

## Article

# Dual-Wavelength (UV and Blue) Controlled Photopolymerization Confinement for 3D-Printing: Modeling and Analysis of Measurements

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**Abstract:** The kinetics and modeling of dual-wavelength controlled photopolymerization confinement (PC) are presented and measured data are analyzed by analytic formulas and numerical data. The UV-light initiated inhibition effect is strongly monomer-dependent and different monomers have different C=C bond rate constants and conversion efficacy. Without the UV-light, for a given blue-light intensity, higher initiator concentration ( $C_{10}$ ) and rate constant ( $k'$ ) lead to higher conversion, as also predicted by analytic formulas, in which the total conversion rate ( $R_T$ ) is an increasing function of  $k'R$ , which is proportional to  $k[gB_1C_1]^{0.5}$ . However, the coupling factor  $b_1$  plays a different role that higher  $b_1$  leads to higher conversion only in the transient regime; whereas higher  $b_1$  leads lower steady-state conversion. For a fixed initiator concentration  $C_{10}$ , higher inhibitor concentration ( $C_{20}$ ) leads to lower conversion due to stronger inhibition effect. However, same conversion reduction was found for the same H-factor of  $H_0 = [b_1C_{10} - b_2C_{20}]$ . Conversion of blue-only are much higher than that of UV-only and UV-blue combined, in which high  $C_{20}$  results a strong reduction of blue-only-conversion, such that the UV-light serves as the turn-off (trigger) mechanism for the purpose of spatial confirmation within the overlap area of UV and blue light. For example, UV-light controlled methacrylate conversion of a glycidyl dimethacrylate resin formulated with a tertiary amine co-initiator, and butyl nitrite, subject to a continuous exposure of a blue light, but an on-off exposure of a UV-light. Finally, we developed a theoretical new finding for the criterion of a good material/candidate governed by a double ratio of light-intensity and concentration,  $[I_{20}C_{20}]/[I_{10}C_{10}]$ .

**Keywords:** kinetic model; dual-wavelength; photopolymerization; spatial confirmation; additive manufacturing; 3D printing

## 1. Introduction

Polymers for 3D printing and customized additive manufacturing (AM) using various materials such as thermoplastics, polymeric resins, and inorganic powders, and under various methods of material extrusion, powder bed fusion and binder jetting have been developed [1-8]. Conventional photolithographic rapid prototyping achieves reaction confinement in depth through patterned irradiation of a resin having a strong absorbing at a specific wavelength and limited to a very thin layer of photo cured material. All contemporary stereo lithographic devices use of a single wavelength light to initiate polymerization patterned in a plane. However, the single-wavelength irradiation suffers the loss of polymerization confinement by accumulation of non-target light exposure [2-6]. To overcome this drawback, two-color (UV and blue lights), direct-write photolithography was reported, in which the UV-light selectively results polymerization inhibition,

concurrent with the blue-light photo-orthogonal, patterned irradiation employed to induce photopolymerization [9-11].

Idea dual-wavelength AM systems require the following conditions: (i) the photoinhibition light (UV) must yield significantly cease polymerization, while keeping sufficiently high polymerization rates produced by the initiation light (blue or red); (ii) a wide range of compatible monomers and co-initiators; (iii) the photoinhibition of free-radical chain-growth can be rapidly switched on and off by cycling the UV light; (iv) the formulated resins can be spatially confined using concurrent blue and UV irradiation; (v) rapid elimination of the inhibitor species in the dark, or after cessation of UV exposure; (vi) large polymerization inhibition depth adjacent to the projection window; and (vi) continuous part production at translation speeds of several hundred millimeters per hour. Above idea conditions allow for rapid, single-exposure fabrication of complex structures which can not be easily achieved by conventional single-wavelength methods.

Previously reported inhibition layers via oxygen inhibition are typically only tens of micrometers thick and thus it requires the use of low-viscosity resins or fabrication of objects with small cross sections. [6,7]. Single-wavelength, conventional and diffusion-reliant methods with a large inhibition thickness (IT) and high photoinitiation rates offer a continuous and rapid object printing [8-10]. However, they suffer the issues of separation resin reflow. In dual-wavelength systems, the IT may be reduced by decreasing irradiation intensity and thus increase the print speeds as polymerization proceeds closer to the projection window. For a given resin composition, the IT depends on the ratio of inhibitor to initiator absorbance, quantum yield of the radicals, reaction rates among the radicals and monomers, initiator, co-initiator and inhibitor, and the dual-wavelength light intensity and dose (or exposure time). The advantages of dual-wavelength concurrent inhibition and initiation photopolymerization include: controllable high vertical print speeds, eliminating the need for thin, oxygen-permeable projection windows, single-step fabrication of cured materials, rapid generation of personalized products, and the reflow into the inhibition volume during printing can be optimized for large cross-sectional area parts,

Dual-wavelength photopolymerization confinement (PC) were reported in both parallel [10] and perpendicular [11] concurrent irradiation schemes. In the parallel scheme, the volumetric PC was achieved by inhibition volume depth controlled by varying the ratio of the intensities of the UV and blue lights, where print speeds of 2 m/hour have been achieved in a wide variety of resins including acrylates, methacrylates, and vinyl ethers. In addition, by varying the intensity of the light source on a per-pixel basis, the system can perform surface topographical patterning in a single exposure/layer with no stage translation [10]. In the perpendicular scheme, photopolymerizations were reported to confine in depth the region polymerized resin, in which two perpendicular blue and UV lights, independently effect polymerization initiation and inhibition, respectively [11].

As reported by van der Laan et al [11], the effectiveness of a photoinhibitor is strongly monomer-dependent, which also requires: (i) a high conversion of blue-photoinitiation in the absence of the UV-active inhibitor; (ii) a strong chain termination with significant reduction of blue and UV conversion in the presence of UV-active inhibitor; and (iii) short induction time or rapid elimination of the inhibitor species in the dark (or absence of UV-light), such that the initiation-inhibition cycles may be switched on and off rapidly. Fast switching-time may be achieved by high conversion rate, or high blue-light intensity, the triple-state quantum yield, and the absorption constant of the monomer resin. However, we note that higher light intensity achieves a faster photoinitiation, but suffers a lower saturation value, or lower conversion. Moreover, a short induction time may be achieved by high UV intensity or large inhibitor-concentration, and it also requires minimum impurity and oxygen which cause a delayed curing of the resin.

The monomer-dependence of a dual-wavelength PC was reported by van der Laan et al [11], in which different monomers have different C=C bond rate constants (K) under the exposure of blue, UV and blue+UV. For example, bisphenol ethoxylate diacrylate (BPAEDA) resins formulated with camphorquinone (CQ) and ethyl 4-(dimethylamino)benzoate (EDAB) have a maximum conversion rate constant  $K_{max} = 0.675$  (at blue+UV) for 0% butyl nitrite (BN), and reduces to 0.0106 (for 1%BN),

a factor of 64 reduction, Therefore, it is a better candidate than trimethylolpropane triacrylate (TMPTA) which only has a 3 times reduction of  $K_{max}$ .

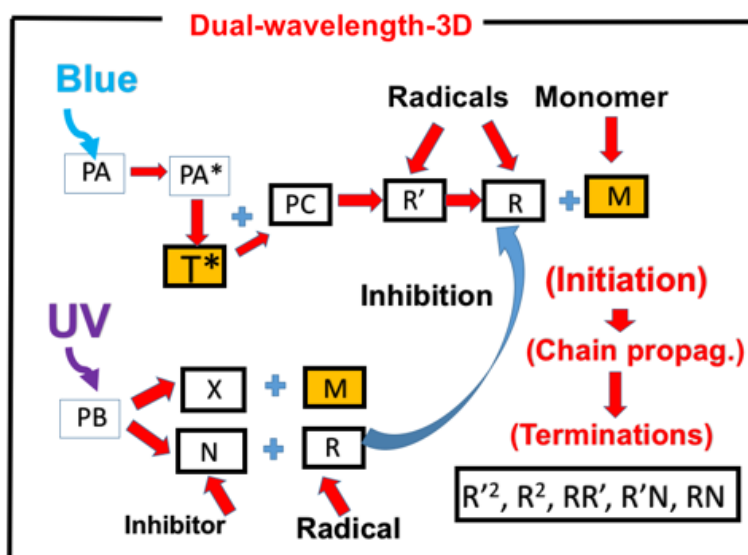
Although the mechanism for radical-mediated polymerization initiation and inhibition in dual-wavelength system and the simple formulas for the associated printing speed and inhibition volume thickness [11] were reported, there is no detailed kinetics or the conversion efficacy have been theoretically reported in a dual-wavelength, thick polymer system. We have previously reported the kinetics and modeling of a single-wavelength radical-mediated photopolymerization in single-initiator [12-14], two-initiator [15] and two-component system [16,17]. This study will extend our previous modeling to a 2-wavelength, 3-initiator system. We will focus on the following issues: the roles of the concentration of the blue-light active initiator and its co-initiator, and the UV-light-active inhibitor (BN); the role of reaction rates and light intensity on the conversion profiles; the polymerization inhibition depth and printing speed; and induction time, which is desired for a fast on-off switching control of PC. Finally, analytic formulas and numerical results will be utilized to analyze the measured data of de Beer et al [10] and van der Laan et al [11].

## 2. Materials and Methods

### 2.1. Photochemical Kinetic

As shown by Fig. 1, a dual wavelength (UV and blue light) radical-mediated system consists of photoinitiator, PA and a co-initiator, PC; and another photosensitizer, PB, which serves as a photoinhibitor for the monomer conversion. The blue-light (at 470 nm) excites photoinitiator (PA) producing excited states,  $PA^*$  and triplet state  $T^*$ ; whereas the UV-light (at 365 nm) photodecomposes the photoinhibitor (PB) to produce an inhibition radical [N], and an initiation radical [X], which could interact with the monomer [M] for crosslinking. The  $PA$  triplet state ( $T^*$ ) could interact with the co-initiator, PC, forming the primary radical ( $R'$ ), which reacts with PA for a chain propagation and produces more radical (R). Bimolecular termination of  $R'$  produces the propagating radical (R) which leads to crosslink; terminations may be also resulted by the recombination of R, interaction of  $R'$  and R, [N] and R' and R.

Example of the above described dual-wavelength system were reported by de Beer et al [10] and van der Laan et al [11], in which methacrylate conversion under the exposure of blue (470 nm) and UV (365 nm) light was measured for the photopolymerization of a methacrylate resin formulated with camphorquinone (CQ, as PA), ethyl 4-(dimethylamino)benzoate (EDAB, as a co-initiator, PC), and butyl nitrite (BN), as the UV-activated initiator, PB). The photochemical decomposition of butyl nitrite (BN) results in nitric oxide (N), an efficient inhibitor and alkoxide radical (X) for extra polymerization initiation, besides the reactive radical (R).



**Figure 1.** Schematics of photochemical pathways of dual wavelength photopolymerization; in which crosslinkers are formed via two pathways, via the photoinitiator PA (under a blue light), and PB (under a UV light). The initiation radicals R and [X] crosslink with the monomer [M]; whereas the inhibition radicals [N] reduces the conversion efficacy by reducing the active radicals (R' and R). Also shown is the co-initiator (PC) which reacts with the triplet state of PA (T\*) forming an intermediate radical (R'). Bimolecular termination of R' produces a propagating radical (R) which leads to crosslinks; terminations could be also resulted by the interaction of R and R', and R and [N].

Using the short-hand notations for the concentration of various components:  $C_1$ ,  $C_2$ ,  $C_3$  for the ground state concentration of PA, PB and PC, respectively, and [M] for the monomer, the kinetic equations for the dual-color and 5 radicals (R', R, [N], [X]) system are derived as follows [12,15,16]

$$\frac{\partial C_1}{\partial t} = -gB_1C_3C_1 \quad (1)$$

$$\frac{\partial C_2}{\partial t} = -g'B_2[M]C_2 \quad (2)$$

$$\frac{\partial C_3}{\partial t} = -gB_1C_3C_1 + 2k_T R^2 + k_{22} R[N] \quad (3)$$

$$\frac{\partial [N]}{\partial t} = B_2C_2 - k_{22}R[N] \quad (4)$$

$$\frac{\partial [X]}{\partial t} = B_2C_2 - (k_6 + k_8[M])[X] \quad (5)$$

$$\frac{\partial R'}{\partial t} = gB_1C_1C_3 - (2k_T R' + k_{12}R)R' \quad (6)$$

$$\frac{\partial R}{\partial t} = -2k_T R^2 - (k'[M] + k_{12}R' + k_{22}[N])R + 2k_T R'^2 \quad (7)$$

$$\frac{\partial [M]}{\partial t} = -(k_8[X] + k'R)[M] \quad (8)$$

where  $B_1 = b_1 I_1(z, t)$ ,  $B_2 = b_2 I_2(z, t) I_3(z, t) / I_{30}$ ,  $B_3 = (b_3 / b_2) B_2$ ;  $g = 1 / (k_{57} + k C_3)$ ;  $g' = 1 / (k_{68} + [M])$ ,  $k_{57} = (k_5 / k_7)$ ,  $k_{68} = (k_6 / k_8)$ ,  $k = (k_3 / k_7)$ .  $R_E$  is the  $C_1$  regeneration term given by  $R_E = k_{22}[N]R + 2k_T R^2$ .  $b_j = 83.6 a_j q_j w_j$ ;  $a_j$  is the extinction coefficient for PA, PB and PC (with  $j=1,2,3$ ); the light wavelength (in cm),  $w_1$  for the blue (at 470 nm), and  $w_2$  for UV (at 365 nm) and light intensity  $I_j(z, t)$  in mW/cm<sup>2</sup>;  $q_j$  is the quantum yields of the PA triplet state and PB radical.

Eq. (6) gives the generation of radical R' via the interaction of T\* and the co-initiator (PC), given by  $k_3 T^* C_3$ , with the steady-state  $T^* = (g / k_7) B_1 C_1$ . The UV light intensity,  $I_2(z, t)$ , is absorbed mainly by PB, in which the UV conversion of PA monomer is reduced by the presence of PB. On the other hand, the blue-conversion of PA could be significantly reduced by the UV-generated radical of PB, such that the inhibition depth may be controlled by the on-off of UV light, to be discussed more later. All the reaction rate constants are defined by the associated coupling terms. For examples, in Eq. (7),  $k'$  is for the reaction of monomer and radical R, which has a relaxation rate  $k_5$ ;  $k_{12}$  is for the radical interaction of R' and R, and both have a bimolecular termination rate of  $k_T$ ; More detail derivation and definition of rate constants in  $g$  and  $g'$ , have been previously published [12,15].

Using the quasi-steady-state conditions of [12,15]  $d[N]/dt = d[X]/dt = d[R']/dt = 0$ , we obtain steady-state radicals given by  $[N] = B_2 C_2 / (k_{22} R)$ ,  $[X] = g' B_2 C_2 / k_8$ , and Eq. (8) becomes

$$\frac{\partial [M]}{\partial t} = -R_T [M] \quad (9)$$

$$R_T = g' B_2 C_2 + k' R \quad (10)$$

Where  $R_T$  is a total rate constant which consists of two crosslink components attributed from the interaction of the monomer and  $[X]$  and  $R$ , respectively. Furthermore, the steady-state radicals,  $R'$  and  $R$ , are given by

$$2k_T R'^2 + k_{12} R' - gB_1 C_1 C_3 = 0 \quad (11)$$

$$2k_T R^2 + (k'[M] + k_{12} R')R - (2k_T R'^2 - B_2 C_2) = 0 \quad (12)$$

Solving for Eq. (11) and (12), we obtain

$$R = \left( \frac{1}{4k_T} \right) [-G + \sqrt{G^2 + 8k_T H}] \quad (13)$$

$$H = gB_1 C_1 C_3 - B_2 C_2 \quad (14)$$

where  $G = k'[M] + 2k_{12} R'$ . Eq. (13) may be further approximated to  $R = (0.5H/k_T)^{0.5} - k'[M](1-d)$ , with  $d = 0.5G^2/(8k_T H)$ , for  $2k_{12} R' \ll k'[M]$ , which shows that  $R$  and efficacy, are increasing function of  $H$ . The balance point of inhibition depth is defined by when  $R=0$ , or  $8k_T H=0$ , or  $gB_1 C_1 C_3 = B_2 C_2$ , in which the PA initiated radical ( $R$ ) is completely inhibited/consumed by the PB's radical,  $[N]$ . We will have more discussion later.

The dynamic light intensity of blue ( $I_1$ ) and UV ( $I_2$ ) are given by, when they are applied to the resin orthogonally and separately, [13,15]

$$I_j(z, t) = I_{j0} \exp[-A_j z] \quad (15)$$

$$A_j(z, t) = 2.3(a_j C_{j0} + Q_j) - A_{j1}t \quad (16)$$

where  $a_j$  is the extinction coefficients of PA (for  $j=1$ ) and PB (for  $j=2$ ) and their photolysis products, respectively;  $Q_j$  is the absorption coefficient of the monomer at the blue and UV wavelength. Most previous modeling [8-13] assumed a constant  $C(z, t)$  in Eq. (2.b). Our analytic formulas in this article will use a time-average of  $A(z, t)$  to count for the dynamic of light intensity due to PA and PB depletions. Accurate solutions of Eq. (1) and (8) require numerical simulations (to be shown later). For analytic formulas, we will use approximated analytic formulas for the light intensity and the PI and PE concentration and the expressive closed forms of  $I_j(z, t)$  and  $C_j(z, t)$  allow us to solve for the first-order and second-order solutions of  $R$ ,  $[M]$  and the conversion efficacy.

## 2.2. Analytic formulas for Efficacy

The monomer conversion efficacy for a bimolecular termination process is given by  $C_{EFF} = 1 - [M]/[M]_0 = 1 - \exp(-S)$ , with  $[M]_0$  being the initial monomer concentration, and the  $S$ -function is given by the time integral of the total rate factor  $R_T$  given by Eq. (1.f),  $d[M]/dt = -R_T[M]$ , in which  $R_T$  has 3 components defined by the coupling of the monomer  $[M]$  and the triplet-state, PA-radical, and PB radical, respectively.

For  $k_{53} \ll k_{C3}$ ,  $k_{68} \ll k''[M]$ ,  $gC_3 = g'[M] = 1$ , the solutions of Eq. (1) to (3) are available by the approximated analytic formulas for  $I_j(z, t)$  and  $C_j(z, t)$ , with  $j=1, 2, 3$ , for PA, PB, PC, as follow [12,13]

$$I_j(z, t) = I_{j0} \exp[-A_j z] \quad (17)$$

$$C_j(z, t) = C_{j0} \exp[-B'_j t] \quad (18)$$

$$A_j(z, t) = 2.3(a_j C_{j0} + Q_j) - A_{j1}t \quad (19)$$

where  $B'_j = b_j I_{j0} \exp(-A'_j z)$ ,  $A_{j1} = 2.3(a_j - b'_j)C_{j0} I_{j0} b_j z$ , with  $A'_j$  is the time-averaged absorption given by  $A'_j = 1.15(a_j + b'_j) + 2.3Q_j$ ,  $b'_j$  is the extinction coefficient of the photolysis products. We note that the  $-A_{j1}t$  term represents the decrease of  $A'_j$ , or increase of light intensity due to concentration depletions of PA, PB and PC.

Using Eq. (17) and (18) for the total rate fraction,  $R_T$ , we solve for Eq. (9) to obtain the total efficacy given by  $C_{EFF} = 1 - [M]/[M]_0 = 1 - \exp(-S)$ , with  $S$  is the time integral of  $R_T$ , which requires a numerical integration, in general. For analytic solutions, two cases are considered. For  $gB_j C_j \ll k'R$ , case (i)  $H \gg G$ ,



$k'R=KH^{0.5}$ , with  $K=0.5k'/k_T^{0.5}$ ; case (ii)  $H \ll G$ ,  $k'R=k'H/G$ ; where  $H^{0.5}$  may be further reduced to  $H^{0.5}=(B_1C_1)^{0.5}-0.5(B_2C_2)/(B_1C_1)^{0.5}$ , for  $(B_2C_2) \ll (B_1C_1)$ , and  $gC_3=1$ , for  $k=k_3/k_7=1$ .

In case (i)  $H \gg G$ ,  $C_{EFF}=1-[M]/[M]_0=1-\exp(-S)$ , with  $S$ -function is given by

$$S=K\sqrt{D_{10}} [E_{11} - 0.5(D_{20}/D_{10})E_{12}] \quad (20)$$

$$E_{11} = [1 - \exp(-G_{31}t)]/G_{31} \quad (21)$$

$$E_{12} = [1 - \exp(-(G_{32} - G_{31})t)]/(G_{32} - G_{31}) \quad (22)$$

where initial values:  $D_{j0}=B_{j0}X_j$ ,  $G_{3j}=0.5(B_{j0}-A'_{1j})$ , with  $B_{j0}=b_jI_{j0}C_{j0}$ ,  $X_j=\exp(-A'_{1j}z)$ ,  $A'_{1j}=1.15(a_j+b_j)C_{j0}+2.3Q_j$ , is a mean value of  $A_j(z,t)$ . We note that Eq. (20) reduces to our previous formula for one-wavelength system with  $B_{20}=0$  and  $H=B_1C_1$ .

For case (ii),  $H \ll G$ , and for  $k'[M] \ll k'R$ ,  $k'R=H/[M]$ , Eq. (9) becomes

$$\frac{\partial[M]}{\partial t} = -H(t) \quad (23)$$

Therefore, the efficacy is given by the time integral of  $H(t)/[M]_0$ , or

$$C_{EFF} = S/[M]_0 \quad (24)$$

with  $S$  given by Eq. (20).

The steady-state of Eq. (21) and (22) are given by when  $E_{11}=1/G_{31}$ ,  $E_{12}=1/(G_{32}-G_{31})$ , whereas transient state is given by  $E_{11}=E_{12}=t$ . Therefore, the inhibition effect given by the second term of Eq. (21) is proportional to  $B_{20}/(B_{10})^{0.5}/(G_{32}-G_{31})$ , with  $B_{j0}=b_jI_{j0}C_{j0}$ , for steady-state; and  $[tB_{20}/(B_{10})^{0.5}]$  for transient state. Numerical data will be shown later. We also note that for a given  $B_1C_1$ , the radical  $R$  is a decreasing function of the ratio of  $R_{AB}=(B_2C_2)/(B_1C_1)^{0.5}$ . Therefore, same  $R_{AB}$  reaches the same efficacy. This feature will be numerically shown later.

### 2.3 The Inhibition depth and Time

Polymerization inhibition depth adjacent to the projection window is a critical parameter for continuous stereolithographic fabrication. One may find the inhibition depth ( $z_H$ ) defined by the balance point of initiation and inhibition rate, or when  $R=0$ , or  $H=0$ . We find from Eq. (14),

$$z_H(t) = \frac{1}{A_2+A_3-A_1} \ln\left(\frac{B_{20}C_2}{gB_{10}C_1C_3}\right) \quad (25)$$

where  $B_{j0}=b_jI_{j0}$ , and  $C_j(t)$  are the  $z$ -averaged function of  $C_j(z,t)$ . We note that Eq. (8) defines an inhibition coefficient defined by  $\beta=(b_2/b_1)[C_2/(gC_1C_3)]$ , which depends on multifactor and rate constants related by  $g=1/(k_{57}+kC_3)$ . Our formula is more general than that of de Beer et al [10] which is our special case when  $A_3=0$ , and  $C_2=gC_1C_3$ , such that Eq. (25) reduces to Eq. (1) of de Beer et al [10]:  $z_H=(1/(A_2-A_1))\ln[\beta I_{20}/I_{10}]$ , with  $\beta=(k_37+kC_3+[M])b_2/b_1$ . We note that  $b_j=83.6a_jq_jw_j$ ; which is defined by the extinction coefficient for PA, PB and PC (with  $j=1,2,3$ ); the light wavelength,  $w_1$  for the blue (at 470 nm), and  $w_2$  for UV (at 365 nm) and the quantum yields ( $q_j$ ). Moreover, in our more general formula,  $\beta$  is also proportional to  $1/g=k_{57}+kC_3$ , defined by the rate constants of  $k_{57}$  and  $k=k_3/7$ .

Due to the time-dependence of  $C_j(z,t)$ , Eq. (25) in general is time-dependent, which was assumed as time-independent by de Beer et al [10], when  $C_j$  reaches a steady-state or remains as its initial value, in which the initiators depletion is ignored. To explore this dynamic feature, one may define an inhibition time ( $T_H$ ) given by when the radical  $R=H=0$ . For a common situation that that  $C_1(t)=C_{10}\exp(-B't)$ , with PA has a depletion rate,  $B'$ , much larger than that of PB and PC, such that  $C_2(t)=C_{20}$ ,  $C_3(t)=C_{30}$ , both are much slowing decay function of time (to be shown by our numerical data later), we obtain an analytic formula

$$T_H = \frac{1}{B'} \ln\left(\frac{I_{10}}{\beta' I_{20}} \exp[(A_2+A_3-A_1)z]\right) \quad (26)$$

where  $\beta'=(b_2/b_1)[C_{20}/(gC_{10}C_{30})]$ . Eq. (26) shows that  $T_H$  is an increasing function of the depth ( $z$ ), but a decreasing function of the concentration ratio  $C_{20}/(C_{10}C_{30})$ , i.e., higher inhibitor concentration  $C_{20}$ , results to a shorter inhibition time, which is desired for a faster on-off switching mechanism.

The thickness of this polymerization inhibition volume adjacent to the projection window is a critical parameter for continuous stereolithographic fabrication. A minimum intensity ratio of UV and blue light,  $R_{\min} = (I_{20}/I_{10})_{\text{crit}}$  defined by which initiation and inhibition rates are balanced to generate an inhibition depth,  $z_H=0$  in Eq. (25), and can be calculated by when  $R_{\min} = 1/\beta' = gC_{10}C_{30}/[(b_2/b_1)C_{20}]$ , which is dependent on resin composition ratios and rate constants. de Beer et al [10] reported  $\beta' = 1$  in a TMPTA-based system.

Above formulas, Eq. (8) are based on  $H = 0$  as also defined by de Beer et al [10]. A more accurate definition should be based on the time integral of  $R_T$ , and defined by the S-function higher than a critical value,  $S > S_T$ , or efficacy  $C_{\text{EFF}} > C_T$ , where  $S_T = \ln [1/(1-C_T)]$ , which can only be calculated numerically (to be shown later).

## 2.4 Print Speed

Based on Eq. (13), the maximum print speed ( $S_{\max}$ ) as defined by de Beer et al [10], when the dose difference of blue light and UV light equals to a critical value ( $E^*$ ), and  $B_1 = \beta B_2$ , we obtain a similar formula

$$S_{\max} = \frac{B_{20} - \beta B_{10}}{E^*} \quad (27)$$

which, however, has a much complex function of  $\beta = (C_2 b_{20}/b_{10})/(gC_1 C_3)$ , than the simplified function of de Beer et al [10], with  $\beta = b_{20}/b_{10}$ . A more accurate definition would be based on the S function, or time integral of Eq. (10), rather than light dose given by Eq. (20). However,  $S_{\max}$  needs numerical result integral of Eq. (10), to be shown later.

## 2.5 Curing depth

There are two ways to define a curing depth of the green light: the simple one is defined by when the blue light dose,  $I_{10}t$ , larger than a threshold value of  $E_{TH}$ . Using the time integral of Eq. (17) with neglected  $A_1 t$ , we obtain,

$$z_C = \frac{1}{A_1} \ln \left( \frac{I_{10}t}{E_{TH}} \right) \quad (29)$$

Above formula is the same as that of de Beer et al [10]. However, the more accurate definition of curing depth is given by when the blue-light conversion efficacy, given by Eq. (20), is higher than a critical value,  $C_{\text{EFF}} > C_T$ , or when  $S > S_T$ , with  $S_T = \ln [1/(1-C_T)]$ . We obtain

$$T_C = \left( \frac{2}{B'} \right) \ln [B' / (K \sqrt{0.5X'B}) - 1] \quad (30)$$

where  $B' = (b_1 I_{10} C_{10})$ , and  $X' = \exp(-A' z_C)$ , with  $A' = 1.15(a'_1 + b'_2)C_{10} + 2.3Q_1$ , is a mean value of  $A_1(z, t)$ .

## 3. Results and Discussion

Numerical results based on Eq. (1) to (8) and the steady-state radical given by Eq. (13), and using the light intensities given by Eq. (17) are shown as follows. We will first show the conversion for the case of blue-light only, i.e., when  $B_2=0$  (no UV light) for various concentration of the initiator,  $C_{10} = (0, 0.5, 1.0, 3.0) \%$ , and coupling parameter  $b_1$  which is given by the absorption coefficient and blue-light intensity; and also the role of the crosslink rate constant ( $k'$ ) which gives the conversion in Eq. (10). For simplicity, our modeling will limit to the surface layer of the resin, that is for  $z=0$ . Spatial conversion profiles can be found in our previous study, but limited to a single-wavelength system [13,14].

We then present the inhibition effect (IBE) on the conversions under the exposure of both UV and blue light for various inhibitor concentration. As shown by Eq. (9), the conversion efficacy is governed by the H-function of Eq. (14), or  $B_1 C_1 - B_2 C_2$ . As suggested by our analytic formulas, Eq. (6),

our numerical input will be the the initial values of  $B_j$  or  $b_j$  (with  $j=1,2$ ) rather than the light intensity or the absorption coefficients. We will also show that the IBE is strongly monomer-dependent, as reported by van der Laan et al [11] by various reaction rate constants ( $k'$ ,  $k_T$ ). The numerically produced temporal profiles will be analyzed by our analytic formulas of Eq. (20). Finally, our numerical profiles will fit to the measured data of de Beer et al [10] and van der Laan et al [11] in an on-off scheme.

### 3.1. Efficacy Temporal Profiles

We will first show the conversion of blue-light only (without the UV-light). Fig. 2 (left Figure) shows that higher initiator concentration,  $C_{10}$ , leads to higher conversion, as also predicted by our Eq. (10), in which the total conversion rate ( $R_T$ ) is an increasing function of  $k'R$ , which is proportional to  $[gB_1C_1]^{0.5}$ . However, the coupling factor  $b_1$  plays a different role that higher  $b_1$  leads to higher conversion only in the transient regime; whereas higher  $b_1$  leads lower steady-state conversion (as shown by right Figure). This unique reverse effect in steady-state is also predicted by our analytic Eq. (20).

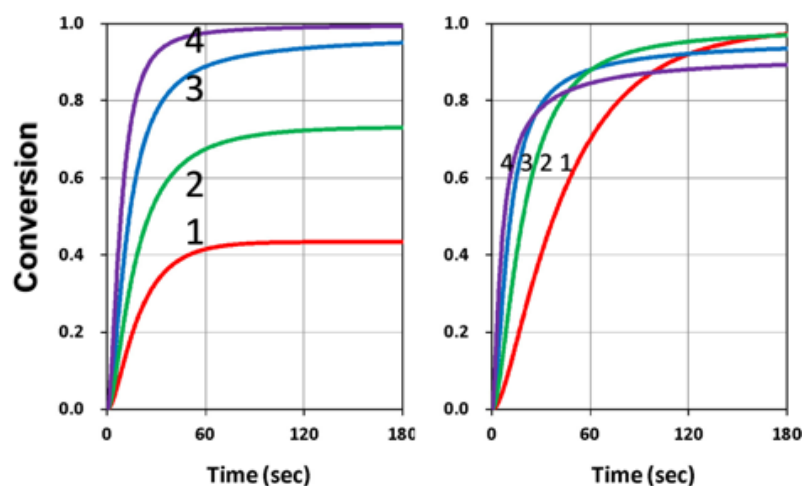
Fig. 3 shows that higher coupling rate constant ( $k'$ ) leads to higher conversion, as also predicted by Eq. (10). These calculated profiles well fit with the measured data of de Beer et al [10], for various resin formations (shown by their Figure 3).

Figure 4 shows the conversion profiles for blue-only, UV-only, and both-light and compared with measured data of de Beer et al [10], with fit parameter of  $k'$ ,  $k_T$  and  $b_j$ . Conversion of blue-only are much higher than that of UV-only and UV-blue combined, in which high inhibitor concentration ( $C_{20}$ ) results a strong reduction of blue-only-conversion, such that the UV-light serves as the turn-off (trigger) mechanism for the purpose of spatial confirmation within the overlap area of UV and blue light. Figure 5 shows the conversion profiles under the same conditions as that of Figure 4, but for different resin formations which are specified by our parameter  $k''$ . As reported by van der Laan et al [11], different monomers have different C=C bond rate constants ( $K$ ) under the exposure of blue, UV and blue+UV. For example, bisphenol ethoxylate diacrylate (BPAEDA) resins formulated with camphorquinone (CQ) and ethyl 4-(dimethylamino)benzoate (EDAB) have a maximum conversion rate constant  $K_{max} = 0.675$  (at blue+UV) for 0% butyl nitrite (BN), and reduces to 0.0106 (for 1%BN), a factor of 64 reduction, Therefore, it is a better candidate than trimethylolpropane triacrylate (TMPTA) which only has a 3 times reduction of  $K_{max}$ .

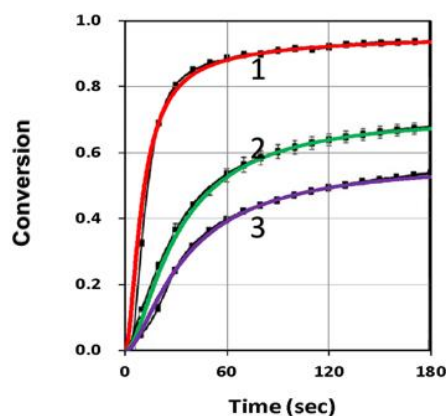
Similar to Figure 4, Figure 5 also shows significant reduction of conversion under the UV-light initiated inhibition of the radical ( $R$ ). This is one of the key factors to achieve efficient photopolymerization confirmation (PC).

Figure 6 shows the initiation radical ( $R$ , left) and conversion (right) profiles in the presence of UV and blue light, for various inhibitor concentration ( $C_{20}$ ), in which, for a fixed initiator concentration  $C_{10}$ , higher  $C_{20}$  leads to lower conversion due to stronger inhibition effect. However, , as shown by Figure 7, same conversion reduction was found for the same H-factor of  $H_0 = [b_1C_{10} - b_2C_{20}]$ . This unique feature is also predicted by Eq. (13) and (20).

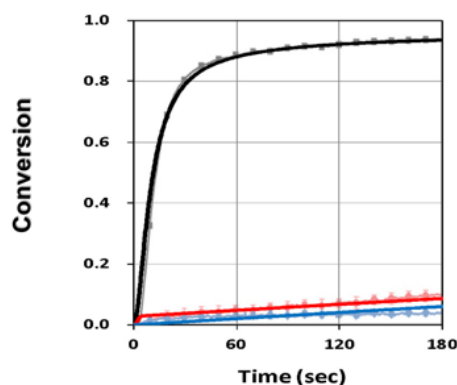




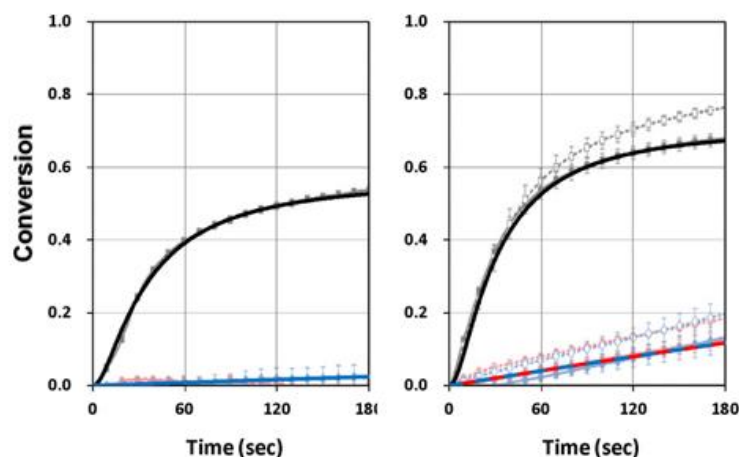
**Figure 2.** Conversion profiles of blue-light (without UV-light) for (left Figure)  $C_{10} = (0.05, 0.1, 0.2, 0.4)\%$ , for curve (1,2,3,4), for fixed  $b_1 = 0.1$ ; and (right Figure)  $b_1 = (0.015, 0.05, 0.15, 0.5)$ , for fixed  $C_{10} = 0.2\%$ ; for  $C_{30} = 0.5\%$ ,  $[M]_0 = 0.2\%$ ,  $k' = 1.0$ ,  $k_T = 0.5$ ,  $k_{57} = (k_5/k_7) = k_{68} = (k_6/k_8) = k'' = 35$  (1/s).



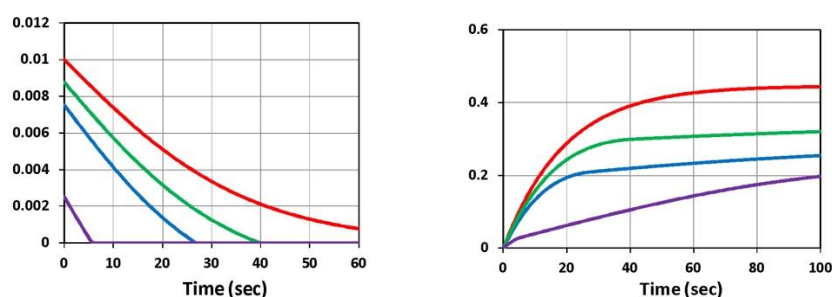
**Figure 3.** Same as Fig.2 but for various  $k' = (1.0, 0.3, 0.19)$ , for curves (1,2,3), for fixed  $C_{10} = 0.2\%$ , and adjusted  $b_1 = (0.15, 0.05, 0.05)$  to fit the measured data of de Beer et al [10].



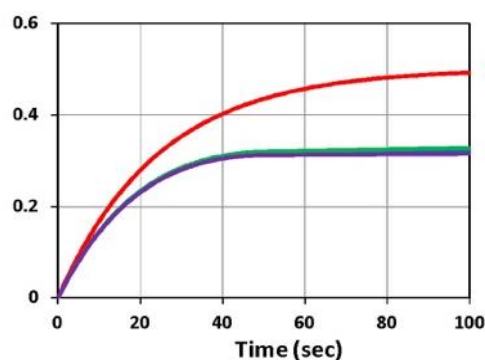
**Figure 4.** Conversion profiles for blue-only (black curve), UV-only (blue curve), and both-light (red curve), for  $C_{10} = 0.2\%$ ,  $C_{30} = 3.0\%$ ,  $b_1 = 0.1$ ,  $b_2 = 0.007$ ,  $k'' = 35$ ; where solid color curves are calculated and bars are measured data of de Beer et al [10].



**Figure 5.** Same as Fig.4, but for different monomers governed by various  $k''=60$  (left Figure) and  $k''=150$  (right Figure), for  $b_1=0.05$ ,  $b_2=0.007$ ,  $k'=1.0$ , where bars are measured data of de Beer et al [10].



**Figure 6.** The initiation radical (R, left) and conversion (right) profiles in the presence of UV light; for various inhibitor concentration,  $C_{20} = (0, 1.0, 2.0, 3.0)$ , for curve (red, green, blue, violet), for  $b_1 = b_3 = 0.1$  (1/s/%),  $C_{10} = 0.2$  (%),  $C_{30} = 0.5$  (%),  $[M]_0 = 0.2$  (%);  $k' = 1.0$  (1/s),  $k_{48} = 1.0$  (1/s),  $k_{37} = 1.0$  (1/s),  $k_{57} = 0.01$  (1/s).



**Figure 7.** Same as Fig. 6, but for a fixed difference of  $[b_1C_{10} - b_2C_{20}] = 0.003$ , showing the overlapping of all curves, for  $C_{20} > 0$ .

### 3.2. Analysis of measured data

Our modeling conversion profiles shown by Fig. 3, 4 and 5 could be compared with the Figure 3 of de Beer et al [10], where to fit their data, we have adjusted the parameters of  $b_1$ ,  $b_2$  and rate constants ( $k'$ ,  $k''$ ,  $k_T$ ), but kept the same initiator concentrations ( $C_{10}$ ,  $C_{20}$ ,  $C_{30}$ ), as the measured data [10]. The monomer-dependence of the conversion for various resin formations are governed by the adjusted rate constants (in a relative amount), because the actual values are not available,

Our modeling conversion profiles shown by Fig. 8 could be compared with the measured data. Figure 2 of van der Laan et al [11]. Our modeling profiles show the similar features as the measured

data that inhibition effects are an increasing function of the inhibitor concentration, although our curves are slightly higher. The difference might be due to the fact that our modeling is based on ideal kinetics and excludes complex factors involved in the measurements. In addition, our presented conversions are for the surface layer only (with  $z=0$ ). Therefore, we expect a better fit if the spatial ( $z$ -dependence) conversion profiles are included, as shown by our Eq. (15). However, most of the features and the roles of the key parameters presented based on surface layer should also apply to a volume conversion. Spatial conversion profiles can be found in our previous study, but limited to a single-wavelength system [13,14].

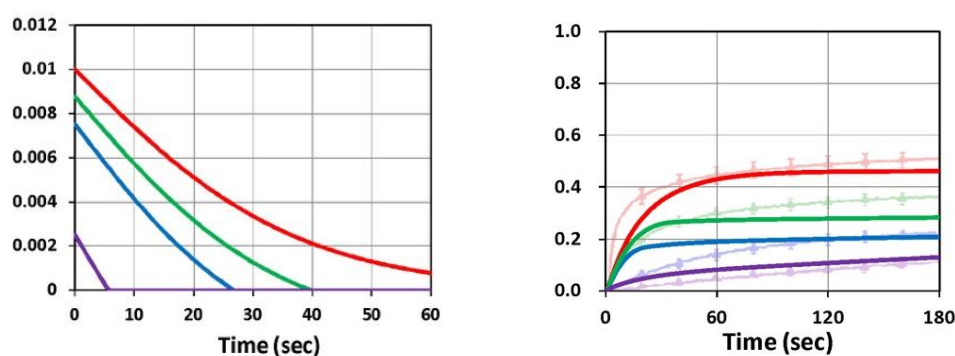
van der Laan et al [11] also reported the UV-light controlled conversion in an on-off scheme, in which methacrylate conversion of a glycidyl dimethacrylate (bisGMA) resin formulated with 0.2 wt% CQ, 0.5wt% EDAB, a tertiary amine co-initiator, and 0.5wt% butyl nitrite (BN). Figure 9 shows the calculated conversion profile of methacrylate subject to a continuous exposure of a blue light, but an on-off exposure of a UV-light for 0.5 min, as indicated by the violet vertical areas. Measured data of van der Laan et al [11], shown by their Figure 4, are well fit to our theoretical curve (in red).

### 3.3. The general criterion for efficient UV-inhibitor

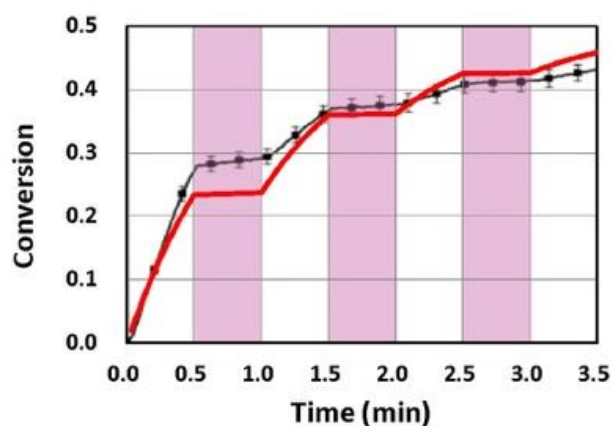
The monomer-dependence of a dual-wavelength PC was reported by van der Laan et al [11], in which different monomers have different  $C=C$  bond rate constants ( $K$ ) under the exposure of blue, UV and blue+UV. For example, bisphenol ethoxylate diacrylate (BPAEDA) resins formulated with camphorquinone (CQ) and ethyl 4-(dimethylamino)benzoate (EDAB) have a maximum conversion rate constant  $K_{max}=0.675$  (at blue+UV) for 0% butyl nitrite (BN), and reduces to 0.0106 (for 1%BN), a factor of 64 reduction. Therefore, it is a better candidate than trimethylolpropane triacrylate (TMPTA) which only has a 3 times reduction of  $K_{max}$ . The above measured feature could be mathematically described by a more general criterion.

Based on our H-factor defined in Eq. (14), which also governs the conversion efficacy, a *general criteria for efficient UV-inhibitor, or a good candidate*, could be mathematically determined as follows. Defining two H factors:  $H_{OFF}$  and  $H_{ON}$  for the H-value without and with-UV, respectively, an efficient candidate (or effective UV-inhibitor) requires two conditions: (i) high enough  $H_{OFF}$  such that the conversion without UV-light larger than 50%; and (ii) high enough  $H_{ON}$  such that the conversion of both blue and UV light is reduced to lower than 20%. This concept could be further described mathematically as follows.  $H = H_{OFF} - H_{ON} = H_{OFF} (1 - H_{ON}/H_{OFF})$ . Therefore, a good candidate requires a large  $H_{OFF}$  and also a high H-ratio,  $R_H = H_{ON}/H_{OFF}$ . For example, for a fixed value of  $H_{OFF} = 10$ , a candidate with  $H_{ON} = 6$  leading to  $R_H = 0.6$ , and  $H = 10 \times (1 - 0.6) = 4$ , is not as good as a candidate having a higher  $H_{ON} = 8$  leading to  $R_H = 0.8$  and  $H = 80$ , which is 4 times higher.

From Eq. (14),  $H = gC_3B_1C_1 - B_2C_2$ , which defines  $R_H = B_2C_2/(gC_3B_1C_1)$ . We note that  $B_j = b_jI_j$ , which is proportional to the light intensities ( $I_1$  for blue-light and  $I_2$  for UV-light) and the effective absorption constant ( $b_j$ ) governed by the quantum yield ( $q$ ) and absorption coefficient at a specific wavelength. Therefore, a high H-ratio ( $R_H$ ) is determined not only by the material properties, but also the ratio of light intensity (UV/blue), and concentration ratio of the initiator and inhibitor,  $C_{20}/C_{10}$ . In addition, it is also rate-constant dependence, because the  $g$ -factor is given by  $g = 1/(k_{57} + kC_3)$ . Therefore, we conclude that the criterion for a good candidate is governed by collective factors, and at least by the double ratio of  $[I_{20}C_{20}]/[I_{10}C_{10}]$ . Above criterion is the important new finding of our theoretical study, which requires further experimental study to confirm.



**Figure 8.** The initiation radical (left) and conversion (right) profiles for  $C_{20} = (0, 0.5, 1.0, 3.0)$ , for curve (red, green, blue, violet), in the presence of both blue and UV light; for  $b_1=0.04$ ,  $b_2=0.002$ ,  $b_3=0.1$  (1/s/%);  $k'=2.0$  (1/s),  $k_{48}=10$  (1/s),  $k_{37}=20$  (1/s),  $k_{57}=0.01$  (1/s). In left figure, the background is measured data of van der Laan et al [11].



**Figure 9.** Methacrylate conversion of a bisGMA/TEGDMA resin formulated with 0.2 wt% CQ/0.5wt% EDAB/0/5wt%BN and subject to a continuous exposure of a blue light, but an on-off exposure of a UV-light for 0.5 min, as indicated by the violet vertical areas; where black bars are measured data of van der Laan et al [11] and red curve is our theoretical simulation.

#### 4. Conclusion

We have theoretically demonstrated that without the UV-light, for a given blue-light intensity, higher initiator concentration ( $C_{10}$ ) and rate constant ( $k'$ ) lead to higher conversion, governed by a scaling law of  $k[gB_1C_1]^{0.5}$ . However, the coupling factor  $b_1$  plays a different role that higher  $b_1$  leads to higher conversion only in the transient regime; whereas higher  $b_1$  leads lower steady-state conversion. For a fixed initiator concentration  $C_{10}$ , higher  $C_{20}$  leads to lower conversion due to stronger inhibition effect. However, same conversion reduction was found for the same  $H_0 = [b_1C_{10} - b_2C_{20}]$ . Conversion of blue-only are much higher than that of UV-only and UV-blue combined, such that the UV-light serves as the turn-off (trigger) mechanism for the purpose of PC. The UV-light initiated inhibition effect is strongly monomer-dependent and different monomers have different C=C bond rate constants and conversion efficacy.

**Conflicts of Interest:** Jui-Teng Lin is the CEO of New Vision, Inc. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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