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Estimation of Performance Parameters of Turbine engine Components Using Experimental Data in Parametric Uncertainty Conditions

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Abstract: Gas Path Analysis and matching turbine engine models to experimental data are inverse problems of mathematical modelling which are characterized by parametric uncertainty. This results from the fact that the number of measured parameters is significantly lower than the number of components' performance parameters needed to describe the real engine. In these conditions, even small measurement errors can result in a high variation of results, and obtained efficiency, loss factors etc. can appear out of the physical range. The current methods of engine model identification have developed considerably to provide stable, precise and physically adequate solutions. Presented in this work is an estimation method of engine components' parameters based on multi-criteria identification which provides stable estimations of parameters and their confidence intervals with the known measurement errors. A priori information about the engine, its parameters and performance is used directly in the regularised identification procedure. The mathematical basis for this approach is the fuzzy sets theory. Forming objective functions and scalar convolutions synthesis of these functions is used to estimate gas-path components' parameters. A comparison of the proposed approach with traditional methods showed that its main advantage is high stability of estimation in the parametric uncertainty conditions. Regularization reduces scattering, excludes incorrect solutions which do not correspond to a priori assumptions, and also helps to implement the Gas Path Analysis at the limited number of measured parameters. The method can be used for matching thermodynamic models to experimental data, Gas Path Analysis and also adapting dynamic models for the needs of the engine control system.

Keywords: gas turbine engine; performance model; gas path analysis; robust estimation; identification; fuzzy set; membership function

1. Introduction

Mathematical models of gas turbine engines, which are based on thermo-gas-dynamic process description, are widely used in the design of the engine and its automatic control and diagnostic systems. These models calculate the gas path parameters such as gas temperature and pressure, speeds of rotors, fuel consumption, thrust, etc. as a function of a set steady-state or transient operating mode and ambient conditions (atmospheric temperature and pressure, flight speed and altitude). These relations are non-linear. The considered models decrease material expenses and man-hours at the design stage and, sometimes, give new information about the object's behaviour.

The models are based on the performance of various engine components (compressor, turbine, main combustion chamber, afterburner, intake, exhaust system, transition channels, secondary air system, etc.). The performance of these components is known with some degree of confidence. For each engine, they have their own individual deviations because of differences in manufacturing and assembling. Additionally, the performance of these components degrades in operation because of wear and failures. These changes are simulated by special parameters (component performance

parameters or engine state parameters) that are included in thermo-gas-dynamic models. They serve to shift the initial (average engine) component performances, thus taking into account the current technical condition of the engine.

The models are subjected to a fitting procedure when the experimentally measured parameters are matched to similar simulated parameters. The identification adjusts the model to make its output parameters as close as possible to the experimental ones. Besides the significant improvement in the gas path simulation, the identification has a significant diagnostic value because the estimated parameters contain information about the health of each component.

The non-linear thermo-gas-dynamic models of turbine engines and the identification procedures have been applied in diagnostics for more than 40 years. Nowadays, due to the progress in computing capability, there are no limits for the direct implementation of these models and identification procedures in the real-time diagnostic systems of engines.

The model calculates parameters of the engine gas path \vec{Y} at steady-state modes depending on a mode, external conditions \vec{U} and component performance $\vec{\Theta}$. Hence, in the general case, it is represented as

$$\vec{Y} = F(\vec{U}, \vec{\Theta}) \quad (1)$$

The linear model may be formulated as

$$\delta\vec{Y} = H\delta\vec{\Theta} \quad (2)$$

which relates small deviations of the gas path parameters $\delta\vec{Y}$ and parameters of components' performances $\delta\vec{\Theta}$ at a single operating point. H is an influence coefficient matrix. The linear model (2) is a component of the identification algorithm for the non-linear model.

The problem of the identification plays a vital role as the mathematical models are used in the engine design and development. A. Tunakov solved it using the Least Squares Method (LSM) [1]. His approach can be considered as a universal one because it uses the information at some off-design modes of the engine operation. S. Yepifanov applied correction of the component performance, which provides not only deviation but also angular deviation of these performance parameters at a multi-mode identification. In fact, this corresponds to variable correction coefficients [2, 3]. Similar methods were used by A. Stamatis, K. Matioudakis et al. [4, 5] and developed by C. Kong, Y. Li, P. Pilidis et al. [6, 7, 8], B. Roth, D. Doel [9], Li, Y. et al. [10, 11], V. Zaccaria et al. [12] and many other researchers. They are based on a minimization of a functional, which is a sum of squared deviations, calculated as differences of calculated and measured values.

The task of the engine mathematical model matching with experimental data is characterized by the presence of multiple parameters which can be used for the model correction. These parameters can be strongly correlated. At the same time, a quantity of measured parameters is strongly limited. These reasons decrease the stability of the correction procedure, which is based on the Least Squares Method, and force researchers to look for methods to improve the stability. For this reason, V. Borovick and Ye. Taran applied the Least Modulus Method [13], which is more robust for the data outliers. S. Yepifanov implemented the Marquardt's method [14] as well as Singular Value Decomposition and ε -structuration [15, 16]. A. Volponi et al. applied the Kalman filter [17] and this approach is followed and developed in many papers, for example [18-22], which authors improved stability of the algorithm and its applicability to a non-linear engine model. In publications [23-25], X. Chang et. al. applied an alternative method based on the non-linear filtration (sliding mode observer).

Stability of all the above-mentioned procedures is provided due to the regularization. It is well-known that any regularization reduces the modification of the above-mentioned functional by adding the regularizing component to it. This causes biased estimates of the model coefficients. B. Roth, D. Doel et al. demonstrated this clearly [26]. Hence, matching engine models needs regularization.

Breikin et al presented regularisation-based approach to the model coefficients estimation which was based on the regularity of the frequency response pattern over the operation range of the engine [27] but that method is not universal. The Marquardt's method of regularization implemented here was also used by Guseynov et al. [28, 29] but they did not analyse the bias nor consider how the regularization coefficient influences the bias and the noise which could impede practical applications. The estimated errors of the model parameters which are due to the bias need to be monitored but formal methods of regularization essentially limit the possibility of this.

This paper presents the regularization of the matching task using a priori information about the engine, its mathematical model and expected performance, and also about the measuring system and the measuring procedure. This information is heterogeneous: partially, it is presented in form of statistic parameters and partially in form of heuristic expert representations. The unified instrument that helps to formalize this information and complete the mathematical formulation of the considered task is the Fuzzy Sets Theory [30]. Due to this modification of the functional, the identification task becomes non-linear. Therefore, traditional methods of its solution are not applicable. This paper considers the solution based on the genetic algorithm [31], which is adapted to specifics of the engine model matching [32]. The proposed approach enables engine users to implement the Gas Path Analysis at the limited number of measured parameters [33, 34].

2. Materials and Methods

2.1 Basic identification procedure

Identification of the non-linear model (1) by measuring results \bar{Y}^* and \bar{U}^* requires determining estimations $\hat{\Theta}$, which are solutions of the following optimization task:

$$\Phi(\bar{\Theta}) = \left\| \bar{Y}^* - Y(\bar{U}^*, \bar{\Theta}) \right\|, \quad \Phi(\hat{\Theta}) = \min \Phi(\bar{\Theta}) \quad (3)$$

where $\| \cdot \|$ is the norm of the vector.

The estimation precision will improve if a priori information is redundant in relation to the number of unknown parameters. So, the identification can be provided by a set of measurements that are done at K different operating modes. In this case, the generalized vector of residuals is minimized

$$\bar{Z}(\bar{\Theta}) = \begin{bmatrix} \bar{Y}_1^* - Y(\bar{U}_1^*, \bar{\Theta}) \\ \bar{Y}_2^* - Y(\bar{U}_2^*, \bar{\Theta}) \\ \dots\dots\dots \\ \bar{Y}_N^* - Y(\bar{U}_N^*, \bar{\Theta}) \end{bmatrix} \quad (4)$$

The model (1) which is included in (4) is numerical. Therefore, the minimization of residuals (4) is a numerical iterative procedure, which in each iteration determines the solution as a sum of the previous solution and current correction:

$$\bar{\Theta}^{n+1} = \bar{\Theta}^n + \Delta\bar{\Theta}^{n+1} \quad (5)$$

and iterations continue till corrections become negligible.

The correction $\Delta\bar{\Theta}^{n+1}$ is determined as a solution of the overdetermined system of linear algebraic equations:

$$C(\bar{\Theta}^n)\Delta\bar{\Theta}^{n+1} = Z(\bar{\Theta}^n) \quad (6)$$

where

$$C(\bar{\Theta}^n) = \begin{bmatrix} H_1(\bar{\Theta}^n) \\ H_2(\bar{\Theta}^n) \\ \dots \\ H_N(\bar{\Theta}^n) \end{bmatrix} \quad (7)$$

is a generalized matrix of influence coefficients, which is composed of elementary matrixes $H_i(\bar{\Theta}^n)$ determined for each operating mode. In accordance with equations (6), for each iteration, such correction $\Delta\bar{\Theta}^{n+1}$ of required parameters is found, which narrows residuals (4) $\bar{Z}(\bar{\Theta})$ down.

The known solution of the linear system (6) by the least squares method takes the form:

$$\Delta\bar{\Theta}^{n+1} = A^{-1}C^T G \bar{Z} \quad (8)$$

where $A = C^T G C$ – Fisher information matrix, G – weight diagonal matrix, whose elements are inverse to dispersions of measuring errors $\sigma_{y_i}^2$.

Unfortunately, the least squares method is sensitive to outliers in the right part of the system (6), which can be related to faults in experimental data. So practical application of the LSM had demonstrated the instability of estimations $\Delta\bar{\Theta}^{n+1}$ and poor convergence of the identification algorithm as a whole. In some cases, the estimations $\hat{\Theta}$ are far from the expected values. The reasons for such results may be lack of empirical information, an excess number of estimated parameters or correlation between two or more state parameters. This causes ill-conditioning of the Fisher Matrix and excessive deviations of estimations. These estimations lose a physical sense (are out of range of possible components' performances parameters variation – for example, efficiency is more than one) and can cause the model calculation program to crash.

Hence, the LSM identification procedure needs modification for providing its stability and physically adequate estimations.

2.2. Regularized identification procedure

In the proposed procedure, the generalized functional is minimized, which besides the residuals by measured parameters includes a norm of the finding parameter vector with a weighting factor α . The identification task takes the form

$$\Phi'(\bar{\Theta}) = \left\{ \left\| \bar{Y}^* - Y(\bar{U}^*, \bar{\Theta}) \right\| + \alpha \left\| \bar{\Theta} \right\| \right\}, \quad \Phi'(\hat{\Theta}) = \min \Phi'(\bar{\Theta}) \quad (9)$$

The new identification procedure is based on the procedure described above. As it is understood from relation T, the minimized functional is extended by elements which are related with parameters $\bar{\Theta}$ to be found. So the main changes in the identification procedure are an extension of the generalized vector of residuals $\bar{Z}(\bar{\Theta})$ and generalized matrix of influence coefficients $C(\bar{\Theta})$. The modified values have the following structure:

$$\bar{Z}'(\bar{\Theta}) = \begin{bmatrix} \bar{Z}(\bar{\Theta}) \\ \alpha \bar{\Theta} \end{bmatrix} \quad C'(\bar{\Theta}) = \begin{bmatrix} C(\bar{\Theta}) \\ \alpha I \end{bmatrix} \quad (10)$$

where I is a unity matrix.

It is understood that the influence of coefficient γ will depend on proportions between components $\left\| \bar{Y}^* - Y(\bar{U}^*, \bar{\Theta}) \right\|$ and $\left\| \bar{\Theta} \right\|$ of the generalized functional $\Phi'(\bar{\Theta})$, hence on conditions of identification: number of measured parameters m , number of operating points N , measuring errors G and number of estimated parameters r .

The proposed identification procedure will provide the possibility to obtain the former non-regularized solution if the regularization coefficient is zero.

Like any new algorithm, the regularized identification procedure needs careful testing. Therefore, we performed two stages of testing using the turbofan engine model: a) initial checking on model information without measuring noise; b) random testing on model information with measuring noise simulation.

Calculations were done for different values of α , which are varied in a range from 0 to 1; corresponding diagrams are shown in Figure 1, which represent a variety of estimations $\hat{\Theta}$ and simulation results $\hat{Y} = F(\bar{U}, \hat{\Theta})$. In Fig.1, ΔY_{av} is the average deviation between initial and estimated models:

$$\Delta Y_{av} = \sqrt{\frac{1}{mN} \sum_{j=1}^N \sum_{i=1}^m [y_i(\bar{U}_j, \hat{\Theta}) - y_i(\bar{U}_j, \bar{\Theta} = 0)]^2} \quad (11)$$

and ΔY_{av}^* is the average deviation between measurements and estimated model:

$$\Delta Y_{av}^* = \sqrt{\frac{1}{mN} \sum_{j=1}^N \sum_{i=1}^m [y_i(\bar{U}_j, \hat{\Theta}) - y_{ij}^*]^2} \quad (12)$$

The following conditions were used:

- 12 operating points defined by fuel flow;
- Simulated changes in the gas path: $\Theta_{1 \text{ sim}} = \Delta W_{\text{HPC}} = -0.03$ (high pressure compressor (HPC) performance deviation by a flow rate); $\Theta_{2 \text{ sim}} = \Delta \eta_{\text{HPC}} = -0.04$ (HPC performance deviation by efficiency);
- Five gas path parameters Y_i , which are initial data for identification of: HPC discharge pressure, high-pressure turbine (HPT) discharge temperature, low-pressure turbine (LPT) discharge temperature, rotational speeds of both rotors;
- Four estimated parameters, including the two above-mentioned simulated parameters $\Theta_1 = \Delta W_{\text{HPC}}$, $\Theta_2 = \Delta \eta_{\text{HPC}}$, and $\Theta_3 = \Delta W_{\text{HPT}}$ – deviation of HPT performance by flow, $\Theta_4 = \Delta \eta_{\text{HPT}}$ – deviation of HPT performance by efficiency.

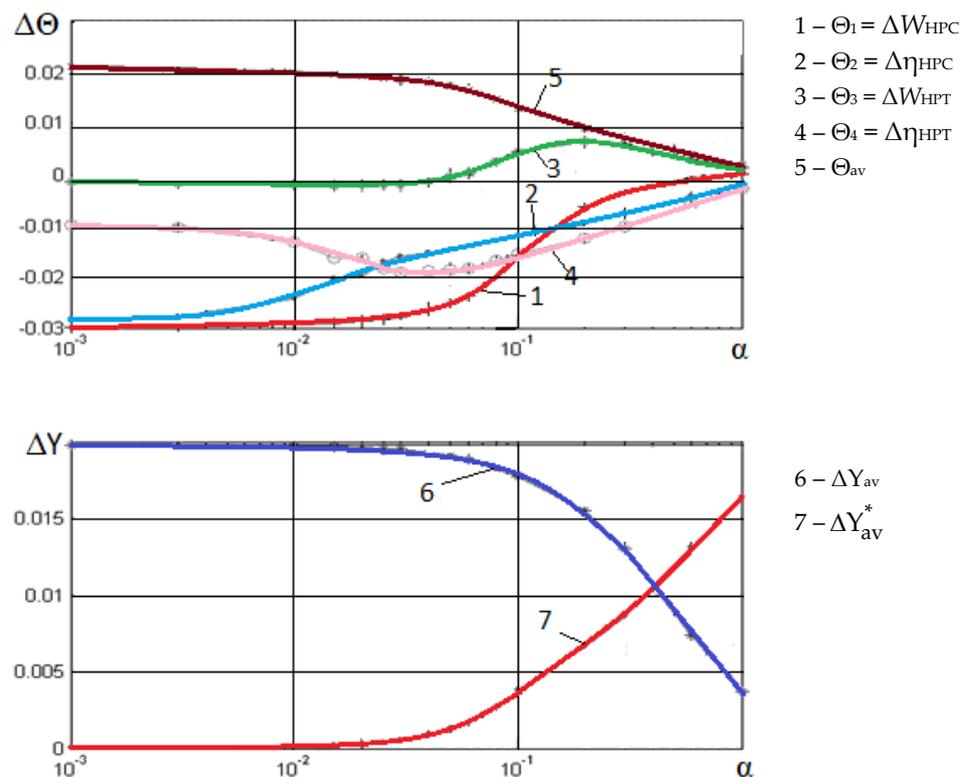
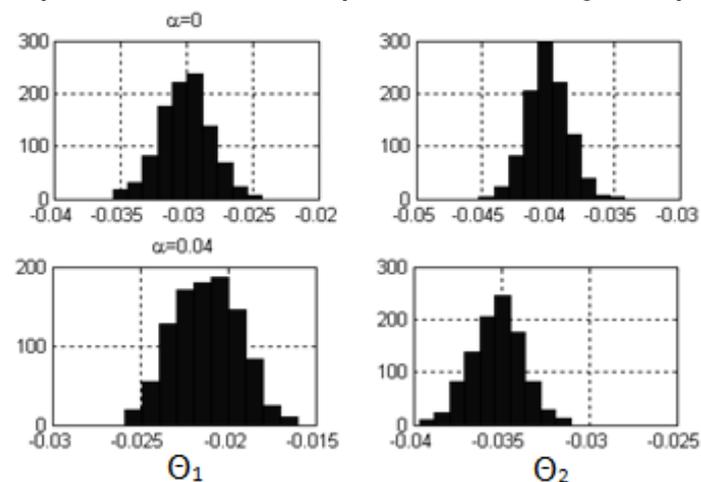
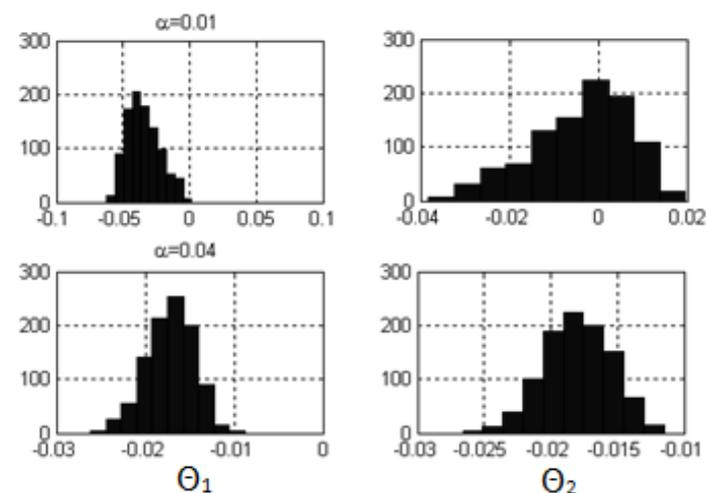


Figure 1. Influence of the regularization coefficient on estimations when noise is absent

Figure 1 shows that, despite the initial growth of Θ_3 and Θ_4 absolute values, the average value Θ_{av} decreases monotonously and the residual between measurements and the corrected model increases. This corresponds to the theoretical concept of the influence of regularization on the identification process. The diagrams in Figure 1 allow for estimating the range of the regularization coefficient. If the deviation of parameters \bar{Y} must be no more than 5%, and the deviation of parameters Θ – 1%, then the value of α must not exceed 0.03.

To obtain precise values of dispersions and to analyse the character of estimations $\hat{\Theta}$ distribution, a cycle of random measuring errors setting with dispersions $\sigma_{Y_i}^2$ and identification was repeated 1000 times. This provided average precision of about 1% of initial residuals for parameters Y and about 0.5% of simulated deviation 0.03 for estimated parameters $\hat{\Theta}$.

Figure 2 represents distributions of errors for calculations with two estimated parameters at $\alpha = 0$ and $\alpha = 0.04$. It is seen that both non-regularized and regularized estimations have distributions close to normal ones. Total scatter of estimations is also saved, but centres of regularized estimations are displaced essentially from their true values by -0.03 and -0.04, respectively.

**Figure 2.** Distributions of estimations when two parameters are estimated**Figure 3.** Distributions of estimations when four parameters are estimated

Estimations $\hat{\Theta}_1$ and $\hat{\Theta}_3$ preserved the abovementioned properties of estimation with two unknown parameters; at the same time estimations $\hat{\Theta}_2$ and $\hat{\Theta}_4$ have significant differences, which are illustrated in Figure 3. At low regularization, centres of distributions are close to true values, but scatter is high, and distributions are strongly non-symmetrical. When α increases then the mean values displace, scatter decreases and distribution shape approaches a normal one. Such abnormal behaviour of parameters Θ_2 and Θ_4 is explained by their correlation.

2.3. Regularized identification procedure development using a priori information

The above-represented regularization procedure is formal. The values of the regularization coefficient have to be set appropriately; therefore, this procedure needs preliminary adjustment and checking the results. The searching for the sources to improve the precision and stability led to an idea to implement a theoretical information about the engine, its parameters and performances. The main difficulty is in the diversity of this information, which is presented in one of the following forms:

- exact statement (for example, a part-load performance in a determined area is smooth);
- statement in the form of limitations of the area of acceptable solutions (for example, the efficiency of the individual compressor cannot differ for more than 3% from the efficiency of an “average” compressor, the performance of which is used in the initial model);
- statement in the form of fuzzy information (for example, the gas temperature in the turbine will rather grow with the engine life);
- statistical form (for example, probability density functions of parameters).

The next difficulty is in formalization of parameters that characterize the model quality. These parameters are set on the basis of subjective preferences of decision makers (DM). The same difficulties appear in the ranking of partial criteria and limitations according to their significance for the model quality estimation. The analysis showed that the main problem is that theoretical-probabilistic methods are difficult to apply for operation with uncertainties, which are related to subjective preferences that nature is not statistical. Actually, the choice of the model's structure is the DM procedure, which in multi-criteria case inevitably contains elements of subjectivity. If the complexity of the task increases, the role of quality factors will grow. Therefore, it is possible to take into account all criteria using the proper mathematical tool.

We propose to use the fuzzy sets theory [20] as this tool. This theory makes the uniform base for the description of information, which is given in all the forms listed above, thus providing the correct mathematical definition of the identification task.

An example of applying the fuzzy set in the GPA and engine models matching is given by M. Zwingenberg et al. [29]. They used fuzzy logic for the evaluation of sensor failures. In contrast, we introduce a priori information about engine performance and experimental data directly into the stabilizing functional through the fuzzy logic approach.

Generalization of the functional (9) gives

$$\Phi(\bar{\Theta}) = \Phi_e(\bar{\Theta}) + \Phi_a(\bar{\Theta}) \quad (13)$$

where $\Phi_e(\bar{\Theta})$ is functional (3), which is considered as the functional of empirical risk; $\Phi_a(\bar{\Theta})$ is stabilizing functional, which is considered as the functional of a priori risk.

The determined a priori information is set as a limitation, which may take the form of an equality or inequality. The more general form of setting a priori information is its representation as a fuzzy set. The fuzzy set of parameter x is represented as a definition domain and a membership function $\mu(x)$ in this domain. For the considered task, the parameters x may be the model parameters $\bar{\Theta}$ or the output (calculated) variables \bar{X} .

Next, we will use limited normal fuzzy sets with limited definition domain (x_{min}, x_{max}) and $0 \leq \mu(x) \leq 1$. We will express each a priori information as a particular functional of a priori risk

$\Phi_{a_q}(\bar{\theta})$, and will determine the general functional of a priori risk as a linear composition of particular functionals:

$$\Phi_a(\bar{\theta}) = \sum_{q=1}^Q \alpha_q \Phi_{a_q}(\bar{\theta}) \quad (14)$$

where α_q are weight coefficients.

Let us consider some types of a priori information and its expression in view of fuzzy sets.

Case 1) Limitations of some parameters are known. For example, it is known that $0 < \eta < 1$ and $0 < \sigma < 1$, etc. Using experience and calculation results, these limits can be significantly reduced; for example, $0.5 < \eta < 0.9$ and $0.9 < \sigma < 0.99$ (Figure 4).

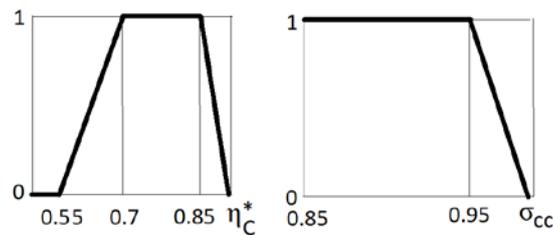


Figure 4. Examples of membership functions

Case 2) The a priori mathematical model is known. This can be the model with design maps of components or the model of the average engine, which is matched with previous testing results.

This information may be expressed as a relationship between μ_a or Φ_a and the difference between parameters that correspond to the matched and a priori models. These parameters may be the model parameters $\bar{\Theta}$ as measured (for example fuel flow) or non-measured calculated parameters (for example thrust). The membership function, in this case, can be of symmetrical triangular shape and the functional of a priori risk $\Phi_{a_q}(\bar{\theta})$ that characterizes the similarity between values of the parameter x_q of matched and a priori models can be formed as

$$\Phi_{a_q}(\bar{\Theta}) = \sum_{j=1}^N \mu_q(\Delta x_{qj}) \cdot (\Delta x_{qj})^2 \quad \Delta x_{qj} = x_q(\bar{U}_j, \hat{\Theta}_j) - x_{q0}(\bar{U}_j, \bar{\Theta}_0) \quad (15)$$

where x_q is calculated parameter of the engine; $x_q(\bar{U}_j, \hat{\Theta}_j)$ – the value calculated by the model to be matched; $x_{q0}(\bar{U}_j, \bar{\Theta}_0)$ – the value calculated by the a priori model; $\bar{\Theta}_0$ – parameters of the a priori model.

Case 3) Information about the confidence in different sets of experimental data obtained in different conditions with a different precision.

Case 4) Confidence in the available maps of the engine components. For example, we know that the compressor map used in the model corresponds to the old version of the engine and is far from the actual map. Thus, we can express this knowledge in view of the confidence functions that are in this case the membership functions.

The empirical risk functional $\Phi_e(\bar{\theta})$ in a form (3) may be considered as a square of Euclidian distance between two sets, one of which contains experimental data and the other is composed of simulation results.

$$\Phi_e(\bar{\theta}) = \frac{1}{mN} \sum_{i=1}^m g_i \sum_{j=1}^N \left[y_i(\bar{U}_j, \bar{\theta}) - y_{ij}^* \right]^2 \quad (16)$$

By analogy, the stabilizing functional can be formed as a second power of a distance from estimated parameter θ_k (or characteristic $x(\bar{\Theta})$ that is determined using the estimated parameters).

The main problem in this analogy is that the fuzzy set that contains the a priori information is infinite so the sum in the above equation must be replaced with integral. For example, the second power of a distance from a value of the engine characteristic parameter $\theta=x$ to the fuzzy set with a given membership function $\mu(\theta)$ is

$$\Phi_a(\bar{\theta}) = \frac{\int_{-\infty}^{\infty} \mu(\theta)(x-\theta)^2 d\theta}{\int_{-\infty}^{\infty} \mu(\theta) d\theta} \quad (17)$$

In the iteration process of the task solution, each integral must be calculated numerically. This requires a lot of time and high computational capacity. The computational problems can be solved for a limited shape of the membership function. We considered the trapezoidal function, the partial cases of which are uniform, triangle and isosceles trapezium (Figure 5).

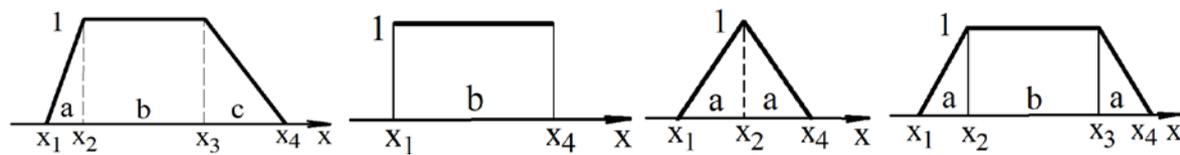


Figure 5. Considered membership functions

Equation derived for the functional in this partial case:

$$\begin{aligned} \Phi_{a_q}(\bar{\theta}) = & X_q^2 - \left\{ X_q \left[x_1(a+2b+c) + \frac{1}{3}(2a^2+3b^2+c^2+6ab+3ac+3bc) \right] + \right. \\ & \left. + \frac{ax_1^2}{2} + \frac{2}{3}a^2x_1 + \frac{a^3}{4} + b \left(x_1^2 + 2ax_1 + bx_1 + a^2 + ab + \frac{b^2}{3} \right) \right\} + \\ & + \frac{c}{12} \left[6x_1^2 + (12a+12b+4c)x_1 + 6a^2 + 6b^2 + 12ab + 4ac + 4bc + c^2 \right] \Big/ \left(\frac{a}{2} + b + \frac{c}{2} \right). \end{aligned} \quad (18)$$

3. Results and Discussion

The designed methods were implemented using experimental data obtained during the test-cell testing of the helicopter turboshaft engine (Figure 6). Table 1 contains the values of measured parameters. The last row represents mean squared measurement errors σ_i , which were used to form the diagonal weight matrix G (Equation 8).

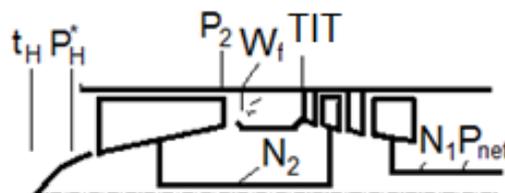


Figure 6. Scheme and measured parameters of the engine

Table 1. Parameters acquired during bench testing

Operating mode	t_{H_2} , °C	$P^*_{H_2}$, Pa	N_2 , %	N_1 , %	P_{net} , kW	W_f , kg/h	TIT, K	CPR
1st cruise	12.0	100192	94.96	94.58	1223	372.5	1017	8.05
2nd cruise	11.6	100192	97.05	96.87	1521	439.0	1073	8.91
Nominal	12.3	100192	98.67	98.51	1729	482.1	1106.5	9.41
Maximum	11.9	100192	102.34	102.02	2203	596.4	1197	10.54
σ_i , %	0.4	0.3	0.085	0.085	0.4	0.3	0.4	0.3

The model parameters to be corrected:

$\pi_C^*(c)$ – CPR (Compressor Pressure Ratio) scaling factor;

$\pi_C^*(a)$ – CPR turning in CPR- W_{Ccor} plane factor;

$\pi_C^*(b)$ – CPR turning in CPR- N_{cor} plane factor;

$W_{HPT}(c)$ – HPT flow coefficient;

$W_{PT}(c)$ – power turbine flow coefficient;

$\eta_C^*(a)$ – compressor efficiency characteristics turning in $\eta_C^*-W_{Ccor}$ plane factor;

$\eta_C^*(c)$ – compressor efficiency characteristics scaling factor;

η_{CC} – combustion efficiency scaling factor.

The initial values of model parameters $\bar{\Theta}_0$ that correspond to the average engine (a priori information) are shown in Table 2.

Table 2. A priori values of the model parameters

$\pi_{HPC}^*(a)$	$\pi_{HPC}^*(b)$	$\pi_{HPC}^*(c)$	$W_{HPT}(c)$	$\eta_{HPC}^*(a)$	$\eta_{HPC}^*(c)$	$W_{HPT}(c)$	$W_{PT}(c)$	$\eta_{PT}^*(c)$	η_{CC}
-0.0788	0.0125	0.023	0.0464	0.0383	0.0548	0.0265	-0.0938	0.01	0.02

In the first step of the iteration procedure, the following corrections to component maps were determined:

$$\delta\pi_C^*(a) = 342\%; \delta W_{HPT} = 0.93\%; \delta W_{PT} = 0.85\%; \delta\eta_C^*(a) = 26\%; \delta\eta_C^*(c) = 1.4\% \quad (19)$$

Such large corrections make it impossible to calculate part-load performance and the matrixes of influence coefficients for the next iteration. Therefore, the calculation procedure was modified: corrections to the component maps were limited. At corrections as high as 7%, the procedure slowly becomes convergent (by 7-9 iterations). The solution corresponds to small (less than 1%) corrections to the component maps.

The results of the 'classic' LSM application show that this method has low stability due to a weak conditionality of the engine model. Therefore, practical application requires improvement of the method's stability.

The same experimental data were processed by the regularized method based on Equation 9. Parameter α was set at 0.01. Table 3 shows corrections $\delta\bar{\Theta}^s$ for the first three iterations while Table 4 presents the parameters calculated by the corrected model.

Table 3. Corrections for three first iterations

Iteration	$\delta\pi_C^*(a)$	δW_{HPT}	δW_{PT}	$\delta\eta_C^*(a)$	$\delta\eta_C^*(c)$
1	0.004	0.17	0.39	-0.033	0.33
2	0.003	0.18	0.15	-0.018	0.08
3	0.003	0.09	0.08	-0.016	0.02

Table 4. Values of measured parameters determined by the corrected model

Operating point	t_{tr} , °C	P^*_{H} , Pa	N_2 , %	N_1 , %	P_{net} , kW	W_f , kg/h	TIT, K	CPR
1st cruise	12.0	100192	92.38	98.42	1223	374.2	1050.3	7.48
2nd cruise	11.6	100192	94.44	98.20	1521	439.8	1105.7	8.30
Nominal	12.3	100192	96.00	98.10	1729	490.1	1146.5	8.84
Maximum	11.9	100192	98.86	98.12	2203	601.0	1224.2	10.18

Table 3 shows the high stability of the procedure: estimations change slowly, without significant oscillations. The changes become smaller from iteration to iteration (this is a sign of stability). Comparison of Tables 4 and 1 shows that parameters determined by the corrected model are well correlated with the measured parameters. Thus, the developed procedure of component performance estimation and the engine model matching can be implemented in practice.

Finally, the above task of multi-mode diagnostics was considered with a priori information about the model of the “average” engine, the parameters of which were estimated by previous testing data and are shown in Table 2. It is also known that a scatter of measured parameters of different engines at $P_{net} = \text{const}$, $N_1 = \text{const}$ follows the normal distribution with the following mean squared error (Table 5):

Table 5. Mean squared error of performance parameters in the engine population

Parameter	N_2	W_f	T_3	π_c
Mean squared error	0.85 %	0.7 %	1.1 %	0.6 %

This information was then transformed into the functional Φ_a (Equation 13), and taking into account that in this case, the functionals Φ_e and Φ_a are homogeneous as they contain residuals by parameters of the same names. This facilitated the setting of weight coefficients in Equation 14. They had to relate to a scatter of measurements and a scatter of the engine parameters by series. This provided the composition of the functionals:

$$\Phi(\bar{\Theta}) = \Phi_e(\bar{\Theta}) + \frac{\sigma_{\text{meas av}}^2}{\sigma_0^2} \Phi_a(\bar{\Theta}) \quad (20)$$

In the considered example $\frac{\sigma_{\text{meas av}}^2}{\sigma_0^2} \approx 8$

Table 6 shows the estimated parameters of the model in subsequent iterations. The final results of the corrected model are presented in Table 7. The comparison of Tables 7, 4 and 1 confirms that the proposed procedure is stable. For the corrected model, deviations of output parameters from experimental data are within the measurement error. The introduction of the a priori information results in a smaller deviation of results compared to the conventional regularization method.

Table 6. Corrections for three first iterations

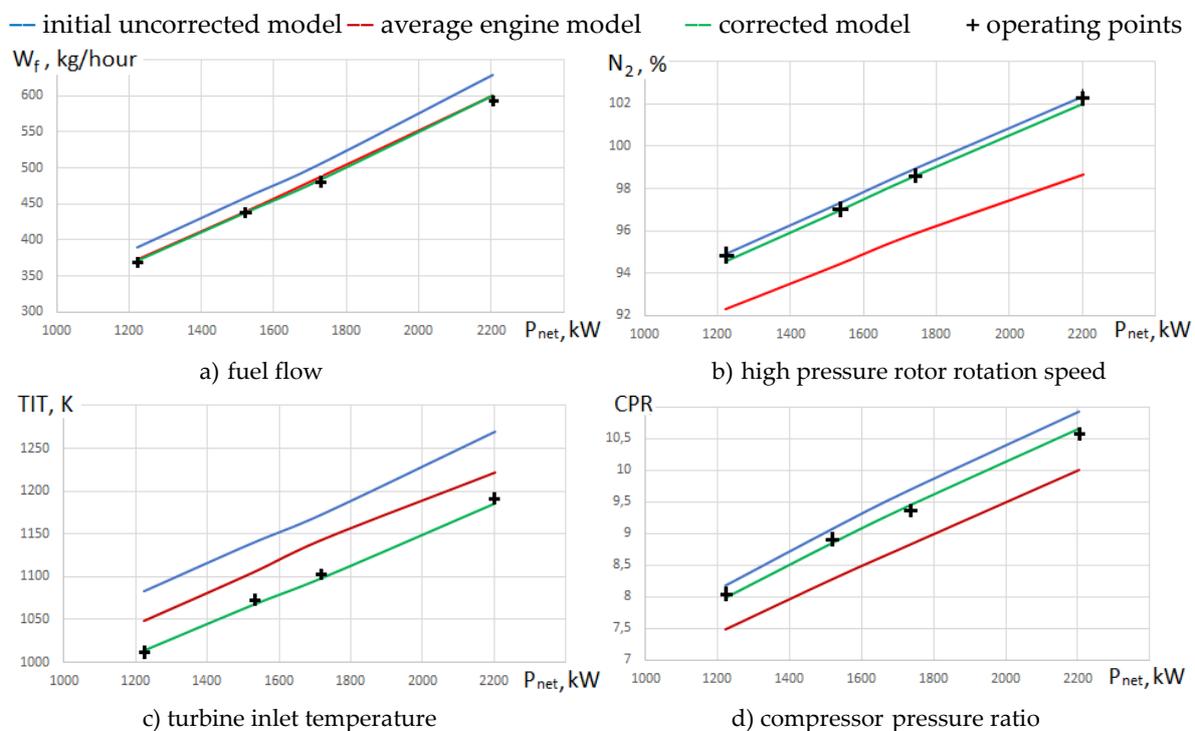
Iteration	$\delta\pi_C^*(a)$	δW_{HPT}	δW_{PT}	$\delta\eta_C^*(a)$	$\delta\eta_C^*(a)$
1	0.005	0.15	0.27	-0.04	0.28
2	0.003	0.12	0.13	-0.02	0.06
3	0.002	0.07	0.06	-0.015	0.02

Table 7. Values of measured parameters determined by the corrected model

Operating point	t_{H_1} , °C	P_{H_1} , Pa	N_2 , %	N_1 , %	P_{net} , kW	W_f , kg/h	TIT, K	CPR
1st cruise	12.0	100192	94.58	94.58	1223	371.7	1013	7.98
2nd cruise	11.6	100192	96.87	96.87	1521	438.2	1066	8.85
Nominal	12.3	100192	98.51	98.51	1729	484.0	1100.5	9.43
Maximum	11.9	100192	102.02	102.02	2203	600.2	1186	10.65

Figure 7 presents the experimental data and modelling results for part-load performance. As the initial model is far enough from the field data, the diagnostic algorithm needs a preliminary adjustment to make the model closer to an "average" engine and to improve the GPA precision. The visible lines correspond to the initial model before the correction, the average engine model and corrected model of the analyzed engine. The deviations of performance parameters from the reference indicate the engine's health status.

Figure 8 illustrates the correction of compressor maps and confirms that the procedure was effective even though the initial model was inaccurate.

**Figure 7.** Experimental data vs models

5. Conclusions

In this paper, a new method for the stable estimation of engine performance parameters is proposed which uses a priori information about the engine and data acquisition system i.e. the expected performance and acceptable variations of parameters. The method aims to improve the accuracy of gas-turbine performance models at off-design conditions. The a priori information is introduced in the procedure in view of fuzzy sets, which are transformed into additional stabilization components of the functional to be minimized.

The integral form of the stabilization functional was proposed which operates with continuous membership functions. To accelerate numerical calculations, the trapezoidal membership function is recommended. The analytical equation is derived for the stabilization functional with this function, which can be used in place of numerical integration, thus decreasing the computing capacity required for the estimation procedure.

Application of the developed approach to field data taken from helicopter turboshaft engine and comparison of this approach with the conventional Least Squares Method and formal regularization method has proved the following:

LSM has low stability that can cause a crash of the engine simulation software.

The formal regularization method is stable but intensive bias of the estimates needs to be excluded in advance.

The proposed method is stable and provides a small bias of the estimates aside a priori information. As this information has a physical sense, it improves the stability and precision of the estimation.

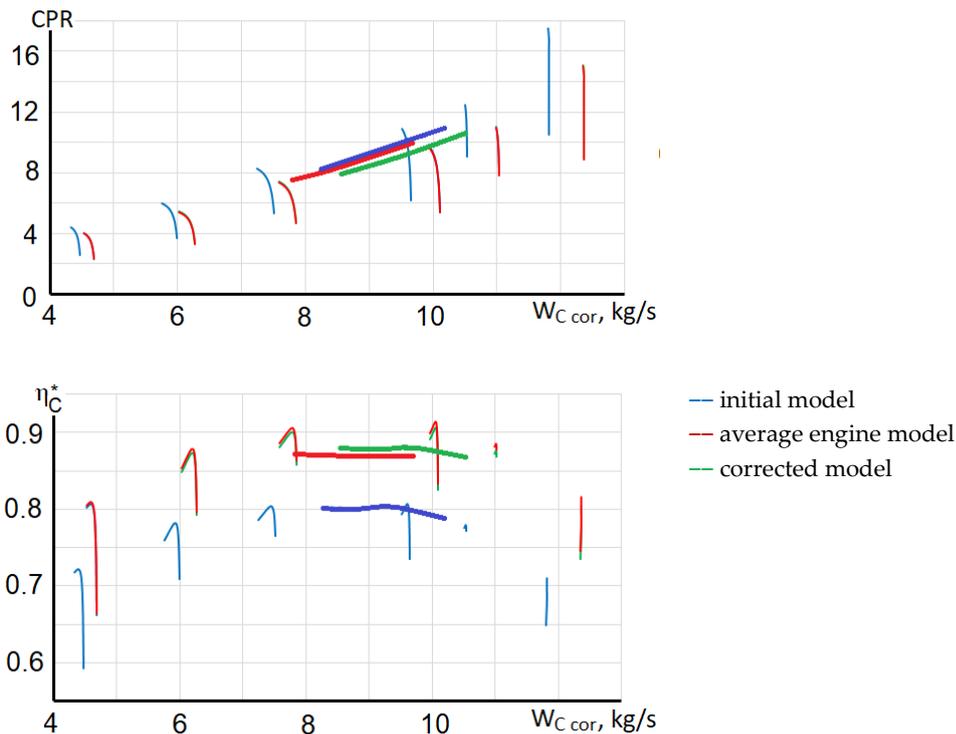


Figure 8. Compressor maps with the operating lines

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Nomenclature

The following abbreviations and symbols are used in this manuscript:

av	average value
C	generalized matrix of influence coefficients compressor
CC	Combustion Chamber
CPR	Compressor Pressure Ratio
G	weight diagonal matrix
GPA	Gas Path Analysis
H	matrix of influence coefficients

HPC	High Pressure Compressor
HPT	High Pressure Turbine
I	unity matrix
N	number of operating points, rotational speed
LPT	Low Pressure Turbine
LSM	Least Squares Method
m	number of measured parameters
OP	Operating Point
P	pressure
P_{net}	net thrust
r	number of estimated parameters
sim	simulated value
TIT	Turbine Inlet Temperature
U	parameters that determine the engine operating conditions
W	air/gas flow
W_f	fuel flow
Y	measured parameters
Z	generalized vector of measured parameters
α	regularization (weighting) factor
Δ	parameter's correction
δ	relative deviation
η	efficiency
σ^2	dispersion
π	pressure ratio
σ	pressure loss coefficient
θ	component's map parameter
\rightarrow	vector
\wedge	estimated parameter
*	measured value

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