

Elasticity Calculation in EKC Literature:

A Cautionary Note

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Abstract

The polynomial functional specification is widely used in environmental and ecological economics. When different powers of the same variable enter to the specification the elasticity of explained variable with respect to that variable should be obtained using the elasticity formula. Since, in this case the elasticity itself is a function, one need to calculate it at a certain point, to have a general idea about the response of explained variable to the change in explanatory variable. The mainly used point is mean of the variables entering to the elasticity formula. Sometimes, the minimum and maximum points also used for comparison purposes. One should careful in interpreting the response of explanatory variable to the change in the variable entering to the specification with its different powers. This study revisits some methodological points regarding the calculation and interpretation of responses in the above-mentioned cases.

Key words: Environmental Kuznets Curve; polynomial specification; elasticity formula; quadratic function; percentage change

Introduction

Although, some authors are against to the use of square (or cub) of the non-stationary variable in EKC literature (see for example, Müller-Fürstenberger and Wagner, 2007, inter alia), the polynomial specification still have been widely used. In addition, the polynomial specification is also used in other circumstances (see Hunt and Lynk, 1993). Some studies employed different methodologies/specifications to avoid or address the environmental degradation-economic growth nexus in a more relevant way (Galeotti, 2009; Moosa, 2015; Apergis 2016, Mikayilov et al., 2018, inter alia).

For the EKC hypothesis and its detailed discussion Shafik and Bandyopadhyay (1992) Lieb (2002, 2003), Dinda (2004, 2005), among others, are valuable papers.

The polynomial specifications, quadratic and cubic, proposed by Shafik and Bandyopadhyay (1992) can be expressed as follow:

$$\ln Y_t = \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln^2 X_t \quad (1)$$

$$\ln Y_t = \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln^2 X_t + \alpha_3 \ln^3 X_t \quad (2)$$

Where Y stands for the environmental indicator, X is income measure, α_i 's are parameters to be estimated and t stands for time (we skipped error terms for the sake of ease). Using the simple calculus it can be shown that the corresponding elasticities have the below forms:

$$E = \frac{\partial Y_t}{\partial X_t} \frac{X_t}{Y_t} = \frac{\partial \ln Y_t}{\partial \ln X_t} = \alpha_1 + 2\alpha_2 \ln X_t \quad (3)$$

$$E = \frac{\partial Y_t}{\partial X_t} \frac{X_t}{Y_t} = \frac{\partial \ln Y_t}{\partial \ln X_t} = \alpha_1 + 2\alpha_2 \ln X_t + 3\alpha_3 \ln^2 X_t \quad (4)$$

Since the right-hand sides of equations (3) and (4) are functions of X, elasticities will take different values at different points. To have an idea about the average response of dependent variable (environmental indicator) to the change in independent variable (proxy for economic development) it is a common practice to calculate elasticity at a certain point (see Gujarati and

Porter, 2009, among others). This point is mainly the mean of the variable (s) entering to elasticity formula. The minimum and maximum values of the variables also can be used for comparison purposes (Jaforullah and King, 2017, among others).

It is worth emphasizing that in the EKC literature some papers, even published in recognized journals such as Energy Economics, Energy Policy, Energy, Ecological Indicators, Renewable Energy the interpretation of elasticity, which included to the functional specification with its powers (as a polynomial), has not been provided in a relevant way. Some authors interpret the coefficient of the linear term (α_1) and coefficient of the leading term (α_2) separately, in the quadratic specification.

Even some studies report the coefficient of the linear term (in logs) as an elasticity, which changes sign and size with rescaling (see Hasanov et al., 2019 for proofs). Since, the impact of the same variable cannot be different on the dependent variable at the same time, in other word, in “stating” the impact of linear income on the dependent variable the square of income cannot be kept constant, the well-known *ceteris paribus* approach is not valid here. Hence, for interpreting the impact of income variable on the dependent variable/environmental indicator one of the measures of change (unit change, relative change or percentage change=elasticity) should be used properly. Since, in the applied work in this framework the elasticity is the widely used measure of change, the elasticity calculation should be done. For calculation of the elasticity the formulas provided above, namely (3) for quadratic specification and (4) for cubic specification should be used. In the econometrics as well as in EKC literature calculation of the elasticity at mean value of the independent variable is the widely used one (Gujarati and Porter, 2009; Onafowora and Owoye, 2014; Baek, 2015; Jaforullah and King, 2017, inter alia). Alternatively, it can be calculated at minimum and maximum values for comparison purposes as well as for the detailed information. Once, the elasticity (E) of environmental indicator with respect to income calculated, one can say that on average, 1 % change in income causes E %

change in environmental indicator. In addition, this statement can be made regardless of the sign of the calculated elasticity, which means, for example in the EKC case, before the turning point 1 % increase in income level causes E % increase in environmental indicator on average, and after the turning point 1 % decrease in income level causes E % decrease in environmental indicator on average. In the linear-logarithmic specification, as it well-known, the coefficient of the logged income variable is the elasticity, and can be interpreted directly as a measure of percentage change (for more information about elasticities of different functional forms (see for example Gujarati and Porter, 2009 pages 162-174 inter alia).

Conclusion

This study revisits the measure of the percentage change, elasticity, from some aspect, which, has been used in inadequate way in recent literature. The point is the calculation and interpretation of the elasticity when the powers of the same variable are present in a given specification. This is a case with a widely used literature related Environmental Kuznets Curve phenomenon, since in the polynomial specification squared and cubed terms are present. Hence, caution is needed in interpretation and usage of the estimated coefficients. In this regard, the use of the coefficient of the linear term (with respect to income measure) as an elasticity measure is misleading, and one should calculate the elasticity using the known formulas for elasticity discussed in literature (Gujarati and Porter, 2009, among others).

Acknowledgments

The views expressed in this paper are those of the authors and do not necessarily represent the views of their affiliated institutions.

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