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Nonsingular model of magnetized black hole based on nonlinear electrodynamics

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Abstract

We find solutions of a magnetically charged non-singular black hole in some modified theory of gravity coupled with nonlinear electrodynamics. The metric of a magnetized black hole is obtained which has one (an extreme horizon), two horizons, or no horizons (naked singularity). Corrections to the Reissner-Nordström solution are found as the radius approaches to infinity. The asymptotic of the Ricci and Kretschmann scalars are calculated showing the absence of singularities. We study the thermodynamics of black holes by calculating the Hawking temperature and the heat capacity. It is demonstrated that phase transitions take place and we show that black holes are thermodynamically stable at some range of parameters.

1 Introduction

It is well-known that the General Relativity (GR) is ultraviolet (UV) incomplete. In addition, there is a problem of singularities in the classical Einstein theory of gravity. Thus, solutions of the Einstein equations for neutral (the Schwarzschild metric), charged (the Reisner-Nordström metric) and rotated (the Kerr metric) black holes (BHs) have curvature singularity in the center ($r = 0$). Therefore, the GR should be modified when the curvature is high. There are some attempts to overcome problems in the classical Einstein theory of gravity. So, if one adds curvature terms of the higher order or terms with higher derivatives, the UV behaviour of the Einstein gravity

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will be improved [1]. But the price for this is the existence of ghosts (non-physical degrees of freedom). A ghost free modification of the GR, which is UV-complete, was considered in [2], [3], [4]. But such theory is non-local and has an infinite number of derivatives. Because the fundamental quantum gravity theory (UV-complete) is absent some phenomenology models can be useful to solve problems of singularities. We suppose that there is a critical energy μ and the corresponding length $l = \mu^{-1}$ in such a way that the metric is modified when the spacetime curvature be in the order of l^2 [5].

In this paper we consider the spherically symmetric nonsingular model of BH based on nonlinear electrodynamics (NED). In some NED, the electric field in the center of point-like charges is finite [6]-[10] and the self-energy of charges is finite unlike classical electrodynamics. It worth mentioning that quantum corrections to Maxwell's electrodynamics lead to NED [11]. The universe inflation also can be explained in the GR coupled with NED [12]-[19].

Here, we study regular BH solutions in some modified theory of gravity coupled with NED proposed in [20]. The BH thermodynamics and phase transitions are investigated. In [21] and [22] authors also considered BH solutions in modified theory of gravity based on NED proposed in [23] and [24], respectively.

The paper is organized as follows. In Section 2 the modified GR with NED is studied and we obtain the asymptotic of the metric and mass functions as $r \to 0$ and $r \to \infty$. Corrections to the Reissner-Nordström (RN) solution are found. The asymptotic of the Ricci and Kretschmann scalars are calculated and we show that curvature singularities are absent. In Section 3 we calculate the Hawking temperature and heat capacity of BHs. We demonstrate that the second-order phase transitions occur. It is shown that in some range of parameters BHs are stable. Section 4 is a conclusion.

2 A regular magnetized BH solution

Let us consider the action of the GR coupled with NED

$$I = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R + \mathcal{L}\right), \tag{1}$$

where $\kappa^2 = 8\pi G_N$ and G_N is the Newton constant. We use the Lagrangian density of NED [20] as follows:

$$\mathcal{L} = -\frac{\mathcal{F}}{\cosh^2 \sqrt[4]{|\beta\mathcal{F}|}}, \tag{2}$$

where $\mathcal{F} = (1/4)F_{\mu\nu}F^{\mu\nu} = (\mathbf{B}^2 - \mathbf{E}^2)/2$ and the field tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The parameter β possesses the dimension of (length)^4 and at the weak field limit when $\beta\mathcal{F} \ll 1$ the Lagrangian density (1) becomes $\mathcal{L} \to -\mathcal{F}$, i.e. the correspondence principle holds. We consider the regular BH with the spherical symmetrical line element

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2), \tag{3}$$

with [25]

$$f(r) = 1 - \frac{2Gm_0r^2}{r^3 + 2Gm_0l^2}, \tag{4}$$

where m_0 being the mass of a neutral BH and l is the fundamental length. The metric (3) corresponds to a solution of a modified theory of gravity. At $l = 0$ we come to the Schwarzschild metric of a BH which is a solution to Einstein's equation without sources. Now we suppose that the BH is magnetically charged and the source of gravity is describes by the Lagrangian density (2). Then the mass m_0 should be replaced by the mass function depending on r,

$$M(r) = m_0 + \int_0^r \rho(r)r^2dr = m_0 + \int_0^\infty \rho(r)r^2dr - \int_r^\infty \rho(r)r^2dr, \tag{5}$$

where $\rho(r)$ is the magnetic energy density and $m_M = \int_0^\infty \rho(r)r^2dr$ is the magnetic mass of the BH. At $\mathbf{E} = 0$ the magnetic energy density is given by [26]

$$\rho(r) = -\mathcal{L} = \frac{\mathcal{F}}{\cosh^2 \sqrt[4]{|\beta\mathcal{F}|}}, \tag{6}$$

where $\mathcal{F} = B^2/2 = q^2/(2r^4)$, and q is a magnetic charge. Then the mass function (5) becomes

$$M(r) = m_0 + m_M - \frac{q^{3/2}}{2^{3/4}\beta^{1/4}}\tanh\left(\frac{\beta^{1/4}\sqrt{q}}{2^{1/4}r}\right), \tag{7}$$

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where the BH magnetic mass is

$$m_M = \int_0^\infty \rho(r) r^2 dr = \frac{q^{3/2}}{2^{3/4}\beta^{1/4}}. \tag{8}$$

The total BH mass is $m = m_0 + m_M$. Then the metric function of a modified theory of gravity corresponding to a charged BH is

$$f(r) = 1 - \frac{2GM(r)r^2}{r^3 + 2GM(r)l^2}, \tag{9}$$

where $M(r)$ is given by (7).

For a convenience we introduce the dimensionless parameter $x = 2^{1/4}r/(\beta^{1/4}\sqrt{q})$. Then from Eqs. (7)-(9) one obtains the metric function

$$f(x) = 1 - \frac{Ax^2 g(x)}{x^3 + Bg(x)}, \tag{10}$$

where

$$A = \frac{\sqrt{2}Gq}{\sqrt{\beta}}, \quad B = \frac{2Gl^2}{\beta}, \quad C = \frac{2^{3/4}\beta^{1/4}m_0}{q^{3/2}}, \quad g(x) = C + 1 - \tanh\left(\frac{1}{x}\right). \tag{11}$$

From Eqs. (10) and (11) we find the asymptotic of the metric function as $r \to \infty$ and $r \to 0$

$$f(r) = 1 - \frac{2Gm}{r} + \frac{Gq^2}{r^2} - \frac{G}{r^4}\left(\frac{\sqrt{\beta}q^3}{3\sqrt{2}} - 4Gl^2m^2\right) + \mathcal{O}(r^{-5}) \qquad r \to \infty, \tag{12}$$

$$f(r) = 1 - \frac{r^2}{l^2} + \frac{r^5}{2Gm_0 l^4} + \mathcal{O}(r^6) \qquad r \to 0. \tag{13}$$

Equation (12) shows the corrections to the RN solution that are in the order of $\mathcal{O}(r^{-4})$. At $l = 0$ and $m_0 = 0$ (when the total BH mass is the magnetic mass) Eq. (12) is converted into the equation obtained in [26] corresponding to Einstein's theory of gravity. As $r \to \infty$ we have $f(\infty) = 1$, and the spacetime becomes Minkowski's space. According to Eq. (13) $\lim_{r\to 0} f(r) = 1$. Thus, the BH is regular. If $\beta = 0$ one has the RN solution. The plot of the function $f(x)$ is depicted in Fig. 1. In Accordance with Fig. 1 at $A < 3.93$ ($B = 1$, $C = 0$) horizons are absent and we have naked singularity. At $A \simeq 3.93$, one horizon occurs (the extreme singularity). If > 3.93, we

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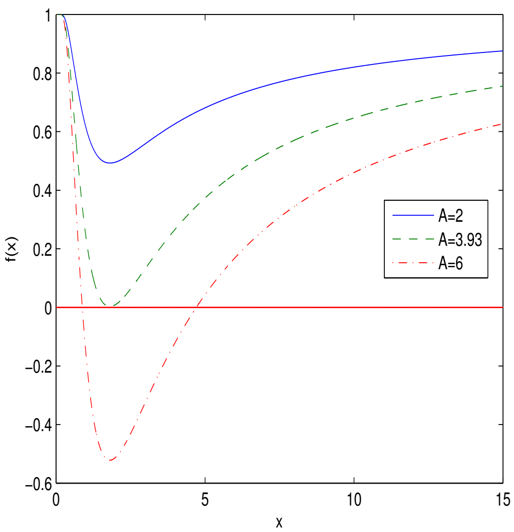

Figure 1: The plot of the function $f(x)$ for $B = 1$ and $C = 0$ ($m_0 = 0$). Dashed-dotted line corresponds to $A = 6$, solid line corresponds to $A = 2$ and dashed line corresponds to $A = 3.93$.

have two horizons. The horizons x_h take place when the equation $f(x_h) = 0$ is satisfied. From Eq. (10), at $B = 1$, $C = 0$ one finds the inner x_- ($x_- = 2^{1/4}r_-/(\beta^{1/4}\sqrt{q})$) and outer x_+ ($x_+ = 2^{1/4}r_+/(\beta^{1/4}\sqrt{q})$) horizon radii of the BH that are given in Table 1.

The asymptotic of the Ricci and Kretschann scalars can be obtained from the relations

$$R = -f''(r) - \frac{4}{r}f'(r) - 2\frac{f(r)-1}{r^2}, \tag{14}$$

$$K = f''^2(r) + \left(\frac{2f'(r)}{r}\right)^2 + \frac{4\left(f(r)-1\right)^2}{r^4}. \tag{15}$$

From Eqs. (14) and (15) we find

$$R = \frac{12}{l^2} - \frac{21r^3}{Gm_0l^4} + \mathcal{O}(r^4) \quad\quad r \to 0, \tag{16}$$

$$R = -\frac{86Gm}{r^3} + \frac{12Gq^2}{r^4} - \frac{34G}{r^6}\left(\frac{\sqrt{\beta}q^3}{3\sqrt{2}} - 4Gl^2m^2\right) + \mathcal{O}(r^{-7}) \quad\quad r \to \infty, \tag{17}$$

Table 1: The BH inner and outer horizon radii ($B = 1$, $C = 0$)

A	4	5	6	7	8	9	10	15
x_-	1.56	1.04	0.88	0.78	0.72	0.68	0.64	0.54
x_+	2.11	3.58	4.71	5.78	6.82	7.84	8.87	13.92

$$K = \frac{24}{l^4} - \frac{84r^3}{Gm_0l^6} + \mathcal{O}(r^4) \qquad r \to 0, \tag{18}$$

$$K = \frac{48G^2m^2}{r^6} - \frac{96G^2mq^2}{r^7} + \mathcal{O}(r^{-8}) \qquad r \to \infty. \tag{19}$$

As $r \to \infty$ the Ricci scalar R vanishes and spacetime becomes flat. Equations (16)-(19) indicate that solutions obtained in modified theory of relativity are regular. In Einstein's theory of gravity ($l = 0$) based on NED [26], singularities are still present.

3 Thermodynamics and phase transitions

Let us study the thermal stability of magnetized BHs and the possible phase transitions. The Hawking temperature is given by

$$T_H = \frac{\kappa_s}{2\pi} = \frac{f'(r_h)}{4\pi}, \tag{20}$$

where κ_s is the surface gravity and r_h is the horizon radius. Making use of Eqs. (10) and (20) we obtain the Hawking temperature

$$T_H = \frac{\sqrt{q}G}{2^{5/4}\pi\beta^{3/4}(x^3+Bg(x))}\Bigg(-2xg(x)$$
$$-\frac{1}{\cosh^2(1/x)} + \frac{g(x)(B+3x^4\cosh^2(1/x))}{(x^3+Bg(x))\cosh^2(1/x)}\Bigg). \tag{21}$$

The plot of the function $T_H(x_h)$ is depicted in Fig. 2 for different values of the parameter B. When the Hawking temperature is negative the BH does not exist. The heat capacity at constant charge is defined by the relation

$$C_q = T_H\left(\frac{\partial S}{\partial T_H}\right)_q = \frac{T_H\partial S/\partial r_h}{\partial T_H/\partial r_h} = \frac{2\pi r_h T_H}{G\partial T_H/\partial r_h}. \tag{22}$$

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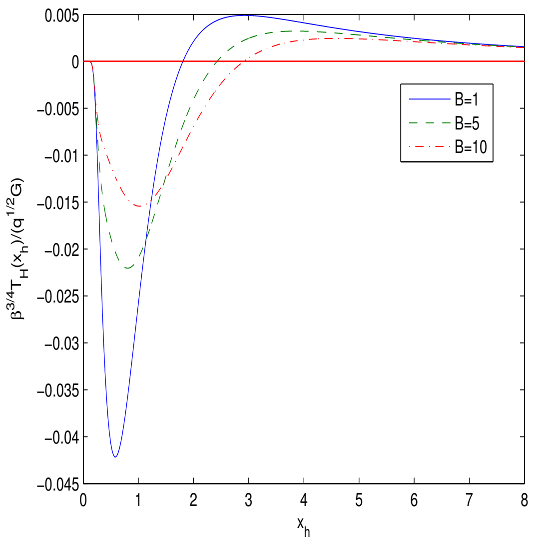

Figure 2: The plot of the function $T_H\sqrt{q}\beta^{1/4}$ vs. horizons x_h for $C = 0$ ($m_0 = 0$). Dashed-dotted line corresponds to $B = 10$, solid line corresponds to $B = 1$ and dashed line corresponds to $B = 5$.

The entropy obeys the Hawking area low $S = A/(4G) = \pi r_h^2/G$. When the Hawking temperature has extremum ($\partial T_H/\partial r_h = 0$) the heat capacity is singular and the second-order phase transition takes place. In Fig. 3 the function $GC_q/(\sqrt{\beta}q)$ vs. the horizon x_h for different values of B for $C = 0$ ($m_0 = 0$) is presented. In accordance with Fig. 3 when the parameter B is bigger the second-order phase transition of the BH (the heat capacity possesses a discontinuity) occurs ant larger value of the horizon x_h. For large value of x_h the BH is unstable ($C_q < 0$).

4 Conclusion

Solutions of a magnetically charged regular BH in modified theory of gravity coupled with NED were obtained. We found the mass and metric functions that can have one (an extreme horizon), two horizons, or no horizons (naked singularity) (see Fig. 1). Corrections to the RN solution that are in the order of $\mathcal{O}(r^{-4})$ were obtained as the radius approaches to infinity. As $r \to \infty$ the spacetime becomes flat. We calculated the asymptotic of the Ricci and

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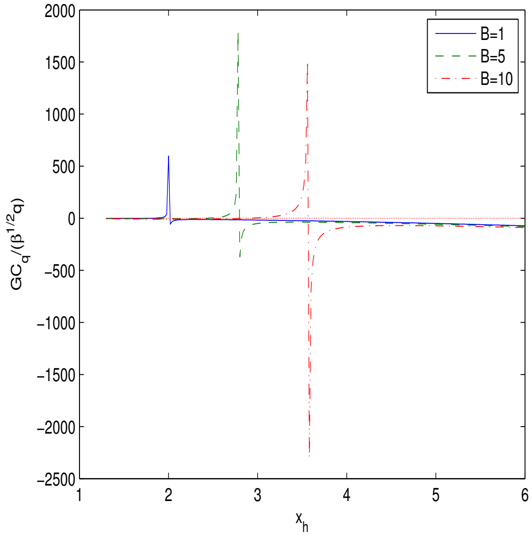

Figure 3: The plot of the function $C_qG/(\sqrt{\beta}q)$ vs. x_h for $C = 0$ ($m_0 = 0$). Dashed-dotted line corresponds to $B = 10$, solid line corresponds to $B = 1$ and dashed line corresponds to $B = 5$.

Kretschmann scalars as $r \to \infty$ and $r \to 0$ showing the absents of singularities. Thus, our solution describes nonsingular BH with the finite curvature everywhere including $r = 0$. The Hawking temperature and heat capacity of the BH were found demonstrating that second-order phase transitions take place. For small values of the horizon radius, depending on the parameters of the model, the Hawking temperature is negative (see Fig. 2) indicating the absence of the BH. The thermodynamic stabilities of black holes were studied and it was shown that in some horizon radii the BHs are stable (the heat capacity is positive) (see Fig. 3).

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