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Article

Modeling Hysteresis Loops of Self-Developed Soft Magnetic Composite Cores Using the Jiles-Atherton-Sablik Theory

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Abstract: Magnetic properties of soft magnetic composites are highly sensitive to the processing conditions. In this paper we focus on the possibility to model this effect using the Jiles-Atherton-Sablik theory. It is assumed that the effect of varying compaction pressure may be described by direct introduction of stress-dependent term in the model equations. The values of model parameters are kept constant. Verification of the proposed approach is carried out using measurement data from self-developed iron-based composite cores.

Keywords: soft magnetic composites; magnetic properties; Jiles-Atherton model

1. Introduction

Energy conversion is of crucial interest to practitioners interested in exploring new possibilities for development of novel sensors and actuators. From the historical point of view, the magnetostriction effect, i.e. the change in shape of a ferromagnetic body under the action of external magnetic field, is the oldest phenomenon, described already by J. P. Joule in 1842 [1,2]. The complementary effect i.e. the change of magnetization in ferromagnetic materials subject to applied forces, either tensile or compressive, was studied by E. Villari [3]. For a long time physicists and engineers have striven to develop new mathematical descriptions of the coupled phenomena. An important step ahead was made in the eighties of the last century, when D. C. Jiles and D. L. Atherton developed a simple model of ferromagnetic hysteresis [4,5]. The model was capable of taking into account the magnetomechanical effect by the introduction of an additional term in the so-called effective field, being an indispensable part of the description.

Subsequently M. Sablik and co-workers have further scrutinized the coupling of the JA theory with the magnetoelastic effect [6-8]. As already stated, the coupling is implemented by an additional term in the effective field, which appears explicitly in model equations. From the engineering perspective the effective field is understood as a means to introduce the results of any phenomenon into a theoretical model. In this way an approximation of the effect is obtained. The effective field should be perceived as a cooperative interaction between numerous contributions that amplify the action of external stimulus. The effective field may include the effects of eddy currents, thermal viscosity, mechanical stresses, demagnetization effects, etc., what may be written as [9]

$$H_{\text{eff}} = H + H_{\omega} + H_{T} + H_{\sigma} + H_{D} \quad (1)$$

In the present paper we consider the effective field as consisting of Weiss' mean field term and the magnetoelastic term i.e. $H_{\text{eff}} = H + \alpha M + H_{\sigma}$, where the last term is attributed in the literature to M. Sablik. It is dependent on the applied stress σ. The Weiss' mean field takes into account mutual interactions between magnetic moments within the material [10-12]. According to the well known monograph [13, p. 130] " … it is of invaluable importance in giving a simple and at the same time deep physical interpretation of the existence of spontaneous magnetization …".

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It should be stated that the Jiles-Atherton (JA) model [4] still remains one of the most commonly used descriptions of hysteresis loops, this is most probably due to its relative simple implementation and the possibility to take different physical phenomena into account. Description of magnetomechanical effect still remains its most important application target [14], yet it should be stated that the JA model is quite versatile. Some of the less common applications include modeling of transient states in electrical grids related to inrush phenomena [15] or calculations of shielding factors [16]. The Jiles-Atherton model including the magnetoelastic term is usually applied for the description of magnetization processes in steels [6-8, 17-23]. Papers devoted to other materials of practical importance like amorphous alloys [24] are less common. In one of the landmark papers Sablik et al. [6] focused on the choice of the most appropriate expression for the $\lambda = \lambda(M, \sigma)$ dependence. The paper [7] attempted to correlate magnetostriction λ to such physical quantities as Burgers' vector, Poisson's ratio and Young's modulus. Later Sablik provided some clues how to combine some ideas inherent in his and the Schneider-Cannell-Watts [25] descriptions of magnetomechanical hysteresis in order to explain the Villari reversal [8]. Jianwei Li and Minqiang Xu followed this line of reasoning and obtained a good agreement with experiment [17]. Lo et al. [18] studied the interrelating effects of plastic deformation and stress on magnetic properties of a series of nickel samples, which were pre-stressed to various plastic strain levels. An important conclusion from their study was that the value of model parameter k was dependent on the applied stress (this parameter is approximately equal to coercive field strength; the aforementioned conclusion was later used in modeling residual stresses in drawn wires [26] and an indirect proof of its correctness may be inferred from the analysis of real-life experimental data in Ref. [27]). The paper [19] included an in-depth analysis of the relationships between strain-hardening stress and micro-structural quantities such as dislocation density and some values of JA model parameters. In a subsequent study Jianwei Li *et al.* [20] suggested that yet another term representing the contribution due to residual stresses in the expression for the effective field should be accounted. Jiancheng Leng et al. used the Jiles-Atherton-Sablik (JAS) model to explain variations of magnetic memory signals caused by early stages of plastic deformation [21]. Singh et al. [22] analyzed the effect of stress on hysteresis loops of non-oriented electrical steel with the JAS model. In their approach magnetostriction was modeled as a product of two functions, $\lambda = f(M)g(\sigma)$. The first function was a polynomial, the second one was hyperbolic tangent with offset. The authors were able to describe the magnetoelastic effects in the examined steel with a reasonable accuracy. Quite recently Hergli et al. [23] suggested that the JA model parameter a might be related to plastic deformation. However in their work they availed of the classical JA model without the Sablik's term in the expression for the effective field.

The JA model was used previously for the SMC materials by Benabou et al. [28], Zidarič and Miljavec [29] and by Ślusarek *et al.* [30]. The paper [28] compared the capabilities of classical JA 9odel to the Preisach approach. The authors of [29] suggested that the reversibility parameter c should be made dependent on the excitation amplitude. The paper [30] analyzed the dependence of model parameters on processing temperature for commercial Somalloy 500 samples. The modified JA model [31] was used in modeling. However, none of the papers [28-30] used the extended expression for the effective field with the magnetoelastic term. The present paper is written to fill the gap.

In this work we attempt to model hysteresis loops of self-developed SMC cores subject to different compaction pressures, whose effect on the loop shapes is assumed to be described with the Sablik's term in the effective field.

2. The JAS model equations

The set of equations considered in this work is as follows:

$$\frac{\mathrm{d}M}{\mathrm{d}H_{\mathrm{eff}}} = \frac{\delta_{\mathrm{M}}\left(M_{\mathrm{an}} - M\right)}{k\delta} \tag{2}$$

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$$H_{\rm eff} = H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{{\rm d}\lambda}{{\rm d}M}\right) \cong H + \alpha M + K\sigma \qquad (3)$$

$$M_{\rm an} = M_{\rm s}\left[\coth\left(H_{\rm eff}/a\right) - a/H_{\rm eff}\right] \qquad (4)$$

The model parameters are $\alpha, a, k, M_{\rm s}, K$. $\delta = \pm 1$ whereas $\delta_{\rm M} = 0.5\left[1 + {\rm sign}\left((M_{\rm an} - M)\cdot {\rm d}H/{\rm d}t\right)\right]$. The reasons for assuming the simplest form of JA model equations [4, 26] are twofold: 1) we want to keep the number of model parameters as small as possible, 2) we do not want to go into details concerning the subtle intricate problems concerning the description of reversible magnetization phenomena in the JA model, since they have been addressed thoroughly elsewhere [32, 33].

After transformations described in detail in Refs. [34, 35] the expression for ${\rm d}M/{\rm d}B$ is derived :

$$\frac{{\rm d}M}{{\rm d}B} = \frac{\delta_{\rm M}\left(M_{\rm an} - M\right)}{\mu_0\left[k\delta + \left(1-\alpha^*\right)\left(M_{\rm an} - M\right)\right]} \qquad (5)$$

where the stress dependent parameter $\alpha^* = \alpha + K\sigma$. This equation may be integrated to yield hysteresis curves taking into account the effect of compaction pressure on the shape of the loops. The values of field strength in the corresponding time instants are computed from the constitutive relationship, $H(t) = B(t)/\mu_0 - M(t)$. The assumption of constant value for coefficient K makes the considered model equivalent to the Schneider-Cannell-Watts description [25]. We assume a linear dependence of magnetostriction on magnetization, since some problems were reported for more complicated relationships [29].

3. Measurements, modeling

Measurements have been carried out for several self-developed SMC cores subject to different compaction pressures. Figure 1 depicts the press device used during sample preparation, whereas Figure 2 presents some of the developed cores.

Figure 1. Mechanical press used for sample preparation.

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Figure 2. Exemplary cores prepared by compacting Fe powder and PVC.

The weight percentage ratio Fe powder vs. PVC was kept constant at 99.5/0.05. We have noticed that for compaction pressure equal to 470 MPa the obtained maximum induction was approximately 1.3 T, what is a value comparable to the one for some permalloys or amorphous alloys. For lower compaction pressures B_{max} values were lower. We have chosen as the representative value B_m = 1.0 T in order to depict the shapes of some measured hysteresis curves in Figure 3. The JAS model parameters were estimated using the robust DIRECT algorithm [37]. Their values as well as some chosen modeled hysteresis curves are shown in Figure 4. The error in determination of coercive field strength did not exceed 2.4%. For remanence point it was around 16.5%. It can be stated that a qualitative change of shape of modeled curve is possible to obtain by updating the value of the effective mean-field parameter.

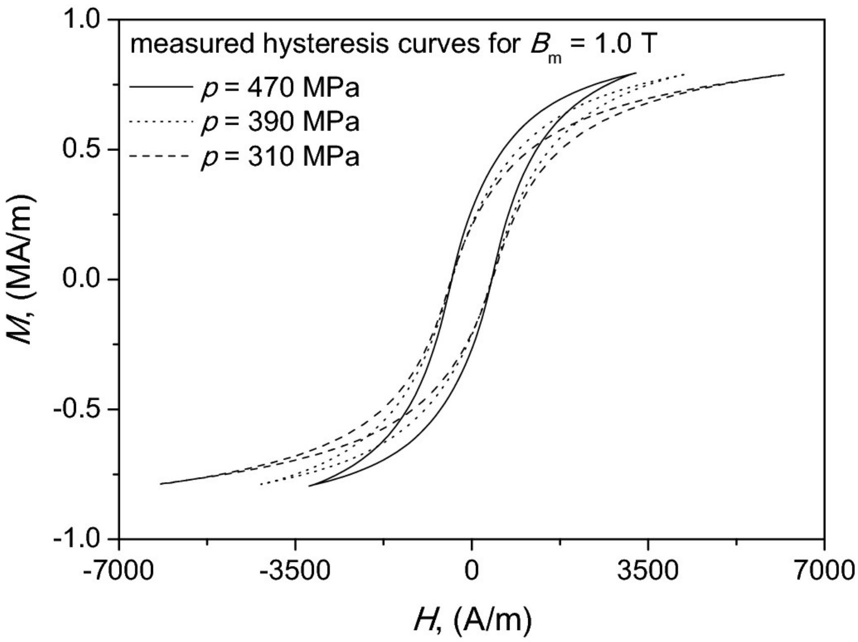


Figure 3. Measured hysteresis curves for chosen values of compaction pressure.

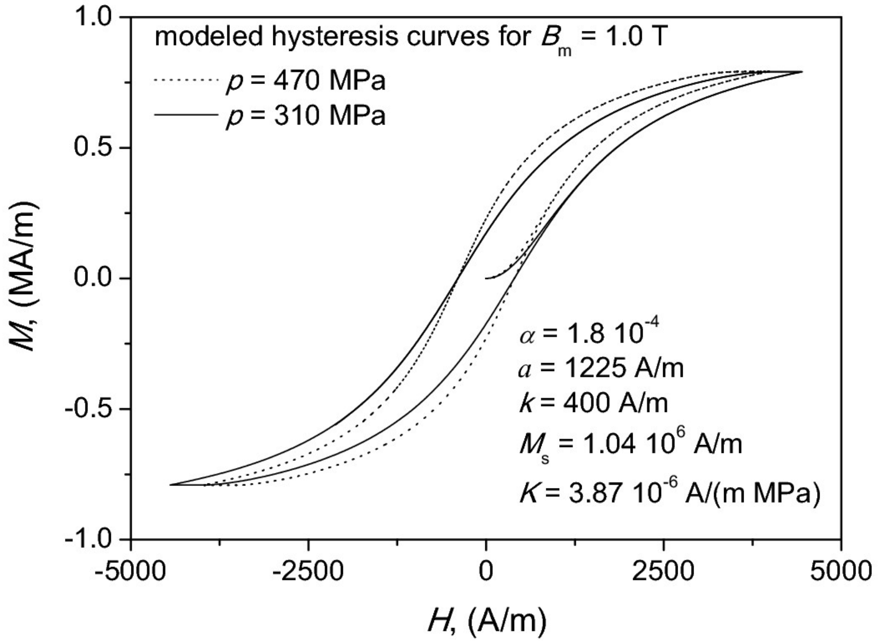
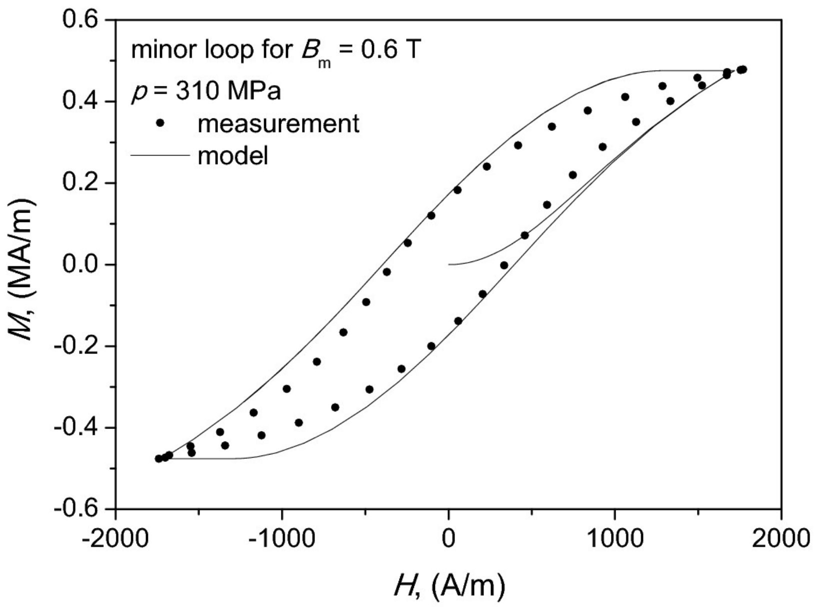


Figure 4. Modeled hysteresis curves for B_m = 1.0 T and the estimated set of JAS model parameters.

Using the same values of model parameters modeling was carried for a lower induction amplitude, B_m = 0.6 T. A well known drawback of the JA model is the necessity to update the values of some parameters upon the excitation amplitude for some parameter sets, this problem was raised in a number of publications, cf. [38-41]. However in the present paper we have assumed the same values of all model parameters for the minor loops as for the major loop. The modeling results are shown in Figure 5. For the considered SMC material a reasonable modeling accuracy was obtained without any parameter value update, what can be qualitatively assessed from the Figure. The modeling error for the characteristic points in the M(H) plane did not exceed 25%.



(a)

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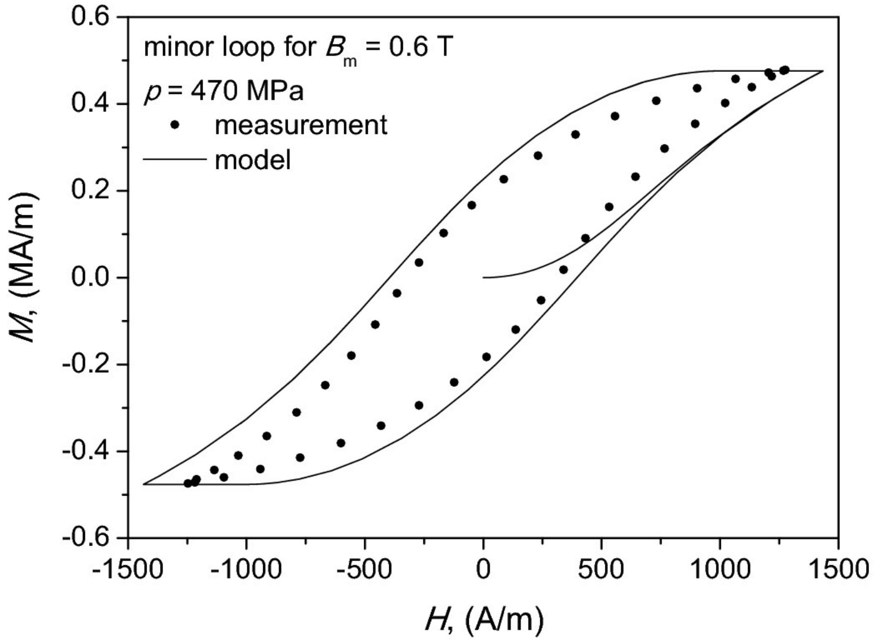

(b)

Figure 5. Modeled hysteresis curves for B_m = 0.6 T (case a) – for 310 MPa, case b) – for 470 MPa). Dots denote measurement points.

Figure 6 depicts the dependence density of the developed SMC cores versus the compaction pressure. In the paper [42] it was indicated that the material density might be a proper quantity of direct interest to the designers of magnetic circuits containing SMC materials. From Figure 6 it follows that as the compaction pressure increases, the material density also increases, what implies better magnetic properties due to a higher filling ratio. The results are consistent with those obtained in [42] for commercial Somaloy. A qualitatively similar dependence $\rho = \rho(p)$ (exhibiting saturation after a certain threshold value) was presented in Ref. [43].

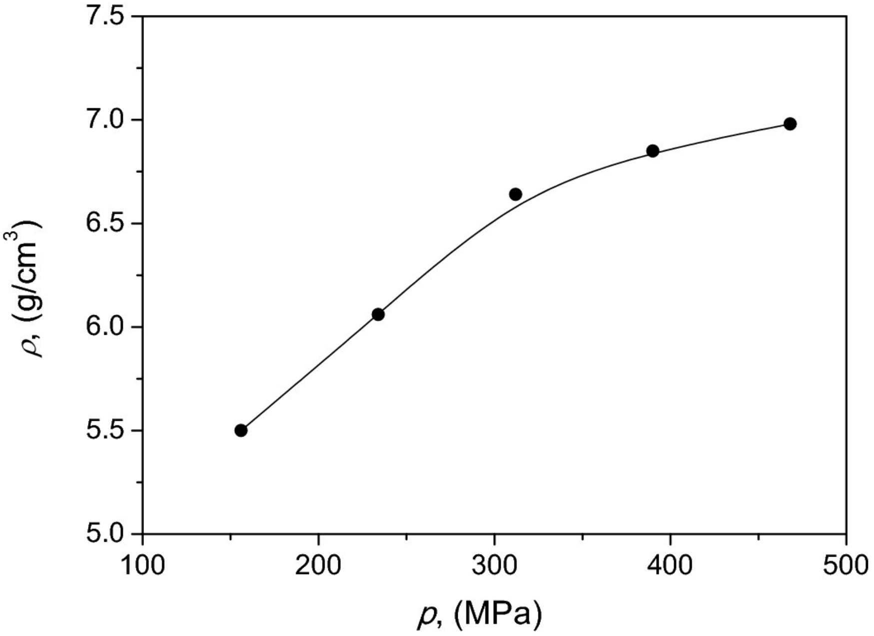

Figure 5. Experimental dependence of material density versus compaction pressure.

4. Conclusions

In the paper we have applied the Jiles-Atherton-Sablik model to describe hysteresis curves of self-developed SMC cores compacted at different pressures. The effect of varying compaction pressure was accounted as an additional term in the so-called effective field. The results might be of interest to the designers of magnetic cores.

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