

# Hypothetical role of large nuclear gravity in understanding the significance and applications of the strong coupling constant in the light of Up and Down quark clusters

U.V.S. Seshavatharam<sup>1</sup> and S. Lakshminarayana<sup>2</sup>

<sup>1</sup>Honorary faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-84, Telangana, INDIA

<sup>2</sup>Dept. of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, INDIA

Emails: seshavatharam.uvs@gmail.com (and) Insrirama@gmail.com

Orcid numbers : 0000-0002-1695-6037 (and) 0000-0002-8923-772X

**Abstract:** As there exist no repulsive forces in strong interaction, in a hypothetical approach, strong interaction can be assumed to be equivalent to a large gravitational coupling. Based on this concept, strong coupling constant can be defined as a ratio of the electromagnetic force and the gravitational force associated with proton, neutron, up quark and down quark. With respect to the product of strong coupling constant and fine structure ratio, we review our recently proposed two semi empirical relations and coefficients 0.00189 and 0.00642 connected with nuclear stability and binding energy. We wish to emphasize that- by classifying nucleons as ‘free nucleons’ and ‘active nucleons’, nuclear binding energy can be fitted with a new class of ‘three term’ formula having one unique energy coefficient. Based on the geometry and quantum nature, currently believed harmonic oscillator and spin orbit magic numbers can be considered as the lower and upper “mass limits” of quark clusters.

**Key words:** Large nuclear gravity, Strong coupling constant; nuclear stability; nuclear binding energy; Free nucleons; Active nucleons; Magic numbers; Up and Down Quark clusters;

## 1. Introduction

With reference to nuclear stability and binding energy, relationship between nuclear force and strong force is still a grey area and is a challenging task for field experts and young scientists [1,2]. It is well established that,

- 1) Less than 0.8 fm, strong force is mediated by gluons.
- 2) At a range of 1 to 3 fm, strong force is mediated by mesons.
- 3) Neutrons, protons, baryons and mesons are made up of quarks.
- 4) Gluons interact with quarks and other gluons and mesons interact with neutrons and protons.
- 5) Strong force that binds quarks into neutrons, and protons can be called as ‘residual strong force’ or simply ‘nuclear force’.
- 6) Within the quark surroundings, force is strong and distance independent.
- 7) Nuclear force is weaker and rapidly decreases with increasing distance among nucleons (bound quarks).
- 8) Even though nuclear force is weaker than the strong force, still it is very energetic in producing gamma rays and holding nucleons with large binding energy.

- 9) Strength of strong interaction is parameterized by strong coupling constant,  $\alpha_s \cong 0.1181$  [3].

In this context, in this paper, with reference to earlier published concept of large nuclear gravitational coupling [5,6,7,8] and our recent paper [9] and references therein, we make an attempt to understand the physical significance and applications of strong coupling constant with respect to nuclear stability and binding energy.

## 2. To understand the physical significance of strong coupling constant

Strong coupling constant ( $\alpha_s$ ) can be defined as a ratio of the electromagnetic force and the gravitational force associated with proton, neutron, up quark and down quark. Mathematically, it can be represented as

$$\alpha_s \cong \frac{e^2}{4\pi\epsilon_0} \left[ \frac{[G_s m_p (2m_u + m_d)]}{+[G_s m_n (m_u + 2m_d)]} \right]^{-1} \quad (1)$$

where  $G_s$  = large nuclear gravitational constant,  
 $(m_p, m_n)$  = Proton and neutron masses and  
 $(m_u, m_d)$  = Up and down quark masses.

In our earlier published papers, we proposed that [4],  
 $G_s \cong 3.329561 \times 10^{28} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$ . With reference to  
particle data group [3],

$$\left. \begin{aligned} m_u &\cong 2.16_{-0.26}^{+0.49} \text{ MeV} \\ m_d &\cong 4.67_{-0.17}^{+0.48} \text{ MeV} \end{aligned} \right\}$$

Based on these values, estimated  $\alpha_s \cong 0.11333$   
and its recommended world average  $\alpha_s \cong 0.1181$ .  
By refining the magnitudes of up and down quark  
masses and the large nuclear gravitational constant,  
absolute value of  $\alpha_s$  can be fixed. Conceptually, it  
seems better to understand that,

$$\frac{e^2}{4\pi\epsilon_0} \cong \alpha_s \left\{ \begin{aligned} &[G_s m_p (2m_u + m_d)] \\ &+ [G_s m_n (m_u + 2m_d)] \end{aligned} \right\} \quad (2)$$

### 3. Two new coefficients and their applications

In our earlier publications, in a quantum gravitational  
approach [9], we have introduced two coefficients  
0.00189 and 0.006423 pertaining to nuclear stability  
and binding energy.

#### A) Coefficient connected with Stability

We noticed that,

- 1) As proton number increases, at stability zone,  
neutron number increases with square of the  
proton number.
- 2) Proportionality coefficient seems to be close to  
a number 0.0064 [5,6,7]. Quantitatively it can  
be fitted with a relation of the form,

$$\begin{aligned} k &\cong \alpha - \alpha\alpha_s \cong (1 - \alpha_s)\alpha \cong 0.00644 \\ \text{where } \alpha_s &\cong 0.1181 \end{aligned} \quad (3)$$

Based on the coefficient,  $k \cong 0.00644$ , neutron  
number close to stability zone can be expressed as,

$$\left. \begin{aligned} N_s &\cong Z + kZ^2 \\ N_s - Z &\cong kZ^2 \end{aligned} \right\} \quad (4)$$

where,  $Z$  = Proton number

$N_s$  = Neutron number close to stability

In terms of nucleon number,

$$\left. \begin{aligned} A_s &\cong 2Z + kZ^2 \\ A_s - 2Z &\cong kZ^2 \end{aligned} \right\} \quad (5)$$

where

$A_s$  = Nucleon number close to stability

$(N_s - Z) \cong (A_s - 2Z) \cong \Delta N_s$

= Excess Neutron number close to stability zone

With this kind of relation, by guessing the proton  
number, corresponding stable zone nucleon number  
can be estimated directly. With even-odd corrections  
and fine tuning the value of  $k$ , better understanding is  
possible. Considering  $k \cong 0.00644$  and by  
considering a simple quadratic equation, relation (5)  
can be derived.

$$\left. \begin{aligned} \text{Let, } X &= \frac{Zk}{2} \\ C &= \frac{Zk}{2} = \frac{Ak}{4} \left( \because \text{Initially, } Z = \frac{A}{2} \right) \\ \text{and } X^2 + X - C &\cong 0 \\ \left\{ \begin{aligned} X^2 \text{ coefficient} &= 1 \\ X \text{ coefficient} &= 1 \end{aligned} \right\} \\ \rightarrow \frac{Zk}{2} &\cong \frac{-1 \pm \sqrt{kA+1}}{2} \end{aligned} \right\} \quad (6)$$

With reference to observational data, it seems  
reasonable to assign the proposed relation (5) with  
mean stable mass number ( $A_m$ ).

$$\boxed{(A_s)_{mean} \cong A_m \cong A_{mean} \cong 2Z + kZ^2} \quad (7)$$

Best lower and upper limits for stable and relatively  
long living mass numbers can be approximated with  
the following relation.

$$\begin{aligned} (A_s)_l &\cong 2Z + (1 - \alpha_s)^2 kZ^2 \\ A_l &\cong 2Z + (1 - \alpha_s)^2 kZ^2 \cong 2Z + (0.78kZ^2) \\ A_u &\cong 2Z + (1 + \alpha_s)^2 kZ^2 \cong 2Z + (1.25kZ^2) \end{aligned} \quad (8)$$

Using these relations as guidelines, long living isotopes of super heavy elements can be estimated.

With reference to the famous stability relation pertaining to semi empirical mass formula [10,11,12],

$$Z \cong \frac{A}{(2.0 + (a_c/2a_a)A^{2/3})} \cong \frac{A}{(2.0 + 0.0153A^{2/3})} \quad (9)$$

where  $a_c \cong 0.71$  MeV and  $a_a \cong 23.2$  MeV  
Based on the proposed quadratic relation (6) and relation(9), it is possible to show that,

$$Z \cong \frac{\sqrt{kA+1}-1}{k} \quad (10)$$

Comparing relations (9) and (10), solution seems to be a relation of the form,

$$0.0153A^{2/3} \approx \sqrt{kA+1}-1 \quad (11)$$

$$\left. \begin{aligned} 12A) \quad Z &\cong \frac{A}{(2.0 + 0.0153A^{2/3})} \\ 12B) \quad Z &\cong \frac{A}{1 + \sqrt{kA+1}} \end{aligned} \right\} \quad (12)$$

$[\sqrt{kA+1}-1]$  seems to be more appropriate than  $[0.0153A^{2/3}]$  and it needs further study. With a curiosity, we noticed that,

$$(\alpha_s^2 + \alpha_s^3 + \alpha_s^4 + \alpha_s^5 + \dots) \cong 0.0158 \approx \frac{a_c}{2a_a} \approx 0.0153 \quad (13)$$

where  $a_c \cong 0.71$  MeV and  $a_a \cong 23.2$  MeV

See Table 1 for a comparison for estimated proton number starting from A=340 to 4.

Mass number	Proton number estimated with relation (12A)	Proton number estimated with relation (12B)	Difference in estimated proton number
340	124	122	2
332	121	120	1
324	119	118	1
316	117	115	2
308	114	113	1

300	112	111	1
292	109	108	1
284	107	106	1
276	104	104	1
268	102	101	1
260	99	99	0
252	97	96	1
244	94	94	0
236	91	91	0
228	89	89	0
220	86	86	0
212	83	84	-1
204	81	81	0
196	78	78	0
188	75	76	-1
180	72	73	-1
172	70	70	0
164	67	67	0
156	64	65	-1
148	61	62	-1
140	58	59	-1
132	55	56	-1
124	52	53	-1
116	49	50	-1
108	46	47	-1
100	43	44	-1
92	40	41	-1
84	37	37	0
76	33	34	-1
68	30	31	-1
60	27	28	-1
52	23	24	-1
44	20	21	-1
36	17	17	0
28	13	13	0
20	9	10	-1
12	6	6	0
4	2	2	0

Based on the data presented in Table 1, workability of the proposed coefficient  $k \cong 0.00644$ , can be validated.

### B) Coefficient connected with free nucleons

With reference to our experience in this field, we understood that,

- 1) All the nucleons are not involving in nuclear binding energy scheme.
- 2) Nucleons that are not involving in nuclear binding energy scheme can be called as 'free nucleons'.

- 3) Number of free nucleons increases with increasing  $A\sqrt{ZN}$ .
- 4) Nucleons that involve in nuclear binding energy scheme can be called as 'active nucleons'.
- 5) In finding the free nucleon number, with trial-error solutions, we are able to come across a number close to 0.00189 [9]. Quantitatively it can be fitted with a relation of the form,

$$f \cong \left(\frac{m_d}{m_u}\right) \alpha \alpha_s \cong 0.001863 \quad (14)$$

where  $\alpha_s \cong 0.1181$

- 6) To a very good approximation, free nucleon number can be expressed with a relation of the form,

$$A_f \cong fA\sqrt{ZN} \cong 0.001863A\sqrt{ZN} \quad (15)$$

- 7) Active nucleon number can be expressed with a relation of the form,

$$A_a \cong A - A_f \cong A(1 - 0.001863\sqrt{ZN}) \quad (16)$$

- 8) By considering the integer form of

$$A_a \cong (1 - 0.001863\sqrt{ZN})A \text{ or}$$

$A_f \cong (0.001863A\sqrt{ZN})$ , to some extent, error can be minimized in the estimation of binding energy.

#### 4. Proposed nuclear binding energy scheme

We propose that, nuclear binding energy [1,2,13,14,15]

- 1) Can be understood with single energy coefficient and three simple terms.
- 2) Decreases with increasing number of free nucleons.
- 3) Increases with increasing number of active nucleons.
- 4) Decreases with increasing radius.
- 5) Of stable and unstable isotopic mass numbers can be estimated with mean stable mass number as a reference line of stability.

Based on these points, for estimating nuclear binding energy, we propose the following semi empirical relation for  $Z=3$  to 118.

$$(BE)_{(z,A)} \cong \left\{ A_a - A^{1/3} - \left( 1 + \frac{(A_m - A)^2}{A_m} \right) \right\} B_0 \quad (17)$$

$$\cong \left\{ (1 - 0.001863\sqrt{ZN})A - A^{1/3} - \left( 1 + \frac{(A_m - A)^2}{A_m} \right) \right\} 10.1 \text{ MeV}$$

$$\text{where } B_0 \cong \frac{G_s m_p (2m_u + m_d)}{R_0} + \frac{G_s m_n (m_u + 2m_d)}{R_0}$$

$$\cong \frac{3}{2} (m_u + m_d) c^2 \cong 10.245 \text{ MeV} \approx 10.1 \text{ MeV}$$

$$\text{and } R_0 \cong \frac{2G_s m_p}{c^2} \cong \frac{2G_s m_n}{c^2} \cong 1.24 \text{ fm.}$$

In this relation (17),

$$\text{First term: } +(1 - 0.001863\sqrt{ZN})A \times 10.1 \text{ MeV} \quad (18)$$

$$\text{Second term: } -A^{1/3} \times 10.1 \text{ MeV} \quad (19)$$

$$\text{Third term: } -\left( 1 + \frac{(A_m - A)^2}{A_m} \right) \times 10.1 \text{ MeV} \quad (20)$$

$\left( \frac{(A_m - A)^2}{A_m} \right)$  can be considered as a representation of 'asymmetry' about the mean line of stability. We are still working on understanding the physical significance of the third term [16,17,18] and it needs in-depth study.

Close to mean stable mass number,

$$(BE)_{(z,A_m)} \cong \left\{ \left( 1 - f \sqrt{ZN_m} \right) A_m - A_m^{1/3} - 1 \right\} 10.1 \text{ MeV}$$

$$\text{where } \begin{cases} A_m \cong (2Z + kZ^2) \text{ and} \\ N_m \cong A_m - Z \cong (Z + kZ^2) \end{cases} \quad (21)$$

See Table 2 nuclear binding energy of isotopes of Z=50 estimated with relations (7) and (17) compared with standard semi empirical mass formulae.

**Note:** To validate the relations (7) and (17), readers are encouraged to see the supplementary file for the whole range of nuclides starting from Z= 3 to 120. (Table 3)

In table 2, (Column-6) – **SEMF – 1 BE**  
 ([https://en.wikipedia.org/wiki/Semi-empirical\\_mass\\_formula](https://en.wikipedia.org/wiki/Semi-empirical_mass_formula))

$$BE \cong (a_v * A) - (a_s * A^{2/3}) - \left( a_c * \frac{Z * (Z-1)}{A^{1/3}} \right) - \left( a_a * \frac{(A-2Z)^2}{A} \right) \pm \left( \frac{a_p}{\sqrt{A}} \right) \quad (22)$$

where  $\begin{cases} a_v \cong 15.8 \text{ MeV}; a_s \cong 18.3 \text{ MeV}; a_c \cong 0.714 \text{ MeV}; \\ a_a \cong 23.2 \text{ MeV}; a_p \cong 12.0 \text{ MeV}; \end{cases}$

In table 2, (Column-7) – **SEMF – 2 BE**  
 (<http://oregonstate.edu/instruct/ch374/ch418518/lecture3-1.pdf>) (Slide-16)

$$BE \cong (a_v * x * A) - (a_s * x * A^{2/3}) - \left( a_c * \frac{Z^2}{A^{1/3}} \right) + \left( a_{pr} * \frac{Z^2}{A} \right) \pm \left( \frac{a_p}{\sqrt{A}} \right) \quad (23)$$

where  $\begin{cases} x \cong \left[ 1 - 1.79 \left( \frac{N-Z}{A} \right)^2 \right] \\ a_v \cong 15.677 \text{ MeV}; a_s \cong 18.56 \text{ MeV}; a_c \cong 0.717 \text{ MeV}; \\ a_{pr} \cong 1.211 \text{ MeV}; a_p \cong 11.0 \text{ MeV}; \end{cases}$

Table 2: Estimated binding energy of isotopes of Z= 50						
Z	A	N	Estimated A mean	Estimated BE (MeV)	SEMF-1 BE (MeV)	SEMF-2 BE (MeV)
50	100	50	116	836.6	810.1	813.0
50	101	51	116	847.4	823.1	825.7
50	102	52	116	858.0	838.0	840.2
50	103	53	116	868.4	850.1	852.1
50	104	54	116	878.6	864.1	865.7
50	105	55	116	888.6	875.4	876.9
50	106	56	116	898.5	888.6	889.8
50	107	57	116	908.1	899.1	900.2

50	108	58	116	917.6	911.5	912.4
50	109	59	116	926.9	921.2	922.1
50	110	60	116	936.0	932.8	933.6
50	111	61	116	944.9	941.9	942.7
50	112	62	116	953.7	952.8	953.6
50	113	63	116	962.3	961.1	962.0
50	114	64	116	970.6	971.4	972.3
50	115	65	116	978.8	979.1	980.1
50	116	66	116	986.9	988.8	989.8
50	117	67	116	994.7	995.9	997.1
50	118	68	116	1002.4	1004.9	1006.1
50	119	69	116	1009.8	1011.4	1012.9
50	120	70	116	1017.1	1019.9	1021.4
50	121	71	116	1024.2	1025.9	1027.7
50	122	72	116	1031.1	1033.8	1035.7
50	123	73	116	1037.9	1039.3	1041.5
50	124	74	116	1044.4	1046.7	1049.0
50	125	75	116	1050.8	1051.6	1054.3
50	126	76	116	1057.0	1058.5	1061.3
50	127	77	116	1063.0	1063.1	1066.1
50	128	78	116	1068.9	1069.5	1072.7
50	129	79	116	1074.5	1073.5	1077.1
50	130	80	116	1080.0	1079.5	1083.3
50	131	81	116	1085.3	1083.2	1087.3
50	132	82	116	1090.4	1088.7	1093.0
50	133	83	116	1095.3	1091.9	1096.6
50	134	84	116	1100.0	1097.0	1101.9
50	135	85	116	1104.6	1099.9	1105.1
50	136	86	116	1108.9	1104.6	1110.0
50	137	87	116	1113.1	1107.1	1112.9
50	138	88	116	1117.1	1111.5	1117.4
50	139	89	116	1121.0	1113.6	1119.9
50	140	90	116	1124.6	1117.6	1124.1
50	141	91	116	1128.1	1119.4	1126.3
50	142	92	116	1131.3	1123.0	1130.1
50	143	93	116	1134.4	1124.5	1132.0
50	144	94	116	1137.3	1127.8	1135.5
50	145	95	116	1140.1	1129.0	1137.0
50	146	96	116	1142.6	1131.9	1140.2
50	147	97	116	1145.0	1132.8	1141.4
50	148	98	116	1147.2	1135.5	1144.3
50	149	99	116	1149.2	1136.1	1145.2
50	150	100	116	1151.0	1138.5	1147.8
50	151	101	116	1152.7	1138.8	1148.5
50	152	102	116	1154.1	1140.9	1150.8
50	153	103	116	1155.4	1140.9	1151.1
50	154	104	116	1156.5	1142.8	1153.2
50	155	105	116	1157.4	1142.5	1153.3
50	156	106	116	1158.1	1144.1	1155.0
50	157	107	116	1158.7	1143.7	1154.9
50	158	108	116	1159.0	1145.0	1156.4
50	159	109	116	1159.2	1144.3	1156.0
50	160	110	116	1159.2	1145.4	1157.2

## 5. About magic numbers and quark clusters

Observed two series of magic numbers 2,8,20,... and 2,28,50,... can be understood with quarks in the following way. Based on the geometry [19] and quantum nature [20-28],

- 1) Nuclear volume constitutes systematically arranged quark clusters.
- 2) Currently believed harmonic oscillator and spin orbit magic numbers can be considered as the lower and upper "mass limits" of the assumed quark clusters.
- 3) Each quark shell is associated with a principal quantum number,  $n = 1, 2, 3, \dots$
- 4) As nucleon constitutes 3 quarks, number of quarks that can be accommodated in  $n^{\text{th}}$  cluster can be represented by,

$$\left. \begin{aligned} 3A_n &\cong \left[ \frac{n^3 + 2n}{3} \pm 3n \right] \\ &\cong 3 \left[ n + \sum_{n=1}^n n(n-1) \right] \pm 3n \end{aligned} \right\} \quad (24)$$

- 5) Corresponding lower and upper limits of the nucleons can be represented by,

$$\left. \begin{aligned} (A_n)_{\text{lower}} &\cong \left[ \frac{n^3 + 2n}{3} \right] - n \\ &\cong \sum_{n=1}^n n(n-1) \end{aligned} \right\} \quad (25)$$

$$\left. \begin{aligned} (A_n)_{\text{upper}} &\cong \left[ \frac{n^3 + 2n}{3} \right] + n \\ &\cong 2n + \sum_{n=1}^n n(n-1) \end{aligned} \right\} \quad (26)$$

See Table 4 for the lower, mean and upper mass limits of quark shells.

Principal quantum number	Mean mass number	Lower mass number	Upper mass number
1	$1 \pm 1$	0	2
2	$4 \pm 2$	2	6

3	$11 \pm 3$	8	14
4	$24 \pm 4$	20	28
5	$45 \pm 5$	40	50
6	$76 \pm 6$	70	82
7	$119 \pm 7$	112	126
8	$176 \pm 8$	168	184
9	$249 \pm 9$	240	258
10	$340 \pm 10$	330	350

- 6) Gap between  $n^{\text{th}}$  cluster  $(A_n)_{\text{lower}}$  and cluster  $(A_n)_{\text{upper}}$  is  $(n-1)(n-2)$ .
- 7) Based on the beta stability relations (12A,12B), based on the capacity of the quark cluster and corresponding to the lower, average and upper limits of the nucleons in the quark cluster, 2 to 4 even proton numbers can be inferred. These inferred even protons seem to have more number of isotopes and magic nature.
- 8) Inferred proton number corresponding to the average mass number of any quantum cluster seems to have more isotopes compared to minimum and maximum mass limits.
- 9) At  $n=6$ , quark cluster lower, average and upper mass limits are 70, 76 and 82 respectively and this mass range seems to be in-line with the whole isotopic mass range of  $Z=34$ . Corresponding even proton numbers seem to be  $34 \pm 2$ . See references [24, 27, 28].
- 10) At  $n=7$ , quark cluster lower, average and upper mass limits are 112, 119 and 126 respectively and this mass range seems to be in-line with the whole isotopic mass range of  $Z=50$ . Corresponding even proton numbers seem to be  $50 \pm 2$ .
- 11) At  $n=(8,9,10)$ , inferred proton numbers are  $Z \approx (70 \text{ to } 74), (92 \text{ to } 96) \text{ and } (120 \text{ to } 124)$  respectively. It needs further investigation.
- 12) At  $n=2$ , quark cluster upper mass limit is 6 and at  $n=3$ , quark cluster upper mass limit is 14. See references [21, 26].

## 6. Conclusion

With reference to the data presented in Tables 1, 2, 3 and 4, our proposed concepts and relations can be recommended for further investigation. By refining the values of  $(G_s, m_u, m_d)$ , magnitudes of  $(\alpha_s, f, k, B_0)$  can be refined.

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