

## Article

# Directional Modulation Technique Using a Polarization Sensitive Array for Physical Layer Security Enhancement

Wei Zhang <sup>1</sup>, Bin Li <sup>1,\*</sup>, Mingnan Le <sup>1</sup>, Jun Wang <sup>1</sup>, and Jinye Peng <sup>1</sup>

<sup>1</sup> School of Information Science and Technology, Northwest University, Xi'an 710127, China;  
zhang\_wei@nwu.edu.cn (W.Z.); lib@nwu.edu.cn (B.L.); lemingnan@nwu.edu.cn (M.L.); jwang@nwu.edu.cn  
(J.W.); pjyxida@nwu.edu.cn (J.P.)  
\* Correspondence: lib@nwu.edu.cn; Tel.: +86-15209210549

**Abstract:** Directional modulation (DM), as an emerging promising physical layer security (PLS) technique at the transmitter side with the help of an antenna array, has developed rapidly over decades. In this study, a DM technique using a polarization sensitive array (PSA) to produce the modulation with different polarization states (PSs) at different directions is investigated. PSA can be employed for more effective DM for an additional degree of freedom provided in the polarization domain. The polarization information can be exploited to transmit different data streams simultaneously at the same directions, same frequency, but with fixed different PSs in the desired directions to increase the channel capacity, and with random PSs off the desired directions to enhance PLS. The proposed method has the capability of concurrently projecting independent signals into different specified spatial directions while simultaneously distorting signal constellation in all other directions. Mathematical analysis and design examples for single-beam and multi-beam DM systems are presented. Simulation results demonstrate that the proposed method is more effective for PLS and the channel capacity is significantly improved compared with conventional antenna arrays.

**Keywords:** directional modulation; physical layer security; polarization sensitive array; polarization state.

## 1. Introduction

Wireless communication technique in many applications to both civil and military fields is attractive mainly due to its inherent flexibility, low installation cost and scalable nature. However, the broadcast nature of the wireless media makes the confidential information transmitted wirelessly vulnerable to interception by malicious eavesdroppers. Therefore, information security is emerging into a hot research field. Traditionally, the security problem of wireless communications has been handled by encrypting confidential messages at the higher layer and conveying the secret keys to the legitimate users (LUs) via secured transmissions. However, malicious eavesdroppers can still decipher the complex encryption mechanism, capture the confidential information and then decode it with large amount of computation resources. As a result, researchers have turned their interests towards physical layer security (PLS) [1]. PLS enables wireless communications to exploit the properties of physical layer to deliver the intact information to LUs, while simultaneously destroy the confidential information potentially intercepted by illegitimate users.

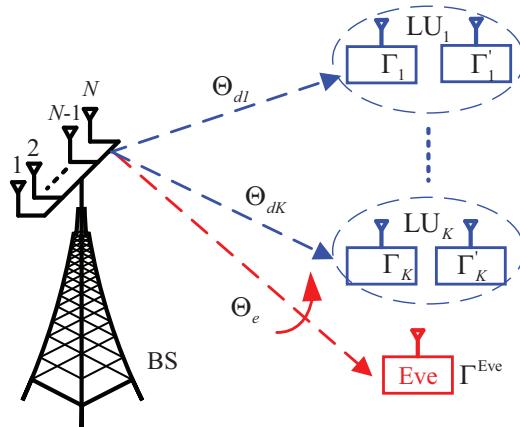
In recent ten years, directional modulation (DM), as an emerging promising keyless PLS technology for wireless communication systems, has attracted lots of attention of researchers. The fundamental concept of DM was first introduced in [2], which implies that the modulation happened at the antenna level, instead of at the baseband. In [2-5], DM transmitters using passive parasitic elements, which rely on the effect of near-field coupling, were described in detail. The complex interactions in the near-field and their spatial dependent transformation into the far-field make the DM design process extremely complicated. The DM technique using actively driven antenna arrays with reconfigurable

phase shifters [6-8] and radiators [9-10] was proposed for simplification of DM signal synthesis. Based on the same DM structures, H. Z. Shi conducted a detailed study of system parameters, such as element spacing [11], the quantization of phase shifter [12], and the mode of active component [13]. In [14], a dual-beam DM technique was introduced, where the I and Q signals are transmitted by two different antennas, such as two four-element arrays or two corner reflector antennas. Antenna subset modulation (ASM) was introduced as a DM synthesis method in [15], and two extensions of the method based on the spread-spectrum DM structure in [4] and the linear sparse arrays (LSAs) in [16] were further developed. A DM transmitter using a four-dimensional (4-D) antenna array was proposed by utilizing the time modulation technique in [17]. A DM transmitter constructed using a circular array was described in [18] to enhance secrecy performance with the help of a Fourier transform network. In [19], a synthesis-free DM transmitter was realized by using retrodirective array.

In addition to the above-mentioned single-beam DM approaches with only one pre-specified spatial direction (i.e., receiver), the multi-beam DM synthesis methods with multiple desired directions (i.e., destinations) have also been extensively investigated in [20-28]. The orthogonal vector approach for synthesis of multi-beam DM transmitters based on artificial noise (AN) was proposed in [20], which is capable of synchronously projecting multiple independent data streams into different specified destinations while concurrently scrambling the signals leaking out in other undesired directions. The authors of [21] derived the array weights using the least-norm solution and directionally modulated symbols towards to two different directions in a multi-user multiple-input multiple-output (MU-MIMO) system. Furthermore, the authors in [22] utilized the dispersive nature of the channel (i.e., multi-path) to provide a location-based secure communication link. The scheme shows the ability to highly degrade the eavesdropper channel, even for the worst-case scenarios. The work in [23] demonstrated a robust synthesis method using optimization algorithms for a multi-beam DM system with imperfect estimations of desired directions. The AN was also applied in [24] and [25] to achieve multi-beam DM systems utilizing precoding and iterative convex optimization, respectively. In [26], ASM was used efficiently for multidirectional broadcasting communication and maintaining its inherent security. A secure multiple access scheme was put forward in [27], which exploits the multi-path structure of the channel to create a multi-user interference environment. The authors in [28] further proposed the zero-forcing (ZF) based approach for multi-beam DM synthesis with the help of AN.

However, the polarization information is not taken into account for the works mentioned above. As we know, the polarization state (PS), as a source of separation, can be utilized to enhance the data rate to make spectrum efficient. For example, in [29], the authors used the antennas to produce the PSs, then combined the PSs with amplitude-phase modulation (APM) together with confidential information, which further enhanced the data rate on the basis of the traditional APM. Similarly, a spectrum efficient polarized phase shifting keying/quadrature amplitude modulation (PSK/QAM) scheme in the wireless depolarized channel was proposed in [30]. The constellation rotation (CR) and weighted fractional Fourier transform (WFRFT) were combined for secure transmission in polarization modulation (PM) based dual-polarized satellite communication in [31]. The authors in [32] proposed a dual-polarized phased array (PA) based PM method to enhance PLS in millimeter-wave (mm-wave) communication systems. In order to increase the secure transmission performance of a dual-polarized satellite MIMO system, a directional PM scheme using a four-element array was put forward in [33], which can be viewed as the first amalgamation of DM and PM. Recently, a combination of DM and polarization design using crossed dipole arrays was put forward in [34]. Nevertheless, the polarization information was directionless for the works mentioned above.

Inspired by the pioneering researches, especially in [22] and [34], a multi-beam DM technique using directional polarization information based on a polarization sensitive array (PSA) is proposed in this paper. The spatial domain and polarization domain are simultaneously introduced into the proposed multi-beam DM system. That can further improve the traditional multi-beam DM systems' spectrum efficiency. In the meantime, that also can provide a more secure communication link for



**Figure 1.** System model.

transmitted data streams. The key contributions of this paper conclude: (1) We realize the multi-beam DM synthesis using a precoding matrix to send independent data streams with different PSs using a PSA to further increase the channel capacity; (2) The proposed method utilizes the directional polarization formation to enhance PLS; (3) We carry out an analysis of the security performance and provide the design examples to verify the effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 introduces the system model and polarized beamforming. In Section 3, the principle of multi-beam DM technique a PSA based is elaborated from the sampling perspective and the signal processing perspective, respectively. Next, the security performance for the proposed methods is analyzed in Section 4. Then, Section 5 provides design examples, simulation results and discussions. Finally, Section 6 concludes the paper.

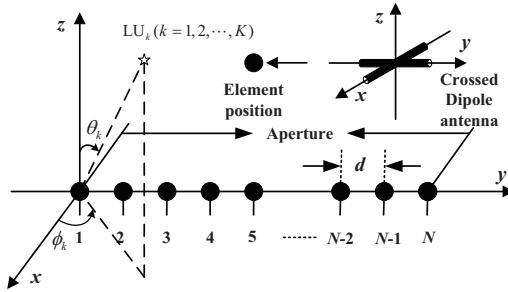
Throughout the paper, the following notations will be used:  $(\cdot)^T$ ,  $(\cdot)^H$ , and  $(\cdot)^+$  designate transpose, complex conjugate transpose, and the Moore-Penrose pseudo inverse, respectively;  $|\cdot|$ ,  $\|\cdot\|_2$ ,  $Q(\cdot)$ , and  $\Xi(\cdot)$  represent modulus,  $\ell_2$ -norm, the scaled complementary error function, and the phase acquisition function;  $\prod$  and  $\sum$  denote quadrature and sum operation; Operator " $\otimes$ " denotes the Kronecker product;  $\mathbf{I}_K$  indicates the identity matrix with size  $K \times K$ .

## 2. System Model and Polarized Beamforming

### 2.1. System Model

The system model is shown in Fig. 1. As depicted in Fig. 1, we consider a multiple-input single-output (MISO) system with  $N$  transmitting antennas for the base station (BS), (i.e., source transmitter) and one single receiving antenna for an LU or an Eavesdropper (Eve). Meanwhile, the BS sends confidential messages to  $K$  LUs along the line-of-sight (LoS) paths in the presence of an Eve. Here, the BS has *a-priori* information about the directions of the  $K$  LUs, but not of the potential Eve. Specifically, in order to utilize the polarization information, a PSA is adopted for the proposed system. However, the idea proposed in this paper can also be extended to a MIMO system for multiple LUs and Eves.

Unless note otherwise, a narrowband linear PSA is assumed for the BS in this paper. A PSA structure with  $N$  crossed dipole antennas is shown in Fig. 2. However, other polarization sensitive antennas also work. The  $N$  polarization sensitive antennas are evenly distributed with an adjacent spacing distance  $d = \lambda/2$  to avoid spatial aliasing. For each polarization sensitive antenna, there are two orthogonally oriented dipoles. One is parallel to  $x$ -axis and the other is parallel to  $y$ -axis. The spacing from the first antenna to itself and its subsequent antennas is represented by  $d_n$  for  $n = 1, 2, \dots$ ,



**Figure 2.** A PSA structure.

$N$ . Also shown is a desired LU with its accurate DOA (direction of arrival) defined by the elevation angles  $\theta \in [-\pi/2, \pi/2]$  and the azimuth angles  $\phi \in [-\pi, \pi]$ .

## 2.2. Polarized Beamforming

In this system, there are  $K$  different desired directions, and  $1 \leq K < N$ . A plane-wave model is assumed, i.e., the LUs are located in the far-field region. The spatial steering vector for the given array geometry is expressed as

$$\mathbf{s}_{spa}(\theta, \phi) = [e^{-j\beta d_1 \sin \theta \sin \phi}, e^{-j\beta d_2 \sin \theta \sin \phi}, \dots, e^{-j\beta d_N \sin \theta \sin \phi}]^T, \quad (1)$$

where  $\beta = 2\pi/\lambda$  is the intrinsic propagation constant, the parameter  $\lambda$  is the wavelength at the carrier frequency of interest, and  $d_n$  is given by

$$d_n = (n-1)d = \frac{(n-1)\lambda}{2}, \quad n = 1, 2, \dots, N. \quad (2)$$

Then, the spatial steering vector for the array above can be given by

$$\mathbf{s}_{spa}(\theta, \phi) = [1, e^{-j\pi \sin \theta \sin \phi}, \dots, e^{-j(N-1)\pi \sin \theta \sin \phi}]^T. \quad (3)$$

Furthermore, for a polarization sensitive antenna, the spatial-polarization coherent vector contains spatial information and polarization information of the signal. This vector is given by [35]

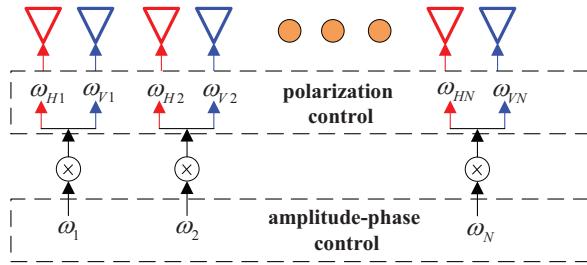
$$\begin{aligned} \mathbf{s}_{pol}(\theta, \phi, \gamma, \eta) &= \begin{bmatrix} -\sin \phi \cos \gamma + \cos \theta \cos \phi \sin \gamma e^{j\eta} \\ \cos \phi \cos \gamma + \cos \theta \sin \phi \sin \gamma e^{j\eta} \end{bmatrix} \\ &= \begin{bmatrix} -\sin \phi & \cos \theta \cos \phi \\ \cos \phi & \cos \theta \sin \phi \end{bmatrix} \begin{bmatrix} \cos \gamma \\ \sin \gamma e^{j\eta} \end{bmatrix} \\ &= \begin{bmatrix} s_{pol,H}(\theta, \phi, \gamma, \eta) \\ s_{pol,V}(\theta, \phi, \gamma, \eta) \end{bmatrix}, \end{aligned} \quad (4)$$

where  $\gamma \in [0, \pi/2]$  represents the auxiliary polarization angle and  $\eta \in [-\pi, \pi]$  is the polarization phase difference.

The polarization constellation point is given by

$$\mathbf{P}_n = \begin{bmatrix} \cos \gamma_n \\ \sin \gamma_n e^{j\eta_n} \end{bmatrix} = \begin{bmatrix} w_{Hn} \\ w_{Vn} \end{bmatrix}. \quad (5)$$

Each  $\mathbf{P}_n$  denotes a polarization state.



**Figure 3.** A schematic diagram of the amplitude-phase and polarization independent control.

For the sake of description and analysis, we first define

$$(\theta, \phi, \gamma, \eta) \stackrel{\Delta}{=} (\Theta, \Gamma), \quad (6)$$

where  $\Theta$  and  $\Gamma$  denote spatial information and polarization information, respectively. Then, the PSA model can be split into two sub-arrays structure, one parallel to each axis. Thus, the steering vectors of the two sub-arrays are given by

$$\mathbf{s}_H(\Theta, \Gamma) = s_{pol,H}(\Theta, \Gamma) \mathbf{s}_{spa}(\Theta, \Gamma), \quad (7)$$

and

$$\mathbf{s}_V(\Theta, \Gamma) = s_{pol,V}(\Theta, \Gamma) \mathbf{s}_{spa}(\Theta, \Gamma), \quad (8)$$

respectively.

Finally, the overall steering vector of the PSA is given by

$$\begin{aligned} \mathbf{s}(\Theta, \Gamma) &= \mathbf{s}_{pol}(\Theta, \Gamma) \otimes \mathbf{s}_{spa}(\Theta, \Gamma) \\ &= \begin{bmatrix} \mathbf{s}_H(\Theta, \Gamma) \\ \mathbf{s}_V(\Theta, \Gamma) \end{bmatrix}. \end{aligned} \quad (9)$$

The beam response of the PSA is given by

$$p(\Theta, \Gamma) = \mathbf{w}^H \mathbf{s}(\Theta, \Gamma), \quad (10)$$

where  $\mathbf{w}$ , a  $2N \times 1$  vector denoting the overall complex valued weight coefficients of the full array, is given by

$$\mathbf{w} = [w_{1,H}, \dots, w_{N,H}, w_{1,V}, \dots, w_{N,V}]^T. \quad (11)$$

Fig. 3 shows a schematic diagram of the amplitude-phase and polarization independent control algorithm for a PSA. For the polarization control portion, each antenna element is separately weighted firstly. After the element weighting is completed, the array factor is controlled by the classical array factor control method. Then, we have

$$\begin{cases} w_{n,H} = w_n \cdot w_{Hn} \\ w_{n,V} = w_n \cdot w_{Vn} \end{cases}, \quad n = 1, 2, \dots, N. \quad (12)$$

### 3. Principle of Multi-beam DM Technique a PSA based

In this section, we formulate the design to achieve directional modulation for  $K$  desired directions for the same data stream or independent data streams. Aiming at each expected direction, different PSs can be adopted to convey independent data streams simultaneously based on a PSA. We will

propose two methods to achieve multi-beam DM synthesis from the sampling perspective in spatial and polarization domain and from the signal processing perspective, respectively.

### 3.1. Principle of Multi-beam DM based on a PSA from the Sampling Perspective

The essence of the DM design for a PSA is to find a set of weight coefficients that give the directional responses. In doing so, a certain constellation with low symbol error rate (SER) can be realized in the desired directions, while the constellation will be scrambled in other undesired directions to result in high SER for illegal users.

Take  $M$ -ary signaling, like multiple phase shift keying (MPSK) for example, there are  $M$  sets of desired array response across one diversity polarization channel in one anticipant direction. Therefore, for DM across one polarization diversity channel in  $K$  different desired directions, i.e., there exist  $K$  beams, the total number of sets of desired response  $p_i(\Theta, \Gamma)$  is given by

$$T = \prod_{j=1}^K M_j, \quad (13)$$

where  $M_j$  denotes that  $M_j$ -PSK signaling is transmitting in the  $j$ th direction. Furthermore, each desired response  $p_i(\Theta, \Gamma)$ ,  $i = 1, 2, \dots, T$ , can be viewed as a function of  $(\Theta, \Gamma)$ .

From the spatial domain prospective, each desired response  $p_i(\Theta, \Gamma)$  can be split into two regions: the spatial mainlobe and the spatial sidelobe according to the azimuthal points of interest or not. Without loss of generality, we assume that  $r$  ( $r \geq 1$ ) points are sampled in each desired direction and  $R$  ( $R \gg 1$ ) points in the sidelobe region. In order to narrow the information beam width to enhance the security,  $r$  is always set to be 1, and  $R$  is usually set to be a large number.

Meanwhile, from the polarization domain prospective, each desired response  $p_i(\Theta, \Gamma)$  can also be split into two regions: the polarization mainlobe and the polarization sidelobe according to the polarization points of interest or not. Like the spatial sampling, we designate fixed PSs for different polarization diversity channels, for all sampled responses in the desired directions. While, the expected responses at different sidelobe directions with different PSs are generated randomly.

Then, the corresponding responses in the mainlobe range and sidelobe region are given by

$$\mathbf{p}_{i,ML} = [p_i(\Theta_1, \Gamma_1), p_i(\Theta_2, \Gamma_2), \dots, p_i(\Theta_K, \Gamma_K)], \quad (14)$$

and

$$\mathbf{p}_{i,SL} = [p_i(\Theta_{K+1}, \Gamma_{K+1}), p_i(\Theta_{K+2}, \Gamma_{K+2}), \dots, p_i(\Theta_{K+R}, \Gamma_{K+R})], \quad (15)$$

respectively.

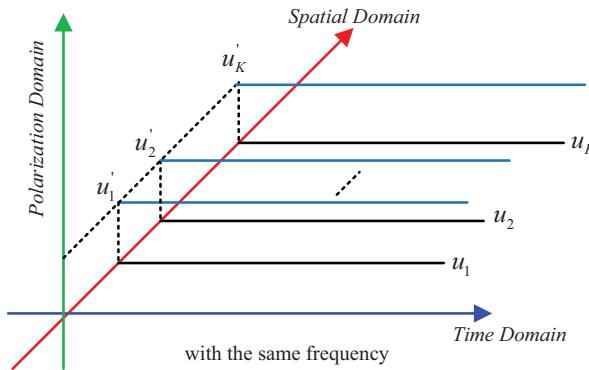
Moreover, before proceeding to the next step, we define a  $2N \times Kr$  matrix  $\mathbf{S}_{ML}$  and a  $2N \times R$  matrix  $\mathbf{S}_{SL}$ , given by

$$\mathbf{S}_{ML} = [\mathbf{s}(\Theta_1, \Gamma_1), \mathbf{s}(\Theta_2, \Gamma_2), \dots, \mathbf{s}(\Theta_K, \Gamma_K)], \quad (16)$$

and

$$\mathbf{S}_{SL} = [\mathbf{s}(\Theta_{K+1}, \Gamma_{K+1}), \mathbf{s}(\Theta_{K+2}, \Gamma_{K+2}), \dots, \mathbf{s}(\Theta_{K+R}, \Gamma_{K+R})], \quad (17)$$

respectively.



**Figure 4.** A schematic diagram of the joint time-polarization-spatial domain.

Next, for the  $i$ th constellation point, its corresponding weight coefficients can be obtained by

$$\begin{aligned} \min \quad & \left\| \mathbf{p}_{i,SL} - \mathbf{w}_i^H \mathbf{S}_{SL} \right\|_2 \\ \text{s.t.} \quad & \mathbf{p}_{i,ML} = \mathbf{w}_i^H \mathbf{S}_{ML}. \end{aligned} \quad (18)$$

The objective function enables a minimum difference between the intended and designated responses in the sidelobe for the different polarization diversity channels. And the constraint condition in Eq. (18) ensures to maintain desired polarization constellation points in desired directions. In order to enhance the security performance of the system, we have to distort the constellation points in the sidelobe. The amplitude, phase and the polarization information of the desired response at different directions in the sidelobe are all generated at random for every constellation point.

The optimization problem in Eq. (18) can be solved using the method of Lagrange multipliers. The optimal solution for the weight coefficient vector  $\mathbf{w}_i$  can be expressed as

$$\mathbf{w}_i = \mathbf{R}^{-1} (\mathbf{S}_{SL} \mathbf{p}_{SL,i}^H - \mathbf{S}_{ML} (\mathbf{S}_{ML}^H \mathbf{R}^{-1} \mathbf{S}_{ML})^{-1} (\mathbf{S}_{ML}^H \mathbf{R}^{-1} \mathbf{S}_{SL} \mathbf{p}_{SL,i}^H - \mathbf{p}_{ML,i}^H)). \quad (19)$$

The proof is seen in Appendix.

### 3.2. Principle of Multi-beam DM based on a PSA from the Signal Processing Perspective

As we all know, besides the carrier's amplitude and phase, the carrier's PS can also be used as the information bearing parameter, which can further improve the conventional DM methods' spectrum efficiency. A schematic diagram of the joint time-polarization-spatial domain is shown in Fig. 4. Each direction can have two desired data streams with the same modulation mode or not by using different PSs, and also sharing the same resources of time slots, frequency bands. Therefore, the spectrum efficiency is significantly improved in our methods.

Due to the introduction of polarization information, we can consider a scenario where the BS is trying to transmit confidential messages to  $K$  different directions, and each direction have two LUs with different PSs, as shown in Fig.1. Assuming that the BS has *a-prior* knowledge of the desired directions and polarization information of all LUs. Before transmitting, the data streams should be processed with a pre-coding matrix  $\mathbf{P}$  to match the  $N$  transmit antennas. In order to obtain the precoding matrix  $\mathbf{P}$ , the spatial positions and polarization information of all LUs are specified, the combined set of which can be expressed as

$$(\Theta^{\text{LU}}, \Gamma^{\text{LU}}) = \{(\Theta_1^{\text{LU}}, \Gamma_1^{\text{LU}}), \dots, (\Theta_K^{\text{LU}}, \Gamma_K^{\text{LU}}), (\Theta_1^{\text{LU}}, \Gamma_1'^{\text{LU}}), \dots, (\Theta_K^{\text{LU}}, \Gamma_K'^{\text{LU}})\}, \quad (20)$$

where  $(\Theta_k^{\text{LU}}, \Gamma_k^{\text{LU}})$ ,  $k \in \{1, 2, \dots, K\}$ , is the specified spatial position and polarization information of the  $k$ th LU. Then, the steering vectors of all LUs can compose a steering matrix, i.e.,

$$\mathbf{H}(\Theta^{\text{LU}}, \Gamma^{\text{LU}}) = \{\mathbf{s}(\Theta_1^{\text{LU}}, \Gamma_1^{\text{LU}}), \dots, \mathbf{s}(\Theta_K^{\text{LU}}, \Gamma_K^{\text{LU}}), \mathbf{s}(\Theta_1^{\text{LU}}, \Gamma_1^{\text{LU}}), \dots, \mathbf{s}(\Theta_K^{\text{LU}}, \Gamma_K^{\text{LU}})\}. \quad (21)$$

Using the steering matrix in Eq. (21), the precoding matrix  $\mathbf{P}$  at the BS can be designed as [28]

$$\mathbf{P} = [\mathbf{H}(\Theta^{\text{LU}}, \Gamma^{\text{LU}})]^+ = \mathbf{H}(\Theta^{\text{LU}}, \Gamma^{\text{LU}})[\mathbf{H}^H(\Theta^{\text{LU}}, \Gamma^{\text{LU}})\mathbf{H}(\Theta^{\text{LU}}, \Gamma^{\text{LU}})]^{-1}, \quad (22)$$

which is normalized to the steering matrix, i.e.,

$$\mathbf{H}^H(\Theta^{\text{LU}}, \Gamma^{\text{LU}})\mathbf{P} = \mathbf{I}_{2K}. \quad (23)$$

The radiating signal  $\mathbf{x} = [x_1, \dots, x_N, x'_1, \dots, x'_N]^T$  after precoding for the  $N$  dipole antenna elements can be obtained by

$$\mathbf{x} = \sqrt{P_t}\mathbf{P}\mathbf{u}, \quad (24)$$

where  $\mathbf{u} = [u_1, u_2, \dots, u_K, u'_1, u'_2, \dots, u'_K]^T$  is the transmitting symbol vector by the  $N$  polarization sensitive antennas located at the  $x$ -axis and  $y$ -axis, i.e., the different data streams,  $P_t$  is the total transmitting power.

#### 4. Security Performance Analysis for the Proposed Methods

In this section, we will analyze the security performance for the proposed method in Section III.

##### 4.1. Security Performance Analysis Known Polarization Information

In this part, assuming that the desired receivers and eavesdroppers know the PSs used and the path loss is neglected.

The signal vector transmitted by an  $N$ -element PSA across one polarization diversity channel is given by

$$\mathbf{D}(i) = [d_1(i), d_2(i), \dots, d_K(i)]^T. \quad (25)$$

Through the LoS channel, the received signal  $\mathbf{y}$  is

$$\mathbf{y}(\Theta, \Gamma, i) = \mathbf{H}(\Theta, \Gamma)\mathbf{W}(i)\mathbf{D}(i) + \mathbf{n}, \quad (26)$$

where  $\mathbf{y}$  is a  $K \times 1$  vector representing the received signals,  $\mathbf{H} = [\mathbf{s}_1, \dots, \mathbf{s}_j, \dots, \mathbf{s}_K]^T$  is a  $K \times 2N$  matrix denoting the channel matrix from transmitter to receiver,  $\mathbf{s}_j$  is a  $2N \times 1$  vector denoting the steering vector.  $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_j, \dots, \mathbf{w}_K]$  is a  $2N \times K$  matrix denoting the antenna weights. The variable  $\mathbf{n}$  with distribution  $\mathcal{CN}(0, \sigma^2 \mathbf{I}_K)$  is the normalized additive white Gauss noise (AWGN), where  $\mathcal{CN}$  denotes a complex and circularly symmetric random variable.

Meanwhile, in order to guarantee that the LUs receive the exact information, the  $j$ th user should receive the  $i$ th complex symbol. Then, letting  $\mathbf{x}(i) = [x_1(i), \dots, x_j(i), \dots, x_K(i)]^T$ , and neglecting the noise, we have

$$\mathbf{x}(i) = \mathbf{H}(\Theta, \Gamma)\mathbf{W}(i)\mathbf{D}(i). \quad (27)$$

Because  $K \geq N$ , Eq. (27) is an overdetermined equation with no exact solution. Therefore, we only consider the scenario  $K < N$ , i.e., the number of LUs is less than that of transmit antennas. If

the receivers with the same DOA and polarization parameters want to receive different information, and Eq. (27) becomes a nonuniform equation with no solution. Therefore, the different receivers with different spatial and polarization parameters are requested. The minimum norm solution for Eq. (27), i.e., the signal transmitted by the antenna array, is given by

$$\mathbf{D}(i) = \left( (\mathbf{H}(\Theta, \Gamma) \mathbf{W}(i))^H \mathbf{H}(\Theta, \Gamma) \mathbf{W}(i) \right)^{-1} (\mathbf{H}(\Theta, \Gamma) \mathbf{W}(i))^H \mathbf{x}(i). \quad (28)$$

In an arbitrary desired direction  $\Theta_j$ , ( $j \in \{1, 2, \dots, K\}$ ), the steering vector can be expressed as

$$\mathbf{H}(\Theta_j, \Gamma) = \mathbf{e}_j \mathbf{H}(\Theta, \Gamma), \quad (29)$$

where  $\mathbf{e}_j \in R^{1 \times K}$  denotes a unit vector, the  $j$ th term of which is one.

Then, the received signal for desired receiver is given by

$$y_j(i) = \mathbf{H}(\Theta_j, \Gamma) \mathbf{D}(i) = x_j(i). \quad (30)$$

Therefore, an arbitrary desired receiver can recover the exact confidential information.

For an Eve in the direction  $\Theta_e$  with a polarization sensitive antenna, the steering vector can be written as

$$\mathbf{H}(\Theta_e, \Gamma) = \mathbf{r} \mathbf{H}(\Theta, \Gamma), \quad (31)$$

where  $\mathbf{r} \in R^{1 \times K}$ , which is given by

$$\mathbf{r} = \left( (\mathbf{H}(\Theta, \Gamma) \mathbf{W}(i))^H \mathbf{H}(\Theta, \Gamma) \mathbf{W}(i) \right)^{-1} (\mathbf{H}(\Theta, \Gamma) \mathbf{W}(i))^H \mathbf{H}(\Theta_e, \Gamma). \quad (32)$$

Then, the received signal for undesired receiver is given by

$$y_e(i) = \mathbf{H}(\Theta_e, \Gamma) \mathbf{D}(i) = \mathbf{r} \cdot \mathbf{x}(i). \quad (33)$$

Assume that the Eve expects to intercept the confidential information for the  $l$ th ( $l \in \{1, 2, \dots, K\}$ ) LU. For the Eve, the information of other LUs can be regarded as noise interference or artificial interference. Then, the received signal for the eavesdropper can be rewritten as

$$y_e(i) = r_l \cdot x_l(i) + \sum_{j=1, j \neq l}^K r_j \cdot x_j(i). \quad (34)$$

Obviously, the modulated signal  $x_l(i)$ , that the eavesdroppers intend to crack, is seriously affected by the variables  $r_l$  and  $\sum_{j=1, j \neq l}^K r_j \cdot x_j(i)$ . This results in that Eves cannot demodulate the exact confidential information. Therefore, PLS is enhanced.

#### 4.2. Security Performance Analysis Unknown Polarization Information

Assuming that PM is adopted for information transmission for the  $K$  LUs using a PSA. Then, we can decompose the signal transmitted into two parts in polarization domain: the horizontal component ( $\mathbf{x}_H(i)$ ) and the vertical component ( $\mathbf{x}_V(i)$ ). They are given by

$$\mathbf{x}_H(i) = [\cos \gamma_{1,i}, \cos \gamma_{2,i}, \dots, \cos \gamma_{L,i}]^T, \quad (35)$$

and

$$\mathbf{x}_V(i) = [\sin \gamma_{1,i} e^{j\eta_{1,i}}, \sin \gamma_{2,i} e^{j\eta_{2,i}}, \dots, \sin \gamma_{L,i} e^{j\eta_{L,i}}]^T, \quad (36)$$

respectively.

When the receiver in the direction  $\Theta$  uses a polarization sensitive antenna, the channel matrix is given by

$$\mathbf{h} = \mathbf{s}(\Theta, \Gamma) \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (37)$$

When the receiver in the direction  $\Theta$  uses horizontally polarized antenna, the channel matrix is given by

$$\mathbf{h}_H = \mathbf{s}(\Theta, \Gamma) \otimes \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (38)$$

Similarly, when the receiver in the direction  $\Theta$  uses vertically polarized antenna, the channel matrix is given by

$$\mathbf{h}_V = \mathbf{s}(\Theta, \Gamma) \otimes \begin{bmatrix} 0 & 1 \end{bmatrix}. \quad (39)$$

Therefore, the signal vector transmitted can be rewritten as

$$\mathbf{D}(i) = \mathbf{D}_H(i) \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \mathbf{D}_V(i) \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (40)$$

$$\mathbf{D}_H(i) = \left( (\mathbf{H}_H W(i))^H \mathbf{H}_H W(i) \right)^{-1} (\mathbf{H}_H W(i))^H \mathbf{x}_H(i), \quad (41)$$

$$\mathbf{D}_V(i) = \left( (\mathbf{H}_V W(i))^H \mathbf{H}_V W(i) \right)^{-1} (\mathbf{H}_V W(i))^H \mathbf{x}_V(i), \quad (42)$$

where

$$\mathbf{H}_H = [\mathbf{h}_H(\Theta_1, \Gamma_1), \dots, \mathbf{h}_H(\Theta_K, \Gamma_K)]^T, \quad (43)$$

$$\mathbf{H}_V = [\mathbf{h}_V(\Theta_1, \Gamma_1), \dots, \mathbf{h}_V(\Theta_K, \Gamma_K)]^T. \quad (44)$$

If the receiver located in the desired direction  $\Theta_j$  ( $j \in \{1, 2, \dots, K\}$ ) uses a polarization sensitive antenna, the received signal can be expressed as

$$\mathbf{E}(\Theta_j, i) = \mathbf{h}(\Theta_j) \cdot \mathbf{D}(i) = \begin{bmatrix} \mathbf{s}(\Theta_j, \Gamma) \cdot \mathbf{D}_H(i) \\ \mathbf{s}(\Theta_j, \Gamma) \cdot \mathbf{D}_V(i) \end{bmatrix} = \begin{bmatrix} \cos \gamma_{j,i} \\ \sin \gamma_{j,i} e^{j\eta_{j,i}} \end{bmatrix}. \quad (45)$$

Therefore, the  $K$  LUs can receive the exact signal.

Assuming that an Eve wants to intercept the confidential information for the  $l$ th ( $l \in \{1, 2, \dots, K\}$ ) LU. When the Eve does not know what antenna is used by the  $l$ th user, the eavesdropper receiver is likely to utilize a polarization sensitive antenna or a single polarized antenna. If the receiving antenna used by the Eve is different from the  $l$ th user, the information intercepted cannot be accurately demodulated.

When the Eve adopts a polarization sensitive antenna to receive the confidential information and does polarization demodulation, the PS received by the Eve can be given by

$$\mathbf{P}_e = \begin{bmatrix} \cos \gamma_e \\ \sin \gamma_e e^{j\eta_e} \end{bmatrix}, \quad (46)$$

$$\gamma_e = \arctan\left(\frac{\mathbf{r}_V \cdot \mathbf{x}_V(i)}{\mathbf{r}_H \cdot \mathbf{x}_H(i)}\right) = \arctan\left(\frac{r_{lV} \cdot x_{lV}(i) + \sum_{j=1, j \neq l}^K r_{jV} \cdot x_{jV}(i)}{r_{lH} \cdot x_{lH}(i) + \sum_{j=1, j \neq l}^K r_{jH} \cdot x_{jH}(i)}\right). \quad (47)$$

$$\begin{aligned}\eta_e &= \mathbb{E}(\mathbf{r}_V \cdot \mathbf{x}_V(i)) - \mathbb{E}(\mathbf{r}_H \cdot \mathbf{x}_H(i)) \\ &= \mathbb{E}(r_{IV} \cdot x_{IV}(i) + \sum_{j=1, j \neq l}^K r_{jV} \cdot x_{jV}(i)) - \mathbb{E}(r_{IH} \cdot x_{IH}(i) + \sum_{j=1, j \neq l}^K r_{jH} \cdot x_{jH}(i)).\end{aligned}\quad (48)$$

Thereinto,

$$\mathbf{r}_H = \mathbf{s}(\Theta, \Gamma) \left( (\mathbf{H}_H W(i))^H \mathbf{H}_H W(i) \right)^{-1} (\mathbf{H}_H W(i))^H, \quad (49)$$

$$\mathbf{r}_V = \mathbf{s}(\Theta, \Gamma) \left( (\mathbf{H}_V W(i))^H \mathbf{H}_V W(i) \right)^{-1} (\mathbf{H}_V W(i))^H. \quad (50)$$

Therefore, the received signal by the Eve is distorted in the polarization domain by the equivalent artificial interference  $\sum_{j=1, j \neq l}^L \frac{r_{jV}}{r_{IV}} \cdot x_{jV}(i)$  and  $\sum_{j=1, j \neq l}^L \frac{r_{jH}}{r_{IH}} \cdot x_{jH}(i)$ . So, it is a very hard task for an Eve to demodulated the useful information without *a-prior* knowledge of polarization parameters, and PLS can also be guaranteed.

#### 4.3. Security Performance Analysis from the Signal Processing Perspective

In this part, we will analyze the security performance for the proposed method from the signal processing perspective.

After passing through the LoS channel, the received signal of the  $k$ th LU is obtained by

$$y_k^{\text{LU}} = y(\Theta_k^{\text{LU}}, \Gamma_k^{\text{LU}}) = s^H(\Theta_k^{\text{LU}}, \Gamma_k^{\text{LU}}) \mathbf{x} + \xi_k^{\text{LU}}, \quad (51)$$

where  $\xi_k^{\text{LU}} \sim \mathcal{CN}(0, \sigma_{\xi^{\text{LU}}}^2)$  is the AWGN with zero mean and variance  $\sigma_{\xi^{\text{LU}}}^2$ . Then, we can express their received signals as a vector, i.e.,

$$\begin{aligned}\mathbf{y}^{\text{LU}} &= \mathbf{y}(\Theta_k^{\text{LU}}, \Gamma_k^{\text{LU}}) \\ &= [y_1^{\text{LU}}, y_2^{\text{LU}}, \dots, y_K^{\text{LU}}, y_1'^{\text{LU}}, y_2'^{\text{LU}}, \dots, y_K'^{\text{LU}}]^T \\ &= \mathbf{H}^H(\Theta^{\text{LU}}, \Gamma^{\text{LU}}) \mathbf{x} + \xi^{\text{LU}},\end{aligned}\quad (52)$$

where  $\xi^{\text{LU}} = [\xi_1^{\text{LU}}, \xi_2^{\text{LU}}, \dots, \xi_K^{\text{LU}}, \xi_1'^{\text{LU}}, \xi_2'^{\text{LU}}, \dots, \xi_K'^{\text{LU}}]^T \sim \mathcal{CN}(\mathbf{0}_{2K \times 1}, \sigma_{\xi^{\text{LU}}}^2 \mathbf{I}_{2K})$  is the AWGN vector.

Substituting Eq. (22) and Eq. (24) into Eq. (52), the received signals can be simplified as

$$\mathbf{y}^{\text{LU}} = [y_1^{\text{LU}}, y_2^{\text{LU}}, \dots, y_K^{\text{LU}}, y_1'^{\text{LU}}, y_2'^{\text{LU}}, \dots, y_K'^{\text{LU}}]^T = \sqrt{P_t} \mathbf{u} + \xi^{\text{LU}}, \quad (53)$$

which is simply the summation of the useful information and AWGN. Then, Eq. (51) can be rewritten as

$$y_k^{\text{LU}} = \sqrt{P_t} u_k + \xi_k^{\text{LU}}. \quad (54)$$

From Eq. (54), each LU can easily recover the confidential information transmitted from the BS.

However, for the Eve with spatial domain and polarization domain information  $(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}})$ , the received signal can be calculated by

$$y^{\text{EVE}} = \mathbf{s}^H(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}}) \mathbf{x} + \xi^{\text{EVE}} = \sqrt{P_t} \mathbf{s}^H(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}}) \mathbf{P} \mathbf{u} + \xi^{\text{EVE}}, \quad (55)$$

where  $\xi^{\text{EVE}} \sim \mathcal{CN}(0, \sigma_{\xi^{\text{EVE}}}^2)$  is the AWGN with zero mean and variance  $\sigma_{\xi^{\text{EVE}}}^2$ ,  $\mathbf{s}(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}})$  is the steering vector of the Eve.

Assuming that an Eve expects to intercept the confidential information of the  $k$ th ( $k \in \{1, 2, \dots, K\}$ ) LU.

When the Eve is far away from the  $k$ th LU, we have

$$\Theta^{\text{EVE}} \neq \Theta_k^{\text{LU}}, k \in \{1, 2, \dots, K\}. \quad (56)$$

Then, regardless of whether or not  $\Gamma^{\text{EVE}} = \Gamma_k^{\text{LU}}, k \in \{1, 2, \dots, K\}$ , we have

$$\mathbf{s}^H(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}})\mathbf{P} \neq e_k. \quad (57)$$

Obviously, we can obtain that

$$y^{\text{EVE}} \neq \sqrt{P_t}u_k + \xi^{\text{EVE}}. \quad (58)$$

Therefore, we can come to a conclusion that the Eve cannot recover the confidential messages when it is far away from the LU.

Next, we consider that an Eve is located at the same position as the  $k$ th LU's as the worst situation. Here, we have

$$\Theta^{\text{EVE}} = \Theta_k^{\text{LU}}, k \in \{1, 2, \dots, K\}. \quad (59)$$

In this scenario, the security can also be guaranteed for our method due to the introduction of polarization information. The Eve has no *a-prior* knowledge of the PS  $\Gamma_k^{\text{LU}}$  for the  $k$ th user, i.e.,  $\Gamma^{\text{EVE}} \neq \Gamma_k^{\text{LU}}, k \in \{1, 2, \dots, K\}$ . Then, we still have

$$\mathbf{s}^H(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}})\mathbf{P} = \mathbf{s}^H(\Theta_k^{\text{LU}}, \Gamma^{\text{EVE}})\mathbf{P} \neq e_k. \quad (60)$$

The received signal for the Eve is given by

$$\begin{aligned} y^{\text{EVE}} &= \sqrt{P_t}\mathbf{s}^H(\Theta^{\text{EVE}}, \Gamma^{\text{EVE}})\mathbf{P}\mathbf{u} + \xi^{\text{EVE}} \\ &= \sqrt{P_t}\mathbf{s}^H(\Theta_k^{\text{LU}}, \Gamma^{\text{EVE}})\mathbf{P}\mathbf{u} + \xi^{\text{EVE}} \\ &\neq \sqrt{P_t}u_k + \xi^{\text{EVE}}. \end{aligned} \quad (61)$$

Therefore, the Eve still cannot recover the confidential information even if it is located at the same position as the  $k$ th LU's. Therefore, the PLS is significantly enhanced.

#### 4.4. Metric

Symbol error rate (SER) is a key criterion to evaluate the performance of a DM system. A closed-form expression is given to calculate the SER of DM systems for Gray-coded QPSK modulation, which is based on the nearest-neighbor approximation method in [36]. In this paper, MPSK modulation is adopted for the DM system; for BPSK,  $M = 2$ , for QPSK,  $M = 4$ . The average SER of DM is given by

$$SER_{DM} = \frac{1}{M} \sum_{m=1}^M SER_{SY_m}, \quad (62)$$

where  $SY_m$  denotes the  $m$ th symbol, the  $SER_{SY_m}$  is obtained by [37]

$$SER_{SY_m} = Q\left(\frac{d_m/2}{\sqrt{N_0/2}}\right), \quad (63)$$

where  $d_m$  is the minimum Euclidean distance between the  $m$ th point and any other constellation point, and  $N_0/2$  is the noise power spectral density over a Gaussian channel.

## 5. Simulation Results and Discussions

In this section, we will provide several design examples based on a  $10\lambda$  uniform linear PSA with a half-wavelength interelement spacing to verify the performance of the proposed method.

Without loss of generality, we assume that the azimuth angles are fixed  $\phi = 90^\circ$  for all design examples.  $10^7$  random symbols are used in SER simulations, the AWGN power is the same for all directions, and signal-to-noise-ratio (SNR) = 12 dB.

### 5.1. Single beam with fixed polarization information

First, we will consider broadside and off-broadside design examples for single beam to transmit two independent data streams with different PSs to verify that the introduction of PS can increase the spectral efficiency. That can also be viewed as the polarization multiplexing. The capacity performance of our method is equal to the polarization multiplexing that two separate signals are transmitted by a PSA. We assume that the signals are Gray-coded QPSK modulated. The PSs are defined by  $\Theta_1 = (45^\circ, 90^\circ)$  for data stream 1 for a left-hand circular polarization, and  $\Theta_2 = (45^\circ, -90^\circ)$  for data stream 2 for a right-hand circular polarization at arbitrary directions. For each data stream, the desired beam pattern is a value of one with  $90^\circ$  phase shift at the desired mainlobe, i.e., symbols "00", "01", "11", "10" correspond to  $45^\circ, 135^\circ, -135^\circ, -45^\circ$ . Meanwhile, the desired beam patterns over the sidelobe region are random complex numbers with the amplitudes being approximate zero.

For broadside design example, the desired direction is  $\theta_{ML} = 0^\circ$ ; for off-broadside design example, the desired direction is set  $\theta_{ML} = 30^\circ$ . The sidelobe regions are sampled every  $1^\circ$  except the mainlobe direction.

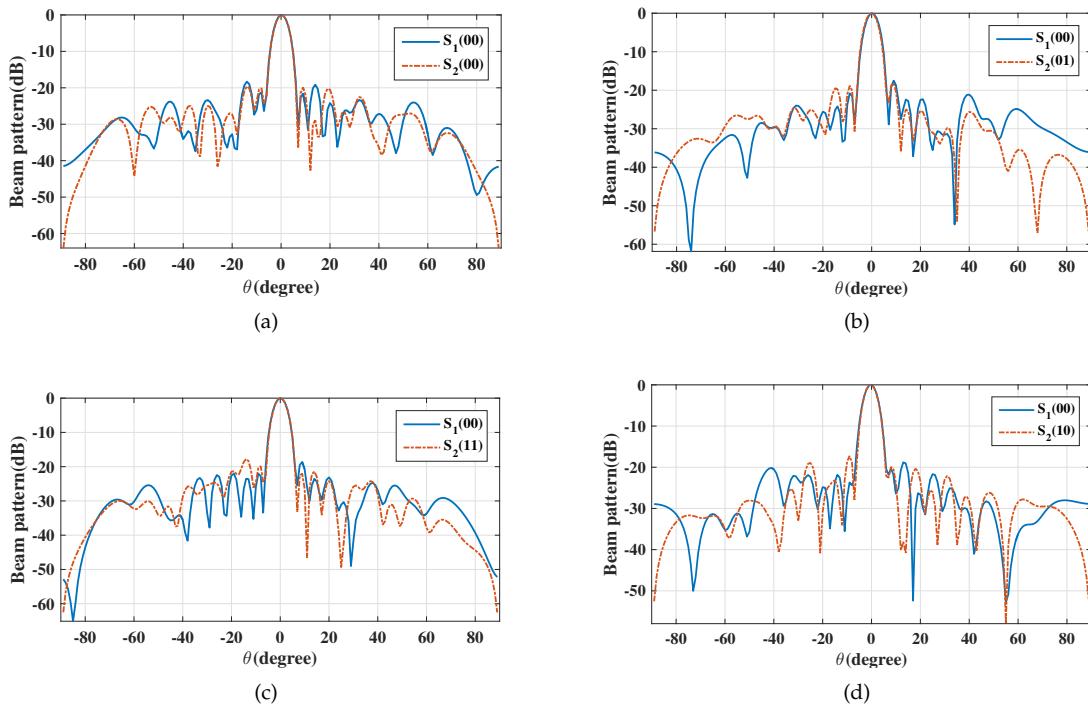
The beam patterns for broadside for symbols "00,00", "00,01", "00,11" and "00,10" are depicted in Fig. 5, where all main beams are exactly pointed to  $0^\circ$  with normalized magnitude 0 dB level, denoting that the amplitude of the desired data streams as expected. The phase patterns for broadside for those symbols are shown in Fig. 6. It is readily to see that the phases of the two data streams in the desired direction are in line with the standard QPSK constellation, while in the sidelobe regions, phases are random enough. The beam and phase patterns for other twelve symbols are not displayed here on account of the similar features as the four symbols mentioned above. The resulting SER curve are demonstrated in Fig. 7. It is indicated that the low SER is achieved in the desired direction, while in other undesired directions, the SER is very high, representing that the directional modulation has been realized effectively.

The beam and phase patterns for off-broadside  $\theta_{ML} = 30^\circ$  for symbols "00,00", "00,01", "00,11" and "00,10" are depicted in Fig. 8 and Fig. 9, respectively. The resulting SER curves are displayed in Fig. 10. It is easy to see that the designed responses are slightly less desirable because the wider mainlobe. Even so, the PLS is still enhanced.

### 5.2. Multiple beams with fixed polarization information

Next, we will consider design examples for multiple beams to transmit different or the same modulation information. Take two beams for example, one data stream transmits the QPSK modulation symbols in two desired directions with a horizontal polarization  $\Theta_1 = (0^\circ, 0^\circ)$ , and another data stream transmits the BPSK modulation symbols in the same two directions with a vertical polarization  $\Theta_1 = (90^\circ, 0^\circ)$ . The simulated far-field (a) magnitude patterns and (b) phase patterns for 100 random symbols are shown in Fig. 11.

Thus, from Fig. 11, it can be observed that standard QPSK and BPSK constellation patterns are only along the prescribed directions,  $0^\circ$  and  $30^\circ$  as expected, with the signal IQ formats along all other directions being distorted, in such a manner to lower the possibility of interception by eavesdroppers located in these regions. Fig. 12 shows the SER performance versus elevation angle for the two data streams transmitted when SNR equals 12 dB. It is obviously to find that the SER performance of the two data streams is the same as the traditional QPSK or BPSK signal at the direction  $0^\circ$  and  $30^\circ$ , while



**Figure 5.** Beam patterns for broadside  $\theta_{ML} = 0^\circ$  for single beam for symbols (a) "00,00", (b) "00,01", (c) "00,11", (d) "00,10".

the SER performance is deteriorated seriously when the elevation angle is off the desired directions. Therefore, the channel capacity is double increased, and this characteristic of the design signal is beneficial for PLS.

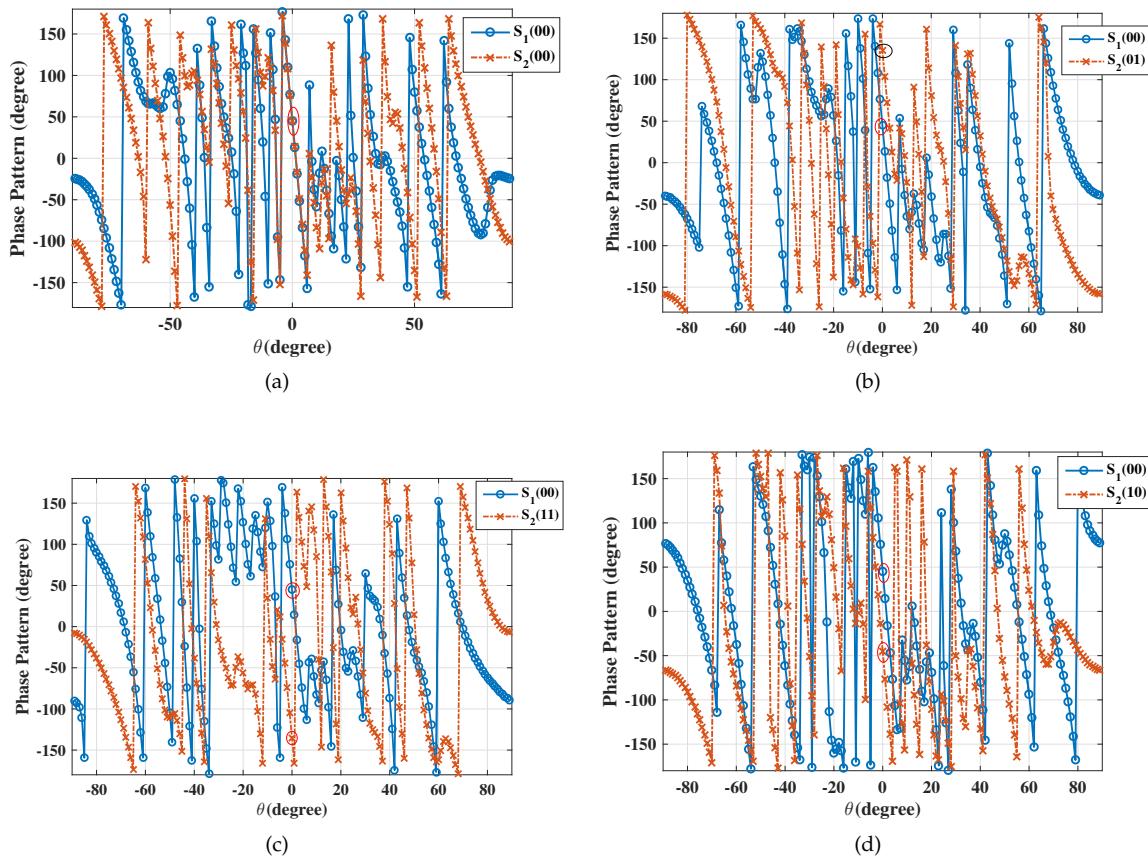
### 5.3. Multiple beams with variable polarization information

In the following design example, it is assumed that a signal stream modulated with QPSK are projected along broadside  $0^\circ$ , while another independent data stream modulated with BPSK, is transmitted along  $30^\circ$  by a 21-element uniform linear PSA. Meanwhile, we designate the polarization  $\Theta_1 = (45^\circ, 45^\circ)$  at the direction  $0^\circ$ , the polarization  $\Theta_2 = (45^\circ, -45^\circ)$  at the direction  $30^\circ$ , and the polarization is generated randomly at other undesired directions by the polarization control unit in Fig. 3. The simulated far-field (a) magnitude patterns and (b) phase patterns for 100 random symbols are shown in Fig. 13.

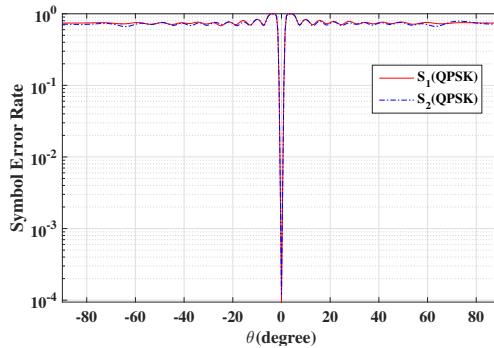
In Fig. 14, the SER simulation results versus elevation angle obtained for receivers using a polarization sensitive antenna and a single-polarized antenna are depicted. The SER performance versus SNR for receivers at the desired direction  $0^\circ$  utilizing a polarization sensitive antenna and a single-polarized antenna is displayed in Fig. 15. We can see that even if the eavesdroppers locate in the desired directions with high SNR enough, the confidential information still cannot be demodulated exactly, when the eavesdroppers use the antennas different from the legal users, i.e.,  $\Gamma^{EVE} \neq \Gamma_k^{LU}$ ,  $k \in \{1, 2, \dots, K\}$ . Therefore, the DM technique based on a PSA is an effective approach to enhance PLS.

## 6. Conclusion

DM technique based on a PSA has been proposed and further studied for PLS enhancement from the sampling perspective in spatial and polarization domain and from the signal processing perspective, respectively. We have formulated a design example to send two different independent data streams

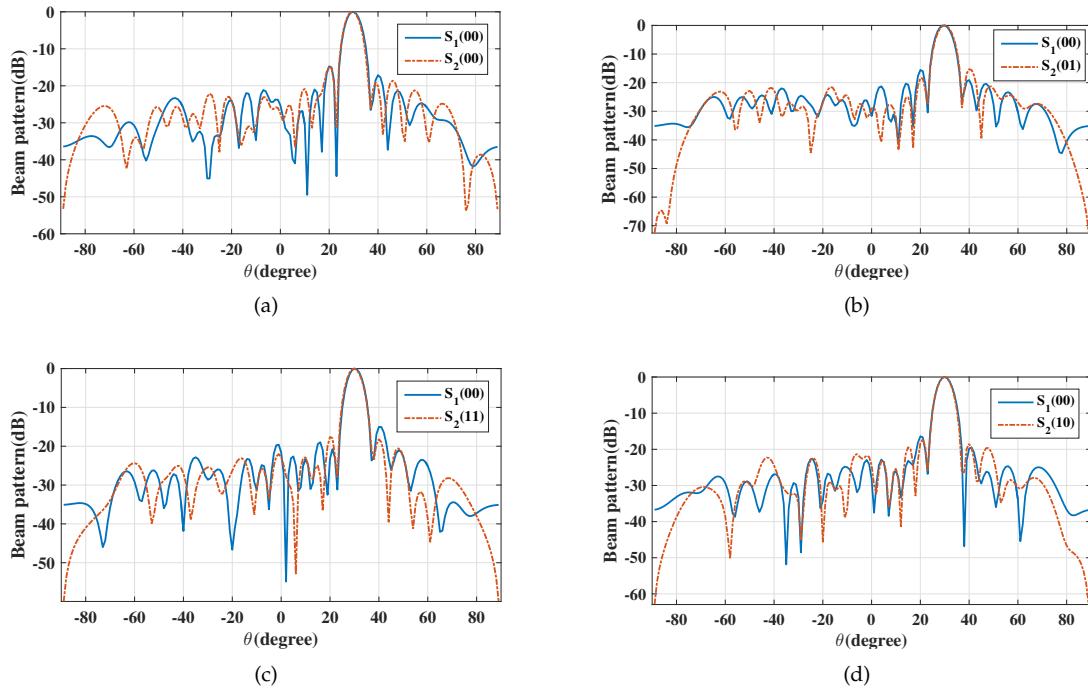


**Figure 6.** Phase patterns for broadside  $\theta_{ML} = 0^\circ$  for single beam for symbols (a) "00,00", (b) "00,01", (c) "00,11", (d) "00,10".

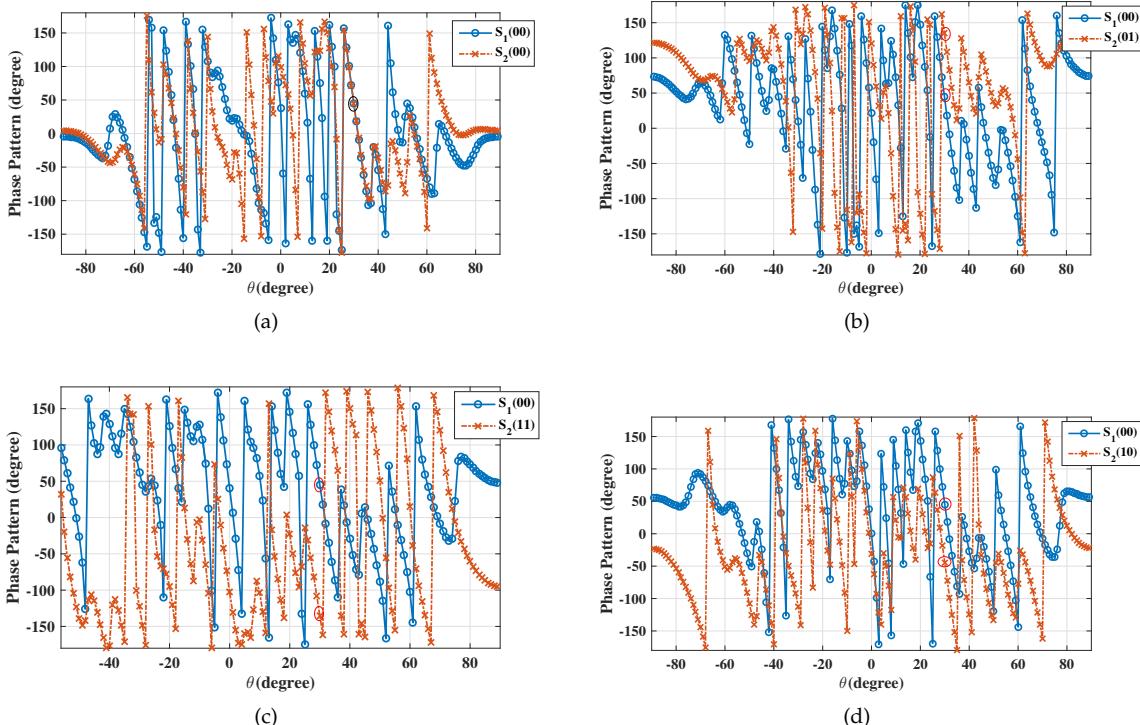


**Figure 7.** The resulting SER curve for broadside  $\theta_{ML} = 0^\circ$  for two data streams.

simultaneously at the same desired directions, same frequency, but with different PSs. It indicates that the channel capacity can be easily increased by the introduction of polarization information. Meanwhile, we have formulated another design example to send two different independent data streams simultaneously at two desired directions, with two fixed PSs, respectively, and off the desired directions, the PSs are distorted randomly. Simulation results verify that the security performance is significantly enhanced.



**Figure 8.** Beam patterns for broadside  $\theta_{ML} = 30^\circ$  for single beam for symbols (a) "00,00", (b) "00,01", (c) "00,11", (d) "00,10".

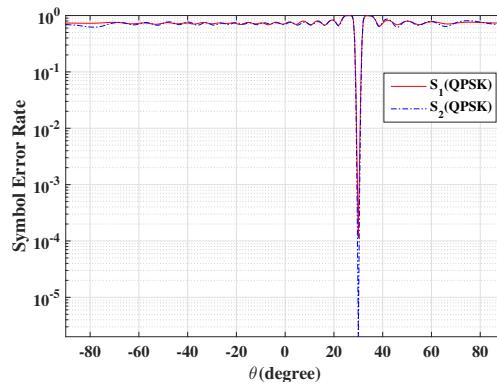


**Figure 9.** Phase patterns for broadside  $\theta_{ML} = 30^\circ$  for single beam for symbols (a) "00,00", (b) "00,01", (c) "00,11", (d) "00,10".

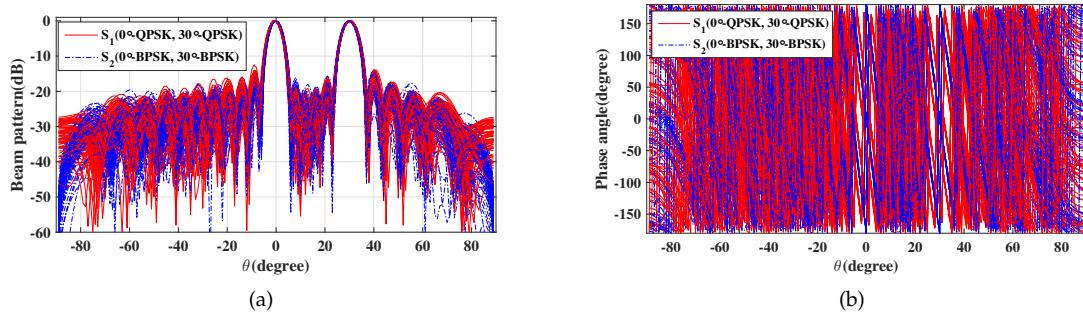
## Appendix

In this appendix, the derivation of the optimal solution for the optimization problem Eq. (18) will be given. For simplicity, we omit the subscript  $i$ . Meanwhile, the optimization problem can be equivalently reformulated as

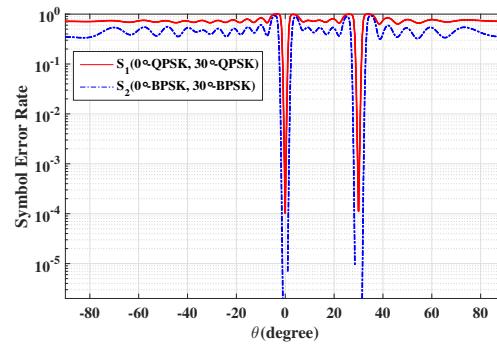
$$\min \quad \left\| \mathbf{p}_{SL} - \mathbf{w}^H \mathbf{S}_{SL} \right\|_2^2 \quad (64)$$



**Figure 10.** The resulting SER curve for off-broadside  $\theta_{ML} = 30^\circ$  for two data streams.



**Figure 11.** The simulated far-field (a) magnitude patterns and (b) phase patterns for 50 symbols.



**Figure 12.** The resulting SER curve versus elevation angle for two data streams in two desired directions.

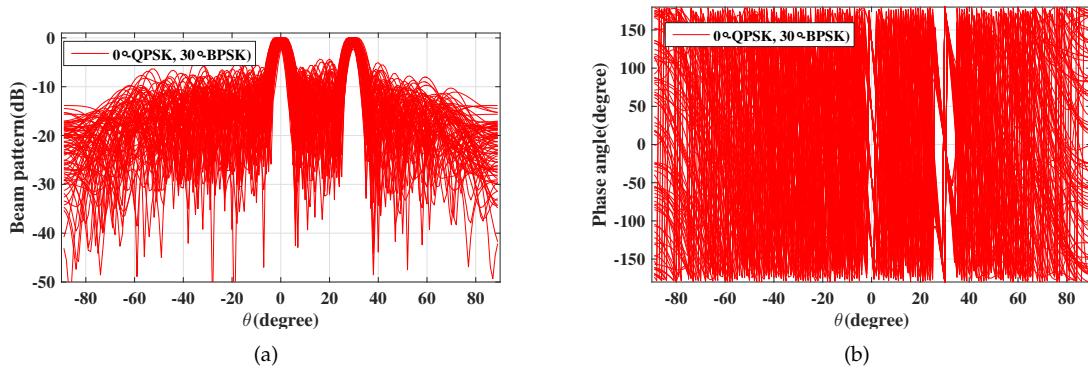
According to the method of Lagrange multipliers, first, we let

$$\mathbf{L}(\mathbf{w}, \mu) = \left\| \mathbf{p}_{SL} - \mathbf{w}^H \mathbf{S}_{SL} \right\|_2^2 + \mu (\mathbf{w}^H \mathbf{S}_{ML} - \mathbf{p}_{ML}). \quad (65)$$

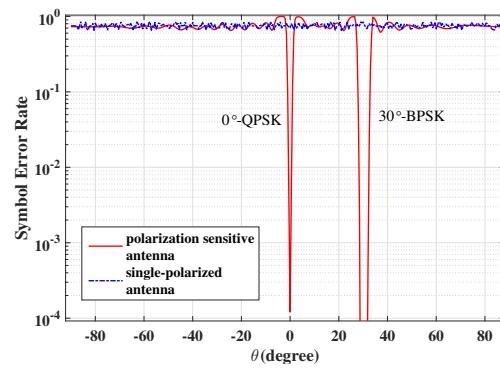
Then, to find the partial derivatives of function  $\mathbf{L}(\mathbf{w}, \mu)$  with respect to  $\mathbf{w}$  and  $\mu$ , respectively.

$$\frac{\partial \mathbf{L}(\mathbf{w}, \mu)}{\partial \mathbf{w}} = -2(\mathbf{p}_{SL} - \mathbf{w}^H \mathbf{S}_{SL}) \mathbf{S}_{SL}^H + \mu \mathbf{S}_{ML}^H, \quad (66)$$

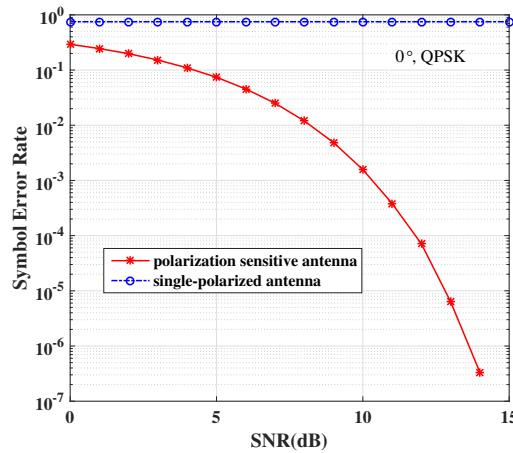
$$\frac{\partial \mathbf{L}(\mathbf{w}, \mu)}{\partial \mu} = \mathbf{w}^H \mathbf{S}_{ML} - \mathbf{p}_{ML}. \quad (67)$$



**Figure 13.** The simulated far-field (a) magnitude patterns and (b) phase patterns for 50 symbols with variable polarization information.



**Figure 14.** The SER simulation results versus elevation angle obtained for receivers using a polarization sensitive antenna or a single-polarized antenna.



**Figure 15.** The SER performance versus SNR for receivers at the desired direction 0° utilizing a polarization sensitive antenna or a single-polarized antenna.

Next, let  $\frac{\partial L(\mathbf{w}, \mu)}{\partial \mathbf{w}} = 0$ , we obtain

$$\mathbf{w}^H = \frac{1}{2} (2\mathbf{p}_{SL} \mathbf{S}_{SL}^H - \mu \mathbf{S}_{ML}^H) (\mathbf{S}_{SL} \mathbf{S}_{SL}^H)^{-1}. \quad (68)$$

Let  $\frac{\partial L(\mathbf{w}, \mu)}{\partial \mu} = 0$ , and inserting Eq. (68) into Eq. (67), we obtain

$$\mu = 2(\mathbf{p}_{SL} \mathbf{S}_{SL}^H (\mathbf{S}_{ML}^H)^{-1} - \mathbf{p}_{ML} \mathbf{S}_{ML}^{-1} \mathbf{S}_{SL} \mathbf{S}_{SL}^H (\mathbf{S}_{ML}^H)^{-1}). \quad (69)$$

Finally, inserting Eq. (69) into Eq. (66), we obtain the optimal solution  $\mathbf{w}$  for the optimization problem Eq. (64), also for the optimization problem Eq. (18), which is given by

$$\begin{aligned} \mathbf{w} = & (\mathbf{S}_{SL} \mathbf{S}_{SL}^H)^{-1} (\mathbf{S}_{SL} \mathbf{p}_{SL}^H - \mathbf{S}_{ML} (\mathbf{S}_{ML}^H (\mathbf{S}_{SL} \mathbf{S}_{SL}^H)^{-1} \mathbf{S}_{ML})^{-1} \\ & (\mathbf{S}_{ML}^H (\mathbf{S}_{SL} \mathbf{S}_{SL}^H)^{-1} \mathbf{S}_{SL} \mathbf{p}_{SL}^H - \mathbf{p}_{ML}^H)). \end{aligned} \quad (70)$$

Let  $\mathbf{R} = \mathbf{S}_{SL} \mathbf{S}_{SL}^H$ , then, Eq. (70) can be simplified as

$$\begin{aligned} \mathbf{w} = & \mathbf{R}^{-1} (\mathbf{S}_{SL} \mathbf{p}_{SL}^H - \mathbf{S}_{ML} (\mathbf{S}_{ML}^H \mathbf{R}^{-1} \mathbf{S}_{ML})^{-1} \\ & (\mathbf{S}_{ML}^H \mathbf{R}^{-1} \mathbf{S}_{SL} \mathbf{p}_{SL}^H - \mathbf{p}_{ML}^H)). \end{aligned} \quad (71)$$

The proof is completed.

**Author Contributions:** All authors contributed extensively to the study presented in this manuscript. Wei Zhang designed the main idea, methods and experiments, interpreted the results and wrote the paper. Mingnan Le and Bin Li supervised the main idea, edited the manuscript and provided many valuable suggestions to this study. Jun Wang, and Jinye Peng carried out the experiments.

**Funding:** This work is supported by Natural Science Basic Research Program of Shaanxi under Grant No. 2019JM-591, and by National Key R&D Program of China under Grant No. 2017YFB1402103 and No. 2017YFB10203104. It is also funded by Open Research Fund of CAS Key Laboratory of Spectral Imaging Technology, Program for Changjiang Scholars and Innovative Research Team in University under Grant No. IRT17R87.

**Acknowledgments:** The authors would like to thank Dr. Feng Liu from Northwestern Polytechnical University for his many constructive suggestions and comments that helped to improve the quality of the paper. The authors also would like to thank the anonymous reviewers and editors for their efforts in reviewing the manuscript and providing many insightful comments and suggestions, which helped a lot for the improvement of the manuscript.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

- Chen, X. M.; Ng, D. W. K.; Gerstacker, W. H.; Chen, H. H. A survey on multiple-antenna techniques for physical layer security. *IEEE Commun. Surv. Tutor.* **2016**, *19*, 1027-1053.
- Babakhani, A.; Rutledge, D. B.; Hajimiri, A. Transmitter architectures based on near-field direct antenna modulation. *IEEE J. Solid-State Circuits.* **2008**, *43*, 2674-2692.
- Babakhani, A.; Rutledge, D. B.; Hajimiri, A. Near-field direct antenna modulation. *IEEE Microw. Mag.* **2009**, *10*, 36-46.
- Hong, T.; Song, M. Z.; Liu, Y. RF directional modulation technique using a switched antenna array for physical layer secure communication applications. *Prog. Electromagn. Res.* **2011**, *116*, 363-379.
- Alrabadi, O. N.; Pedersen, G. F. Directional space-time modulation: a novel approach for secured wireless communication. in Proc. IEEE International Conference on Communications (ICC 2012), Ottawa, ON, Canada, November 2012, pp. 3554-3558.
- Daly, M. P.; Bernhard, J. T. Directional modulation technique for phased arrays. *IEEE Trans. Antennas Propag.* **2009**, *57*, 2633-2640.
- Shi, H. Z.; Alan, T. Direction dependent antenna modulation using a two element array. in Proc. IEEE European Conference on Antennas and Propagation (EuCAP 2011), Rome, Italy, May 2011, pp. 812-815.
- Shi, H. Z.; Alan, T. An experimental two element array configured for directional antenna modulation. in Proc. IEEE European Conference on Antennas and Propagation (EuCAP 2012), Prague, Czech Republic, June 2012, pp. 1624-1626.
- Daly, M. P.; Bernhard, J. T. Beamsteering in pattern reconfigurable arrays using directional modulation. *IEEE Trans. Antennas Propag.* **2010**, *58*, 2259-2265.

10. Daly, M. P.; Daly, E. L.; Bernhard, J. T. Demonstration of directional modulation using a phased array. *IEEE Trans. Antennas Propag.* **2010**, *58*, 1545-1550.
11. Shi, H. Z.; Alan, T. Characteristics of a two element direction dependent antenna array. in Proc. IEEE The Loughborough Antennas and Propagation Conference (LAPC 2011), Loughborough, UK, December 2011, pp. 1-4.
12. Shi, H. Z.; Alan, T. Secure physical-layer communication based on directly modulated antenna arrays. in Proc. IEEE The Loughborough Antennas and Propagation Conference (LAPC 2013), Loughborough, UK, January 2013, pp. 1-4.
13. Shi, H. Z.; Alan, T. Secure communications based on directly modulated antenna arrays combined with multi-path. in Proc. IEEE The Loughborough Antennas and Propagation Conference (LAPC 2014), Loughborough, UK, January 2014, pp. 582-586.
14. Hong, T.; Song, M. Z.; Liu, Y. Dual-beam directional modulation technique for physical-layer secure communication. *IEEE Antennas Wireless Propag. Lett.* **2011**, *10*, 1417-1420.
15. Valliappan, N.; Lozano, A.; Heath, R. W. Antenna subset modulation for secure millimeter-wave wireless communication. *IEEE Trans. Commun.* **2013**, *61*, 3231-3245.
16. Liu, F.; Wang, L.; Xie, J. Directional modulation technique for linear sparse arrays. *IEEE Access.* **2019**, *7*, 13230-13240.
17. Zhu, Q. J.; Yang, S. W.; Yao, R. L. Directional modulation based on 4-D antenna arrays. *IEEE Trans. Antennas Propag.* **2013**, *62*, 621-628.
18. Ding, Y.; Fusco, V.; Anil, C. Circular directional modulation transmitter array. *IET Microw. Antennas Propag.* **2017**, *11*, 1909-1917.
19. Ding, Y.; Fusco, V. A synthesis-free directional modulation transmitter using retrodirective array. *IEEE J. Sel. Top. Signal Process.* **2016**, *11*, 428-441.
20. Ding, Y.; Fusco, V. Orthogonal vector approach for synthesis of multi-beam directional modulation transmitters. *IEEE Antennas Wireless Propag. Lett.* **2015**, *14*, 1330-1333.
21. Hafez, M.; Arslan, H. On directional modulation: an analysis of transmission scheme with multiple directions. in Proc IEEE International Conference on Communications Workshops, (ICCW 2015), London, UK, June 2015, pp. 459-463.
22. Hafez, M.; Khattab, T.; Elfouly, T. Secure multiple-users transmission using multi-path directional modulation. in Proc IEEE International Conference on Communications (ICC 2016), Kuala Lumpur, Malaysia, May 2016, pp. 1-5.
23. Shu, F.; Wu, X.M.; Li, J.; Chen, R.Q.; Vucetic, B. Robust synthesis scheme for secure multi-beam directional modulation in broadcasting systems. *IEEE Access.* **2016**, *4*, 6614-6623.
24. Shu, F.; Xu, L.; Wang, J.Z.; Zhu, W.; Zhou, X.B. Artificial-noise-aided secure multicast precoding for directional modulation systems. *IEEE Trans. Veh. Technol.* **2018**, *67*, 6658-6662.
25. Christopher, R. M.; Borah, D. K. Iterative convex optimization of multi-beam directional modulation with artificial noise. *IEEE Commun. Lett.* **2018**, *22*, 1712-1715.
26. Akl, A.; Elnakib, A.; Kishk, S. Broadcasting multi-beams antenna subset modulation for secure millimeter-wave wireless communications. *Wireless Pers. Commun.* **2017**, *12*, 1-15.
27. M. Hafez, M. Yusuf, T. Khattab, et al, "Secure spatial multiple access using directional modulation," *IEEE Trans. Wireless Commun.*, vol. 17, no. 1, pp. 563-573, Jan. 2018.
28. Xie, T.; Zhu, J.; Li, Y. Artificial-noise-aided zero-forcing synthesis approach for secure multi-beam directional modulation. *IEEE Commun. Lett.* **2018**, *22*, 276-279.
29. Wei, D.; Feng, C.; Guo, C. An optimal pre-compensation based joint polarization-amplitude-phase modulation scheme for the power amplifier energy efficiency improvement. in Proc IEEE International Conference on Communications (ICC 2013), Budapest, Hungary, June 2013, pp. 4137-4142.
30. Wei, D.; Zhang, M.; Fan, W. A spectrum efficient polarized PSK/QAM scheme in the wireless channel with polarization dependent loss effect. in Proc IEEE International Conference on Telecommunications (ICT 2015), Sydney, NSW, Australia, June 2015, pp. 249-255.
31. Luo, Z. K.; Wang, H. L.; Zhou, K. Combined constellation rotation with weighted FRFT for secure transmission in polarization modulation based dual-polarized satellite communication. *IEEE Access.* **2017**, *5*, 27061-27073.
32. Luo, Z. K.; Wang, H. L. Dual-polarized phased array based polarization state modulation for physical-layer secure communication. *IEICE Trans. Fundamentals Elec. Commun. Computer Sci.* **2018**, E101-A, 740-747.

33. Luo, Z. K.; Wang, H. L.; Lv, W. H. Directional polarization modulation for secure transmission in dual-polarized satellite MIMO systems. in Proc IEEE International Conference on Wireless Communications and Signal Processing (WCSP 2016), Yangzhou, China, October 2016, pp. 1-5.
34. Zhang, B.; Liu, W.; Lan, X. Directional modulation design based on crossed-dipole arrays for two signals with orthogonal polarisations. in Proc. IEEE European Conference on Antennas and Propagation (EuCAP 2018), London, UK, April 2018, pp. 1-5.
35. Compton, R. The tripole antenna: an adaptive array with full polarization flexibility. *IEEE Trans. Antennas Propag.* **1981**, *29*, 944-952.
36. Ding, Y. Establishing metrics for assessing the performance of directional modulation systems. *IEEE Trans. Antennas Propag.* **2014**, *2*, 2745-2755.
37. Goldsmith, A. Wireless communications. 1st ed. New York, NY: *Cambridge University Press*, 2005.