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Entropy, Information and Symmetry: Ordered is Symmetrical

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Abstract: Entropy is usually understood as the quantitative measure of "chaos" or "disorder". However, the notions of "chaos" and "disorder" are definitely obscure. This leads to numerous misinterpretations of entropy. We propose to see the disorder as an absence of symmetry and to identify "ordering" with symmetrizing of a physical system; in other words, introducing the elements of symmetry into initially disordered physical system. We demonstrate with the binary system of elementary magnets that introducing elements of symmetry necessarily diminishes its entropy. This is true for 1D and 2D systems of elementary magnets. Imposing symmetry does not influence the Landauer Principle valid for the addressed systems. Imposing the symmetry restrictions onto the system built of particles contained within the chamber divided by the permeable partition also diminishes its entropy.

Keywords: Entropy; symmetry; ordering; Landauer principle.

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1. Introduction

The notion of entropy, introduced by Rudolf Clausius [1-2], is considered to be one of the important, but also most abstract and least visualisable quantities of physics [3]. Rudolf Clausius (1867) coined the term entropy from the Greek word *Entropein* for transformation and change [4]. The classical thermodynamics macroscopic definition of the entropy change within a reversible process is $\Delta S = \frac{\Delta Q_{rev}}{T}$, where ΔQ_{rev} is the heat flow which took place under the reversible process, and T is the temperature [5]. The statistical definition of the entropy is:

$$S = k_B ln\Omega, \qquad (1)$$

Where Ω is the multiplicity of a certain state, i.e. the number of different configurations that a system defined by macroscopic variables could assume (or in other words, the number of states accessible to the system) [6-8]. The widespread illustrative interpretation of entropy is "the measure of disorder" in macroscopic systems built from, a large

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particles [8]. However equating entropy with disorder was criticized recently [4, 9, 10]. Indeed, the multiplicity has a well-defined statistical meaning; whereas, "disorder" is an obscure notion, deeply rooted in the human psychology, namely in the human perception of ordered and disordered patterns [11-13]. We propose the hypotheses that "ordering" may be strictly related to the symmetry, and may be quantified by symmetry. In turn, "chaos" or disorder is an absence of symmetry. It seems, that the idea that just a symmetry is an adequate characteristics of ordering for various patterns was suggested in ref. 14 and elaborated further in refs. 15-16. We demonstrate that introducing symmetry into physical systems necessarily orders them and decreases their entropy. Symmetry considerations play a key role in modern science [17-19], giving rise to the conservation laws in physics, and being dominating in quantum theory [20], crystallography [21] and condensed-matter physics. We show that they are crucial for constituting thermodynamics and informational properties of physical systems.

2. Symmetry and entropy of binary systems

Consider first a binary 1D system illustrated in **Figure 1A**. We assume that there are N separate and distinct states fixed in a space and aligned as shown in **Figure 1A** [8]. Attached to each site is an elementary magnet, which can point only up or down. Thus, a binary system is created. The total number of arrangements of the N magnets is $\Omega = 2^N$ if no external restrictions are imposed on the system and all of the states are equally accessible to the system. Thus, the entropy of the system is given by:

$$S = k_B N ln2 \qquad (2)$$

If the addressed binary system is in the thermal equilibrium with a thermal reservoir T the modulus of the term of the Helmholtz Free Energy $|F_S|$ arising from entropy contributions is supplied by:

$$|F_S| = k_B T N ln2, \qquad (3)$$

thus, illustrating the Landauer principle [23-25]. Indeed, if erasing of one bit of information is performed under re-orientation of the single elementary magnet, the energy cost of such an erasure equals $k_B T ln2$ in a strict accordance with the Landauer Principle [23-25].

Now let us restrict the possible configurations of elementary magnets by introducing the symmetry axis, shown with the dashed line in **Figure 1B**. After introducing the symmetry axis only the symmetric configurations of the elementary magnets are available, as depicted in **Figure 1B**. The total number of arrangements of the N magnets is $\Omega = 2^{\frac{N}{2}}$. Hence, the entropy of the symmetrized, ordered binary system is given by:

$$S = k_B \frac{N}{2} ln2 \qquad (4)$$

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It is seen that introducing symmetry orders the binary system, and consequently decreases the entropy. It seems plausible to equate "ordering" with symmetrizing of the system. If this identifying is accepted, the entropy is indeed the measure of disorder, in other words, an absence of symmetry. Obviously the axis of symmetry may be replaced by the center of symmetry [17].

The modulus of the entropy related term of the Helmholtz Free Energy $|F_S|$ for the symmetrized system shown in **Figure 1B** equals $|F_S| = k_B T \frac{N}{2} ln2$. Thus, the Landauer Principle remains untouched. Indeed, in this case erasing of one bit of information is performed under simultaneous re-orientation of two symmetric elementary magnet, and the energy cost of such an erasure again equals $k_B T ln2$ in an accordance with the Landauer Principle [23-25].

Consider now the 2D system of elementary magnets depicted in **Figure 1C**. Generalization of the aforementioned approach for 2D systems is straightforward. If the system is not symmetrized (ordered) and all of the arrangement of elementary magnets are equally available, the entropy of the system is still given by $S = k_B N ln2$. If the system is symmetrized and it possesses two axes of symmetry, as shown in **Figure 1C**, its entropy is $S = k_B \frac{N}{4} ln2$. Additional elements of symmetry in this 2D case (i.e. the axes or centers of symmetry) result is the larger decrease in the entropy of system. This supports the idea that ordering (understood as symmetrizing) necessarily decreases the entropy.

Consider one more example shown in **Figure 2**. Four molecules of two different kinds (i.e. blue and red ones) are located within a chamber divided equally by the permeable partition. The particles of the same kind are considered as indistinguishable. Consider the states, within which two molecules may be located simultaneously on the one side of the partition. These states may be realized by 3 various arrangements (recall, that the particles of the same sort are identical and indistinguishable). Hence, the entropy of such a system is $S = k_B ln3$. Now let us introduce "ordering" in our system. Impose the demand that the particles should be necessarily located symmetrically relative to the axis OO' passing over the partition as depicted in **Figure 2**. Introducing symmetry diminishes the multiplicity to $\Omega = 1$. Consequently, the entropy of the symmetrized system of particles equals to zero. The generalization of the suggested approach to the system of 2N particles is straightforward.

It is noteworthy, that the notion of the Voronoi entropy fails to quantify the ordering of points constituting the 2D pattern [26-28]. It was demonstrated that the symmetric patterns may be characterized by the values of the Voronoi entropy being very close to those inherent to random ones [29].

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Conclusions

Misinterpretations of entropy are ubiquitous among scientists and non-scientists [4, 10]. The widespread interpretation of entropy is "the measure of disorder" [4, 10]. However this understanding of entropy was criticized recently and equating of entropy to "disorder" or "chaos" was considered as a misconception [4, 10]. Indeed, "disorder" and "chaos" are obscure notions, actually rooted in the human psychology [11-13], and hardly could be accurately defined. We demonstrate how this misconception may be avoided. The misunderstanding is eliminated, if the "ordering" is identified with symmetrizing of the system. Thus, "chaos" or "disorder" is understood as an absence of symmetry. We demonstrate that introducing of elements of symmetry orders the system and consequently diminishes its entropy. The idea is illustrated with the binary system built from elementary magnets. 1D and 2D exemplifications of the binary systems are treated. We also demonstrate that introducing of symmetry does not influence the Landauer Principle for the binary systems. We also consider the system built of particles contained within the chamber divided by the permeable partition. Again imposing the symmetry restrictions diminishes the entropy of the system.

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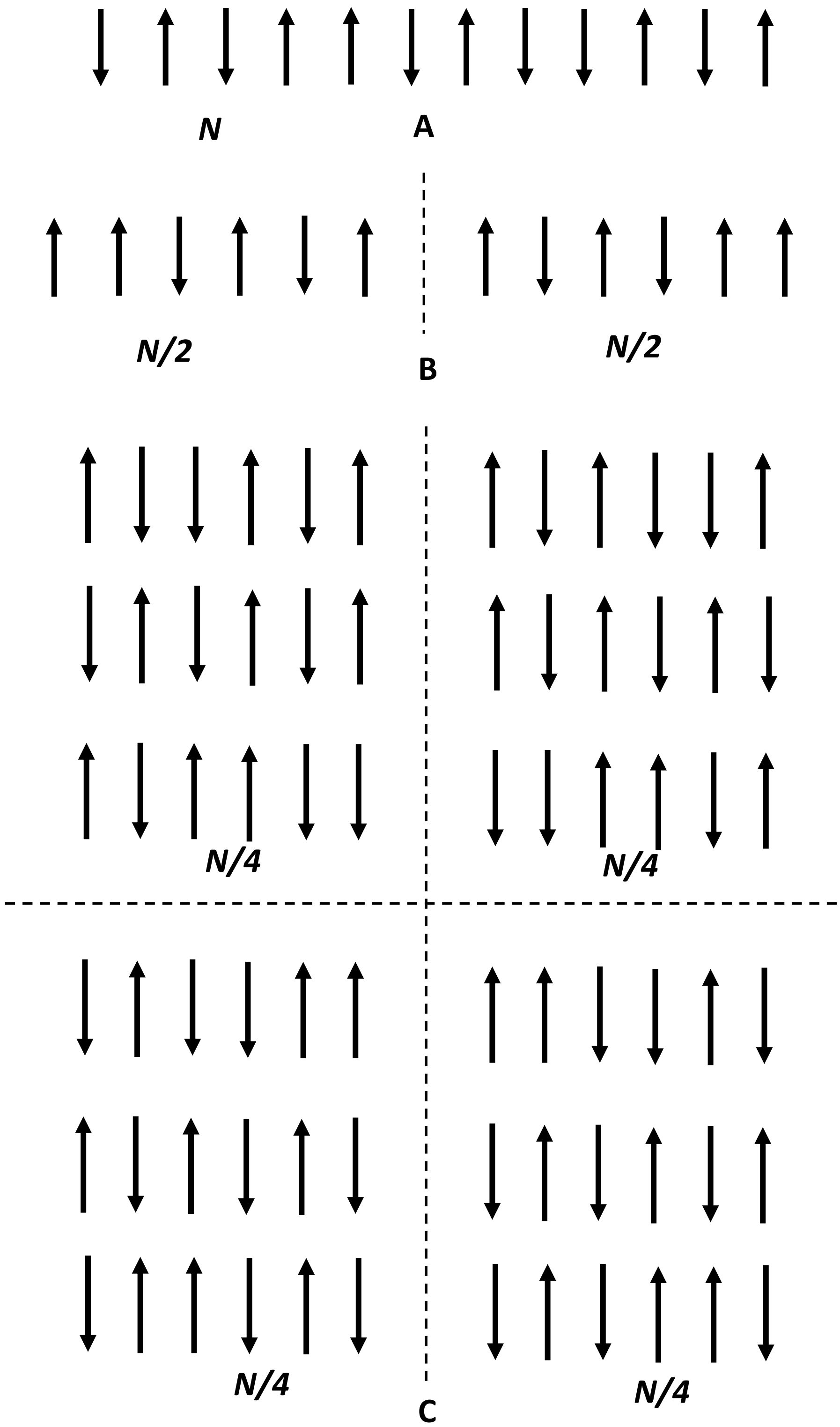

Figure 1. A The binary 1D system of *N* elementary magnets is shown. All of up/down arrangements of the magnets are available. **B** The axis of symmetry shown with the dashed line restricts the number of available configurations of magnets. **C** 2D binary system of elementary magnets is shown. Axes of symmetry shown with dashed lines restrict the available arrangements of the magnets.

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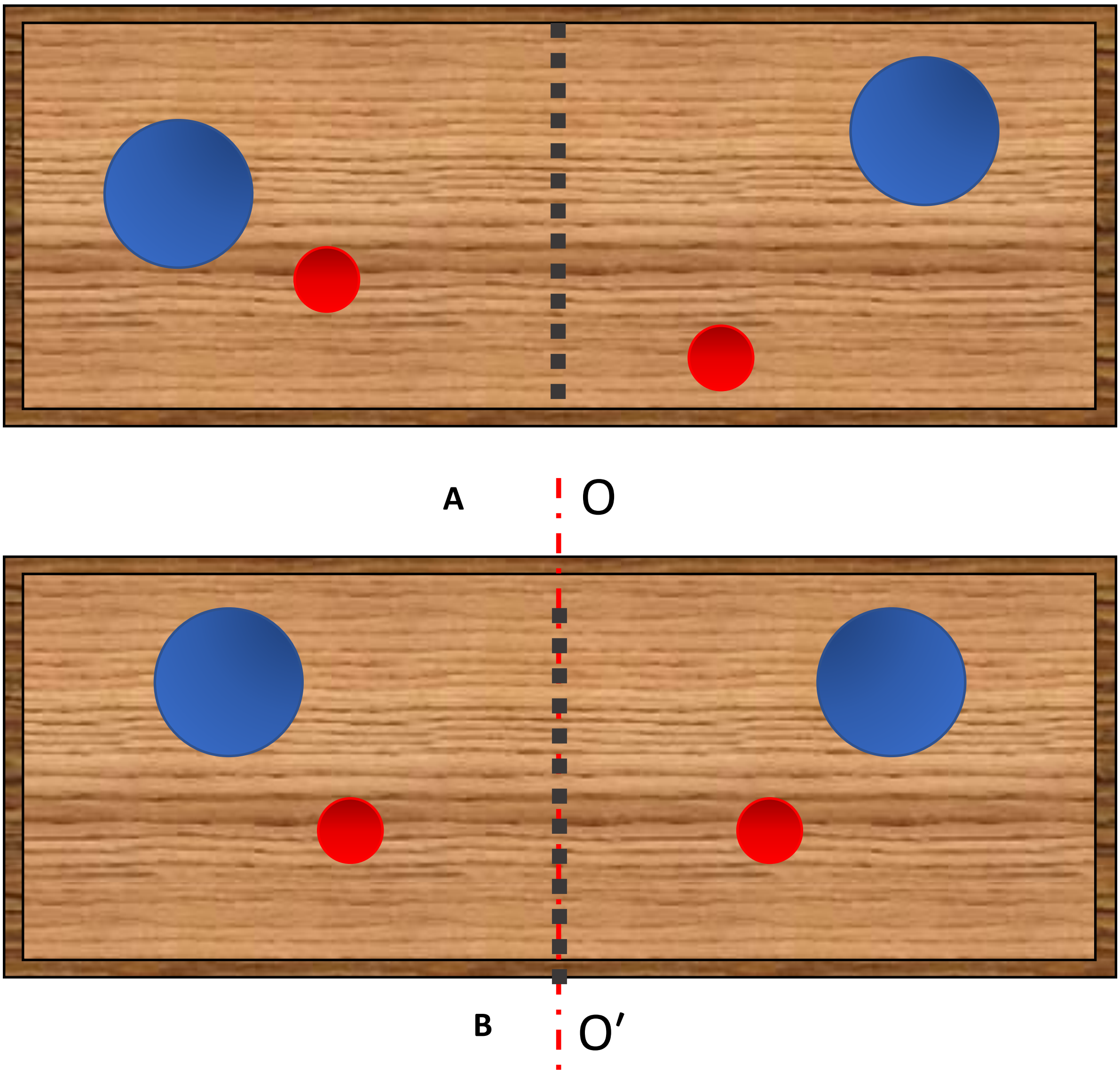

Figure 2. A Particles of two kinds (blue and red) are located within the chamber divided equally by the permeable partition. The arrangements at which two particles are simultaneously located at one side of the partition are permitted. **B** Particles of two kinds (blue and red) are located within the chamber divided equally by the permeable partition. Only the arrangements symmetric relatively to axis OO' shown with the red dashed line are permitted.