Generalization of the Landauer Principle for Computing Devices Based on Many-Valued Logic

Edward Bormashenko

[1] Department of Chemical Engineering, Biotechnology and Materials, Engineering Sciences Faculty, Ariel University, Ariel, Israel 407000

* Correspondence: edward@ariel.ac.il; Tel.: +972 074 729 68 63

Abstract

The Landauer principle asserts that "the information is physical". In its strict meaning Landauer's principle states that there is a minimum possible amount of energy required to erase one bit of information, known as the Landauer bound $W = k_B T ln2$ where T is the temperature of a thermal reservoir used in the process and k_B is Boltzmann's constant. Modern computers use the binary system in which a number expressed in the base-2 numeral system. We demonstrate that the Landauer principle remains valid for the physical computing device based on the ternary and more generally N-based logic. The energy necessary for erasure of one bit of information (the Landauer bound) $W = k_B T ln2$ remains untouched for the computing devices exploiting a many-valued logic.

Keywords: Landauer principle; binary logic; ternary logic; Landauer bound; trit.

1. Introduction

Modern computers use the binary system in which a number is expressed in the base-2 numeral system. The base-2 numeral system is a positional notation with a radix of 2. Each digit is referred to as a bit. The base-2 is ubiquitous in computing devices because of its straightforward implementation in digital electronic circuitry using binary logic gates. However, one of the first computing machines was based on the ternary logic. In 1840, Thomas Fowler, a self-taught English mathematician and inventor, created a unique ternary mechanical calculating machine, completely manufactured of wood [1]. Ternary logic based computers based in the "trit" unit of information were successfully developed in Soviet Union by Nicolay Brousentsov [2]. The Setun computer, based on the ideas of ternary logic, ternary symmetrical number system and ternary memory element ("flip-flap-flop") was designed in 1958 in Moscow University [2-3]. In principle, computer may be based on a many-valued logics, exposed in recent years to a growing interest due to the fundamental aspects and numerous applications [4-5].

The present paper does not come into the mathematical details of the ternary (or another) many-valued logics, but extends the Landauer principle to the erasing of the information by the computing machine, based on the many-valued logics. Informational theory is usually supplied in a form that is independent of any physical realization. In contrast, Rolf Landauer in his papers argued that "information is physical" and it has an energy equivalent [6-8]. It may be stored in physical systems such as books and memory chips and it is transmitted by physical devices exploiting electrical or optical signals [6-8]. Therefore, he concluded, it must obey the laws of physics, and first and foremost the laws of thermodynamics. The Landauer principle [6-8] establishing the energy equivalent of information remains in the focus of investigations in the last decade [9-17]. In its strict, tight and simplest meaning the Landauer principle states that the erasure of one bit of information requires a minimum energy cost equal to $k_B T ln2$, where T is the temperature of a thermal reservoir used in the process and k_B is Boltzmann's constant [6-13]. The Landauer principle is usually demonstrated with the computers, based on the binary logic. We demonstrate how it may be extended to devices, exploiting a many-valued logics.

1. Discussion.

Consider the computing device exploiting a particle enclosed within a chamber (cylinder) divided by half by a partition, as shown in **Figure 1**. Finding of the particle M in the certain (left or right) half of the chamber corresponds to the recording of 1 bit of information. When the partition is removed, the location of particle is uncertain, and this corresponds to the erasure of 1 bit of information. Location of a particle on the certain half of the chamber corresponds to "1", and the uncertain location of the particle corresponds to "0", thus our particle based computer is based on the binary logical system. The work of this computer may be exemplified by the single-particle thermal engine, suggested by Leo Szilard in 1929 [18] and depicted in **Figure 2**. The smallest possible thermodynamic machine consists of a single particle of mass m in a closed cylinder, contacting with a thermal reservoirs. Consider the "evergreen" Carnot cycle performed by the minimal engine, depicted in **Figure 1** from the informational point of view [9, 19-20]. At the first stage the particle contacts the thermal reservoir (bath) T_1 and undergoes a reversible isothermal expansion doubling its available volume [20]. Note, that the particle initially occupies the left side of the cylinder. Heat k_BT_1 ln2 is drawn from the bath and work k_BT_1ln2 is extracted. This process is equivalent to the removal of the partition at the midpoint of the cylinder, thus one bit of information is *erased*, if one bit as seen as finding particle m at the certain side (left in our case) of the cylinder, as shown in **Figure 2** [9]. The first stage (isothermal expansion) of the Carnot cycle may be understood as follows: one bit of information, erased within the engine, was converted into the work $k_BT_1$ln2 [19]. At the second stage our engine is exposed to the adiabatic expansion and the additional mechanical work is made. At this stage the entropy of the working body and the thermal reservoir remain unchanged, and there is no informational change in both of them (thermal reservoir T_1 is disconnected from the engine at this stage). At the next stage the engine is connected to the thermal bath T_2 and the reversible isothermal compression to half volume takes place. A piston reversibly and isothermally compresses the space occupied by the particle m from full to half volume. One bit of information is recorded by the engine. Heat $Q_2 = k_BT_2$ ln2, is delivered to the heat bath, and work kT_2 ln2 is consumed. At the last stage of the cycle the engine is disconnected from the reservoir T_2 and the system is adiabatically heated to the temperature T_1. No entropy and informational changes take place at this stage. The work of the minimal Carnot engine illustrates the Landauer principle: recording/erasing of one bit of information demands k_BTln2 units of energy.

The non-trivial problems of "thermalization" of the motion of the particle in the minimal Carnot engine are out of the scope of our paper [9, 19-20]. The Carnot engine is fully reversible; actually, the erasure/recording of information is asymmetric and it was shown that there may be no entropy cost to the acquisition of information, but the destruction of information does involve an irreducible entropy cost [21]. This erasure/recording asymmetry is essential [9, 21], however it is not in the focus of the present paper. Note, that the efficiency of the engine equals $\eta = 1 - \frac{T_2}{T_1}$, as demonstrated in ref. 20. This result is quite expectable due to the fact that the efficiency of the Carnot machine is insensitive to working substance in the engine and depends only on the temperatures of the thermal reservoirs [20].

The additional exemplification of the Landauer principle is supplied by the Brownian particle in a double-well potential, as shown in **Figure 3** and discussed in detail in refs. 7, 9. When the barrier is much higher than the thermal energy, the particle will remain in either well for a long time [7, 9, 19]. Thus, the particle being in the left or right well can serve as the stable informational states, "0" and "1" of a bit. A Brownian particle trapped in either left or right

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 October 2019 doi:10.20944/preprints201910.0095.v1

well represents the informational states $m = 0$ and $m = 1$, as shown in **Figure 3**, where m is the parameter, characterizing the statistical state of the system. The average work W to change the statistical state of a memory from the state Ψ with the distribution p_m to Ψ' with distribution p'_m is given by Eqs. 1(a-b):

$$W \geq F(\Psi') - F(\Psi) \quad (1a)$$

$$F(\Psi) = \sum_m p_m F_m + k_B T \sum_m p_m ln p_m \,, \quad (1b)$$

where $F_m = E_m - TS_m$ is the free energy of the conditional state [9]. For a symmetrical well and a random bit $p_0 = p_1 = \frac{1}{2}$, and we immediately recover the Landauer bound $W = k_B T ln2$, as shown in ref. 9 and checked experimentally in refs. 22-24. The analysis of the asymmetrical potential well, performed in ref. 9, is out of the scope of our paper.

Now consider the computing device based on the ternary logic, and using the "trit" computing element, presented in **Figure 4** and discussed in refs. 1-3. Finding of the particle m in the certain one third part of the chamber corresponds to the recording of 1 bit of information. When both of partitions are removed the location of particle is uncertain, and this corresponds to the erasure of 1 bit of information. The analysis of the minimal Carnot engine which work is analogical to removing/introducing the partition immediately yields that the work necessary for erasing of the "trit" of information equals $W = k_B T ln3$. The same conclusion arises from the analysis of the "trit" based on the Brownian particle in a triple-well symmetrical potential, analogical to that depicted in **Figure 3** and shown in **Figure 5**. Indeed, in this case $p_0 = p_1 = p_2 = \frac{1}{3}$, and again, we obtain for the Landauer bound $W = k_B T ln3$. It seems from the first glance that the ternary computer device is well-expected to be energetically unfavorable, when compared to the computing device based on the binary logic. However, this conclusion is erroneous. Indeed, "trit" equals to $log_2 3$ bits of information [25]. Thus, an energy bound for erasing of one bit of information for the ternary computers equals:

$$W_{bit} = \frac{k_B T ln3}{log_2 3} = k_B T ln2 \quad (2)$$

It is recognized from Eq. 2 that the erasing of 1 bit of information for the ternary computer equals to that inherent for the binary-memory-based one. Generalization of Eq. 2 for the *N*-based memory is straightforward:

$$W_{bit} = \frac{k_B T lnN}{log_2 N} = k_B T ln2 \quad (3)$$

We conclude that the Landauer bound, necessary for erasing of one bit of information $W = k_B T ln2$ remains the same for the computers based on a many-valued logic.

Conclusions

The physical roots, justification and precise meaning of the Landauer principle remain debatable and were exposed to the turbulent discussion recently [6-11, 26-28]. The present paper is devoted to the very particular question: if we assume that the Landauer principle holds for the binary-logic based computing device, should it hold for the many-valued logic computer? In other words, if we adopt that a minimum possible amount of energy required to erase one bit of information within a binary logic computer equals the Landauer bound $W = k_B T ln2$, what minimal energy should be spent for the same purpose within the many-valued-logic-based computer? Starting from the analysis of ternary-logic-based computing device [1-3, 29] we demonstrated that the Landauer limit, necessary for erasing of one bit of information $W = k_B T ln2$ remains the same for the computers based on a many-valued logic. Thus, the universality of the Landauer principle is shown.

References

1. Glusker, M.; Hogan, D. M.; Vass, P. The ternary calculating machine of Thomas Fowler. *IEEE Annals of the History of Computing* **2005**, *27* (3), 4-22.
2. Brousentsov, N. P. An experience of the ternary computer development. *Bulletin of Moscow University, Mathematics and Mechanics*, **1965**, 2, 39–48.
3. Stakhov, A. Brousentsov's ternary principle, Bergman's number system and ternary mirror-symmetrical arithmetic. *Computer Journal* **2005**, *45* (2), 221-236.
4. Gottwald, Z. A Treatise on Many-Valued Logics (Studies in Logic and computation). Kings College London, London, UK, 2001.
5. Chang, C. C. Algebraic analysis of many valued logics. *Transactions American Mathematical Society* **1958**, *88* (2), 467-490.
6. Landauer, R. Dissipation and heat generation in the computing process. *IBM Journal of Research and Development* **1961**, *5*, 183.
7. Landauer, R. Information is physical. *Physics Today* **1991**, *44*, 5, 23-29 .
8. Landauer, R. Minimal energy requirements in communication. *Science* **1996**, *272*, 1914-1918 .
9. Parrondo, J. M. R.; Horowitz, J. M.; Sagawa, T. Thermodynamics of information. *Nature Phys*. **2015**, *11*, 131–139.
10. Reeb, D.; Wolf, M. N. An improved Landauer principle with finite-size corrections. *New J. Phys*. **2004**, *16*, 103011.
11. Esposito, M.; Van den Broeck, C. Second law and Landauer principle far from equilibrium. EPL **2011**, *95*(4), 40004.
12. Goold, J.; Paternostro, M., Modi, K. Nonequilibrium quantum Landauer Principle. *Phys. Rev. Lett*. **2015**, *114*, 060602.
13. Hilt, S., Shabbir, S., Anders, J., Lutz, E. Landauer's principle in the quantum regime. *Phys. Rev. E*. **2011**, *83*, 030102(R).
14. Herrera, L. The Gibbs Paradox, the Landauer Principle and the irreversibility associated with tilted observers. *Entropy* **2017**, *19*, 110.
15. Herrera, L. The mass of a bit of information and the Brillouin's Principle. *Fluctuation & Noise Lett*. **2014**, *13* (1), (2014) 14500
16. Daffertshofer, A.; Plastino, A. R. Forgetting and gravitation: From Landauer's principle to Tolman's temperature. *Phys. Lett. A* **2007**, *362*, 243–245.
17. Vopson, M. M. The mass-energy-information equivalence principle. *AIP Adv*. **2019**, *9*, 095206.

18. Szilard, L. über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Zeitschrift für Physik*, **1929**, *53* (11–12), 840–856.

19. Lutz, E.; Ciliberto, S. Information: From Maxwell's demon to Landauer's eraser. *Physics Today* **2015**, *68* (9), 30-35 .

20. Bormashenko, Ed.; Shkorbatov, A.; Gendeman O. The Carnot engine based on the small thermodynamic system: Its efficiency and the ergodic hypothesis. *Am. J. Physics* **2007**, *75*, 911-915 .

21. Bub, J. Maxwell's Demon and the thermodynamics of computation. Maxwell's Demon and the thermodynamics of computation. *Studies in History & Philosophy of Science B* **2000**, *32* (4), 569-579.

22. Bérut; A.; Arakelyan; A.; Petrosyan, A.; Ciliberto, S.; Dillenschneider, R.; Lutz, E. Experimental verification of Landauer's principle linking information and thermodynamics, *Nature* **2012**, *483*, 7388.

23. Jun; Y.; Gavrilov; M.; Bechhoefer, J. High-Precision test of Landauer's Principle in a feedback trap. *Phys. Rev. Lett*. **2014**, *113* (19), 190601.

24. Gaudenzi, R.; Burzuri, E.; Maegawa, S.; van der Zant, H.; Luis, F. Quantum Landauer erasure with a molecular nanomagnet. *Nature Physics* **2018**, *14* (6), 565–568.

25. Knuth, D. E. The Art of Computer Programming: Seminumerical algorithms. 2 volume, 3-rd Ed., Addison Wesley, Boston, USA, 1998.

26. Norton, J. D. Eaters of the lotus: Landauer's principle and the return of Maxwell's demon. *Studies in History & Philosophy Sci. B* **2005**, *36* (2), 375-411.

27. Norton, J. D. Waiting for Landauer. *Studies in History & Philosophy Sci. B* **2011**, *42*, 184-198.

28. Bennett, Ch. H. Notes on Landauer's principle, reversible computation, and Maxwell's Demon. *Studies in History & Philosophy of Modern Physics* **2003**, *34*, 501–510.

29. Nosonovsky, M.; Breki, A, D. Ternary logic of motion to resolve kinematic frictional paradoxes. *Entropy* **2019**, *21*(6), 620.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 October 2019 doi:10.20944/preprints201910.0095.v1

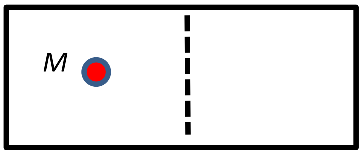

Figure 1. Finding of the particle *M* in the certain (left or right) half of the chamber corresponds to the recording of 1 bit of information.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 October 2019 doi:10.20944/preprints201910.0095.v1

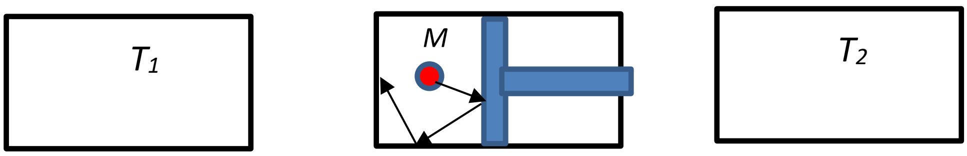

Figure 2. Sketch of the minimal single-particle thermal machine is depicted. Particle M moves the piston. The machine works between the hot (T_1) and cold (T_2) thermal reservoirs which may be finite. The conditions of "thermalization" (randomization) of the particle motion are discussed

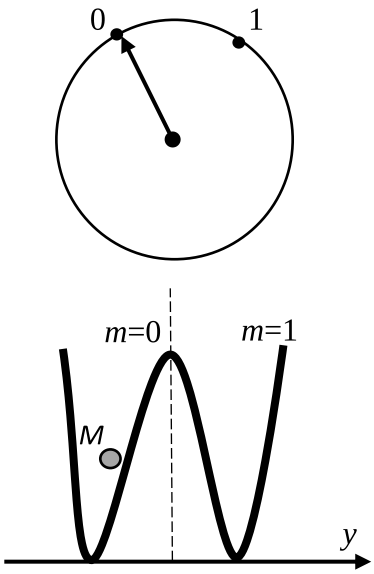

Figure 3. The qubit model of a memory exploiting a Brownian particle M in a symmetrical double-well potential with position y which can be stably trapped in either left or right well, corresponding to informational states $m = 0; m = 1$ (see ref. 9).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 October 2019 doi:10.20944/preprints201910.0095.v1

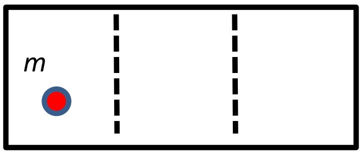

Figure 4. Finding of the particle *m* in the certain one-third part of the chamber corresponds to the recording of 1 bit of information. Thus, the “trit”- based computation becomes possible.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 October 2019 doi:10.20944/preprints201910.0095.v1

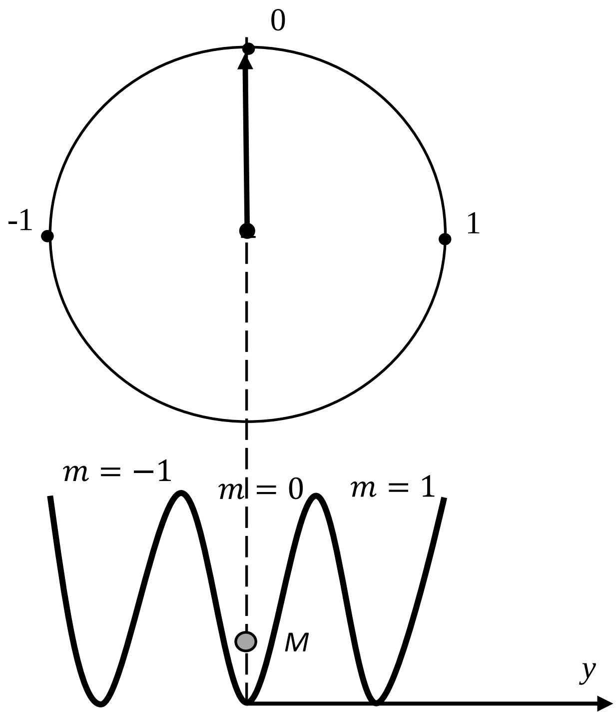

Figure 5. The trit-based model of a memory exploiting a Brownian particle M in a symmetrical triple-well potential with position y which can be stably trapped in either central, left or right well, corresponding to the informational states, namely: $m = -1; m = 0; m = 1$.