

## Article

# Fuzzy PI<sup>λ</sup> Position Control Method for Permanent Magnet Synchronous

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**Featured Application:** This method can be used for motor position control and precise control of industrial robot trajectory.

**Abstract:** Permanent magnet synchronous motor (PMSM) AC servo system shows the characteristics of uncertainty, time-varying and non-linearity, which makes it difficult for traditional PID control to achieve ideal control effect. Fuzzy control has strong adaptability to the problems of parameter variation, non-linearity and model inaccuracy of the controlled object. Vector control strategy is used to study its control principle and the realization of SVPWM. Because the motor object has certain fractional order characteristics, the fuzzy parameter self-tuning PI<sup>λ</sup> control is chosen as the position regulator of servo motor. It combines the accuracy of fractional order PI control and the adaptability of fuzzy control. A simulation model of PMSM three-closed-loop system is built in MATLAB/Simulink environment. The results show that the control method is effective and can satisfy the trajectory tracking of servo control.

**Keywords:** permanent magnet synchronous motor (PMSM); fuzzy control; SVPWM; Fractional order; PI<sup>λ</sup>

## 1. Introduction

The existing systems are more or less affected by non-integer order, especially the dynamic processes of large-scale diffusion or heat conduction with memory and heredity. The researches prove that the actual capacitance and inductance are fractional in nature[1]. Permanent magnet synchronous motor (PMSM) is widely used as controlled object in small and medium capacity servo control system because of its simple physical structure, small installation space, low torque ripple and high efficiency. But its strong coupling, non-linearity and fractional order characteristics make AC servo system uncertain and non-linearity[2,3]. Generally, only approximate mathematical model can be established. The traditional PI controller needs to establish accurate mathematical model of the controlled object, which is often difficult to achieve the desired control effect. Fuzzy control transforms the strategy of fuzzy control into a computer-realizable control algorithm. It has strong adaptability to the parameter variation, non-linearity and model inaccuracy of the control object.

Considering the control performance index, PMSM control strategy needs to achieve the goals of small current harmonic component, fast speed response and high steady-state accuracy. Liu[4] changed the traditional position, speed and current three-loop control from structure to position-current double-loop control, and proposed a control strategy based on ADRC, which simplified the debugging process. Zuo[5] synthesized the position and speed controllers into the outer loop controllers of the system. An integrated design method of position and speed based on second-order auto-disturbance rejection was proposed, which not only retained the advantages of auto-disturbance rejection control, but also solved the problem of uncontrollable speed. The precise

positioning of the servo system was realized. In view of the strong coupling, slow time-varying and uncertain disturbances in high-power AC servo system, Hou[6,7] respectively combined the fuzzy method with self-wavelet sliding mode control and wavelet neural network to design an adaptive control method; Zhang[8] considered the external disturbance when the manipulator was working, designed a disturbance observer based on the non-singular terminal sliding mode to realize the external disturbance. Estimation and compensation for system input.

However, previous studies mostly focus on integer-order systems, ignoring the fractional-order characteristics of the motor, and the robustness of the designed system needs to be further improved. ZHANG et al. [9] proposed a fractional sliding mode control scheme based on self-tuning of fuzzy parameters, which is robust to external load disturbance and parameter change in speed control of permanent magnet synchronous motor. SUN et al. [10] proposed an adaptive fractional-order (FO) terminal sliding mode control (SMC) strategy for linear motor tracking control, which is similar to the traditional fast nonsingular matrix control method. By comparison, the proposed method has feed-forward integral sliding surface and adaptive switching input, and achieves higher convergence accuracy in the presence of system uncertainties in the motion control system. YUE [11] applies fractional-order PID control algorithm to direct torque control (DTC) scheme and combines it with support vector machine pulse width modulation (SVPWM) to improve the dynamic performance and anti-interference ability of PMSM speed control. XIE [12] presents a data-driven Adaptive Fractional Order proportional integration (AFOPI) control method. The closed-loop process data are used to design AFOPI controllers with unknown noise distribution and data loss probability. The convergence and stability of the algorithm are verified.

In recent years, fractional-order fuzzy systems and fractional-order fuzzy control have attracted extensive attention of scholars at home and abroad. Combining fuzzy control with fractional sliding mode control, Delavari H. et al. studied the design of fuzzy fractional sliding mode controller for integer-order nonlinear systems[13]. Lin et al. introduced the adaptive fuzzy control method into fractional-order chaotic systems  $H^\infty$  synchronization problem[14]. However, due to the problems in the process of calculating fractional derivative of energy function, an Iranian scholar commented on and revised part of the paper [15]. Based on integer order T-S fuzzy model and LMI square, Zhong et al. studied the impulse control problem of fractional order chaotic systems[16]. The above literature was concerned with the concepts of fuzzy system and fuzzy control when dealing with fractional order system problems.

In this paper, permanent magnet synchronous motor (PMSM) is taken as the object. Firstly, the structure and mathematical model of PMSM are studied, and the SVPWM control technology is selected to study the fractional order PI lambda controller. The self-tuning PI lambda controller with fuzzy parameters is designed to realize the self-tuning of the three parameters: proportion, integral and integral order. The SIMULINK simulation model is established, and the fuzzy PI lambda is compared with the traditional PI controller. The simulation results show that the design is effective, achieves the desired control performance, responds faster and has higher precision. Finally, according to the actual trajectory characteristics, speed feed-forward and acceleration feed-forward are added to verify the effectiveness of the composite control method.

## 2. Permanent Magnet Synchronous Motor Control System

### 2.1. PMSM AC Servo Control System

The control principle diagram of three closed-loop PMSM AC servo control system is shown in Figure 1. The fractional  $PI^\lambda$  regulator realizes the control of current loop and speed loop, and the fractional  $PI^\lambda$  controller realizes the control of position loop based on the parameter self-tuning of fuzzy logic.

The speed loop has the ability to enhance the anti-load disturbance and robustness of the servo system. The component of the torque and current generated by the output of the speed loop is the given value of the current loop. The output of the current loop directly affects the drive of the inverter to the motor, and the setting of the parameters of the current loop directly affects the control

accuracy and response speed of the system. PMSM AC servo system requires current loop to satisfy the performance of small harmonic component of output current and fast response speed. Current regulator needs to satisfy the response speed of inner loop control. The function of position loop regulator is to ensure the static accuracy and dynamic tracking performance of the system.

For permanent magnet synchronous motor servo system, in addition to its fast dynamic response and stable steady-state operation, it also requires that the system can achieve accurate positioning and fast tracking. Position controller directly affects the positioning control and position tracking performance of the system. Position servo system requires fast dynamic response, no overshoot and high steady-state accuracy. The traditional PID controller has simple structure and strong robustness, but because of the linear combination of three components in itself, there are contradictions between the static and dynamic performance of the controller, the tracking setting value and the ability to suppress disturbance; moreover, due to some uncertain factors in the system, such as the non-linearity of the characteristics of the control object. The time-varying parameters and disturbance factors require the speed controller to have strong adaptive ability (fast tracking when the speed deviation is large, smooth operation and accurate positioning without error when the speed deviation is small).

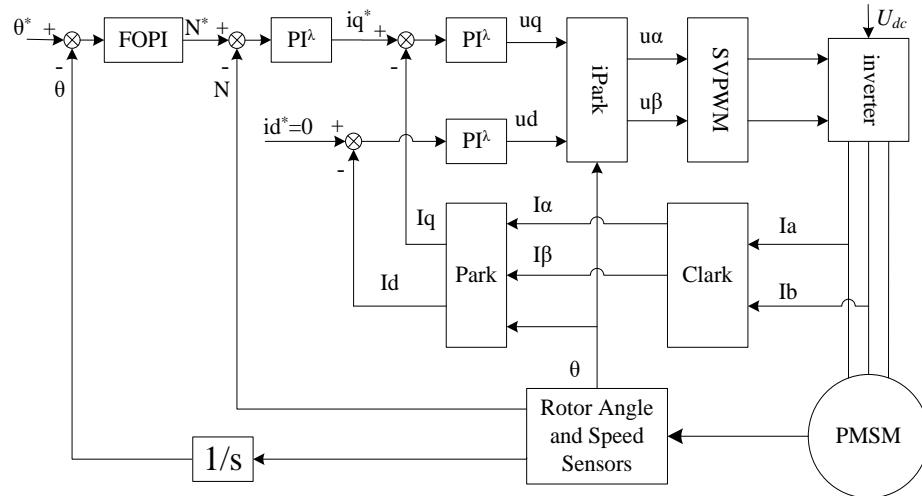


Figure 1. Principle diagram of three closed loop permanent magnet synchronous motor vector control

## 2.2. Mathematical model of permanent magnet synchronous motor

The mathematical model is a mathematical expression describing the relationship between the actual system performance and each physical quantity. For the strong coupling nonlinear system such as permanent magnet synchronous motor, its mathematical model is the basis of analyzing motor performance, designing motor and its control, and also provides theoretical basis for improving the performance of control system [18].

Before discussing the mathematical model of permanent magnet synchronous motor, it is considered that the stator and rotor of the motor have smooth surface, ignoring the core magnetic saturation, eddy current and hysteresis loss, the conductivity of permanent magnet material is zero, the stator is three-phase symmetrical, the rotor has no damping winding and is symmetrical in structure, and the induced electromotive force waveform in the three-phase winding is sinusoidal.

The circuit equation of the mathematical model based on the rotor rotation coordinate system d-q is as follows:

$$\begin{cases} \frac{d}{dt}i_d = \frac{1}{L_d}U_d - \frac{R}{L_d}i_d + \frac{L_q}{L_d}\omega_r i_q \\ \frac{d}{dt}i_q = \frac{1}{L_q}U_q - \frac{R}{L_q}i_q + \frac{L_d}{L_q}\omega_r i_d - \frac{\lambda_0\omega_r}{L_q} \end{cases} \quad (1)$$

Where:  $L_d$  and  $L_q$  are the main inductors on d and q axis;  $R$  is the internal resistance of stator;  $i_d$  and  $i_q$  are the current components on d and q axis;  $U_d$  and  $U_q$  are the voltage components on d and q axis;  $\omega_r$  is the angular speed of rotor;  $\lambda_0$  is the electromagnetic torque coefficient.

The torque equation is as follows:

$$T_e = \frac{3}{2}p_n[\psi_f i_q - (L_d - L_q)i_d i_q] \quad (2)$$

The equation of motion is as follows:

$$T_e - T_L = J \frac{d\omega_r}{dt} \quad (3)$$

Among them,  $T_e$  is the electromagnetic torque,  $T_L$  is the motor torque,  $J$  is the moment of inertia,  $p_n$  is the pole pairs of the rotor,  $\psi_f$  is the flux produced by the permanent magnet.

### 2.3. Vector Control of Permanent Magnet Synchronous Motor

In vector control of AC motor, in order to simulate the control of DC motor, the coordinate transformation of AC motor is needed first, that is, from three-phase stator ABC coordinate system of AC motor to two-phase rotor d-q coordinate system that can be controlled by DC motor, and then the control quantity is obtained by DC motor control theory. In order to control AC motor by Inverse Coordinate transformation, this paper adopts SVPWM method to control the three-phase power supply of the motor.

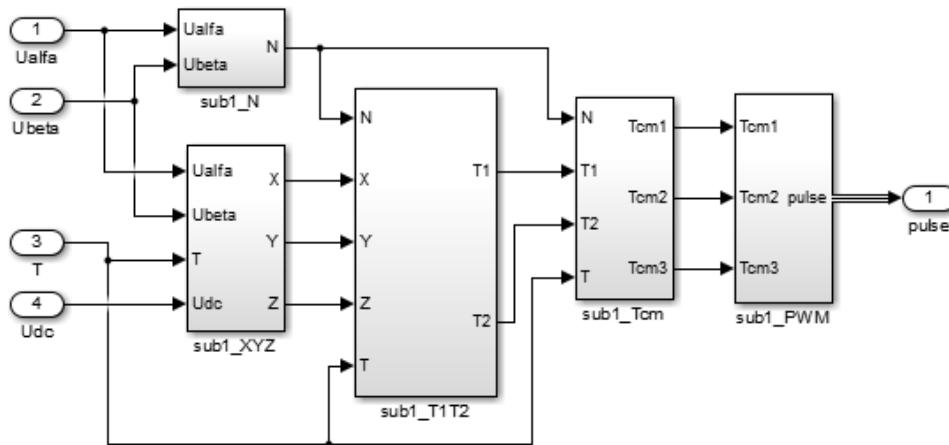


Figure 2. Structure of space vector pulse width modulation algorithm

According to the calculation steps of SVPWM algorithm and module building method [21], the structure diagram of space vector pulse width modulation algorithm is established as shown in Figure 2. Sector judgment module sub1\_N determines the sector where the vector is located, adjacent action time module sub1\_XYZ determines the action time of adjacent two vectors, T1 and T2 generation module sub1\_T1T2 simulates the action time ratio of adjacent vectors, vector switching point realization module sub1\_Tcm determines the switching point of the comparator, and SVPWM waveform generation module sub1\_PWM generates six PWM waves.

#### 2.4. Fuzzy $PI^\lambda$ Position Control (FOPI)

Fuzzy fractional  $PI^\lambda$  controller combines fuzzy control with fractional  $PI^\lambda$  control. According to the actual operation, the parameters are self-tuned online according to the rules of parameter adjustment. The position controller consists of two parts, i. e. fractional  $PI^\lambda$  control part and parameter setting part of fuzzy reasoning. Its structure is shown in Figure. 3.

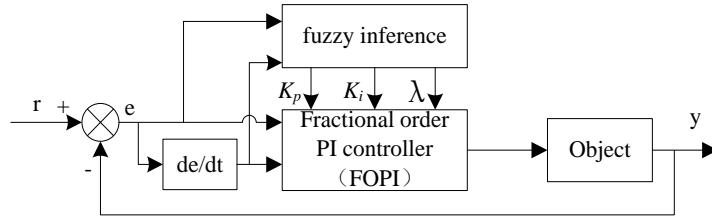


Figure 3. Principle Diagram of Fuzzy Parameter Self-tuning FOPI Controller

### 3. Fuzzy Parameter Self-tuning $PI^\lambda$ Controller

#### 3.1. Fractional-Order Operator

Fractional calculus is essentially a non integer order calculus, or any order calculus, the order number can be real or even for the complex number. The basic operation of fractional calculus is  ${}_a D_t^\alpha$  [22], it can be defined as

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, R(\alpha) > 0 \\ 1, R(\alpha) = 0 \\ \int_a^t (d\tau)^{(-\alpha)}, R(\alpha) < 0 \end{cases} \quad (4)$$

Where a and t are the upper and lower bounds of the operators,  $\alpha$  is the order, which can be any complex number, assuming a real number in the paper. Where  $R(\alpha)$  is the real part of  $\alpha$ .

There are several famous definitions for fractional derivatives. Among them, the most frequently used definitions are Riemann-Liouville definition and Caputo definition [22].

The fractional-order differ-integral operator satisfies the exchange law and the superposition principle [22]

$$D^\alpha (D^\rho f(t)) = D^\rho (D^\alpha f(t)) = D^{\alpha+\rho} f(t) \quad (5)$$

#### 3.2. Fractional-Order System

Fractional order system is the object model based on fractional differential equation. Some basic theory of fractional differential equations is also the basis of fractional order systems. The general form of the fractional order linear differential equation:

$$a_n D^{\alpha_n} f + \dots + a_1 D^{\alpha_1} f + a_0 D^{\alpha_0} f = b_m D^{\beta_m} g + \dots + b_1 D^{\beta_1} g + b_0 D^{\beta_0} g \quad (6)$$

Where,  $f = f(x, y)$  and  $g = g(x, y)$  are the functions of variables X and Y, which is the output and input respectively. The value of  $a_i$  and  $b_j$  are real number,  $a_i$  ( $i = 0, 1, \dots, n$ ) and  $b_j$  ( $j =$

0,1,...,m) can be fractional or decimal, and meet the conditions  $\alpha_n > \alpha_{n-1} > \dots > \alpha_1 > \alpha_0$  and  $\beta_m > \beta_{m-1} > \dots > \beta_1 > \beta_0$ .

For zero initial conditions, the Eq. 6 for Laplace transform, the transfer function is obtained:

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0 s^{\alpha_0}} \quad (7)$$

The fractional order system identification methods include two kinds of time domain method and frequency domain method. In recent years, the intelligent optimization method is applied to the identification of fractional order systems.

#### Oustaloup Approximation Algorithm

Oustaloup's approximation method uses a band-pass filter to approximate the fractional-order operator  $s^\lambda$  based on frequency domain response. The approximate transfer function of a continuous fractional-order operator  $s^\lambda$  with Oustaloup Algorithm is as follows [23]:

$$G_f(s) = K \prod_{k=-N}^N \frac{s + \omega'_k}{s + \omega_k} \quad (8)$$

Where the zeros, poles and the gain can be evaluated, respectively, as:

$$\omega'_k = \omega_b \left( \frac{\omega_h}{\omega_b} \right)^{\frac{k+N+\frac{1}{2}(1-\lambda)}{2N+1}} \quad (9)$$

$$\omega_k = \omega_b \left( \frac{\omega_h}{\omega_b} \right)^{\frac{k+N+\frac{1}{2}(1+\lambda)}{2N+1}} \quad (10)$$

$$K = \left( \frac{\omega_h}{\omega_b} \right)^{-\frac{\lambda}{2}} \prod_{k=-N}^N \frac{\omega_k}{\omega'_k} \quad (11)$$

In our simulation, for approximation of  $s^\lambda$ , frequency range is closed as:  $\omega \in [\omega_b, \omega_h]$  and  $\omega_b = 0.001$ ,  $\omega_h = 1000$ ,  $N=4$ .

### 3.3. Fractional Order PI Controller( $PI^\lambda$ )

Fractional order PID controller is a generalization of traditional PID controller [17]. Differential control is not usually used for permanent magnet synchronous motor control, so the transfer function of PI lambda controller [20] is as follows:

$$G_c(s) = k_p + \frac{k_i}{s^\lambda} \quad (\lambda > 0) \quad (12)$$

Where  $s = j\omega$  is complex frequency,  $k_p$  is proportional constant,  $k_i$  is integral constant, and  $\lambda$  is real number. Consider using the fuzzy logic algorithm to adjust the parameters in (1). The proportional coefficient  $k_p$ , integral coefficient  $k_i$  and integral order  $\lambda$  are determined by the following items:

$$k_p = k_{p0} (1 + \alpha \{E\}_P) \quad (13)$$

$$k_i = k_{i0} (1 + \beta \{E, EC\}_I) \quad (14)$$

$$\lambda = \lambda_0 + \{E, EC\}_\lambda \quad (15)$$

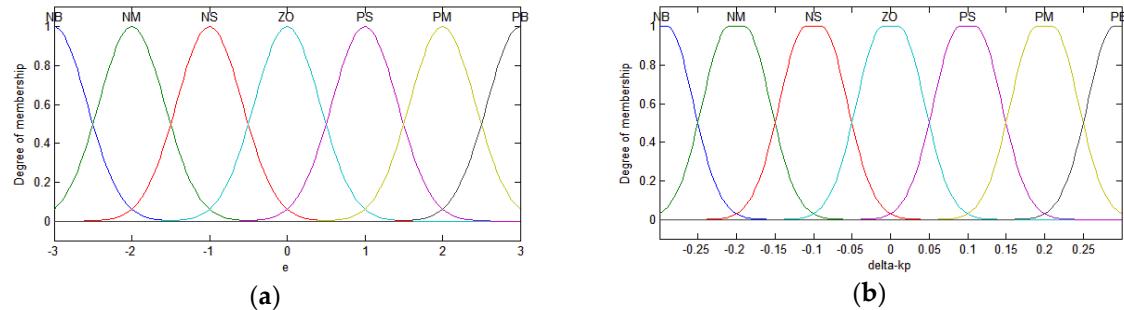
Among them,  $\{E\}P$ ,  $\{E, EC\}I$ ,  $\{E, EC\}\lambda$  are the results of fuzzy reasoning.  $kp_0$ ,  $ki_0$  and  $\lambda_0$  are the initial values of the parameters respectively. As for the determination of initial value, the requirement of self-adjusting controller parameters for initial value is not high. Because of the robustness of PID control and the flexibility of fuzzy control, conventional PID method can be used to control the system first, without requiring that all performance indicators of the system meet the requirements. As long as the system reaches a stable state, these parameters can be used as the initial value of  $PI^\lambda$  parameter adjustment. Then the  $PI^\lambda$  parameters are fine-tuned on-line by using fuzzy control, so that the system performance index can meet the requirements.

### 3.4. Design of Fuzzy Parameter Self-tuning Controller

Fuzzy parameter self-tuning  $PI^\lambda$  controller is designed. Based on fractional order PI controller (FOPI), the parameters  $kp$ ,  $ki$  and  $\lambda$  of fractional order PI controller are tuned online according to the idea of fuzzy reasoning, taking the deviation  $E$  and deviation change rate  $EC$  of feedback value and given value of controlled object as input, in order to meet the need of control accuracy of different control systems. The fuzzy controller has two-input and three-output .

#### 3.4.1. Determination of Membership Function

The determination of membership function of input and output variables is a key link. In this paper, the input variables of the fuzzy controller are deviation  $E$  and deviation rate  $EC$ , and the output variables are  $\Delta kp$ ,  $\Delta ki$ ,  $\Delta \lambda$ . The word sets describing input and output variables are {negative large, negative medium, negative small, zero, positive small, positive medium, positive large} and set their universe as  $\{E, EC\} = \{-3, -2, -1, 0, 1, 2, 3\}$  and the fuzzy subset as {NB, NM, NS, ZO, PS, PM, PB}. The fields of  $(-0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6)$ ;  $(-0.06, -0.04, -0.02, 0.02, 0.04, 0.06)$ ;  $(-0.3, -0.2, -0.1, 0.1, 0.2, 0.3)$  of  $(\Delta kp)$ ; and  $(-0.06)$  of  $(-0.3, -0.2, -0.1, 0.1, 0.2, 0.3)$  of  $(-0.6)$  of  $(-0.04, -0.04, -0.06)$  of  $(-0.06)$ . The membership function is Gauss membership function with overlapping symmetric distribution. The membership curves of input and output variables are shown in Figure 4.



**Figure 4.** Membership Curve of Input and Output Variables.

(a) Membership Curves of E and EC; (b) .Membership Curves of  $\Delta kp$

#### 3.4.2. Parameter Tuning Rule of $PI^\lambda$ Controller

According to the real-time deviation  $E$  and the deviation change rate  $EC$  of the system, according to the experience, according to the setting principle and adjustment test, the rules are set in the fuzzy logic toolbox, and 49 control rules are obtained. The rule surface diagrams of the output and input variables are shown in Figure 5.

- 1.  $(e==NB) \& (ec==NB) \Rightarrow (\Delta kp=PB)(\Delta ki=NB)(\Delta \lambda=NB)$  (1)
- 2.  $(e==NB) \& (ec==NM) \Rightarrow (\Delta kp=PB)(\Delta ki=NB)(\Delta \lambda=NB)$  (1)
- 3.  $(e==NB) \& (ec==NS) \Rightarrow (\Delta kp=PM)(\Delta ki=NB)(\Delta \lambda=NB)$  (1)

- 4. ( $e==NB$ ) & ( $ec==ZO$ ) => ( $\delta_{kp}=PM$ )( $\delta_{ki}=NM$ )( $\delta_{lamda}=NM$ ) (1)
- 5. ( $e==NB$ ) & ( $ec==PS$ ) => ( $\delta_{kp}=PS$ )( $\delta_{ki}=NM$ )( $\delta_{lamda}=NM$ ) (1)
- 6. ( $e==NB$ ) & ( $ec==PM$ ) => ( $\delta_{kp}=PS$ )( $\delta_{ki}=ZO$ )( $\delta_{lamda}=ZO$ ) (1)
- 7. ( $e==NB$ ) & ( $ec==PB$ ) => ( $\delta_{kp}=ZO$ )( $\delta_{ki}=ZO$ )( $\delta_{lamda}=ZO$ ) (1)
- .....
- 43. ( $e==PB$ ) & ( $ec==NB$ ) => ( $\delta_{kp}=ZO$ )( $\delta_{ki}=ZO$ )( $\delta_{lamda}=ZO$ ) (1)
- 44. ( $e==PB$ ) & ( $ec==NM$ ) => ( $\delta_{kp}=NS$ )( $\delta_{ki}=ZO$ )( $\delta_{lamda}=ZO$ ) (1)
- 45. ( $e==PB$ ) & ( $ec==NS$ ) => ( $\delta_{kp}=NS$ )( $\delta_{ki}=PS$ )( $\delta_{lamda}=PS$ ) (1)
- 46. ( $e==PB$ ) & ( $ec==ZO$ ) => ( $\delta_{kp}=NM$ )( $\delta_{ki}=PM$ )( $\delta_{lamda}=PM$ ) (1)
- 47. ( $e==PB$ ) & ( $ec==PS$ ) => ( $\delta_{kp}=NM$ )( $\delta_{ki}=PB$ )( $\delta_{lamda}=PB$ ) (1)
- 48. ( $e==PB$ ) & ( $ec==PM$ ) => ( $\delta_{kp}=NB$ )( $\delta_{ki}=PB$ )( $\delta_{lamda}=PB$ ) (1)
- 49. ( $e==PB$ ) & ( $ec==PB$ ) => ( $\delta_{kp}=NB$ )( $\delta_{ki}=PB$ )( $\delta_{lamda}=PB$ ) (1)

All figures and tables should be cited in the main text as Figure 1, Table 1, etc.

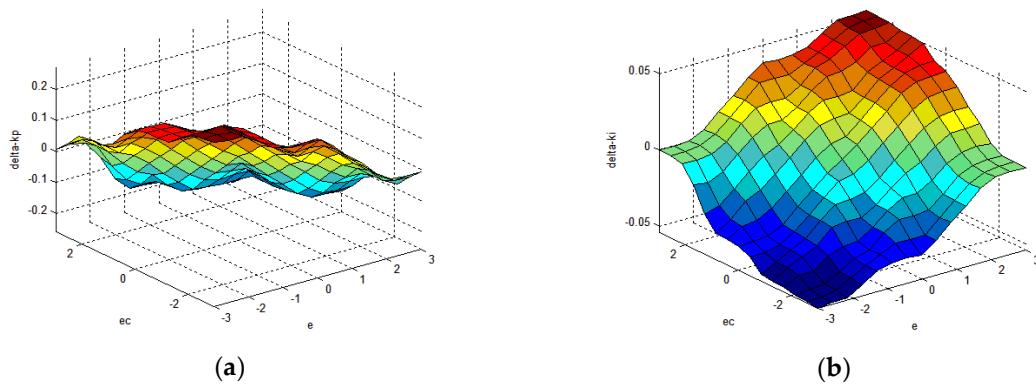


Figure 5. Rule Surface Map of Output Variables

#### 4. Simulation Research and Result Analysis

##### 4.1 Three-Closed-Loop PMSM Simulation Model

Using the Permanent Magnet Synchronous Motor (PMSM) module of Simulink/SimPowerSystem/Machines sublibrary, the simulation model of the system is established. The motor parameters are shown in Table 1.

Tab.1 Parameters of permanent magnet synchronous motor

Parameter name (unit)	Value
stator resistance $R$ ( $\Omega$ )	0.975
Stator inductance $L_d$ , $L_q$ ( $H$ )	0.0085
Flux of Permanent Magnet Rotor in Stator Loop $\lambda$ ( $Wb$ )	0.175
Rotating inertia of rotor and load $J$ ( $kg \cdot m^2$ )	0.00008
Viscous friction coefficient $F$ ( $N \cdot m \cdot s$ )	4.047e-4
Pole logarithm of motor $p$	4

The simulation model of three closed-loop permanent magnet synchronous motor in Simulink is established as shown in Figure 6. The simulation system consists of position regulator, speed regulator, current regulator, Park inverse conversion module, inverter module, SVPWM module, permanent magnet synchronous motor, parameter measurement module, Clark conversion module and Park conversion module. The system adopts three closed-loop control mode of position, speed and current. Both internal control loops adopt digital PI controller, and the outer control loop is position loop. Two different controllers, digital PI and Fuzzy Adaptive Fractional PI lambda, are used as position regulators for simulation.

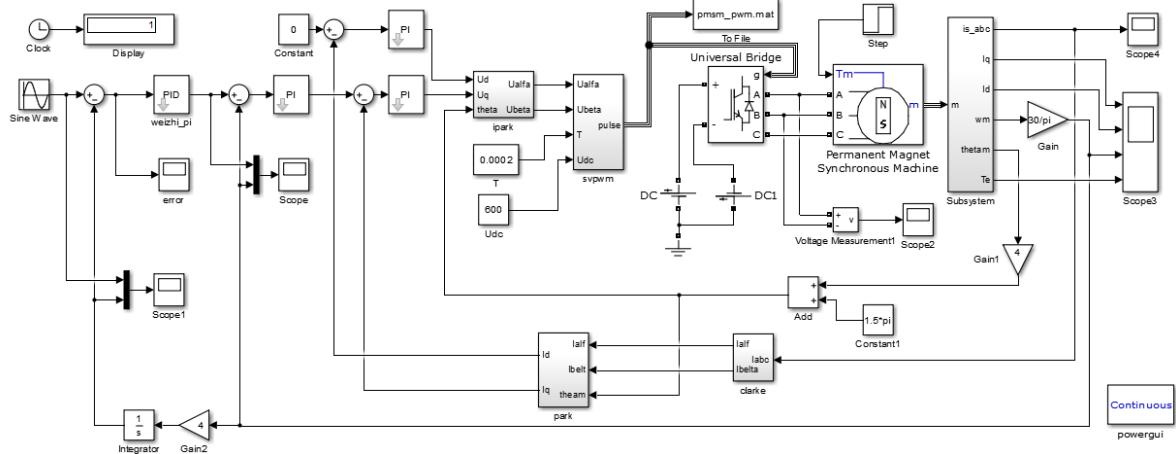


Figure 6 Simulation block diagram of AC servo system

#### 4.2 Simulation of current loop and velocity loop

Through the simulation of current loop and speed loop, the simulation waveform of AC servo system is obtained as shown in Figure 4. The given speed is 1000 rad / s, no-load operation, and the simulation time is 0.05s.

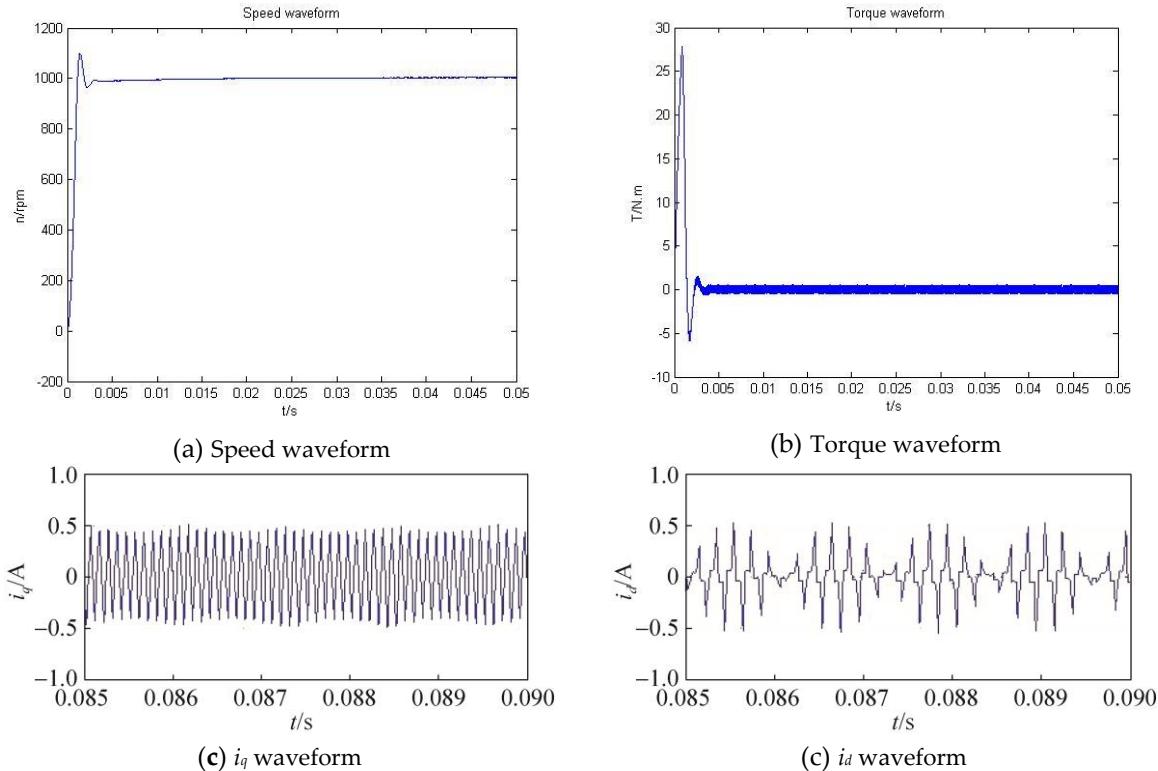


Figure 7. Simulation results of AC servo system

(a) Speed waveform; (b) Torque waveform; (c).  $i_q$  waveform; (d)  $i_d$  waveform

Figure 4 (a) shows the speed waveform. It can be seen that the maximum speed of the system reaches 1100 rad, the overshoot is 10%, and the adjustment time is 0.026 s ( $\pm 2\%$  error band). The system can follow the given speed better, which indicates that the system has good follow performance. Figure 4 (b) shows the torque waveform. It can be seen that in the early stage of motor starting, the torque fluctuates greatly, the maximum torque reaches 27 N  $\cdot$  m, and the later stage is basically stable. Figure 4 (c) and (d) are the waveforms of  $i_q$  and  $i_d$  respectively. The waveforms of [0.085, 0.09] interval are intercepted, which

are consistent with the results of theoretical analysis. It can be seen that the simulation model of servo AC system is basically realized, and the established model can truly reflect the characteristics of the system.

#### 4.3 Position PI Control

The position loop, current loop and speed loop are all controlled by PI, and the parameters are shown in Table 2.

Tab.2 Parameters of the system regulator

Regulator	Proportional P	Integral I
Location loop	100	5
Speed loop	0.1	16
Id current loop	30	1900
iqcurrent loop	30	1900

#### 4.4 Fuzzy Fractional PI Control

The position loop is replaced by a fractional PI parameter self-tuning controller based on fuzzy logic. The subsystem is shown in Figure 8. The fuzzy controller is designed with Fuzzy Logic Toolbox. The initial parameters of the fractional PI controller are  $kp0=100$ ,  $ki0=5$ , and  $\lambda=0.8$ . The input-output proportional coefficients of the fuzzy controller are 1, 2 and 10, respectively.

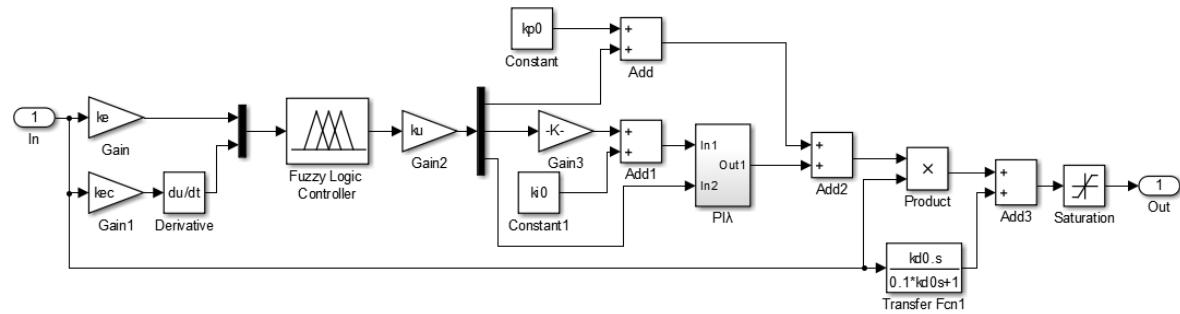


Figure8 Simulation Model of Fuzzy  $PI^{\lambda}$  Controller

The traditional PI control strategy and the fuzzy fractional PI control strategy are used to simulate the three closed-loop control system of PMSM. The sinusoidal input with 10 rad/s amplitude and 10 rad/s frequency is selected as the given trajectory. The simulation time is 1.5 s, and the simulation algorithm uses ode23tb. First, the system starts without load and runs with 0.5s load (5N.m).

The results of the two control strategies are shown in Figure 9. Figure (a) is position tracking, figure (b) is velocity tracking, figure (c) is trajectory error, and figure (d) is velocity error. It can be seen from figure (a) that the trajectory of the motor achieves the tracking of the input signal, and the error curve is shown in figure (c). The traditional PI controller is used as the position loop regulator in the simulation experiment. The position tracking process has certain fluctuation, the rising time is longer and the regulating time is longer. After the sudden load disturbance, the position response fluctuation is larger, the recovery time is longer, the speed fluctuates greatly in a short time, and the speed deviation is larger in the start-up process.

It can be seen that the trajectory tracking error is small by using the fuzzy PI\_lambda controller, and it has stronger robustness to load disturbance and better speed response in steady state.

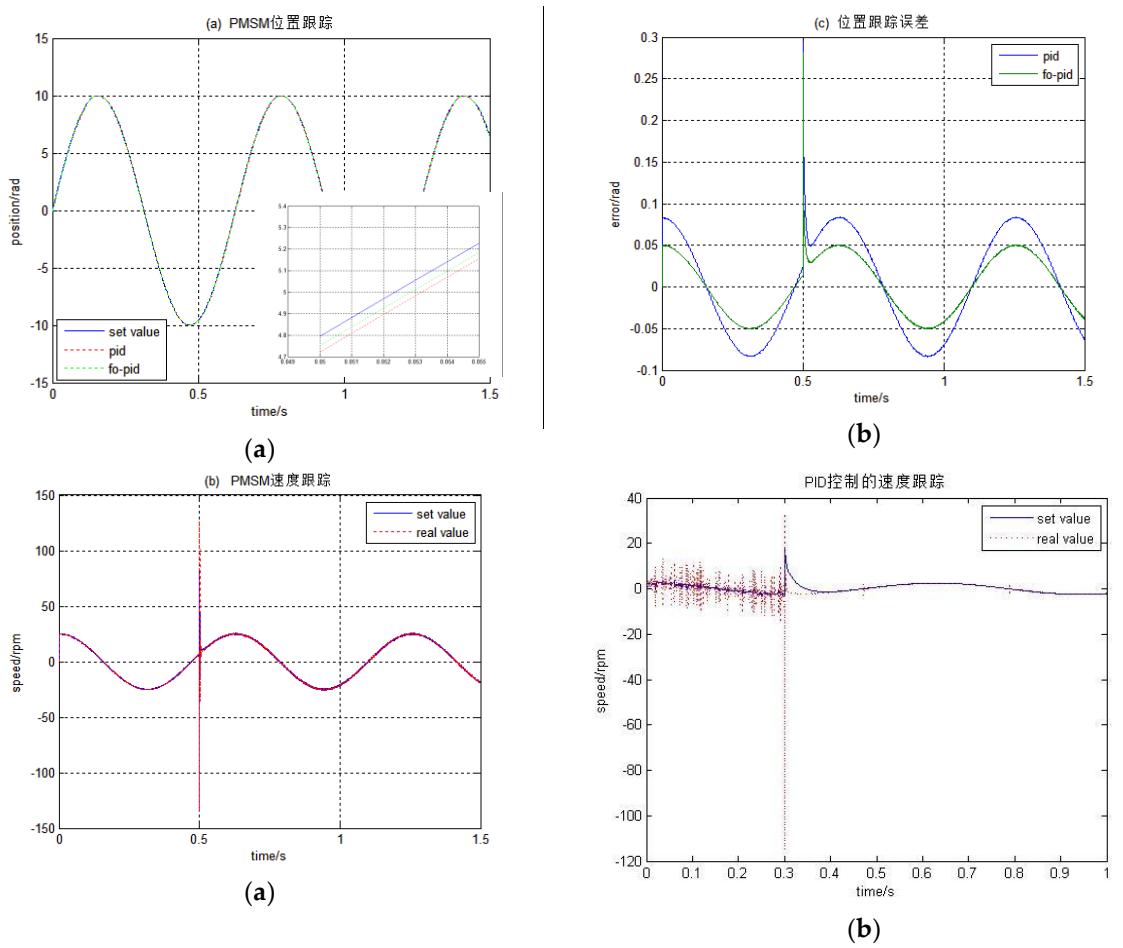


Figure 9 Simulation results of fuzzy  $\text{PI}^\lambda$  position Controller

#### 4.5 Compound Control with Speed Feed-forward and Acceleration Feed-forward

Generally, the random signal of system response is composed of step, constant velocity and sinusoidal signal. When the system is moving on a constant velocity slope, there will be a large tracking error. At this time, speed feed-forward signal should be added to implement the composite control of adaptive fuzzy  $\text{PI}^\lambda$  and feed-forward compensation [21]:

$$u_1 = u + k_{sq} * v_c \quad (16)$$

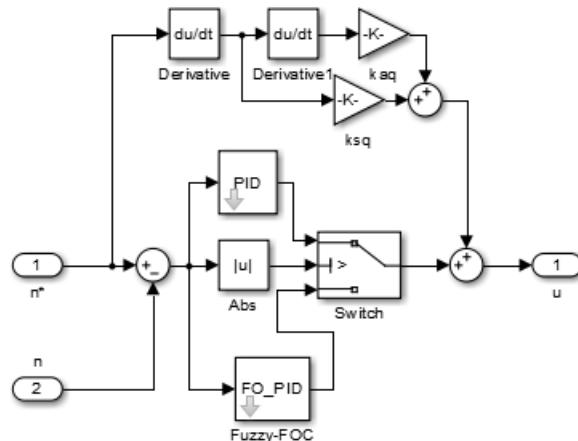
In the formula:  $v_c$  is a velocity value generated by differential position command signal,  $v_c = (x_g(n) - x_g(n-1))/T$ ,  $T$  is the sampling period,  $n$  is the current moment, and  $k_{sq}$  is the velocity feed-forward gain. .

When the system moves sinusoidally, because the input signal is a second-order signal, only adding speed feed-forward compensation can not achieve good tracking effect. Therefore, acceleration feed-forward signal should be added to compensate the system, and the composite control of adaptive fuzzy  $\text{PI}^\lambda$ , speed feed-forward and acceleration feed-forward is implemented.

$$u_1 = u + k_{sq} * v_c + k_{aq} * v_a \quad (17)$$

Among:  $k_{aq}$  is the acceleration feed-forward gain, and  $v_a$  is the acceleration value differentiated by the velocity signal,  $v_a = (v_c(n) - v_c(n-1))/T$ .

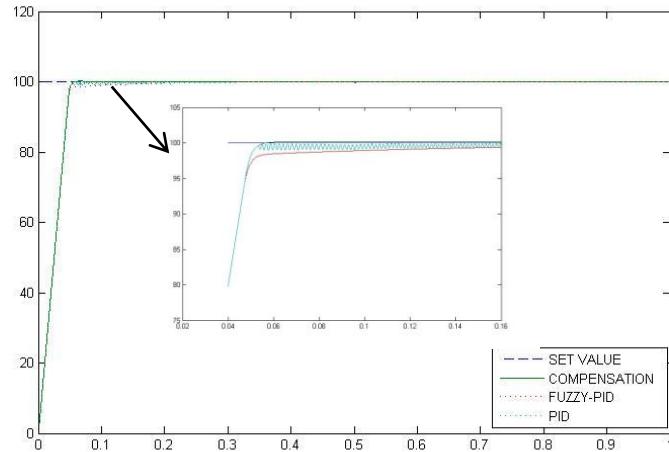
The simulation model of the compound control of fuzzy  $\text{PI}^\lambda$  and feed-forward compensation is established as shown in Figure 10.



**Figure 10 Compound Control Model of Fuzzy  $\text{PI}^\lambda$  and Feed-forward Compensation**

#### 4.6 Performance Analysis of Three Position Control Strategies

The step signal is used as the input signal. Given the position of 100, the simulation time is chosen to be 0-1s, and the load moment of 10N.M is suddenly applied to the system when the simulation time is 0.5s. The step response curves of servo system position control under three kinds of controllers are shown in figure 11. According to the data obtained from each curve, the performance comparison of the three control modes is calculated and listed as shown in Table 3.



**Figure 11 Step Response Curves of Three Control Strategies**

**Tab.3** Performance comparison table of three position controllers

Position regulator	Rise time/s	Overshoot/%	Settling time/s
PI controller	0.0450	0.15	0.051
Fuzzy $\text{PI}^\lambda$ Compound Controller	0.0452	0	0.053
Compound Controller with Feed-forward Compensation	0.0445	0	0.050

The simulation results show that the three control strategies can reach the set value, PI control has 0.15% overshoot, and the fuzzy PI controller and composite control have no overshoot. From the time analysis, the rise time and adjustment time of the composite control are 0.0445 and 0.05, respectively, which are smaller than the response time of the other two methods. Considering the comprehensive

performance, the effect of compound control is the best.

## 5. Conclusion

The application of fuzzy  $PI^\lambda$  control in permanent magnet synchronous motor servo system is studied. The disadvantages of traditional PID control and the disadvantages of variable parameters, non-linearity, time delay and time variation of servo system are overcome by using the characteristics of fuzzy control such as non-linearity, variable structure, self-adaptation, self-tuning and strong robustness. Fuzzy parameter self-tuning fractional-order PI controller, simulation results show that the control effect is better than the traditional PID control, and the desired system control effect can be obtained. After adding feed-forward compensation, the response speed is accelerated, the overshoot is small, and the parameters of the PID controller can be adjusted online and real-time according to the changes of the controlled object, which can reduce the system error; moreover, the system has strong robustness and is not susceptible to external interference and changes in system parameters. It can be seen that for AC servo system with non-linearity and time-varying parameters, the compound position control with fuzzy  $PI^\lambda$  and feed-forward compensation can meet the requirements of system control and improve the dynamic and static characteristics of the system.

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