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Analysis of Energy Flux Vector on Natural Convection Heat Transfer in Porous Wavy-Wall Square Cavity with A Partially-Heated Surface

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Abstract: The study utilizes the energy-flux-vector method to analyze the heat transfer characteristics of natural convection in a wavy-wall porous square cavity with a partially-heated bottom surface. The effects of the modified Darcy number and modified Rayleigh number on the energy-flux-vector distribution and mean Nusselt number are examined. The results show that when a low modified Darcy number with any value of modified Rayleigh number is given, the recirculation regions are not formed in the energy-flux-vector distribution within the porous cavity. Therefore, a low mean Nusselt number is obtained. The recirculation regions do still not form and thus the mean Nusselt number has a low value when a low modified Darcy number with a high modified Rayleigh number is given. However, when the values of the modified Darcy number and modified Rayleigh number are high, the energy flux vectors generate recirculation regions and thus a high mean Nusselt number is obtained.

Keywords: energy-flux-vector; porous cavity; natural convection; wavy-wall

1. Introduction

The plot of the heat energy flow paths is important since it can provide physical insights into the energy transport process in detail. To achieve this purpose, Kimura and Bejan [1] have suggested a heatline visualization technique. Following the study of the Kimura and Bejan [1], numerous researchers have explored the transport process of heat energy within thermal-fluid systems by utilizing the technique [2-4].

Hooman [5, 6] has presented an energy-flux-vector method, which is basically similar to the heatline technique, for visualizing the transport process of heat energy. Comparing the two visualization methods, Hooman [5, 6] has pointed out that the energy-flux-vector method is better than the heatline visualization technique since the algebraic equations do not require to be solved [5-8]. In the literature, numerous researchers [7-9] have demonstrated that the transport process of heat energy can be completely explained by using the energy-flux-vector method.

Natural convection phenomenon in porous cavities has numerous practical applications in engineering fields, including biomedical engineering, geothermal engineering, thermal insulation, solar collection, and so on [10, 11]. To achieve the purpose of heat transfer enhancement, wavy-surface geometries often imposed [7, 8]. In the literature, the investigation into the natural convection heat transfer behavior in a porous cavity with wavy surfaces has been widely discussed [12-14]. The results have demonstrated that the use of the wavy-surface geometries can improve the heat transfer effect within a porous cavity. Recently, Biswal et al. [15] have utilized the energy-flux-vector method for explaining the transport process of heat energy of natural convection within a porous cavity with curved side walls. Their results have shown that given suitable curved-sidewall forms with appropriate flow conditions, the heat transfer effect can be enhanced.

In the current study, the energy-flux-vector method is utilized to analyze the heat transfer characteristics of natural convection within a porous square cavity bounded by a partially-heated flat bottom wall, low temperature left and right wavy-walls, and a insulated flat top wall. The

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simulations focus particularly on the effects of the modified Darcy number and modified Rayleigh number on the energy-flux-vector distribution and mean Nusselt number, respectively.

2. Mathematical Formulation

2.1. Governing Equations and Boundary Conditions

Figure 1 illustrates the studied porous square wavy-wall cavity with a partially-heated bottom surface and a characteristic length of L_c. As shown, the partially-heated wall surface has a length of $0.3L_c$, and it is placed on the center of the bottom wall. Meanwhile, the left and right walls are assumed to have a constant low temperature and a complex-wavy surface.

It is assumed that the working fluid is Newtonian, incompressible, and the flow and temperature fields are two-dimensional, laminar and steady state. In addition, it is assumed that the Boussinesq approximation is imposed, and local thermal equilibrium condition is achieved.

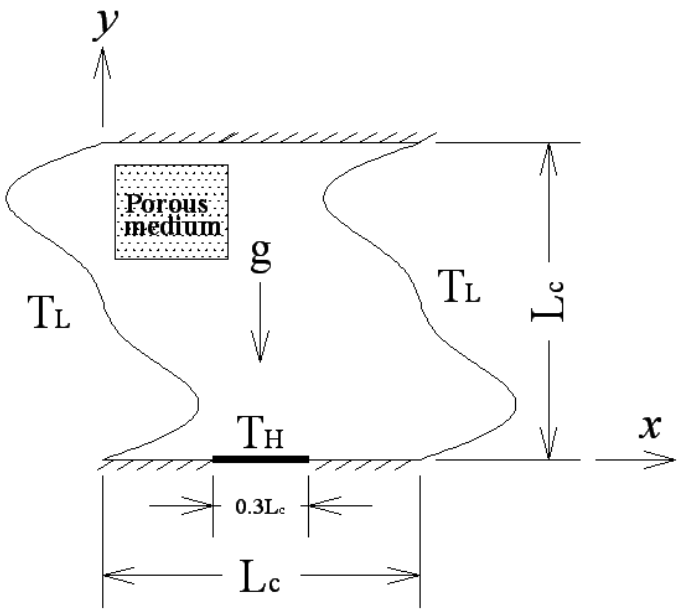

Figure 1. Geometry of partially-heated wavy-wall porous cavity.

Let the following non-dimensional quantities be introduced [16]:

$$x^* = \frac{x}{L_c},\quad y^* = \frac{y}{L_c},\quad \alpha_{eff} = \frac{k_{eff}}{\varepsilon\rho C_p},\quad u^* = \frac{uL_c}{\alpha_{eff}},\quad v^* = \frac{vL_c}{\alpha_{eff}},\quad p^* = \frac{pL_c^2}{\rho\alpha_{eff}^2},\quad \theta = \frac{T-T_L}{T_H-T_L},$$

$$\mathrm{K}_m = \frac{K}{\varepsilon},\quad \mathrm{Pr}_m = \frac{\mu}{\rho\alpha_{eff}},\quad Da_m = \frac{K_m}{L_c^2},\quad Ra_m = \frac{\rho g\beta L_c^3(T_H-T_L)}{\mu\alpha_{eff}} \tag{1}$$

where α_{eff} is the effective thermal diffusivity; k_{eff} is the effective thermal conductivity; ε is the porosity; ρ is the density; C_p is the specific heat; u and v are the velocity components along x- and y-axes, respectively; p is the pressure; θ is the dimensionless temperature; T is the temperature; T_H and T_L are the high temperature and low temperature, respectively; K is the permeability; Pr_m is the modified Prandtl number; Da_m is the modified Darcy number; Ra_m is the modified Rayleigh number; g is the gravitational acceleration; β is the thermal expansion coefficient; subscripts m indicates the modified value; and superscript $*$ denote the non-dimensional quantity. After the viscous dissipation and thermal radiation effects are ignored, the continuity, momentum and energy equations described the flow behavior and heat transfer

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characteristics of natural convection within the porous cavity can be written in the following non-dimensionalized forms [16]:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{2}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}-\frac{\mathrm{Pr}_m}{Da_m}u^*+\mathrm{Pr}_m\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-\frac{1.75}{\sqrt{150}}\frac{\sqrt{u^{*2}+v^{*2}}}{\sqrt{Da_m}}u^* \tag{3}$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}-\frac{\mathrm{Pr}_m}{Da_m}v^*+\mathrm{Pr}_m\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)-\frac{1.75}{\sqrt{150}}\frac{\sqrt{u^{*2}+v^{*2}}}{\sqrt{Da_m}}v^*+Ra_m\cdot\mathrm{Pr}_m\cdot\theta \tag{4}$$

$$u^*\frac{\partial \theta}{\partial x^*}+v^*\frac{\partial \theta}{\partial y^*}=\frac{\partial^2 \theta}{\partial x^{*2}}+\frac{\partial^2 \theta}{\partial y^{*2}} \tag{5}$$

The dimensionless boundary conditions are expressed as follows:

Bottom partially-heat wall: $u^*=v^*=0,\ \theta=1$

Bottom other walls: $u^*=v^*=0,\ \partial\theta/\partial\vec{n}^*=0$

Left and right wavy walls: $u^*=v^*=0,\ \theta=0$

Top wall: $u^*=v^*=0,\ \partial\theta/\partial\vec{n}^*=0$

where $\vec{n}^*$ is the normal vector.

2.2. Energy Flux Vectors and Nusselt Number

The energy flux vectors ($\vec{E}$) are defined as follows [5-9]:

$$\vec{E}=\frac{\partial H^*}{\partial y^*}\vec{i}-\frac{\partial H^*}{\partial x^*}\vec{j}, \tag{6}$$

where

$$\frac{\partial H^*}{\partial y^*}=u^*\theta-\frac{\partial \theta}{\partial x^*}, \tag{7a}$$

$$-\frac{\partial H^*}{\partial x^*}=v^*\theta-\frac{\partial \theta}{\partial y^*}. \tag{7b}$$

Note that H^* is the non-dimensional heat function, and $\vec{i}$ and $\vec{j}$ are the unit components in *x*- and *y*-directions, respectively.

The mean Nusselt number (Nu_m) along the partially-heated bottom surface is defined as follows:

$$Nu_m=\int_{0.35}^{0.65} Nu\,d\xi, \tag{8}$$

where $Nu\left(=-\left(\frac{\partial\theta}{\partial\vec{n}^*}\right)\right)$ is the Nusselt number.

2.3. Geometric Description and Numerical Method

In this study, the left and right walls of the porous cavity are assumed to have a complex-wavy surface, which is formulated as follows [7, 8]:

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$$y^* = \alpha_w Sin(2\pi x^*) + \frac{\alpha_w}{2.5} Sin(4\pi x^*), \quad (9)$$

where α_w is the wave amplitude of the wavy surface.

In the current study, the generalized coordinate transform technique, finite volume method and SIMPLE algorithm are used to solve the governing equations presented in Eqs. (2)~(5). The numerical methods are identical to those used in our previously studies. A detail description for the used numerical methods can be found in [7, 8].

2.4. Numerical Validation and Grid Independence Evaluation

The current used numerical models and numerical methods were valid by comparing the current results with those presented by Singh et al. [16]. Table 1 shows these results. Note that the Rayleigh number of $Ra = 10^5$, porosity of $\varepsilon = 0.6$ and Prandtl number of $\Pr = 1$ are given in the case. It is shown that the current results are identical with those presented in Singh et al. [16].

The mesh sizes of 101×201, 101×501, 101×1001, 201×1001 have been examined for the variation of mean Nusselt number, respectively, and the results showed that the mesh size of 101×1001 has a grid-independent solution.

Table 1. Comparison of current results for mean Nusselt number with published results for two different Darcy numbers.

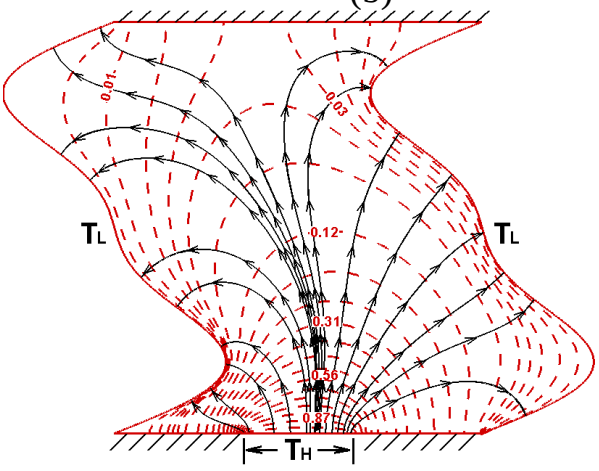
	$Da = 10^{-4}$	$Da = 10^{-2}$
Current results	3.441	1.067
Singh et al. [16]	3.461	1.067
Error (%)	0.6	0.0

3. Results and discussion

In the study, the wavy amplitude of the wavy surface is set to $\alpha_w = 0.25$, and the modified Prandtl number is given as $\Pr_m = 1.0$.

Figures 2 and 3 illustrate the distributions of the energy-flux-vector within the porous cavity for various modified Darcy numbers and modified Rayleigh numbers. Figure 4 shows the effects of the modified Darcy number and modified Rayleigh number on the mean Nusselt number. According to the definition of the energy flux vectors given in Eq. (6), the conduction mechanism dominates if the flow of energy flux vectors is directly from the bottom partially-heated high-temperature wall to

(a) (b)



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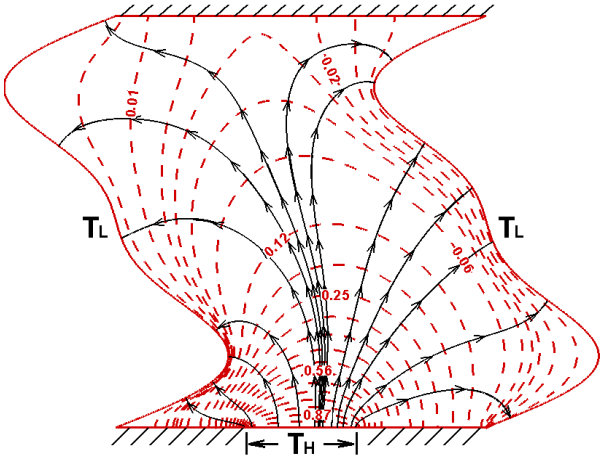

Figure 2. Distributions of energy flux vectors and isotherms within partially-heated porous cavity given modified Darcy number of $Da_m = 10^{-5}$ and modified Rayleigh number of: (a) $Ra_m = 10^2$ and (b) $Ra_m = 10^5$. Note that the black solid lines indicate the flow direction of energy-flux-vector, while the red dashed lines indicate the isothermal contours.

(a) (b)

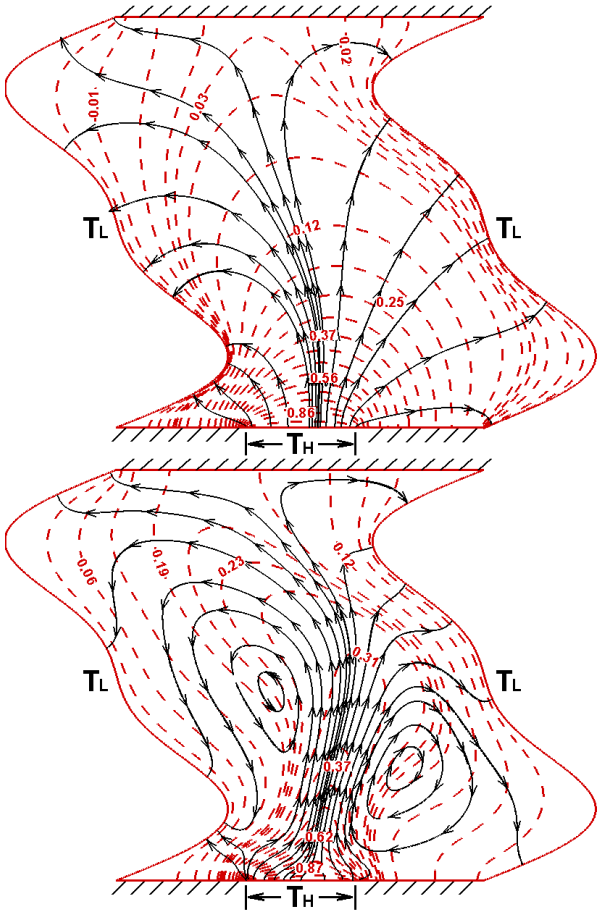

Figure 3. Distributions of energy flux vectors and isotherms within partially-heated porous cavity given modified Darcy number of $Da_m = 10^{-2}$ and modified Rayleigh number of: (a) $Ra_m = 10^2$ and (b) $Ra_m = 10^5$. Note that the black solid lines indicate the flow direction of energy-flux-vector, while the red dashed lines indicate the isothermal contours.

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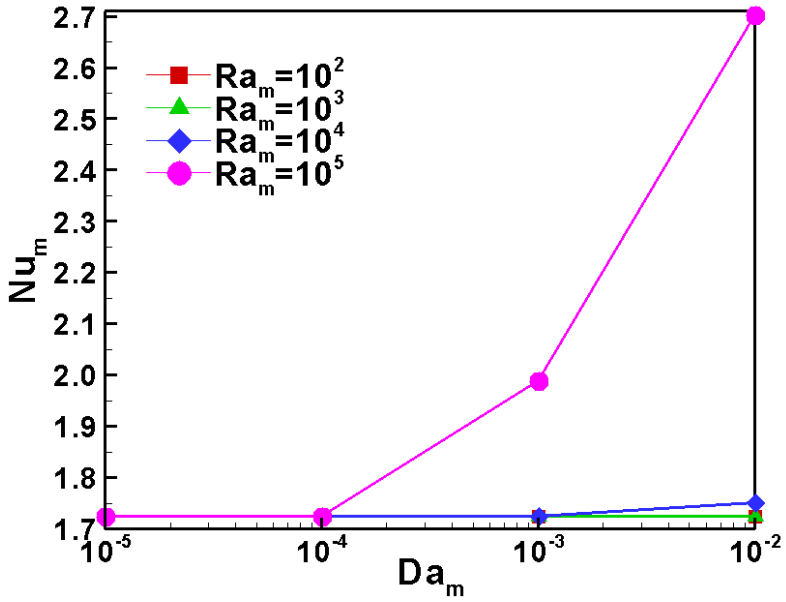

Figure 4. Variation of mean Nusselt number with modified Darcy number as function of modified Rayleigh number.

the left and right low-temperature wavy-walls, while the convection mechanism dominates if the energy flux vectors generate closed recirculation regions [5-9]. Therefore, as shown in Fig. 2(a), when the values of the modified Darcy number and modified Rayleigh number are both low, the heat transfer effect within the porous partially-heated cavity is dominated by a pure conduction mechanism since the closed recirculation regions in the energy-flux-vector distribution are not created. In addition, a low modified Darcy number represents a low permeability due to the present of a high flow resistance [16]. Therefore, although a larger buoyancy effect induced by giving a high modified Rayleigh number is presented, a high flow resistance is also generated within the porous partially-heated cavity. As a result, the energy flux vectors do still not form closed recirculation regions and thus conduction mechanism continually dominates heat transfer behavior (see Fig. 2(b)). Under conduction-domination, the high-temperature fluid heated by the partially-heated bottom surface is slowly diffused to left and right low-temperature wavy-wall surfaces to be dissipated. Consequently, under a low modified Darcy number, a low mean Nusselt number is presented, irrespective of the value assigned to the modified Rayleigh number (see Fig. 4).

When a high modified Darcy number and a low modified Rayleigh number is given, the closed recirculation regions of the energy flux vectors are still not formed within the porous partially-heated cavity (see Fig. 3(a)). Although a high modified Darcy number has a high permeability and a low flow resistance, a low modified Rayleigh number induces a small buoyancy effect. Therefore, the convection effect is still weak, and the conduction mechanism continues to dominate the heat transfer effect. As a result, a low mean Nusselt number is presented (see Fig. 4).

When the modified Darcy number and the modified Rayleigh number are both high, a low flow resistance and a high buoyancy effect occur within the porous partially-heated cavity. Therefore, the closed energy-flux-vector recirculations are formed, resulting in the domination of the convection effect (see Fig. 3(b)). The high-temperature fluid on the partially-heated bottom wall is promptly driven to the left and right low-temperature walls to be dissipated. As a result, a high mean Nusselt number is obtained (see Fig. 4).

4. Conclusions

This paper has analyzed the heat transfer behavior of natural convection in a porous square wavy-wall cavity with a partially-heated bottom surface. To the purpose of visualization of the heat energy transport processes, the contours of energy flux vectors have been plotted. The effects of the

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modified Rayleigh number and modified Darcy number on the energy-flux-vector distribution and mean Nusselt number have been discussed. The studied results are summarized as follows:

1. Given a low modified Darcy number, the energy flux vectors did not generate recirculation regions within the square porous cavity, irrespective of the value assigned to the modified Rayleigh number. The conduction mechanism dominated the heat transfer effect, and thus the mean Nusselt number was low.

2. Given a high modified Darcy number and a low modified Rayleigh number, the heat transfer effect was dominated by the conduction mechanism since no recirculation region was formed in the energy flux vectors. Therefore, a low mean Nusselt number was obtained.

3. Given a high modified Darcy number with a high modified Rayleigh number, recirculation regions in the energy-flux-vector distribution were produced, resulting in a convection-domination. Consequently, a high mean Nusselt number was presented.

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