} . Because the new term is dispersionless like $\epsilon_0 \mathbf{E}$, the electric field can be rescaled to formally recover the initial equation. We suggest that the polarization of the virtual pairs should instead be identified with the first term $\epsilon_0 \mathbf{E}$. This is a paradigm shift in our physical picture of the vacuum. One consequence of this procedure is that in the

¹ In Gaussian units the permittivity of the vacuum is inconspicuously "1". But whatever the unit system, this permittivity calls for a physical interpretation.

really bare vacuum light would travel infinitely fast, but this is only an academic consideration without practical relevance. On the positive side this procedure allows one to relate the value of ϵ_0 to the properties of the modern quantum vacuum.

We will show here that this interpretation of ϵ_0 is consistent with QED. Vacuum has a crucial property that it does not share with dielectric and magnetic materials: it is Lorentz invariant. Due to this property Michelson and Morley failed to detect the motion of the Earth through the "ether", which, in the present context, is the quantum vacuum [25]. Empty spacetime is homogeneous so variations of ϵ_0 and μ_0 can occur only in the presence of charged matter. The linear response of vacuum must be Lorentz invariant, so in reciprocal space the susceptibility of vacuum must be a function of $k^2 = \omega^2/c^2 - \mathbf{k}^2$. The condition $k^2 = 0$, describing a freely propagating photon, is referred to as on-shellness in QED: a real on-shell photon verifies then $\omega^2 = \mathbf{k}^2 c^2$.

This paper is organized as follows: Section 2 describes vacuum polarization due to creation of virtual particle-antiparticle pairs in the framework of QED. In Section 3, we show how the oscillator model can be derived from the results in QED and how \mathbf{D} can be indeed interpreted as vacuum polarization. The Gottfried-Weisskopf dielectric model is explored in Section 4, whereas our conclusions are summarized in Section 5.

2. Vacuum polarization in QED

As heralded in the Introduction, the vacuum in QED acts like a dielectric medium where the virtual pairs shield the original point charges. In this Section, the vacuum contribution to the dielectric permittivity ϵ_0 and the magnetic permeability μ_0 will be calculated by incorporating them into the electromagnetic Lagrangian. In QED the bare potentials A_0^μ and charge e_0 are rescaled to give the physical four-potential A^μ and the physical electron charge e ; that is,

$$A_0^\mu \equiv \sqrt{Z_3} A^\mu, \quad e_0 \equiv \sqrt{Z_3} e. \quad (1)$$

This rescaling is at the basis of the renormalization program. The renormalized QED Lagrangian density will be written as

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{int}}. \quad (2)$$

Details of $\mathcal{L}_{\text{Dirac}}$ are omitted because renormalization of the masses will not be discussed here. The interaction term is

$$\mathcal{L}_{\text{int}} = -j^\mu A_\mu. \quad (3)$$

Here, j^μ is the current due to real charges, while the current density j_ν^μ due to the creation of virtual pairs and the $Z_3 - 1$ counterterm are incorporated into $\mathcal{L}_{\text{Maxwell}}$. Together, they describe the reduction in vacuum polarization relative to its maximum value at $k^2 = 0$. The current induced in the vacuum by the four-potential A_ν due to virtual pairs of type \mathfrak{s} (where \mathfrak{s} corresponds to the possible different leptons) is

$$j_{\nu,\mathfrak{s}}^\mu(k) = c^2 \epsilon_0 \Pi_\mathfrak{s}(k^2) (k^2 g^{\mu\nu} - k^\mu k^\nu) A_\nu(k). \quad (4)$$

where $g^{\mu\nu}$ is the metric tensor, with diagonal $(1, -1, -1, -1)$, and $\Pi_\mathfrak{s}(k^2)$ is the QED vacuum "polarization" of a \mathfrak{s} -type particle. If $A_\nu(k)$ describes real photons, the on-shell condition $k^2 = 0$ is satisfied. This is a generalization of the usual textbook treatment of electron-positron virtual pairs [26,27], extended to other fermions.

The Feynman diagram in Fig. 1 is a pictorial representation of vacuum polarization in the one-loop approximation. The wavy lines represent an electromagnetic field, while a vertex represents the interaction of the field with the fermions, which are represented by the internal lines. The loop labelled 1 represents a virtual electron-positron pair created at space-time point $x_1 = (ct_1, \mathbf{x}_1)$ and annihilated at $x_2 = (ct_2, \mathbf{x}_2)$. The loop labelled 2 represents creation of a muon-antimuon pair and so on. The sum over single virtual pairs in the linear approximation, where Π is A -independent, is

$$\Pi(k^2) = \sum_{\mathfrak{s}}^{\text{e.p.}} \Pi_\mathfrak{s}(k^2). \quad (5)$$

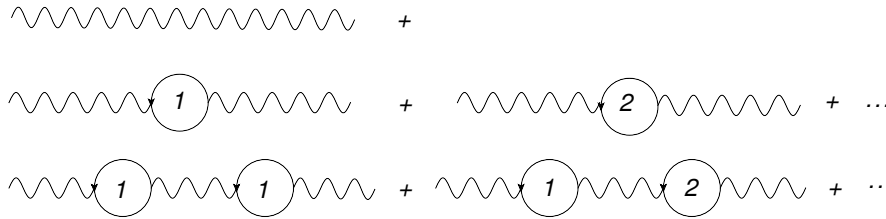


Figure 1. Vacuum polarization in the one-loop approximation. All types of virtual pairs in Nature (labeled as 1, 2, ...) polarize the vacuum. This polarization is maximal for a free electromagnetic field for which $\omega = |\mathbf{k}|c$.

In the on-shell counterterm

$$Z_3 - 1 = \Pi(0). \quad (6)$$

The vacuum polarization $\Pi_s(k^2)$ is a divergent sum over fermion momenta and naive introduction of a cut-off leads to a physically unreasonable result (the photon mass is infinite) [26,28]. Since observations are made near $k^2 = 0$, the vacuum polarization relative to its on-shell value,

$$\hat{\Pi}(k^2) \equiv \Pi(k^2) - \Pi(0) \quad (7)$$

is calculated instead. Writing Z_3 as $1 + (Z_3 - 1)$ to separate it into fully polarized and polarization-reduction terms and integrating $j_V^\mu A_\mu$ by parts to get its $E^2 - c^2 B^2$ form [27], the Maxwell Lagrangian density becomes

$$\mathcal{L}_{\text{Maxwell}} = \frac{1}{2} \epsilon_0(k^2) (\mathbf{E}^2 - c^2 \mathbf{B}^2), \quad (8)$$

where

$$\epsilon_0(k^2) = \epsilon_0 [1 + \hat{\Pi}(k^2)], \quad (9)$$

47 and we have used $\mu_0^{-1} = c^2 \epsilon_0$.

To second order in perturbation theory, the vacuum polarization relative to its maximum values at $k^2 = 0$ can be approximated by

$$\hat{\Pi}_2(k^2) = -\frac{6}{12\pi^2 \hbar c} \sum_s q_s^2 \int_0^1 dx x(1-x) \ln \left[1 - \frac{\hbar^2 k^2}{m_s^2 c^2} x(1-x) \right] \simeq -\frac{1}{12\pi^2 \hbar c} \sum_s q_s^2 \ln \left(\frac{\hbar^2 k^2}{Am_s^2 c^2} \right), \quad (10)$$

48 where the rightmost equation is valid when $\hbar^2 |k^2| \gg m_s^2 c^2$.

49 In classical electromagnetism $\mathbf{D} = \epsilon_0 \mathbf{E}$. This relationship is maintained here except that, with running,
50 $\epsilon_0(k^2) \leq \epsilon_0$. The linear response of the vacuum is then described in reciprocal k space by

$$\mathbf{D}(k) = \epsilon_0(k^2) \mathbf{E}(k), \quad \mathbf{H}(k) = c^2 \epsilon_0(k^2) \mathbf{B}(k). \quad (11)$$

51 The Maxwell equations in vacuum can be derived from the Lagrangian (2), following the standard method. They
52 take the usual form in k -space:

$$i\mathbf{k} \cdot \mathbf{D}(k) = \rho(k), \quad i\mathbf{k} \times \mathbf{H}(k) + i\frac{\omega}{c} \mathbf{D}(k) = \mathbf{j}(k). \quad (12)$$

The photon two-point correlation function is found by solving (4) for A_V [27]. Using Eq. (9), it can be written as

$$iD_F^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{\epsilon_0(k^2)(k^2 + i\eta)}. \quad (13)$$

This D_F is the response to a δ -function source in real space and hence to a constant driving force in k -space. Equation (13) is then a Green function satisfying Maxwell's wave equation. In the Lorenz gauge, and in k -space this is

$$c^2 \epsilon_0(k^2) k^2 A^\mu(k) = -j^\mu \quad (14)$$

where j^μ is k independent. The matter-field coupling constant is $\alpha = e^2 / (4\pi\hbar c\epsilon_0)$. Here, we hold e constant and incorporate running into $\epsilon_0(k^2)$, so that

$$\alpha(k^2) = \frac{e^2}{4\pi\hbar\epsilon_0(k^2)}. \quad (15)$$

53 Since $\epsilon_0(k^2)^{-1}$ contains all powers of e^2 , it incorporates summation over all numbers of pairs as sketched in
54 Fig. 1 and used in the calculation of (13). When restricted to an energy scale \mathcal{E}_{\max} , the sum is over all fermions
55 of mass less than \mathcal{E}_{\max}/c^2 [29–31]. Running of $e^2/\epsilon_0(k^2)$ is in most ways equivalent to running of the square of
56 effective charge in conventional QED, but the physical interpretation is different. In a dielectric it is possible to
57 have $\epsilon_0 < 0$, but $e_{\text{eff}}^2 < 0$ makes no physical sense.

58 The dielectric properties of vacuum differ from those of a material medium in two important ways: $\ln(k^2)$
59 dependence replaces the usual ω dependence and Lorentz invariance requires that $\epsilon_0(k^2)\mu_0(k^2) = 1/c^2$. The
60 speed c is a universal constant² whereas the coupling constant $\alpha(k^2)$ runs. On the photon mass shell $k^2 = 0$,
61 so a free photon always sees ϵ_0 and there is no running. Both the polarization \mathbf{D} and the magnetization \mathbf{H} are
62 nonzero; however, $\mathbf{j}_v = 0$ as it must for propagation in free space. A free electromagnetic wave polarizes and
63 magnetizes the vacuum but the polarization current exactly cancels the magnetization current³.

64 3. Connection with an oscillator model

65 An individual loop in Fig. 1 is analogous to a single polarizable atom with center of mass momentum
66 $\hbar k$. If, for simplicity, we set $k = 0$, the computation of the Feynmann diagrams involve integrals of the form
67 $\int d^4q [q^2 + (\frac{m_s c}{\hbar})^2]^{-2}$, which entail an exponential decay, $\exp[-(m_s c/\hbar)|\mathbf{x}|]$, in real space. Therefore, the
68 "radius" of a virtual atom is of order $\hbar/m_s c$. All in all, this suggests that the virtual pairs can be modelled as
69 oscillating dipoles with frequency mc^2/\hbar and volume of order $(\hbar/m_s c)^3$.

70 When a virtual pair is created by a photon with frequency ω its excess energy is, according the Heisenberg
71 uncertainty principle, $\Delta E \geq 2mc^2 - \hbar\omega$, so it can exist only for a time $t_2 - t_1 \leq \hbar/\Delta E$.⁴ Similar restrictions
72 apply to the distance that the virtual particles can travel.

Based on the uncertainty principle and this simple oscillator model, one can readily find that the permittivity
of vacuum can be expressed as [12,13]

$$\epsilon_0 = f \frac{1}{\hbar c} \sum_s^{\text{e.p.}} q_s^2, \quad (16)$$

73 where f is a geometrical factor of order unity, q_s is charge and the sum is over all elementary charged particles.

This is consistent with QED, as discussed in the previous Section. Indeed, at large k^2 and to second order in
perturbation theory as in Eq. (10), we have that

$$\epsilon_0(k) \simeq \epsilon_0 - \frac{1}{12\pi^2\hbar c} \sum_s^{\text{e.p.}} q_s^2 \ln\left(\frac{\hbar^2 k^2}{m_s^2 c^2}\right). \quad (17)$$

We know that at high momentum (or energy) scale, the coupling constant $\alpha(k)$ in QED becomes infinity [34,35].
If Λ is the value of that momentum (which is usually called the Landau pole [36]), then

$$\epsilon_0 = \frac{1}{12\pi^2\hbar c} \sum_s^{\text{e.p.}} q_s^2 \ln\left(\frac{\hbar^2 \Lambda^2}{m_s^2 c^2}\right), \quad (18)$$

² Interestingly, even without imposing this requirement and when using the oscillator model in next section to independently calculate μ_0 ,
the resulting speed of light is independent of how many types of elementary particles contribute and agrees with the limiting speed in
Lorentz's equations [13].

³ This bears some resemblance with the interference between the emitted electric and magnetic dipole waves leading to forward scattering
only and no back scattering, the property of Huygens waves which typically appear in materials with comparably strong electric and
magnetic interaction, which holds for the vacuum [32] and was recently rediscovered for metamaterials [33].

⁴ While a real pair cannot be created by absorbing a photon due to simultaneous conservation of energy and momentum, this restriction
does not apply to the ephemeral creation of virtual pairs.

and hence the fudge factor f in the oscillator model is given by

$$f = \frac{1}{12\pi^2} \frac{\sum_s^{\text{e. p.}} q_s^2 \ln\left(\frac{\hbar^2 \Lambda^2}{m_s^2 c^2}\right)}{\sum_s^{\text{e. p.}} q_s^2}. \quad (19)$$

74 For the standard model $\sum q_s^2/e^2 = 9$ and with two additional charged Higgs particles of $mc^2 \simeq 5 \times 10^{11}$ eV, $f \simeq 1$
75 and all of ϵ_0 is vacuum polarization if $\log(\hbar\Lambda/c) = 35$.

If running is neglected, the on-shell Maxwell equations in a medium with charges and current sources are

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0, \quad \nabla \times \mathbf{E}(\mathbf{r}, t) + \partial \mathbf{B}(\mathbf{r}, t) / \partial t = 0, \quad (20)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t), \quad \nabla \times \mathbf{H}(\mathbf{r}, t) - \partial \mathbf{D}(\mathbf{r}, t) / \partial t = \mathbf{j}(\mathbf{r}, t). \quad (21)$$

76 as in classical electromagnetic theory. Equations (20) allow us to define the four-vector potential that drives the
77 creation of the pairs. Equations (21) simply say that charge density is the divergence of polarization and the
78 current is the sum of its polarization and magnetization parts.

79 Zeldovich considered a Lagrangian in which all of the electromagnetic term comes from the interaction
80 of the particle with the field and came to an interesting conclusion [37]: In the absence of vacuum polarization
81 electric and magnetic fields act on Dirac fermions, but there is no field energy and no electromagnetic wave
82 propagation. In an absolute void it makes no sense to talk about Maxwell's equations or light propagation. Only
83 after vacuum polarization is introduced does the effective Lagrangian give the ME, EM waves travelling at the
84 speed of light and photons.

85 4. Gottfried-Weisskopf dielectric model

86 Gottfried and Weisskopf [38] introduced a toy model to understand the physical mechanisms involved in
87 vacuum polarization. In this simple picture, one assumes that the bare charge e_0 is uniformly distributed in a
88 small sphere of radius a [$a \ll \hbar/(m_e c)$]. When this charge distribution is surrounded by a spherical shell of inner
89 radius $r > a$, filled with a dielectric medium permittivity ϵ , an induced charge of opposite sign appears on the
90 inner surface of the shell at radius r , canceling part of the charge e_0 for an observer at a distance larger than r .
91 Obviously, at the outer surface of the dielectric medium, an equal charge of opposite sign appears. However, if
92 the charge is measured from within the medium it will appear to be reduced, precisely by the permittivity ϵ [39].

93 When one considers the vacuum, one has to take into account that it extends to infinity. Now, the
94 permittivity depends on the distance r to the charge. This is so because at r only those virtual pairs having
95 Compton wavelengths $\lambda \lesssim r$ contribute. We can interpret his model from a QED viewpoint. To this end, we
96 define the relationship between \mathbf{D} and \mathbf{E} at charge separations $r \gg \hbar/(m_e c)$ where the coupling strength can be
97 measured; that is, $\epsilon_0(r \gg \hbar/(m_e c)) \equiv \epsilon_0$ at large distances or, equivalently, small momenta, where the vacuum
98 is maximally polarized. With polarization included the dielectric permittivity is ϵ_0 at the physical scale. In
99 consequence, $\hat{\Pi}$ is the reduction in polarization.

Since $\epsilon_0(k^2)$ is not a constant, the exact relationship between \mathbf{D} and \mathbf{E} is nonlocal in r -space. It is not, in
general, correct to write the potential as $\Phi = e/4\pi\epsilon_0(r)r$. The simplest example in which running coupling can
be expressed as an explicit local function of r is a static charge, say $+e$. In the Coulomb gauge $\mathbf{E}(\mathbf{k}^2) = -\mathbf{k}\Phi(\mathbf{k}^2)$
and $\omega = 0$. Equation (12), with $k^2 = -\mathbf{k}^2$, then gives

$$\Phi(\mathbf{k}^2) = \frac{e}{\mathbf{k}^2 \epsilon_0(\mathbf{k}^2)}, \quad (22)$$

which in \mathbf{r} -space reads

$$\Phi(r) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{e}{\mathbf{k}^2 \epsilon_0(\mathbf{k}^2)} \exp(i\mathbf{k} \cdot \mathbf{r}), \quad (23)$$

100 equivalent to Eq. (7.93) in [27].

For electron-positron pairs alone the magnitude of $\hat{\Pi}$ is very much less than 1 so that $\epsilon_0^{-1}(k^2)$ is approximately $[1 + \hat{\Pi}(k^2)]/\epsilon_0$ and [40]

$$\Phi(r) \simeq \frac{e}{4\pi\epsilon_0 r} \times \begin{cases} 1 + \frac{2\alpha}{3\pi} \ln\left(\frac{\hbar}{m_e c r}\right) - \gamma - \frac{5}{6} & r \ll \hbar/(m_e c), \\ 1 + \frac{\alpha}{4\sqrt{\pi}} \frac{e^{-2m_e c r/\hbar}}{(m_e c r/\hbar)^{3/2}} & r \gg \hbar/(m_e c), \end{cases} \quad (24)$$

101 where m_e is the mass of the electron, r is the distance from the fixed charge and $\gamma = 0.577$ is Euler's constant.
102 The dielectric constant ϵ_0 decreases with increasing k^2 or decreasing r . For $r < \hbar/(m_e c)$ the Coulomb interaction
103 becomes stronger as the charges approach each other.

104 5. Conclusion

105 We have verified that QED vacuum is a polarizable medium with dielectric constant $\epsilon_0(k^2) \leq \epsilon_0$. Lorentz
106 invariance requires $\mu_0(k^2)\epsilon_0(k^2) = 1/c^2$, where the speed of light is a universal constant, which is also implied
107 by the dielectric/diamagnetic model of the vacuum [13]. For a free photon $k^2 = 0$ so there is no change in $\epsilon_0(k^2)$
108 with photon energy; it does not run. Since $E = B/c \neq 0$ for any free EM wave it will polarize and magnetize the
109 vacuum but their contributions to the electric current cancel.

110 This dielectric model predicts no new observable results but it suggests a paradigm shift in our physical
111 picture of the vacuum. Any electromagnetic field creates virtual pairs of all charged elementary particle types in
112 Nature. At the scale of classical electromagnetism and quantum optics $\epsilon_0 = \epsilon_0(0)$ and the vacuum is maximally
113 polarized.

114 GL acknowledges financial support by the Ministry of Education and Science of the Russian Federation
115 megagrant No. 14.W03.31.0032. LLSS acknowledges financial support from Spanish MINECO (Grants
116 FIS2015-67963-P and PGC2018-099183-B-I00).

117

- 118 1. Borchers, H.J.; Haag, R.; Schroer, B. The vacuum state in quantum field theory. *Il Nuovo Cimento* **1963**, *29*, 148–162.
119 doi:10.1007/BF02750157.
- 120 2. Sciama, D.W., *The Philosophy of Vacuum*; Clarendon: Oxford, 1991; chapter The Physical Significance of the
121 Vacuum State of a Quantum Field, pp. 137–158.
- 122 3. Milonni, P.W., *The Quantum Vacuum*; Academic Press: San Diego, 1994.
- 123 4. Lamb, W.E.; Retherford, R.C. Fine Structure of the Hydrogen Atom by a Microwave Method. *Phys. Rev.* **1947**,
124 *72*, 241–243. doi:10.1103/PhysRev.72.241.
- 125 5. Moore, G.T. Quantum Theory of the Electromagnetic Field in a Variable-Length One-Dimensional Cavity. *J. Math.*
126 *Phys.* **1970**, *11*, 2679–2691. doi:10.1063/1.1665432.
- 127 6. Unruh, W.G. Notes on black-hole evaporation. *Phys. Rev. D* **1976**, *14*, 870–892. doi:10.1103/PhysRevD.14.870.
- 128 7. Hawking, S.W. Particle creation by black holes. *Commun. Math. Phys.* **1975**, *43*, 199–220. doi:10.1007/BF02345020.
- 129 8. Furry, W.H.; Oppenheimer, J.R. On the Theory of the Electron and Positive. *Phys. Rev.* **1934**, *45*, 245–262.
130 doi:10.1103/PhysRev.45.245.
- 131 9. Pauli, W.; Weisskopf, V. Über die Quantisierung der skalaren relativistischen Wellengleichung. *Helv. Phys. Acta*
132 **1934**, *7*, 709–731.
- 133 10. Weisskopf, V.F. Über die Elektrodynamik des Vakuums auf Grund der Quanten-Theorie des Elektrons. *Kgl. Danske*
134 *Videnskab Selskab Mat.-Fys. Medd.* **1936**, *14*, 1–39.
- 135 11. Dicke, R.H. Gravitation without a Principle of Equivalence. *Rev. Mod. Phys.* **1957**, *29*, 363–376.
136 doi:10.1103/RevModPhys.29.363.
- 137 12. Leuchs, G.; Villar, A.S.; Sánchez-Soto, L.L. The quantum vacuum at the foundations of classical electrodynamics.
138 *Appl. Phys. B* **2010**, *100*, 9–13. doi:10.1007/s00340-010-4069-8.

- 139 13. Leuchs, G.; Sánchez-Soto, L.L. A sum rule for charged elementary particles. *Eur. Phys. J. D* **2013**, *67*, 57.
140 doi:10.1140/epjd/e2013-30577-8.
- 141 14. Urban, M.; Couchot, F.; Sarazin, X.; Djannati-Atai, A. The quantum vacuum as the origin of the speed of light. *Eur.*
142 *Phys. J. D* **2013**, *67*, 58. doi:10.1140/epjd/e2013-30578-7.
- 143 15. AlMosallami, A. Time Contraction: The Possibility of Faster Than Light without Violation of Lorentz Transformation
144 or Causality and the Vacuum Energy Dependent. *Int. J. Modern Theo. Phys.* **2014**, *3*, 44–73.
- 145 16. Shivalingaswamy, T.; Rashmi, P. I am the speed of light c , you 'see'! *EJPE* **2017**, *5*, 51–58.
- 146 17. Mainland, G.B.; Mulligan, B. Theoretical calculation of the fine-structure constant and the permittivity of the vacuum.
147 *arXiv:1705.11068* **2017**.
- 148 18. Braun, D.; Schneiter, F.; Fischer, U.R. Intrinsic measurement errors for the speed of light in vacuum. *Class. Quantum*
149 *Gravity* **2017**, *34*, 175009. doi:10.1088/1361-6382/aa8058.
- 150 19. Zhu, Y. Gravitational-magnetic-electric field interaction. *Results Phys.* **2018**, *10*, 794–798.
151 doi:https://doi.org/10.1016/j.rinp.2018.07.029.
- 152 20. Razmi, H.; Baramzadeh, N.; Baramzadeh, H. Dispersive property of the quantum vacuum and the speed of light.
153 *Mod. Phys. Lett. A* **2019**, *34*, 1950035. doi:10.1142/S0217732319500354.
- 154 21. Mainland, G.B.; Mulligan, B. How vacuum fluctuations determine the properties of the vacuum. *J. Phys.: Conf.*
155 *Series* **2019**, *1239*, 012016. doi:10.1088/1742-6596/1239/1/012016.
- 156 22. Udem, T.; Maisenbacher, L.; Matveev, A.; Andreev, V.; Grinin, A.; Beyer, A.; Kolachevsky, N.; Pohl, R.; Yost,
157 D.C.; Hänsch, T.W. Quantum Interference Line Shifts of Broad Dipole-Allowed Transitions. *Ann. Phys.* **2019**,
158 *531*, 1900044. doi:10.1002/andp.201900044.
- 159 23. Ruark, A.E. Positronium. *Phys. Rev.* **1945**, *68*, 278–278. doi:10.1103/PhysRev.68.278.
- 160 24. Wheeler, J.A. Polyelectrons. *Ann. N.Y. Acad. Sci.* **1946**, *48*, 219–238. doi:10.1111/j.1749-6632.1946.tb31764.x.
- 161 25. Wilczek, F. *The Lightness of Being*; Basic Books: New York, 2008.
- 162 26. Itzykson, C.; Zuber, J.B. *Quantum Field Theory*; Dover: New York, 1980.
- 163 27. Peskin, M.E.; Schroeder, D.V. *An Introduction to Quantum Field Theory*; CRC Press: Boca Raton, 2018.
- 164 28. Bogoliubov, N.N.; Shirkov, D.V. *The Theory of Quantized Fields*; Interscience: New York, 1959.
- 165 29. Eidelman, S.; Jegerlehner, F. Hadronic contributions to $(g-2)$ of the leptons and to the effective fine structure constant.
166 *Z. Phys. C* **1995**, *67*, 585–601. doi:10.1007/BF01553984.
- 167 30. Hogan, C.J. Why the universe is just so? *Rev. Mod. Phys.* **2000**, *72*, 1149–1161. doi:10.1103/RevModPhys.72.1149.
- 168 31. Hoecker, A. The hadronic contribution to the muon anomalous magnetic moment and to the running electromagnetic
169 fine structure constant at M_Z —overview and latest results. *Nucl. Phys. B Proc. Suppl.* **2011**, *218*, 189–200.
170 doi:https://doi.org/10.1016/j.nuclphysbps.2011.06.031.
- 171 32. Kerker, M.; Wang, D.S.; Giles, C.L. Electromagnetic scattering by magnetic spheres. *J. Opt. Soc. Am.* **1983**,
172 *73*, 765–767. doi:10.1364/JOSA.73.000765.
- 173 33. Decker, M.; Staude, I. Resonant dielectric nanostructures: a low-loss platform for functional nanophotonics. *J. Opt.*
174 **2016**, *18*, 103001. doi:10.1088/2040-8978/18/10/103001.
- 175 34. Gell-Mann, M.; Low, F.E. Quantum Electrodynamics at Small Distances. *Phys. Rev.* **1954**, *95*, 1300–1312.
176 doi:10.1103/PhysRev.95.1300.
- 177 35. Leuchs, G.; Hawton, M.; Sánchez-Soto, L.L. Quantum field theory and classical optics: determining the fine structure
178 constant. *J. Phys.: Conf. Ser.* **2017**, *793*, 012017.
- 179 36. Landau, L.D.; Abrikosov, A.A.; Khalatnikov, I.M. On the removal of infinities in quantum electrodynamics. *Dokl.*
180 *Akad. Nauk SSSR* **1954**, *95*, 497–502.
- 181 37. Zel'dovich, Y.B. Interpretation of Electrodynamics as a Consequence of Quantum Theory. *JEPT Lett.* **1967**,
182 *6*, 345–347.
- 183 38. Gottfried, K.; Weisskopf, V.F. *Concepts of Particle Physics*; Vol. II, Oxford University Press, 1986.
- 184 39. Ashcroft, N.V.; Mermin, N.D. *Solid State Physics*; Hartcourt: Fort Worth, 1976.
- 185 40. Landau, L.D.; Lifshitz, E.M.; Pitaevskii, L.P. *Relativistic Quantum Theory (part I)*; Vol. 4, *Course of Theoretical*
186 *Physics*, Pergamon: London, 1973.