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Growth Rate of Gravity Wave Amplitudes Observed in Sodium Lidar Density Profiles and Nightglow Image Data

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Abstract: Amplitude growth rates of monochromatic gravity waves were estimated and compared from multiple instrument measurements carried out in Brazil. Wave dynamic parameters were obtained from sodium density profiles from lidar observations carried out in Sao Jose dos Campos (23°S, 46°W), while all-sky images of multiple airglow layers provided amplitudes and parameters of waves over Cachoeira Paulista (23°S, 45°W). Growth rates of gravity wave amplitudes from lidar and airglow imager data were consistent with dissipative wave behavior. Only a small amount of the observed wave events presented freely propagating behavior. Part of the observed waves presented saturated amplitude. The general saturated or damped behavior is consistent with diffusive filtering processes imposing limits to amplitude growth rates of the observed gravity waves.

Keywords: all-sky imager; sodium lidar; gravity waves; mesospheric nightglow; amplitude growth rate; wave dissipation

1. Introduction

Gravity waves play an important role in atmosphere dynamics due to their ability to transport momentum and energy from the lower to the upper atmosphere. Their influence on the mesospheric region (80-100 km) include heating through turbulence generated by breaking waves, transport and mixing of constituents, reversal of the zonal mean jets and mean flow acceleration through momentum flux transfer to the mean flow, modifying the dynamical conditions at those altitudes [1,2].

Freely propagating gravity waves (no dissipative waves) are expected to increase their amplitudes as $\sim \exp(\alpha z)$, where $\alpha = \frac{1}{2H}$ is the growth rate of freely propagating gravity waves, z is the altitude and H is the atmospheric scale height. The wave amplitude increases in order to conserve kinetic energy in response to the atmospheric density decreasing with the altitude [3]. Typical value of H is ~ 6 km, and a wave generated at an altitude of 10 km is expected to have an amplitude ~ 349 times larger at the mesospheric region (~ 90 km) than that measured at the generation altitude.

Frequently, instability processes (i.e. convective and/or dynamical), or diffusion (atmospheric viscosity) impose limits to the amplitude growth of gravity waves. Thus, departures from α (the amplitude growth of freely propagating waves) are observed, indicating that the wave is being dissipated. [4] have investigated high frequency gravity waves (<1 hour) disturbing the mesopause temperature by using wind/temperature lidar measurements. They have shown that gravity waves are basically saturated (no change in the wave amplitude over the observed altitude range) to over-damped below 100 km of altitude, while they are unsaturated to freely propagating above that level.

31 Also, [5] have shown that small period waves (<12 h) observed in rotational temperature of
 32 OH(6-2) and O₂(0-1) emissions tend to be strongly dissipated throughout the year. Gravity wave
 33 characterization has been also carried out using simultaneous measurements of the airglow intensity
 34 and temperatures by [6], and simultaneous measurements of the OH and O₂ emission layers were
 35 utilized to infer wave growth and dissipation. They have reported a high variability in the wave
 36 amplitude growth within a short altitude range of 7 km, i.e., the spatial separation between OH and
 37 O₂ layer centroids.

38 In this paper we use two different instruments (a Na lidar system and a nightglow all-sky imager)
 39 to estimate wave amplitudes and growth rates of gravity waves modulating the atmospheric fields
 40 at different altitudes in the mesopause region. The lidar and the imager sample different regions of
 41 the gravity wave spectra and provide complementary information about gravity wave modes present
 42 in Na density and airglow intensity data. The obtained results also give insights about the limiting
 43 processes taking place in the atmosphere in response to increasing wave amplitudes.

44 2. Instrumentation and Methodology

45 Gravity wave intrinsic parameters, amplitudes, and growth rates were obtained from lidar
 46 and all-sky imager data in this study. As both instruments provide wave amplitudes at different
 47 altitudes, amplitude growth rate of waves may be estimated by $\beta = \frac{\ln A_1/A_2}{\Delta z}$, where A_1 and A_2
 48 are the amplitudes of a gravity wave at the altitude levels 1 and 2, respectively, and Δz is the distance
 49 between these levels. Here we refer to β as the growth rate of monochromatic waves in general, to
 50 distinguish from $\alpha = \frac{1}{2H}$, the growth rate of freely propagating waves (non dissipative waves), where
 51 H is the scale height.

52 A Na lidar system located in Sao Jose dos Campos (23°S, 46°W) provided sodium density vertical
 53 profiles from where 45 monochromatic vertically propagating gravity wavers were observed from
 54 1994 to 2004. Na lidar measurements of these events have been reported by [7], while the gravity wave
 55 parameter estimation from the sodium profiles have been carried out by [8].

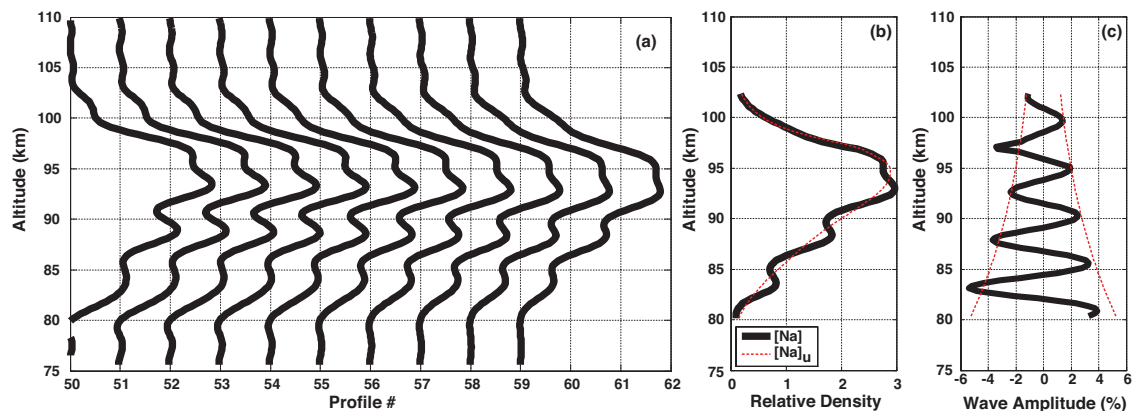


Figure 1. (a) Observed sequence of sodium density profiles taken on May 30, 1996 by a Na lidar system.
 (b) A single Na density profile superposed by an estimated background profile. (c) Wave amplitude
 obtained from (b). Notice the wave amplitude decreasing with altitude indicating that $\beta < 0$.

56 Fig. 1 shows how the monochromatic waves were identified in sodium lidar data. Fig. 1(a) shows
 57 a temporal series of vertical sodium profiles from 75-110 km, with temporal (spatial) resolution of 3 min
 58 (250 m). The sodium density profiles are first spatially and temporally low-pass filtered with cutoffs of
 59 about 1.5 km and 20 minutes, respectively. Coherent downward phase progression can be seen in Fig.
 60 1. Additionally, Fig. 1(b) shows a single $[Na]$ profile superposed to an estimated unperturbed $[Na]_U$
 61 profile. The relative wave amplitude perturbing the Na layer is given in Fig. 1(c), showing a decreasing
 62 wave amplitude as it propagates upward. For this specific case, the wave presents vertical wavelength

63 $\lambda_z = 4.6$ km, amplitude of 2.46% relative to the ambient density (at 90km), and inverse growth rate
 64 $\frac{1}{\beta} = 24$ km. Wave periods, horizontal wavelengths, and phase velocities can be also estimated by using
 65 the technique described by [8].

66 On the other hand, a multicolor nightglow imager operating at Cachoeira Paulista (23°S, 45°W)
 67 provided images of the mesospheric nightglow layers for three emissions during 1999, 2000, 2004 and
 68 2005. A description of this imaging system is given in [9].

69 In order to obtain dynamic parameters of observed gravity waves, we first preprocess the image
 70 dataset by performing usual corrections in every image (i. e., unwarping, star removal, coordinate
 71 transformation, detrending, and filtering). [10] present the preprocessing methodology used in this
 72 study. We focus in wave events occurring quasi-simultaneously in two or three nightglow layers. Fig.
 73 2(a) shows an example of a strong gravity wave perturbing simultaneously the central area of images
 74 of three nightglow emissions. We have spatially filtered the image set in order to increase the contrast
 75 of wave crests by using the Butterworth filter with cutoff spatial frequencies at $\frac{1}{100}$ km⁻¹ and $\frac{1}{10}$ km⁻¹.
 76 The result of filtering operation is presented in 2(b).

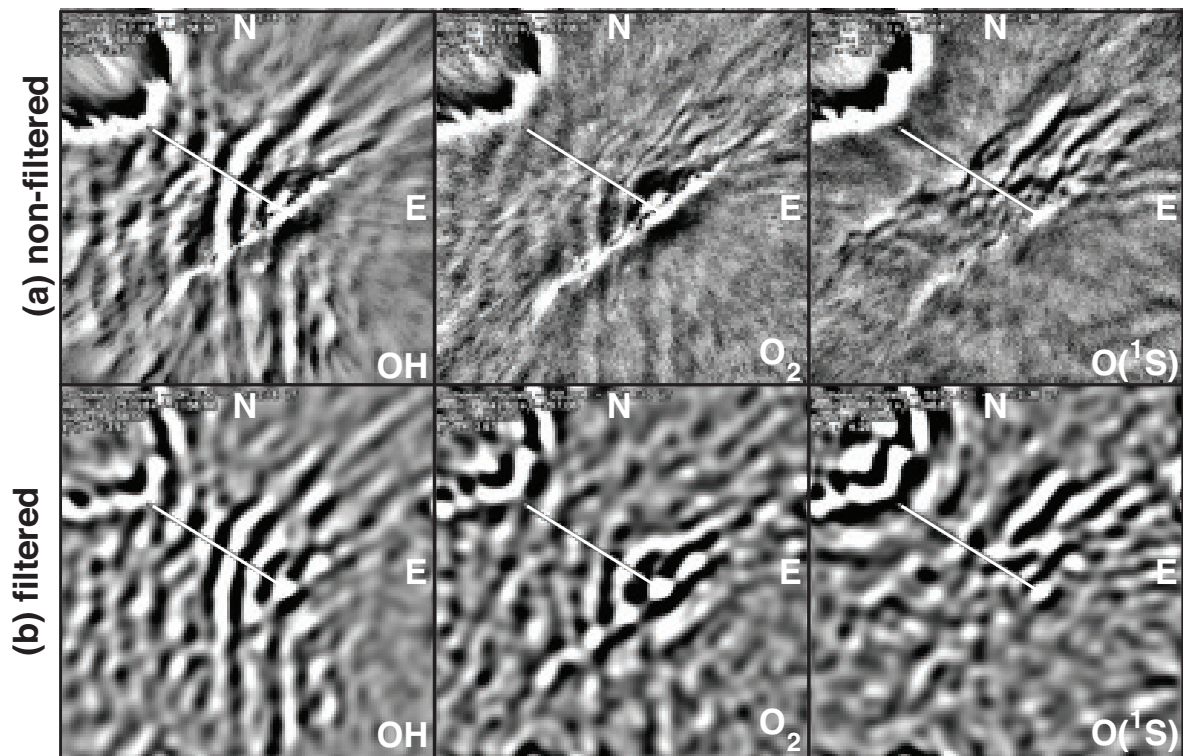


Figure 2. (a) a set of unwarping, non-filtered all-sky images of the OH, O₂ and O(¹S) airglow layers taken quasi-simultaneously on June 30, 2000 at Cachoeira Paulista, Brazil. A large amplitude gravity wave is perturbing all three layers. (b) The same set of images smoothed by a Butterworth spatial band pass filter with cutoff spatial frequencies at $\frac{1}{100}$ km⁻¹ and $\frac{1}{10}$ km⁻¹. The straight lines indicate the pixels whose relative intensity values were extracted to estimate the wave amplitude for each layer.

77 Images of the OH, O₂ and O(¹S) emissions showing simultaneously prominent gravity wave
 78 events are then submitted to 1D cross-spectral analysis in order to deduce the wave horizontal
 79 wavelength, phase difference at different layer, relative amplitude and growth rate, propagation
 80 direction, phase velocity, and period. Due to differences in integration times of every emission and
 81 filter wheel sequence cycle in our imaging system, we have only been able to identify 52 wave events
 82 disturbing simultaneously the layers in four years of observations.

83 The wave amplitude is obtained in one layer by extracting relative intensity $\frac{\Delta I}{I}$ along a straight
 84 line drawn perpendicularly to the wave fronts (see Fig. 2). Spatial series of pixel intensities along that
 85 straight line extracted from the images of each layer can be seen in Fig. 3(a). Pair of these series are

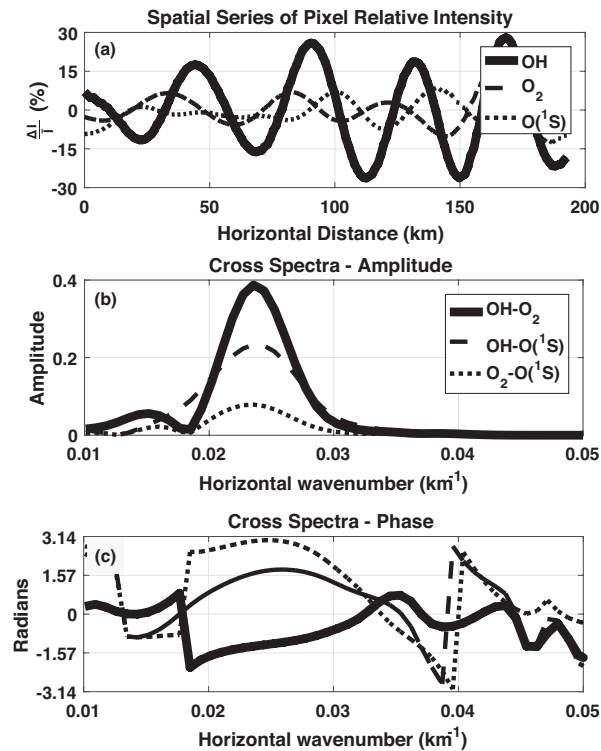


Figure 3. (a) spatial series of pixel of relative intensity extracted along the wave front disturbing the OH, O₂ and O(¹S) airglow layers in Fig. 2. (b) Amplitude and (c) phase cross-spectra calculated for each pairs of spatial series of the emissions. A prominent peak is evident around 0.025 km⁻¹, indicating a gravity wave of ~40 km horizontal wavelength as the cause of the perturbations.

86 then subjected to cross-spectral analysis from where amplitude and phase periodograms are obtained.
 87 Fig. 3(b) and Fig. 3(c) show the amplitude and phase periodograms of the spatial series extracted from
 88 the images in Fig. 2.

89 The location of a spectral maximum in the amplitude periodogram indicates the horizontal
 90 wavelength of the perturbing wave. By integrating below that maximum value we obtain an estimation
 91 of the relative wave amplitude. Because the vertical distance Δz between the centroid of two given
 92 airglow layers is known, the amplitude growth rate is estimated by solving $\beta = \frac{\ln(A_2/A_1)}{\Delta z}$. As the wave
 93 perturbs all three layers at the same time, we observe a finite phase difference for every spatial series
 94 pair (Fig. 3(c)).

95 By applying the procedure above to the images in Fig. 2, we have obtained the following
 96 dynamical parameters for the observed gravity wave event: horizontal wavelength of ~40 km, period
 97 of ~30 minutes, propagation direction of 160°, apparent phase speed of ~20 m/s, and amplitude of
 98 15%, 7% and 5% in OH, O₂ and O(¹S) layers, respectively, indicating a dissipative wave.

99 3. Results

100 [8] identified 45 gravity events from analysis of ten years of sodium density profiles recorded
 101 by lidar, and we have identified 52 gravity events events from analysis of 4 years of airglow images.
 102 These two instruments sample distinct ranges of the gravity wave spectra.

103 Larger vertical scales accessed from lidar measurements are limited by the sodium layer thickness
 104 (~15 km) and the shortest vertical wavelength is basically limited by the signal shot noise [11]. For
 105 this reason waves identified in lidar data by [8] presented vertical wavelengths ranging from 2.4 km
 106 to 9.3 km, with most of these waves (~40%) ranging from ~3 to ~4 km. Observed wave periods
 107 ranged from 63 min to ~20 hours, with maximal occurrence (66%) in the 100-300 min range. Gravity
 108 waves from lidar measurements presented long horizontal wavelengths ($32 < \lambda_h < 1887$ km), but with

109 a tendency of dominance of waves presenting $\lambda_h < 200$ km. Wave amplitudes ranged from 0.77% to
 110 8.4% of the ambient density, with an average value of 2.7%.

111 Gravity wave vertical wavelengths from imager measurements are larger than the airglow layer
 112 thicknesses [12]. Typical layer thickness varies from 8 to 10 km. Because of the observed airglow
 113 intensity is given by vertical integration of the volume emission rate of the emission, short vertical
 114 scale waves ($\lambda_z < 15$ km) are difficult to observe once they self-interfere within the layer. The wave
 115 intensity perturbation is strongly attenuated for ground-based observations in that case. Imagers are
 116 able to observe short period waves ($\tau < 1$ hour) and fast phase speeds ($c_o > 40$ m/s). The horizontal
 117 wavelength accessed with imager is limited by the field of view of the instrument. The lower limit is
 118 determined by the spatial resolution (ds) of each pixel, which is 1 km/pixel in this study. Spectral
 119 analysis of the events studied in here showed λ_h ranging from ~ 14 to ~ 78 km. The analysis of spatial
 120 series extracted from images revealed relative wave amplitudes ($\frac{\Delta I}{I}$) ranging from 0.6% to 15% for the
 121 OH, from 0.5% to 8.5% for O_2 , and from 0.5% to 8.5% for $O(^1S)$ emissions, respectively.

122 Fig. 4 shows histograms for the amplitude growth rate (β) for waves observed in both imager
 123 and lidar data. Positive values of β (under-saturated region) indicate amplitude amplification, while
 124 negative values (over-damped region) indicate decreasing wave amplitudes. Values of β close to
 125 zero indicate that the amplitude does not change much as the wave propagates upward (saturated
 126 wave). Also, it is considered here that waves presenting $\beta > 7$ are freely propagating waves, i.e., their
 127 amplitude increases as $\sim \exp(\alpha z)$.

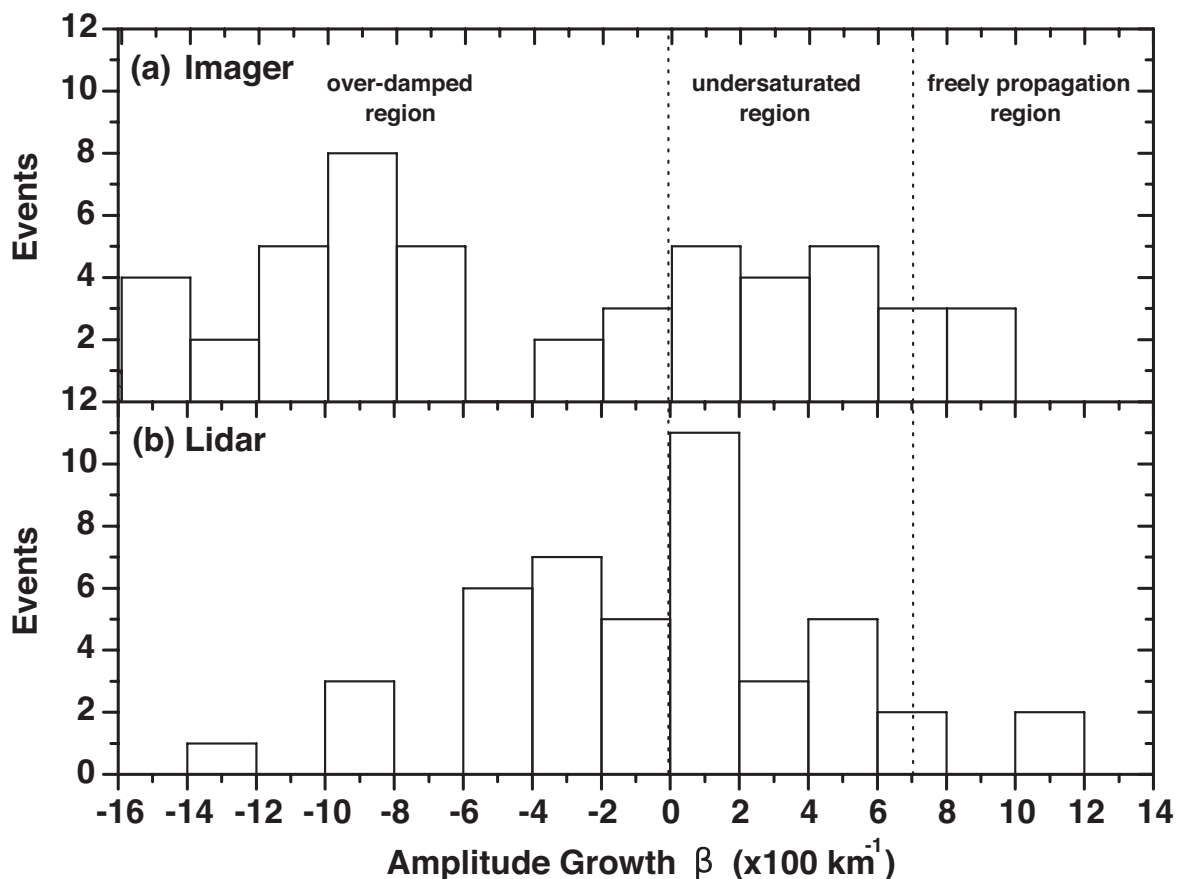


Figure 4. Amplitude growth of waves observed by (a) all-sky imager and (b) sodium lidar. Positive values of the growth rate indicate increasing gravity wave amplitudes. Negative values of β indicate amplitude attenuation as the wave propagates upward. Regions of distinct amplitude growth characteristics are indicated in the diagram.

128 Growth rates obtained for waves from lidar measurements present 48.9% of negative values
129 and 51.1% of positive values, showing a somewhat symmetric β distribution. We observe a maximal
130 occurrence of waves in the range of $0 < \beta < 2$ (under saturated region) that represents 24% of the
131 wave events observed in Na lidar profiles. Those waves presented amplification, but not as rapidly
132 as $\sim \exp(\alpha z)$. It is also observed that $\sim 35.6\%$ of waves in lidar profiles are close of being saturated
133 ($\beta \sim 0$).

134 For waves observed in imager data, we have found 61.5% of negative values and 38.5% of positive
135 values, indicating larger amount of over-damped events. About 51.6% of these waves show strong
136 attenuation ($\beta < -6$), while only $\sim 9\%$ of the waves observed in lidar dataset have similar growth rates.
137 That difference may be caused by the method of analysis used by [8], which is biased towards waves
138 that propagate normal to the wind flow, or are experiencing uniform Doppler shift along the Na layer.
139 Also, $\sim 15.4\%$ of the waves are close to the saturation limit ($\beta \sim 0$), in contrast with waves observed in
140 lidar data. Finally, the growth rate of imager-viewed AGWs show maximal occurrence in the interval
141 of $-10 < \beta < -8$ (over-damped region), which corresponds to 15.4% of the events.

142 4. Discussion

143 While freely propagating waves ($\beta > 7$) correspond to 8.9% and 11.5% of the events observed in
144 lidar and imager, respectively, about 90% of waves observed in both instruments show dissipative
145 behavior (departures from the freely propagating wave growth rate α). The wave energy transferred
146 to the media due to dissipative wave processes may cause several effects in the atmosphere, as mean
147 flow acceleration and local heating. In general, hydrodynamic instabilities and diffusion processes are
148 responsible to limit the wave amplitude.

149 The linear saturation theory (LST) predicts that the wave amplitude will reach the saturation
150 limit when the horizontal perturbation velocity u' equals the intrinsic horizontal phase velocity of the
151 wave c_i . The amplitude is then limited by convective or shear instabilities [13]. On the other hand,
152 the diffusive filtering theory (DFT) states that waves will be severely damped by diffusion when the
153 effective vertical diffusion velocity mD of the particles experiencing the wave motion exceeds the
154 vertical phase velocity of the wave ωm^{-1} [14]. Here, D , m , and ω are the total effective atmospheric
155 diffusivity, the vertical wavenumber and the wave frequency, respectively.

156 In this meaning, waves presenting $\omega > m^2 D$ propagate without attenuation, while waves
157 presenting $\omega < m^2 D$ are removed from the spectra by diffusion. Our study as well as [8] suggest
158 that gravity waves observed in lidar measurements are in accordance with DFT, while rules out the
159 predictions of LST. However, some observed wave events in the lidar dataset presented peculiar
160 behavior, suggesting that processes other than diffusivity have to be considered in order to explain the
161 observed wave amplitude characteristics and growth rates.

162 5. Conclusion

163 Atmospheric gravity waves observed in lidar and imager measurements were analyzed in this
164 study. An amount of 45 monochromatic waves were identified in lidar data, while 52 waves were
165 obtained in images of mesospheric nightglow layers, respectively. The results showed that while
166 each instrument samples a distinct region of the gravity wave spectra, about 90% of the events are of
167 dissipative waves (for both datasets).

168 Growth rate distributions are distinct for waves observed in different lidar to those observed in
169 imager data. The maximal occurrence (24%) of lidar-observed waves is located in the under-saturated
170 region where $0 < \beta < 2$, while the maximal occurrence of waves observed in the imager dataset (15.4%)
171 is between $-10 < \beta < -8$ within the over-damped region of the distribution.

172 Also, 51.6% of imager-observed waves were found in the strong dissipation region ($\beta < -6$),
173 against only $\sim 9\%$ of these type of waves in the lidar dataset. Gravity waves observed in lidar density
174 profiles support the diffusive filtering theory, which states the dissipation of wave energy is mainly
175 due to diffusivity processes acting on the wave amplitude.

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177 G.Y., P.B. and D.G.; Writing – original draft, F.V.; Writing – review editing, F.V.

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