Application of Poisson Autoregressive and Poisson Exponential Weighted Moving Average Models on the Prevalence of some Infectious Diseases in Jos, Nigeria.

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Abstract

This research work examined the trend of HIV/AIDS, Tuberculosis, and Hepatitis diseases in Plateau state. Annual data from 2003 to 2018 was collected from the department of biostatistics at Plateau State Specialist Hospital (PSSH), Jos. The methods of analysis used are the Poisson Autoregressive Model (PAR(1)) and the Poisson Exponentially Weighted Moving Average Model (PEWMA). The results revealed a significant annual decrease of 23.9% and 4% in Tuberculosis and HIV/AIDS respectively. Furthermore, the results showed a significant annual increase of 46% in Hepatitis. The PEWMA model used revealed that TB increased by 0.02% when there is an increase in HIV but not significant, while Hepatitis significantly aggravates TB by at least 0.24%. Also, there is a significant rise in HIV by 0.85% when TB increases but Hepatitis has no such effect on HIV. Lastly, PEWMA model indicated a rise of 0.5% in Hepatitis cases when there is an increase in TB, but a surge in HIV has no such effect on Hepatitis cases in Jos. The study recommended that fight against TB should be intensified since TB cases significantly affect both HIV and Hepatitis in Jos, Nigeria.

Keywords: Tubercolusis (TB), Poisson Autoregressive (PAR), Poisson Exponentially Weighted Moving Average Model (PEWMA), Hepatitis, Human Immunodeficiency Virus (HIV), Acquired Immune Deficiency Syndrome (AIDs).

1. Introduction

Hepatitis and Tuberculosis are the two most common co-infections in people infected with HIV/AIDS. Infection with hepatitis B (HBV) and hepatitis C viruses (HCV) are especially more

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common and significant in HIV patients. (Padmapriyadarsini *et al*., 2006). Tuberculosis is the commonest coinfections amongst HIV/AIDS positive people in Nigeria. Globally, an estimated number of 1.4 million people are living with HIV/AIDS and also have TB, with 2-4 million living with both HIV/AIDS and HBV, likewise 4-5 million having HIV/AIDS and HCV coinfection (WHO, 2011). Disease such as tuberculosis, HIV/AIDs and Hepatitis falls into the class of epidemiology and number of affected patient do follow a counting process. Standard epidemiological and mathematical models have been proposed to studying disease dynamics; some of which can be implemented using deterministic or stochastic approaches. Most natural phenomenon such as disease dynamics are stochastic in nature and in view of that, studying them deterministically may not always be appropriate. The dynamics of such deterministic and stochastic models are described by system of differential equations which requires initial conditions of the disease states before modeling (Twumasi, 2018). Aalen *et al* (1997) anticipated the use of Markov modeling for HIV incidence taking into accounts the effect of HIV diagnosis and treatment in England and Wales. A major limitation of some Markov chain models is the problem of lack of memory or "memorylessness" (Twumasi, 2018). Verma *et al* (1983) estimated some epidemiological parameters of malaria transmission using stochastic process or first-order Markov chain from a longitudinal data.

The Poisson distribution is used to describe discrete quantitative data such as counts in which the population size n is large, the probability of an individual event is small, but the expected number of events, n, is moderate (say five or more).

This study, therefore, aims at using the Poisson Autoregressive Model (PAR) and the Poisson Exponentially Weighted Moving Average (PEWMA) to determine the dynamic interrelationship and effect among HIV/AIDS, Tuberculosis and Hepatitis diseases. The methods

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were used to determine the trend of the diseases, the effect and relationship among the diseases under study.

2. Model Specification

A. Poisson Autoregressive Model PAR(p)

This is a model for count data. It is an autoregressive AR(p) model for event counts and it is constructed based on an approach similar to the PEWMA. Instead of employing the multiplicative transition equation used in the PEWMA model, one then replace the transition with a linear autoregressive process (AR(p)) of which this can be extended by Kalman filter for the PAR(p) and its likelihood function which are discussed in Grunwald *et al* (1997a) and Grunwald *et al* (1997b).

Consider $Pr[y_t | Y_{t-1}]$ and that $E[Y_0] = \mu < \infty$.

Let the conditional expectation $E[y_t | Y_{t-1}] = m_t$ at time t have a finite mean.

Then y_t is a p–th order linear autoregressive process if

$$E[y_t | Y_{t-1}] = \sum_{i=i}^{p} p_i Y_{t-i} + \lambda \quad (1)$$

Where, p_i and λ are any real numbers. This specification of a linear AR(p) process places no restriction on the density $Pr[y_t | Y_{t-1}]$. The choice of this density for y_t places constraints on the admissible values of λ and p.

This can be generalized by

$$E\,[E[y_t | Y_{t-1}]\,] = E[\,\sum_{i=i}^{p} p_i Y_{t-i} + \lambda] \quad (2)$$

$$E[Y_t] = \sum_{i=i}^{p} p_i E\,[Y_{t-i}\,] + \lambda \quad (3)$$

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Noting that (3) is a geometric series for p_i, then

$$\lim_{t\to\infty} E[Y_t] = \frac{\lambda}{(1-\sum_{i=1}^{p} p_i)} = \quad (4)$$

Since $E[Y_t] = \mu$ (by definition), equation (1) can be written as

$$E[y_t | Y_{t-1}] = \sum_{i=i}^{p} p_i Y_{t-i} + (1 - \sum_{i=1}^{p} p_i)\,\mu \quad (5)$$

This is a stationary linear AR(p) process.

This Poisson autoregressive or PAR(p) model can be defined as follows:

$$P_r(y_t/m_t) = \frac{m_t^{y_t \sigma} - m_t}{y_t!} \quad (6)$$

Suppose that the observed event counts, y_t for t=1,2,...,T are drawn from a Poisson distribution conditional on m_t with a prior, so

$$Pr[m_t | Y_{t-1}] = \Gamma(\sigma_{t-1}\, m_{t-1}, \sigma_{t-1})\, m_{t-1}>0, \sigma_{t-1} >0, \quad (7)$$

With $m_{t-1} = E[y_t | Y_{t-1}]$ and $\sigma_{t-1} = Var[y_t | Y_{t-1}]$.

The prior is constructed using the observed data. The prior distribution follows a gamma distribution with mean m_{t-1} and variance. These computations demands filtering the data as in the PEWMA model using equations (5),(6),and(7).

Since the prior is a gamma distribution, using an extended Kalman filter, the conditional distribution at time t given t—1 is also gamma:

$$m_{t/t-1} \sim \Gamma((m_{t/t-1}\sigma_{t/t-1}), \sigma_{t/t-1}).$$

This provides an estimate of the posterior for time t:

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$$\Pr(y_t|Y_{t-1}) = \int_{\theta} \Pr(y_t|\theta_t)\Pr(\theta_t|Y_{t-1})\, d\theta$$

$$= \int_{\theta} \frac{\theta_t^{y_t} e^{-\theta_t}}{y_t!} \cdot \frac{e^{-\theta_{t|t-1}\theta}\theta_t^{\sigma_{t|t-1}m_{t|t-1}} \upsilon_t^{\sigma_{t|t-1}m_{t|t-1}}}{\Gamma(\sigma_{t|t\ \ 1}m_{t|t\ \ 1})}$$

Where Γ(.) denotes the gamma function. This is a negative binomial

$$= \frac{\Gamma(\sigma_{t|t-1}m_{t|t-1})+y_t}{\Gamma(y_t+1)\Gamma(\sigma_{t|t-1}m_{t|t-1})} (\sigma_{t|t-1})^{\sigma_{t|t\ \ 1}m_{t|t\ \ 1}} \times (1+\sigma_{t|t-1})^{(\sigma_{t|t\ \ 1}m_{t|t\ \ 1}+y_t)} \quad (8)$$

distribution. Based on this distribution, we can construct the log–likelihood for the PAR(p) as follows:

$$\mathcal{L}(m_{t-1}, \sigma_{t-1}|y_t,\ldots, y_T; Y_{t-1}) = \mathrm{In}\prod_{t=1}^{T} \Pr(y_t|y_{t-1})$$

$$= \sum_{t=1}^{T} In\,\Gamma(\sigma_{t-1}m_{t-1} + y_t) - In\,\Gamma(y_t + 1) - In\,\Gamma(\sigma_{t-1}m_{t-1}) \quad (9)$$

$$+\sigma_{t-1}m_{t-1}In\,(\sigma_{t-1}) - (\sigma_{t-1}m_{t-1} + y_t)\,In(1+\sigma_{t-1})$$

Substituting the linear AR (1) process for m_t yields a PAR(1) model with a negative binomial predictive distribution.

The one–step ahead conditional forecast function for the PAR(p) model is given as ,

$$E[y_{t+1}|Y_t] = m_{t+1}|_t = \sum_{i=1}^{p} m_{t|t-1} + (1 - \sum_{i=1}^{p} p_i)\mu \quad (10)$$

The forecast variance is:

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$$Var[y_{t+1}|Y_t] = \frac{1+\sigma_{t+1|t}}{\sigma_{t+1|t}} m_{t+1|t} \quad (11)$$

For the PAR(p), this derivative is

$$\frac{\partial m_t}{\partial X_t} = \frac{\partial(\sum_{i=1}^{p} p_i Y_{t-i} + (1 - \sum_{i=1}^{p} p_i)\exp(X_t\delta))}{\partial X_t}$$

$$= (1-\sum_{i=1}^{p} p_i)\exp(X_t\delta).\delta \quad (12)$$

where the instantaneous (and long-run) estimated effect would be $\exp(X_t\delta).\delta$.

Then the long run multiplier is

$$\frac{\frac{\partial m_t}{\partial X_t}}{(1-\sum_{i=1}^{p} p_i)} = \frac{(1-\sum_{i=1}^{p} p_i)\exp(X_t\delta)\delta}{(1-\sum_{i=1}^{p} p_i)} = \exp(X_t\delta).\delta \quad (13)$$

B. The Poisson Exponentially Weighted Moving Average Model (PEWMA)

To address the problem of modeling persistent time series event counts, one can use a structural time series model. In general, a structural time series model specifies two equations;

1) The measurement density or system equation and

2) The state density or transition equation to describe the evolution of a time series. The specification and estimation of the model is done using Kalman filter (Harvey 1989; Hamilton 1994).

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For event count data, specifying a structural time series is more problematic. To deal with such problems, the approach of Harvey and Fernandes (1989) can be adopted. They employ natural conjugate densities to simplify the development of the model and numerical calculation to specify a functional form as the likelihood function (Judge et.al (1988), DeGroot (1970)). Also see Gelman et al. (1995).

The Poisson-gamma exponentially weighted moving average model for count data (PEWMA) is built around the following three assumptions that characterize the mean and dynamics of the process.

1. measurement Equation: We assume the observed counts at time t are drawn from Poisson marginal distribution,

$$P_r(y_t/\mu_t) = \frac{\mu_t^{y_t} e^{-\mu_t}}{y_t!} \qquad (14)$$

2. Transition Equation (Smith and Miller 1986; Shephard 1994).

$\mu_t = e^{rt}\, \mu_{t-1}\eta_t$, t = 1, 2, …,T (16)

Where $\eta_t \sim$ Beta ($\omega\, \alpha$ t-1, (1- ω) a_{t-1}).

The parameter $0 < \omega \leq 1$, captures discounting of the observations in computing the mean and η_t and rt parameterize the growth rate in period t. the beta distributed variable η_t captures the proportional stochastic shift in the mean form t-1 to time t. From the properties of the beta distribution, $E[\eta_t] = \omega\ \forall$ t. The parameter rt describes the growth in the series and insures that $\mu_t > 0$.

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3. Conjugate Prior: The gamma f density is given by;

$$F(\mu; a, b) = \frac{e^{-b\mu}\mu^{a-1}b^{a}}{\Gamma(a)} \quad (17)$$

Where $a = a_{t-1}$, and $\mu = \mu^*_{t-1}$. These values are computed from the previous t-1 observations, Y_{t-1}.

Application of PEWMA model can be seen in modeling political dynamics (Brandth et al. 2000) and modeling of human environmental interaction (Carleton et al. 2018) and in modeling infectious diseases of which this study falls into.

3. Method, Source and presentation of Data

For this research, dataset on HIV/AIDS, TB and Hepatitis was sourced from Plateau State Specialist Hospital (PSSH) Jos, Plateau State from 2003 to 2018. The data used in this research work is a secondary data presented below in Figures 1, 2 and 3.table 1 below.

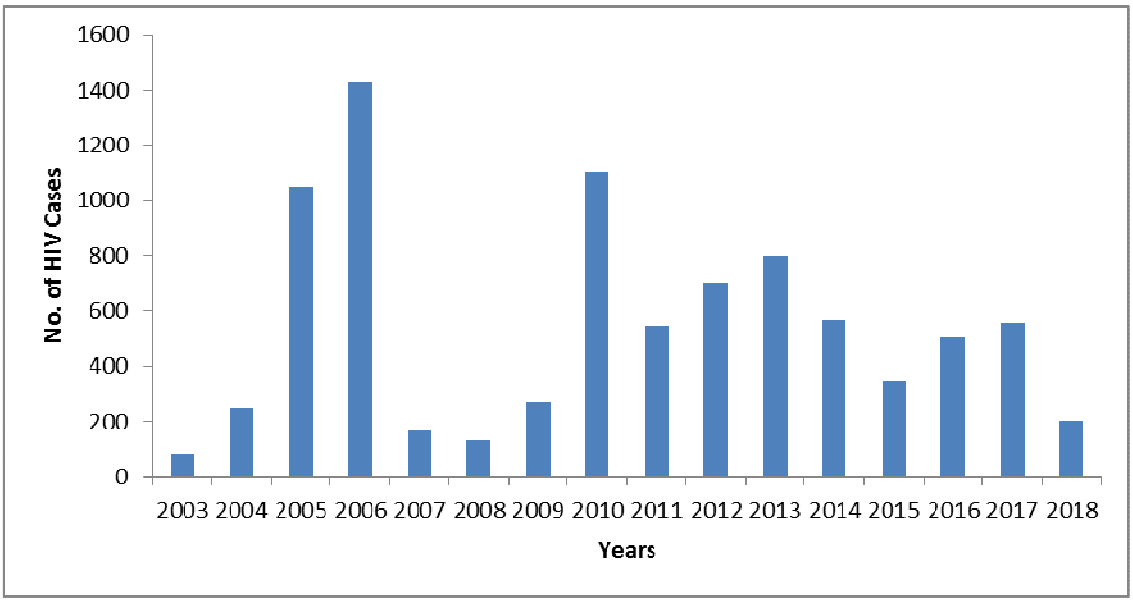

Fig. 1: Bar Chart on Annual Cases of HIV in Jos from 2003 to 2018

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The Fig. 1 shows the distribution of HIV cases in Jos from 2003 to 2018. The highest case of HIV is recorded in 2006 while the lowest case of HIV is recorded in 2003.

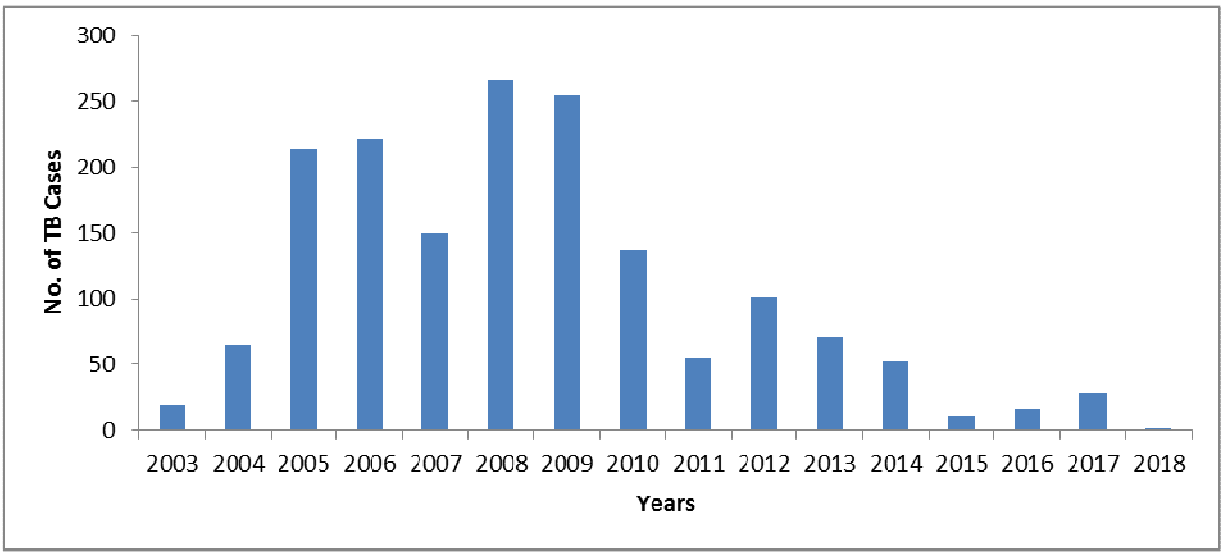

Fig. 2: Bar Chart on Annual Cases of TB in Jos from 2003 to 2018

The Fig. 2 shows the distribution of TB cases in Jos from 2003 to 2018. The highest case of TB is recorded in 2008 while the lowest case of TB is recorded in 2018.

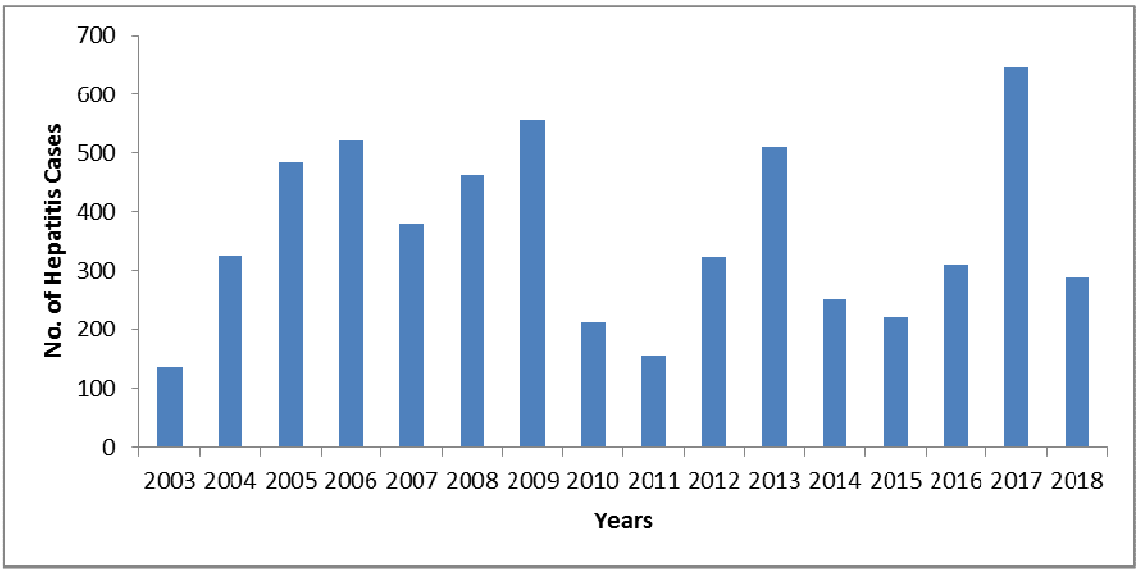

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Fig. 3: Bar Chart on Annual Cases of Hepatitis in Jos from 2003 to 2018

The Fig. 3 shows the distribution of Hepatitis cases in Jos from 2003 to 2018. The highest case of Hepatitis was recorded in 2017 while the lowest case of Hepatitis was recorded in 2003.

4. Data Analysis and Results

This section present results and interpretation of trend analysis using Poisson Autoregressive (PAR) model and the dynamic interrelationship among the infectious diseases using Poisson Exponential Weighted Moving Average (PEWMA) model.

Table 1: Trend analysis of Tuberculosis using PAR(1)

	Parameters	Std. Errors	Z-score
rho	0.264789	0.055490	4.7718
(Intercept)	6.377093	0.082440	77.3543
time	-0.239356	0.012484	-19.1732

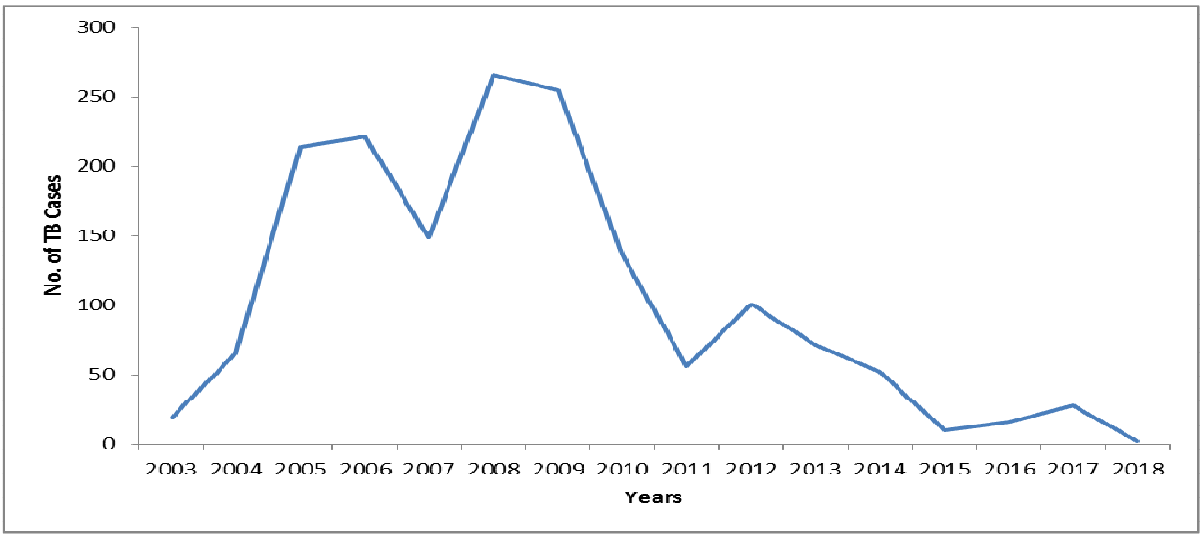

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Fig 4: Graphical representation of TB Cases in Jos from 2003 to 2018.

From Table 1 above present the result from PAR(1) of TB cases which shows a significant annual decrease of 23.9% in tuberculosis cases in Plateau State Specialist Hospital, Jos from 2003 to 2018, (|Z-score|= 19.17>1.96). This result is further amplified by Fig 4 above. In addition, the intercept and the autoregressive coefficient are significant in the PAR model.

Table 2: Trend analysis of HIV using PAR(1)

	Parameters	Std. Errors	Z-score
rho	0.0937946	0.0242319	3.8707
(Intercept)	6.6971332	0.0326401	205.1813
time	-0.0429798	0.0035282	-12.1817

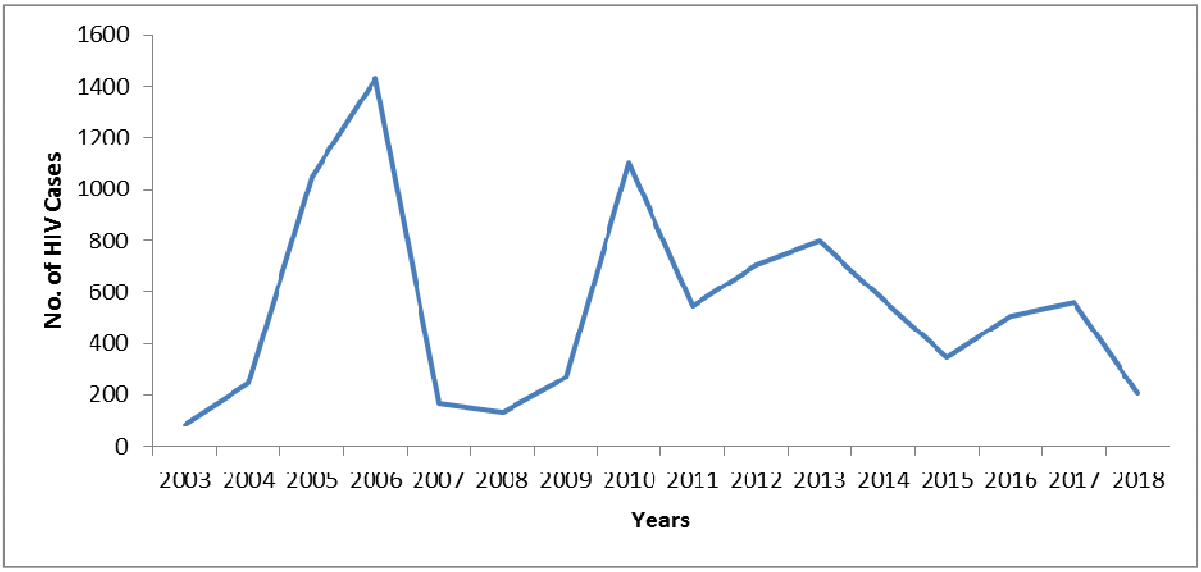

Fig 5: Graphical representation of HIV Cases in Jos from 2003 to 2018.

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From Table 2 above presented the result from PAR(1) of HIV cases which shows a significant annual decrease of 4% in HIV cases in Plateau State Specialist Hospital, Jos from 2003 to 2018 (|Z-score|= 12.18>1.96). This is result is further amplified in Fig 5 above. In addition, the intercept and the autoregressive coefficient are significant in the PAR(1) model.

Table 3: Trend analysis of Hepatitis using PAR(1)

	Parameters	Std. Errors	Z-score
Rho	0.8772796	0.0165413	53.036
time	0.4639210	0.0092304	50.260

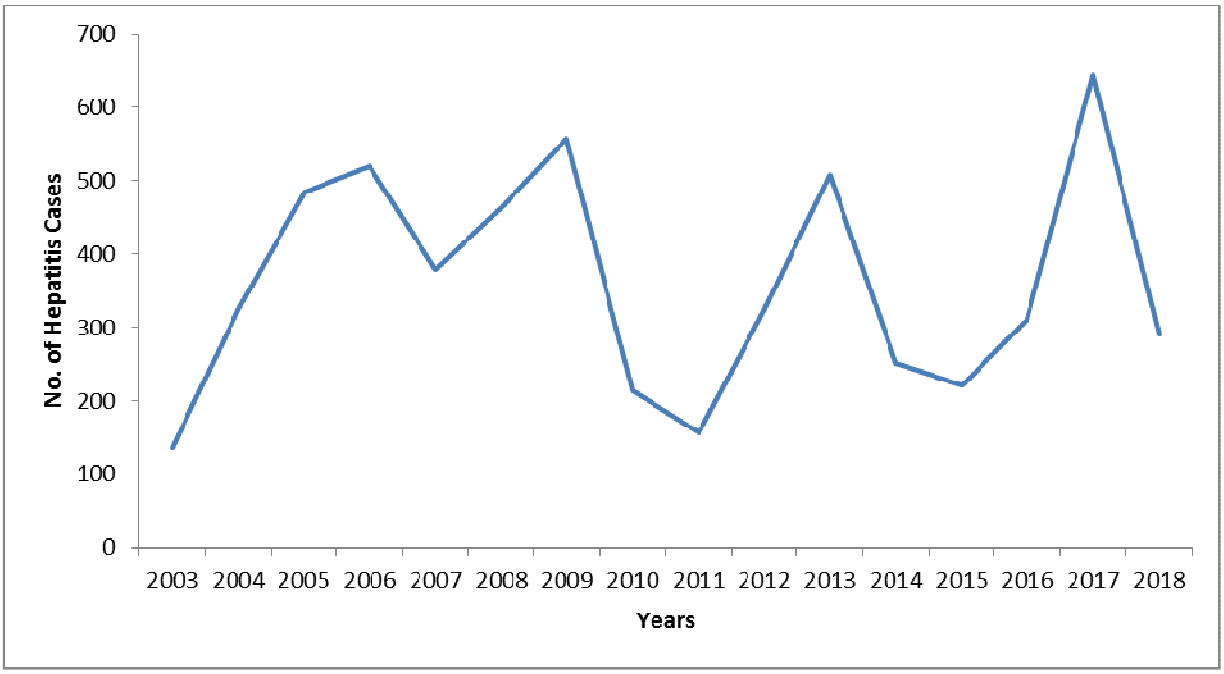

Fig 6: Graphical representation of Hepatitis Cases in Jos from 2003 to 2018.

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The Table 3 above presented the result from PAR(1) model of Hepatitis which shows an annual increase of 46% in Hepatitis cases in Plateau State Specialist Hospital, Jos from 2003 to 2018 which is very significant (|Z-score| = 50.26 > 1.96). This result is amplified by fig 6 above.

Table 4.: Effect of HIV and Hepatitis on Tuberculosis using PEWMA

	Parameters	Std. Errors	Z-Score
Omega	0.05710687	0.01889756	3.0219
Time	-0.01451032	0.10132390	-0.1432
HIV/AIDS	0.00021422	0.00014316	1.4964
HPT	0.00244513	0.00058743	4.1624

The Table 4 above presented the dynamics of the effects of time, HIV and Hepatitis on TB cases using the PEWMA. The results shows negative trend of 1.4% in tuberculosis cases in Plateau State Specialist Hospital, Jos from 2003 to 2018, although not significant (|Z-score|= 0.14<1.96). Furthermore, there was increase in TB cases by 0.02% when there is an increase in HIV, although not significant (|Z-score|= 1.496<1.96), likewise there was a significant increase in TB cases by 0.24% when there is increase in Hepatitis cases (Z-score= 4.16>1.96).

Table 5: Effect of TB and Hepatitis cases on HIV using PEWMA

	Parameters	Std. Errors	Z-Score
Omega	4.1443e-03	7.1370e-04	5.8067
(Intercept)	5.7610e+00	1.6777e+04	0.0003
Time	4.7178e-01	2.0876e-01	2.2599
TB	8.4798e-03	3.3035e-03	2.5669

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HPT	-1.5550e-03	1.1706e-03	-1.3284

The Table 5 shows positive significant trend of 47.2% in HIV cases in Plateau State Specialist Hospital, Jos from 2003 to 2018 (|Z-score|= 2.26>1.96). There was a significant increase in HIV cases by 0.85% when there is an increase in TB (|Z-score|= 2.57>1.96), likewise a negative relationship exists with HIV by 0.16% when there is increase in Hepatitis cases although not significant (|Z-score|= 1.3284<1.96).

Table 6: Effect of HIV and TB cases on Hepatitis using PEWMA

	Parameters	Std. errors	Z-Score
Omega	1.1270e-02	4.1208e-03	2.7350
(Intercept)	5.0673e+00	2.7962e+03	0.0018
Time	1.8315e-02	1.2922e-01	0.1417
TB	5.0132e-03	2.1841e-03	2.2953
HIV	-4.2021e-05	2.2010e-04	-0.1909

The Table 6 shows positive trend of 1.83% in Hepatitis cases in Plateau State Specialist Hospital, Jos from 2003 to 2018, although not significant (|Z-score|= 0.14<1.96). There was a significant increase in Hepatitis cases by 0.5% when there is an increase in TB (|Z-score|= 2.29>1.96), likewise a negative relationship exists with Hepatitis by 0.16% when there is increase in HIV cases although not significant (|Z-score|= 0.19<1.96).

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5. Discussion of Results

The trend analysis from PAR(1) model shows a significant annual decrease of 23.9% in tuberculosis cases (|Z-score|= 19.17>1.96) in Plateau State Specialist Hospital, Jos from 2003 to 2018. This agrees with WHO Global TB report, 2017 which indicated that TB globally is dropping at about 2% per year. Similarly, the result from PAR(1) shows a significant annual decrease of 4% in HIV cases in Plateau State Specialist Hospital, Jos from 2003 to 2018 (|Z-score|= 12.18>1.96). This is also in line with UNAIDS 2014 Global Report on AIDS with 3.2% decline and Nigeria National HIV/AIDS Indicator and Impact Survey (NAIIS) 2018 which showed a decrease of 1.4% in prevalence rate. Also the result from PAR(1) shows an annual increase of 46% in Hepatitis cases in Plateau State Specialist Hospital, Jos from 2003 to 2018 which is very significant (Z-score = 50.26 > 1.96). This is contrary to research work by Musa *et al* (2015) which suggested a sustained decline in the prevalence of hepatitis.

The PEWMA model used revealed that TB increased by 0.02% when there is an increase in HIV but not significant, while Hepatitis significantly aggravates TB by at least 0.24%. Also, there is a significant rise in HIV by 0.85% when TB increases but Hepatitis has no such effect on HIV. Lastly, PEWMA model indicated a rise of 0.5% in Hepatitis cases when there is an increase in TB, but a surge in HIV has no such effect on Hepatitis cases in Jos. These results are similar to the study of Sama et al. (2017).

6. Conclusion and Recommendations

Based on the findings of this research work, the study concluded that a significant decrease in TB and HIV cases in Jos of about 23.9% and 4% respectively while a significant increase of 47%

in Hepatitis cases in Jos. Lastly Hepatitis cases significantly affect TB patient, TB cases significantly affects HIV patients and TB cases significantly affect Hepatitis patient.

Based on the findings, the following is recommended;

1. Since Hepatitis rate isn't declining as reflected in the data set, the government and stakeholders in the health sector should create enlightenment campaigns on vaccination and possible preventive measures.
2. Fight against TB should be intensified since TB cases significantly affect both HIV and Hepatitis in Jos.
3. Fight against HIV should be maintain to ensure permanent reduction in the cases of HIV in Jos, Nigeria.

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