doi:10.20944/preprints201911.0096.v1

Black Hole Entropy from Non-Commutative Geometry and Spontaneous Localisation

Palemkota Maithresh[a] **and Tejinder P. Singh**[b,*]

[a] *UM-DAE Centre for Excellence in Basic Sciences, Mumbai, 400098, India*

[b] *Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India*

*Corresponding: tpsingh@tifr.res.in

ABSTRACT: In our recently proposed theory of quantum gravity, a black hole arises from the spontaneous localisation of an entangled state of a large number of atoms of space-time-matter [STM]. Prior to localisation, the non-commutative curvature of an STM atom is described by the spectral action of non-commutative geometry. By using the techniques of statistical thermo-dynamics from trace dynamics, we show that the gravitational entropy of a Schwarzschild black hole results from the microstates of the entangled STM atoms and is given (subject to certain assumptions) by the classical Euclidean gravitational action. This action, in turn, equals the Bekenstein-Hawking entropy $(\mathrm{Area}/4{L_P}^2)$ of the black hole. We argue that spon-taneous localisation is related to black-hole evaporation through the fluctuation-dissipation theorem.

Keywords: black hole entropy; quantum gravity; non-commutative geometry; spontaneous localisation; trace dynamics

This paper proposes a new derivation of black hole entropy, based on our recent papers which propose a new quantum theory of gravity [1–3]. These papers will be hereafter referred to as I, II and III respectively. We will also frequently refer to Adler's book *Quantum theory as an emergent phenomenon* [4], hereafter referred to as QTEP.

In I, we have introduced the concept of an atom of space-time-matter [STM], which is described by the spectral action of non-commutative geometry. The spectral action, in the presence of a Riemannian manifold, is equal to the Einstein-Hilbert action of classical general relativity, after a heat kernel expansion of square of Dirac operator is carried out, and truncated at the second order in an expansion in L_p^{-2}. We also introduced there the four levels of gravitational dynamics. In II, we used the Connes time parameter, along with the spectral action, to incorporate gravity into trace dynamics. We then derived the

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

spectral equation of motion for the gravity part of the STM atom, which turns out to be the Dirac equation on a non-commutative space. In III, we proposed how to include the matter (fermionic) part and gave a simple action principle for the STM atom. This leads to the equations for a quantum theory of gravity, and also to an explanation for the origin of spontaneous localisation from quantum gravity. We used spontaneous localisation to arrive at the action for classical general relativity [including matter sources] from the action for STM atoms. See Fig. 1 below for the four layers of gravitational dynamics, first introduced in paper I. In the present paper we use the results of III to derive the Bekenstein-Hawking entropy for a Schwarzschild black hole, from the entangled microstates of the STM atoms.

I. CLASSICAL GENERAL RELATIVITY FROM SPONTANEOUS LOCALISATION IN NON-COMMUTATIVE MATTER-GRAVITY

In III, we have proposed the following action principle for the STM atom described by the operator $q = q_B + q_F$, evolving in a Hilbert space with respect to the Connes time-parameter τ:

$$\frac{L_P}{c}\frac{S}{C_0} = \frac{1}{2}\int d\tau\ Tr\left[\frac{L_P^2}{L^2c^2}\left(\dot{q}_B + \beta_1\dot{q}_F\right)\left(\dot{q}_B + \beta_2\dot{q}_F\right)\right] \tag{1}$$

Here β_1 and β_2 are two constant fermionic matrices. This trace Lagrangian can be expanded and written as

$$\frac{L_P}{c}\frac{S}{C_0} = \frac{a}{2}\int d\tau\ Tr\left[\dot{q}_B^2 + \dot{q}_B\beta_2\dot{q}_F + \beta_1\dot{q}_F\dot{q}_B + \beta_1\dot{q}_F\beta_2\dot{q}_F\right] \tag{2}$$

where we have denoted $a \equiv L_P^2/L^2c^2$. The first term inside the trace Lagrangian has the familiar structure of a kinetic energy, and in any case is what gives rise to the Einstein-Hilbert action in the heat kernel expansion of D_B^2, where as in III, $D_B \equiv dq_B/Lc\ d\tau$.

There are three universal constants in the theory: Planck length L_P and Planck time $\tau_{Pl} = L_P/c$, where the speed of light c should be thought of as the ratio L_p/τ_{Pl}. The third universal constant C_0 has dimensions of action, and at Level I it is identified with Planck's constant $\hbar$ (up to a numerical factor). Newton's gravitational constant G and Planck mass m_{Pl} are emergent only at Level I in the four layers of gravitational dynamics [1]. In fact the concepts of mass and spin themselves emerge only at Level I, and are not present at Level 0. We associate only a length scale (more precisely an area L^2) with the STM atom, but not mass nor spin, at Level 0.

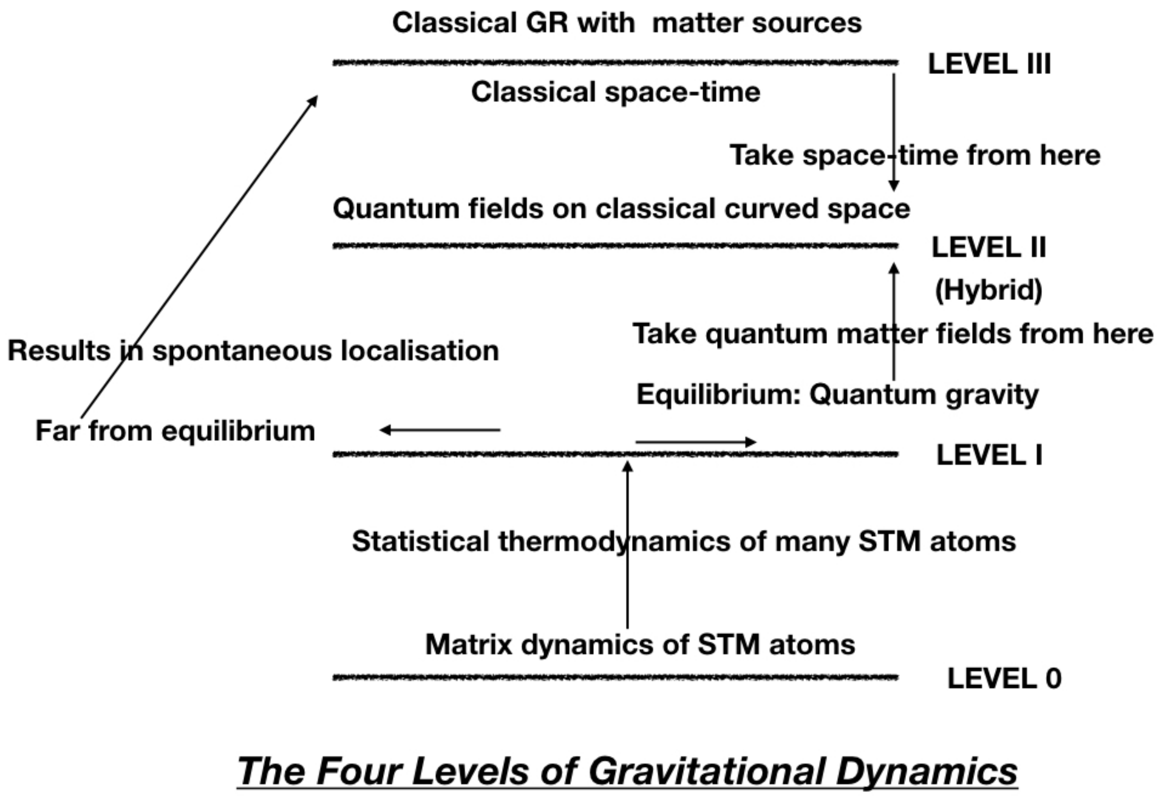

FIG. 1. The four levels of gravitational dynamics. In this bottom-up theory, the fundamental Level 0 describes the 'classical' matrix dynamics of atoms of space-time-matter (STM). This level operates at the Planck scale. Statistical thermodynamics of these atoms brings us below Planck scale, to Level I: the emergent equilibrium theory is quantum gravity. Far from equilibrium, rapid spontaneous localisation results in Level III: emergence of classical space-time, obeying classical general relativity with matter sources. Level II is a hybrid level built by taking classical space-time from Level III and quantum matter fields from Level I, while neglecting the quantum gravitation of Level I. Strictly speaking, all quantum field dynamics takes place at Level I, but we approximate that to Level II.

In III, we have discussed the Lagrangian and Hamiltonian dynamics of the STM atom, based on the above action principle. We also discussed the adjointness properties of the conserved Adler-Millard charge. Next, in the spirit of trace dynamics, one assumes that the Hilbert space is populated by a large collection of STM atoms, whose dynamics is described in the operator phase space. For a discussion of the statistical mechanics of the underlying matrix dynamics, see Chapter 4 of QTEP [4]. We assume that we are examining the system over time-scales much larger than Planck time, and that the system has no memory of its initial conditions, and that an equilibrium state has been reached in the operator phase space. The system is described, as in statistical mechanics, by a phase space density distribution ρ, and the equilibrium state is obtained by maximising the entropy

$$\frac{S_E}{k_B} = -\int d\mu\ \rho\ \ln\rho \tag{3}$$

with $d\mu$ being the measure defined in operator phase space [4]. As shown in QTEP, the emergent theory at equilibrium, upon performing the statistical thermodynamics of the underlying matrices, is quantum field theory on a Minkowski space-time background. (Trace dynamics, as described in QTEP, deals with matter fields, on a flat space-time. Gravity is not included). We showed in III, how to use non-commutative geometry to include gravity in trace dynamics. Thus, in our case the statistical mechanics of the underlying STM atoms yields a quantum theory of gravity, at equilibrium, and under suitable approximations this quantum theory of gravity reduces to the trace dynamics (flat classical space-time) discussed in QTEP. Also, in III, following the philosophy of QTEP, we argued that large statistical fluctuations away from equilibrium give rise to classical space-time and classical matter, via the mechanism of spontaneous localisation. In fact, as argued in III, the action for a large collection of STM atoms, reduces by way of spontaneous localisation, to the action for classical general relativity with point sources. To write this down explicitly, we first write, using (2) above, the total action for a large collection of STM actions as the sum of their individual actions:

$$S_{tot} = \sum_i S_i = \sum_i \frac{acC_0}{2L_p}\int d\tau\ Tr\left[\dot{q}_B^2 + \dot{q}_B\beta_2\dot{q}_F + \beta_1\dot{q}_F\dot{q}_B + \beta_1\dot{q}_F\beta_2\dot{q}_F\right]_i \tag{4}$$

$$\equiv \frac{C_0}{\tau_{Pl}}\int d\tau\ \sum_i Tr L_P^2 D_i^2 \tag{5}$$

The result about the emergence of classical general relativity from this action, upon spontaneous localisation, is that

$$S_{tot} = \frac{C_0}{\tau_{Pl}}\int d\tau\ \sum_i Tr L_P^2 D_i^2 \quad\longrightarrow\quad \frac{1}{\tau_{Pl}}\int d\tau\ \int d^4x\ \sqrt{g}\left[\frac{c^4}{16\pi G}R + c^2\ \sum_i m_i\delta^3(\mathbf{x}-\mathbf{x_0})\right] \tag{6}$$

For an explanation on how this result comes about, see III. The expression on the right

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

hand side of the above arrow should be compared with Eqn. (62) of III. Compared to that equation, we have pulled out a c factor from the space-time volume element, and restored the correct numerical factor of $1/16\pi G$ in the Einstein-Hilbert action (the exact numerical factor was not relevant in III). This correct numerical factor comes from the universal constant C_0 which upon the emergence of Level I is set proportional to Planck's constant $\hbar$.

The origin of this emergence can be traced back to the remarkable result about the spectral action for gravity, in the context of non-commutative geometry. Given the Dirac operator D_B on a Riemannian manifold, the Einstein-Hilbert action can to related to the eigenvalues of D_B:

$$Tr[L_p^2 D_B^2] \propto \int d^4x\, \sqrt{g}\, \frac{1}{L_p^2} R \tag{7}$$

The significance being that the left hand side is spectral, and hence provides a definition of curvature even if the Riemannian manifold is replaced by a non-commutative space-time. This becomes highly relevant for us because we have argued that the Riemannian manifold and its overlying Einstein-Hilbert action arises only after spontaneous localisation of (the fermionic part) of a large number of STM atoms, with one atom localising to one eigenvalue of D_B (see III). Thus for us, the correct interpretation of the above equation is

$$\sum_i Tr[L_p^2 D_{Bi}^2] \longrightarrow \int d^4x\, \sqrt{g}\, \frac{1}{L_p^2} R \tag{8}$$

where the arrow represents spontaneous localisation. That is, it is not as if the spectral action of just one STM atom equals the classical Einstein-Hilbert action. Rather, each of a large collection of STM atoms localises to one or the other eigenvalue of D_B, and the net contribution of all the atoms to the trace adds up to the Einstein-Hilbert action. Thus,

$$\sum_i Tr[L_p^2 D_{Bi}^2] \longrightarrow Tr[L_p^2 D_B^2] \propto \int d^4x\, \sqrt{g}\, \frac{1}{L_p^2} R \tag{9}$$

with the arrow again denoting spontaneous localisation.

II. A DERIVATION OF BLACK HOLE ENTROPY

We have a precise description of the microstates of STM atoms, because we have a well-defined action principle for them, from which their matrix dynamics can be obtained. Thus, in principle, the eigenstates and eigenvalues of the Hamiltonian are known, as in III.

The mathematical structure of our theory provides evidence that these microstates can yield the correct black hole entropy. First and foremost, both quantum theory as well as classical general relativity emerge as thermodynamic approximations to the (deterministic) underlying matrix dynamics. Also, the gravitational part of the trace Lagrangian / Hamiltonian of the STM atoms is given by the sum of the squared eigenvalues of the Dirac operator, which in the presence of a Riemannian manifold equals the Einstein-Hilbert action. This same Hamiltonian occurs in the partition function (for a canonical ensemble of the STM atoms) from which the entropy is calculated. Such suggestive evidence motivates us to the following estimate of the black hole entropy, subject to a few assumptions, which we hope to justify rigorously in future work.

A Schwarzschild black hole, say of mass M much greater than Planck mass, results from the spontaneous localisation of the entangled state of a very large number of STM atoms, each of which has a mass m_i much smaller than Planck mass. Entanglement is essential, so as to speed up the rate of spontaneous collapse (same effect as amplification of spontaneous collapse in collapse models). For instance, if $\psi_{A1,2,3,...N}$ and $\psi_{B1,2,3,...N}$ are two eigenstates of the quantum gravitational system Hamiltonian at equilibrium (Level I), then the entangled state $\psi_{A1,2,3,...N} + \psi_{B1,2,3,...N}$ is also an allowed state, but because N is extremely large, the superposition is rapidly lost, and one gets a macroscopic state. Because of entanglement, collapse of any one STM atom collapses them all. In particular, the Schwarzschild black hole of mass M results from one such process, where all the entangled atoms collapse to the centre of the black hole, as described by Eqn. (6) above, and the sum of the masses of the atoms is M. We recall that an STM atom is nothing but an elementary particle (fermionic) which at Level 0 is inseparable from its non-commutative geometry (bosonic). Spontaneous localisation at Level I separates the fermionic part (which collapses to the centre of the black hole) from its bosonic part, which does not collapse, thus becoming the gravitational field of the black hole at Level III.

The entropy of the black hole results from the microstates of the STM atoms. The calculation follows the methods of standard statistical mechanics (Chapter 4 of QTEP) to compute the entropy given by Eqn. (3) above, at equilibrium. As shown in QTEP, the equilibrium phase space distribution is obtained by maximising the entropy, subject to the constraints that ρ is normalised, and the canonically averaged trace Hamiltonian $\mathbf{H}$,

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

the averaged Adler-Millard charge $\tilde{C}$, and the averaged trace fermion number $\mathbf{N}$ are all conserved. These constraints are implemented by introducing the Lagrange multipliers $\tilde{\tau}$ for $\mathbf{H}$, the matrix $\tilde{\lambda}$ for $\tilde{C}$, and η for $\mathbf{N}$. Our $\tilde{\tau}$ is the same as the τ in QTEP; we have already used up the symbol τ for Connes time. The Lagrange multiplier $\tilde{\tau}$ has dimensions of (1/Energy), and is the analog of inverse temperature, $\beta = 1/k_B T$, in ordinary statistical mechanics. η is the analog of the chemical potential, whereas $\tilde{\lambda}$ has no analog in conventional statistical mechanics. As we will justify below, we will set $\tilde{\tau}$ to the Planck energy scale; the other two Lagrange multipliers will not concern us in the present paper, and the question of their relevance for the calculation of black hole entropy is left for a future investigation.

In III, we have computed the trace Hamiltonian for our fundamental action for the STM atom. Since the Lagrangian does not depend explicitly on q, it coincides with the Hamiltonian. As shown in III, the trace Hamiltonian for a single STM atom is

$$\mathbf{H} = \mathrm{Tr}[p_F \dot{q}_F] + \mathrm{Tr}[p_B \dot{q}_B] - \mathrm{Tr}\ \mathcal{L} \tag{10}$$

which becomes, after substituting for momenta and the Lagrangian,

$$\mathbf{H} = \mathrm{Tr}\left[\frac{a}{2}(\dot{q}_B + \beta_1 \dot{q}_F)(\dot{q}_B + \beta_2 \dot{q}_F)\right] \tag{11}$$

and in terms of the momenta

$$\mathbf{H} = \mathrm{Tr}\frac{2}{a}\left[(p_B\beta_1 - p_F)(\beta_2 - \beta_1)^{-1}(p_B\beta_2 - p_F)(\beta_1 - \beta_2)^{-1}\right] \tag{12}$$

For many atoms, we simply sum over the corresponding Hamiltonians of the above form, one for each atom. It is clear that upon spontaneous localisation of a large number of STM atoms, their net trace Hamiltonian will reach the same classical limit as the trace Lagrangian; the latter limit having been shown in Eqn. (6) above. Thus we conclude the important result that, upon spontaneous localisation,

$$\mathbf{H} \longrightarrow \frac{1}{\tau_{Pl}} \int d^4x\ \sqrt{g}\ \left[\frac{c^4}{16\pi G}R + c^2\ \sum_i m_i \delta^3(\mathbf{x} - \mathbf{x_0})\right] \tag{13}$$

which will be crucial below.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

The equilibrium phase space density distribution is given, as in QTEP, by

$$\rho = Z^{-1} \exp\left(-\mathrm{Tr}\tilde{\lambda}\tilde{C} - \tilde{\tau}\mathbf{H} - \eta\mathbf{N}\right) \tag{14}$$

$$Z = \int d\mu \, \exp\left(-\mathrm{Tr}\tilde{\lambda}\tilde{C} - \tilde{\tau}\mathbf{H} - \eta\mathbf{N}\right) \tag{15}$$

$\tilde{\lambda}$ is a traceless operator parameter, and when $\tilde{C}$ has a self-adjoint part, as is true in our case, $\mathrm{Tr}\tilde{\lambda}\tilde{C}$ in the above equation is modified as

$$\mathrm{Tr}\tilde{\lambda}\tilde{C} \to \mathrm{Tr}\tilde{\lambda}^{\mathrm{sa}}\tilde{C}^{\,\mathrm{sa}} + \mathrm{Tr}\tilde{\lambda}^{\mathrm{asa}}\tilde{C}^{\,\mathrm{asa}} \tag{16}$$

with the superscripts "sa" and "asa" denoting self-adjoint and anti-self adjoint parts respectively. The partition function should be dimensionless; so we divide it by a fiducial volume element ΔV^N in phase space:

$$Z = \frac{1}{\Delta V^N} \int d\mu \, \exp\left(-\mathrm{Tr}\tilde{\lambda}\tilde{C} - \tilde{\tau}\mathbf{H} - \eta\mathbf{N}\right) \tag{17}$$

The entropy S_E is given by

$$\frac{S_E}{k_B} = \log Z - \mathrm{Tr}\tilde{\lambda}\frac{\partial \log Z}{\partial \tilde{\lambda}} - \tilde{\tau}\frac{\partial \log Z}{\partial \tilde{\tau}} - \eta\frac{\partial \log Z}{\partial \eta} \tag{18}$$

(We will suppress k_B in subsequent expressions). We will now estimate the partition function for a large number N of STM atoms which constitute the spontaneously collapsed black hole of mass M. As we have argued in III, spontaneous localisation of the (fermionic part of) an STM atom occurs to one or the other eigenvalues of D_B; equivalently one or the other eigenvalues of the Hamiltonian for the atom. This allows us to express the form of Z, by labelling various contributing terms by the corresponding eigenvalue of the Hamiltonian. Assuming the eigenvalues to be discrete (this assumption is not compulsory; it only makes representation simpler), and ignoring for now the contributions from the Adler-Millard charge and from the trace fermion number, we assume that the partition function for the n-th atom can be written as

$$Z_n = \sum_i \exp(-\tilde{\tau}\mathbf{H}_{ni}) \tag{19}$$

where $\mathbf{H}_{ni}$ is the contribution from the i-th eigenvalue of the n-th atom. The transition from (17) to this form of Z_n above involves a transition from Grassmann integration to an ordinary sum. How this comes about, remains to be proved. The full partition function is the product of the partition functions of the individual STM atoms:

$$Z = Z_1 \times Z_2 \times ..... \times Z_n = \left[\sum_i \exp(-\tilde{\tau}\mathbf{H}_{1i})\right] \times \left[\sum_i \exp(-\tilde{\tau}\mathbf{H}_{2i})\right] \times ..... \times \left[\sum_i \exp(-\tilde{\tau}\mathbf{H}_{Ni})\right] \tag{20}$$

This expression for the partition function can in principle now be used in Eqn. (18); again retaining only the first and third terms in (18), to get

$$S_E = \log Z - \tilde{\tau}\frac{\partial \log Z}{\partial \tilde{\tau}} \tag{21}$$

with Z given as above.

It suffices to consider first the contribution to entropy from Z_n of the n-th atom, and then multiply by the total number of atoms, to get the net entropy of the black hole. The dominant contribution comes from the term

$$-\tilde{\tau}\frac{\partial \log Z_n}{\partial \tilde{\tau}} = \tilde{\tau}\ Z^{-1} \sum_i \mathbf{H}_{ni} \exp(-\tilde{\tau}\mathbf{H}_{ni}) \tag{22}$$

We now note that the underlying matrix dynamics of the STM atoms at Level 0 takes place at the Planck scale. Moreover, the emergence of quantum theory at Level I takes place only on time scales much larger than Planck time. This is confirmed also in QTEP, where in Section. 5.1 Adler writes, and we quote: "We identify the time-scale τ and mass τ^{-1} with the "fast" or "high" physical scale given by the Planck scale, and we assume that the underlying theory develops a mass hierarchy, so that observed physics corresponds to "slow" components ....that are very slowly varying in comparison to time τ". Thus, analogously, in our case too, we set $\tilde{\tau} = \tau_{Pl}$. Since the $\mathbf{H}_{ni}$ are much smaller than $1/\tilde{\tau}_{Pl}$ the exponent above can be set to unity to a very good approximation, and we get the contribution to the

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

entropy from the n-th atom to be, with N_0 being the number of states,

$$S_{En} = \tau_{Pl} N_0^{-1} \sum_i \mathbf{H}_{ni} = \tau_{Pl} N_0^{-1} \times \frac{\hbar}{\tilde{\tau}_{Pl}} Tr L_p^2 D_n^2$$
$$= \frac{1}{N_0} \int d^4x \, \sqrt{g} \left[\frac{c^4}{16\pi G} R + c^2 \sum_i m_i \delta^3(\mathbf{x} - \mathbf{x_0}) \right] \quad (23)$$

The net entropy is NS_{En}, with N being the number of STM atoms, and *if we assume* that N equals N_0 to a very high accuracy, the black hole entropy is then given by

$$S_E = \int d^4x \, \sqrt{g} \left[\frac{c^4}{16\pi G} R + c^2 \sum_i m_i \delta^3(\mathbf{x} - \mathbf{x_0}) \right] \quad (24)$$

Next, we recall that we are working in a Euclidean 4-d space-time. Now, in this case, it is known from the literature that for a Schwarzschild black hole sourced by a point mass source at the centre, the Euclidean gravitational action is precisely equal to one-fourth the black-hole area: see Eqn. (2.7) in [5]

$$\int d^4x \, \sqrt{g} \left[\frac{c^4}{16\pi G} R \right] = \frac{\text{Area}}{4L_P^2} \quad (25)$$

In writing this relation it has been assumed that the Euclidean time t_E has been compactified along a circle and is related to the Hawking temperature of the black hole as $2\pi t_E = \hbar / k_B T_H = 8\pi GM/c^3$. This immediately implies that the gravitational contribution to the entropy is the same as in the Bekenstein-Hawking formula.

Strictly speaking, we should be able to derive the time-temperature relation in our theory, as well as prove the existence of Hawking radiation, without having to appeal to Level II physics - i.e. quantum field theory in curved space-time. In fact, we can infer Hawking radiation in our approach, on the basis of the fluctuation-dissipation theorem. The formation of a black hole of mass M from a large number of entangled STM atoms, is a dissipative process (drag) which takes the atoms away from equilibrium, via spontaneous localisation. Hawking radiation (fluctuation) is the opposite of spontaneous localisation, and it takes the STM atoms back towards equilibrium, via black hole evaporation. Assuming that the black hole emits black-body radiation at a temperature T, we can heuristically relate T to M

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 8 November 2019 doi:10.20944/preprints201911.0096.v1

through the Einstein-Smoluchowski relation:

$$D = \mu k_B T \tag{26}$$

with D being the (quantum) diffusion constant, and the mobility μ is the ratio of the particle's terminal drift velocity to an applied force. Naively, a Newtonian estimate of the force on a test particle of unit mass would be $GM/R^2 = c^4/4GM$ which as is known, is also the surface gravity of the black hole. Assuming that the terminal drift speed is c, and assuming that the diffusion constant per unit mass is $\hbar$, we get the Hawking temperature of the black hole to be $\hbar c^3/4GM$ which matches with the correct result up to a factor of 2π.

There is an instructive analogy between the fluctuation-dissipation theorem when applied to the Brownian motion of a particle in a fluid, and when applied to a black hole resulting from spontaneous collapse of STM atoms. The STM atoms are like the molecules of the fluid. The black hole is like the particle. Molecules move around in physical space, and interact via collisions. STM atoms evolve in Hilbert space and interact via entanglement. When the particle moves through the fluid, it experiences a drag, because of the dissipative frictional force caused by its collision with the molecules. A quantitative measure of the drag is the mobility μ. When a highly entangled state of many STM atoms evolves unitarily in Hilbert space, it experiences a non-unitary stochastic noise (see III) which causes it to spontaneously collapse to a black hole. This non-unitary noise is the drag / dissipation whose strength (in collapse models) is measured by the collapse rate $M\gamma$, where γ is the spontaneous collapse rate per unit mass. In the case of the fluid, the particle, if initially at rest, experiences Brownian motion (fluctuation) because of random collisions with the molecules. Thus the dissipation and fluctuation have the same origin - collision with molecules of the fluid - and are hence related to each other. Correspondingly, in the case of matrix dynamics, the highly entangled state (i.e. the black hole), if not changing with Connes time (i.e. at rest) in Hilbert space, experiences the self-adjoint part of the stochastic noise (see III), which is then possibly the cause of black hole evaporation. Thus, both Hawking radiation and spontaneous collapse have the same origin - the stochastic (real + imaginary) noise which results from the STM atoms deviating from equilibrium - and hence the two are related.

III. CONCLUDING REMARKS

Spontaneous localisation was originally proposed as an ad hoc but falsifiable mechanism for explaining the puzzling absence of position superpositions in the macroscopic world, and to understand the quantum-to-classical transition. In order to overcome the phenomenological nature of the proposal, the theory of trace dynamics derives quantum field theory, as well as spontaneous localisation, as the statistical thermodynamics of an underlying classical matrix dynamics. However, trace dynamics is formulated on a classical Minkowski space-time and does not include gravity. We have exploited the spectral action in non-commutative geometry, as well as the absolute Connes time parameter, to incorporate gravity into trace dynamics. This has led us to the present quantum theory of gravity, in which spontaneous localisation is an inevitable consequence of the underlying non-commutative gravity. Black holes arise from spontaneous localisation of many entangled STM atoms, and in the present paper we have attempted to estimate black hole entropy from the microstates of the STM atoms. This has involved certain assumptions, which we hope to address rigorously in future work.

Another line of reasoning also suggests that spontaneous localisation is an inevitable consequence of quantum gravity. The absence of macroscopic position superpositions of material bodies is a pre-requisite for the existence of classical space-time (manifold as well as geometry, this being a consequence of the Einstein hole argument). If we assume space-time to be emergent from quantum gravity, then the latter must also explain why macroscopic bodies do not exhibit position superpositions. This is a challenging task for theories of quantum gravity which are arrived at by quantizing classical gravity (unless one invokes the many-worlds interpretation). That essentially leaves spontaneous localisation as the only option for achieving a classical universe from quantum gravity. The same process which solves the quantum measurement problem, also explains the emergence of classical space-time. We can say that the quantum measurement problem is solved by quantum gravity. The subject of quantum foundations is not disconnected from quantum gravity; rather the two are directly related.

REFERENCES

[1] Tejinder P. Singh, "Proposal for a new qantum theory of gravity," Zeitschrift für Naturforschung A **74**, 617, DOI: https://doi.org/10.1515–zna–0079, arXiv:1903.05402 (2019).

[2] Tejinder P. Singh, "Proposal for a new quantum theory of gravity ii: equation of motion for an atom of space-time-matter," Zeitschrift für Naturforschung A **74**, DOI: 10.1515/zna–2019–0211 arXiv:1906.08248 (2019).

[3] Palemkota Maithresh and Tejinder P. Singh, "Proposal for a new qantum theory of gravity iii: Equations for quantum gravity, and the origin of spontaneous localisation," Zeitschrift für Naturforschung A **DOI:10.1515/zna-2019-0267 arXiv:1908.04309** (2019).

[4] Stephen L. Adler, *Quantum theory as an emergent phenomenon* (Cambridge University Press, 2004).

[5] Carlos Castro, "The euclidean gravitational action as black hole entropy, singularities, and space-time voids," J. Math. Phys. **49**, 042501 DOI: 10.1142/S0217751X08039669 (2008).