

Review

Recent Advances in Anomaly Detection Methods applied to Aviation

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Abstract: Anomaly detection is an active area of research with numerous methods and applications. This survey reviews the state-of-the-art of data-driven anomaly detection techniques and their application to the the aviation domain. After a brief introduction to the main traditional data-driven methods for anomaly detection, we review the recent advances in the area of neural networks, deep learning and temporal-logic based learning. We cover especially unsupervised techniques applicable to time series data because of their relevance to the aviation domain, where the lack of labeled data is the most usual case, and the nature of flight trajectories and sensor data is sequential, or temporal. The advantages and disadvantages of each method are presented in terms of computational efficiency and detection efficacy. The second part of the survey explores the application of anomaly detection techniques to aviation and their contributions to the improvement of the safety and performance of flight operations and aviation systems. As far as we know, some of the presented methods have not yet found an application in the aviation domain. We review applications ranging from the identification of significant operational events in air traffic operations to the prediction of potential aviation system failures for predictive maintenance.

Keywords: anomaly detection; aviation; trajectory; time series; machine learning; deep learning; predictive maintenance; prognostics and health management; condition monitoring; air traffic management

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52 1. Introduction

53 1.1. Anomaly detection

54 Anomaly detection is an active area of research encompassing a significant number of techniques
 55 developed in diverse fields such as statistics, process control, signal processing and machine learning.
 56 The goal is to be able to identify data deviating from or not being in agreement with what is considered
 57 normal, expected or likely in terms of the data probability distribution, or the shape and amplitude of
 58 a signal in time series.

59 Another commonly used term for anomaly is *outlier* and both are sometimes used interchangeably.
 60 Also, Pimentel [1] prefers the term *novelty detection* to anomaly detection when the goal is to identify
 61 data differing in some degree from the data previously observed, even though the underlying detection
 62 methods are often the same. The distinction between novel data and anomalies is that the former is
 63 usually considered as normal data after being detected [2].

64 One of the main challenges in anomaly detection is the difficulty to clearly distinguish normal
 65 instances from anomalous ones, as the boundary between the two is usually imprecise and evolves
 66 over the time in some application domains. In addition, anomalies are often rare events, so labelled
 67 datasets for model training and validation are either unavailable or severely imbalanced in favor of
 68 normal instances. As a consequence, semi-supervised or unsupervised learning is more frequently
 69 used than supervised learning. In a semi-supervised approach it is assumed that the training set
 70 contains only normal data. On the other hand, unsupervised learning techniques assume only that
 71 there is a small enough fraction of anomalies in the data so as to avoid a high rate of false alarms. A
 72 final consideration is that even though anomaly detection is often based on unsupervised learning,
 73 Erhan et al. [3] explain how unsupervised methods can be of significant help in building supervised
 74 predictive models.

75 Chandola et al. [2] identify three main types of anomalies:

- 76 • *Point anomalies*. A data point that differs significantly from the rest of the data points in the dataset
77 considered. For instance, in a time series of French temperatures in summer, a temperature of
78 40°C can be considered as an anomaly even with the undergoing climate change.
- 79 • *Contextual anomalies*. When a data point is an anomaly only in a particular context. The context
80 is defined by the contextual attributes, which usually refer to time (time series) or location.
81 For instance, in a time series of summer temperatures by country, a temperature of 40°C is
82 an anomaly in France, but it might be not in hotter countries like Libya where temperatures
83 in summer are commonly around 40°C. Attributes (e.g. temperature) indexed by contextual
84 attributes (e.g. country) are called behavioural attributes. Not only anomalies in spatial data but
85 also in time series fall into this category, e.g. 40°C can be an anomaly in Libya from October to
86 April, as at this time average temperatures range from 15°C to 30°C.
- 87 • *Collective anomalies*. When a group of data in a dataset is an anomaly as a whole, but the
88 individual instances in that group (or subsets of them) might be not on their own. In time series,
89 this would correspond for instance to a situation or condition persisting over an abnormal long
90 time. Collective anomalies can only be detected in datasets where data is related somehow, i.e.
91 sequential, spatial or graph data.

92 Detection techniques for contextual and collective anomalies are particularly relevant in our
93 survey since they are applicable to time series data. The adoption of a particular method depends on
94 the nature of the anomaly, the characteristics of data (existence of labels, number and types of data
95 attributes, data volume) and the expected output (label or score, need for result interpretability). For
96 instance, the lack of labels or the presence of just normal data in the training set requires unsupervised
97 or semi-supervised learning techniques. On the other hand, different statistical models or distance
98 functions are used for continuous or categorical data. As another example, some techniques do not
99 work well with high-dimensional data, e.g. data sparsity can be a real issue for both statistical and
100 clustering techniques: the amount of data needed for statistical significance grows exponentially with
101 the dimensions and data instances appear all far away from each other. Finally, the adoption of a
102 particular method will also depend on whether the domain experts require an understanding of how a
103 model produces the results. If so, methods learning human-readable logical expressions from data such
104 as temporal logic-based models are a better option than black-box models such as neural networks.

105 1.2. Previous surveys on anomaly detection

106 Previous surveys in the literature offer a comprehensive and structured review of anomaly
107 detection methods. A first survey in two parts has been published in 2003 by Markou and Singh [4,5],
108 the first focusing on the statistical approaches and the second one on neural networks approaches.
109 Chandola et al. [2] provided in 2009 a good understanding of the subject and a relevant taxonomy
110 of the different techniques. A more recent and extensive survey on novelty detection by Pimentel et
111 al. [1] provides more than 300 references classified in five main categories.

112 More specific surveys, as Zimel et al. [6] (2012), focus on the challenges of unsupervised outlier
113 detection algorithms applied to high-dimensional data. Aggarwal [7] (2013) reviews the techniques in
114 the literature for outlier ensembles and the principles underlying them. Xu et al. [8] (2019) provide
115 a more recent review on the progress made in anomaly detection with a focus to high-dimensional
116 and mixed types. On a side topic, Långkvist et al. [9] (2014) present a more general review on
117 unsupervised machine learning applied to time series. Akoglu et al. [10] provide a general overview
118 of the state-of-the-art methods for anomaly detection in graph data.

119 1.3. Motivation and organisation of the survey

120 The complexity of aviation systems and traffic operations makes the use of model-based anomaly
121 detection techniques difficult due to insufficient model fidelity and over-simplified assumptions.
122 Indeed, a significant research effort have been dedicated to the development of data-driven approaches,
123 which have notably benefited from significant advances in machine learning and the availability of

124 massive amounts of sensor-generated data. However, some of the classical statistical and machine
125 learning techniques for anomaly detection do not scale well with large datasets or perform poorly
126 with high-dimensional data, which is usually the kind of data available in aviation. In this context,
127 recent advances in the deep learning field should significantly improve the performance of anomaly
128 detection with large-scale high-dimensional data.

129 The motivation of this survey is to review the state-of-the-art in data-driven anomaly detection
130 methods and their application to the aviation domain: special attention is given to the techniques
131 applicable to large-scale high-dimensional time-series data, i.e. flight trajectories and sensor-generated
132 data for prognostics and health management (PHM) purposes, widely applied in the predictive and
133 condition-based aircraft fleet maintenance.

134 Recent advances in neural networks and deep learning as well as on anomaly detection
135 using temporal logic based learning justify an up-to-date review of the taxonomy of classical
136 anomaly detection techniques covered in the previous mentioned surveys. This need has been
137 recently addressed in part by Chalapathy et al. [11] with a detailed survey on the state-of-the-art of
138 deep-learning based anomaly detection, but only for domains other than aviation. Concerning the
139 aviation domain, the survey of Gavrilovski et al. [12] focuses indeed on data-mining anomaly detection
140 techniques specifically applied to flight data, but does not cover any of the recent advances on anomaly
141 detection.

142 Therefore, the goal of the present survey is to complete the previous contributions by proposing a
143 review of anomaly detection techniques applied to aviation, including the recent advances on neural
144 networks and deep learning as well as temporal logic based learning. The review of the recent advances
145 on temporal-logic based learning offer a more complete picture of the available anomaly detection
146 techniques by providing an alternative to black-box models for applications where domain experts
147 need to be able to interpret the results.

148 This contribution is organised as follows. Section 2 reviews the big picture of the already published
149 surveys and the taxonomies used for grouping the main anomaly detection methods. Section 3 reviews
150 the latest publications with a particular focus on categories of methods that recently become popular,
151 namely recurrent neural networks (3.1), convolutional neural networks (3.2), autoencoders (3.3),
152 generative models (3.4) and temporal-logic based learning (3.5). Section 4 focuses on how these
153 data-driven methods have been recently employed on two aviation-related domains of application,
154 namely the identification of significant flight operational events in air traffic operations (4.1) and the
155 prediction of aviation system faults for predictive maintenance (4.2).

156 2. Taxonomy of classical methods in previous surveys

157 In this section, we introduce some of the classical anomaly detection techniques already reviewed
158 in previous surveys (see Figure 1). We focus on the main methods, in particular the ones that have
159 been applied to aviation. For a more extensive review, the reader is referred to the previous surveys.

160 2.1. Distance-based methods

161 This category identified by Pimentel et al. [1] includes both nearest neighbour-based and
162 clustering-based anomaly detection approaches, which are two approaches identified as two separated
163 main categories in the taxonomy by Chandola et al. [2]. All the methods in this group rely on the
164 definition of a distance/similarity function between two data instances, which is not always evident
165 when data instances are not points but more complex data like time series. Most of the techniques
166 discussed here do not require the distance/similarity function to be strictly a metric, but at least to be
167 positive-defined and symmetric (triangle inequality not required).

168 2.1.1. Nearest neighbour-based methods

169 In this category, we include the methods capable of detecting an anomalous data point based on
170 either its distance to the neighbour points or its relative data density.

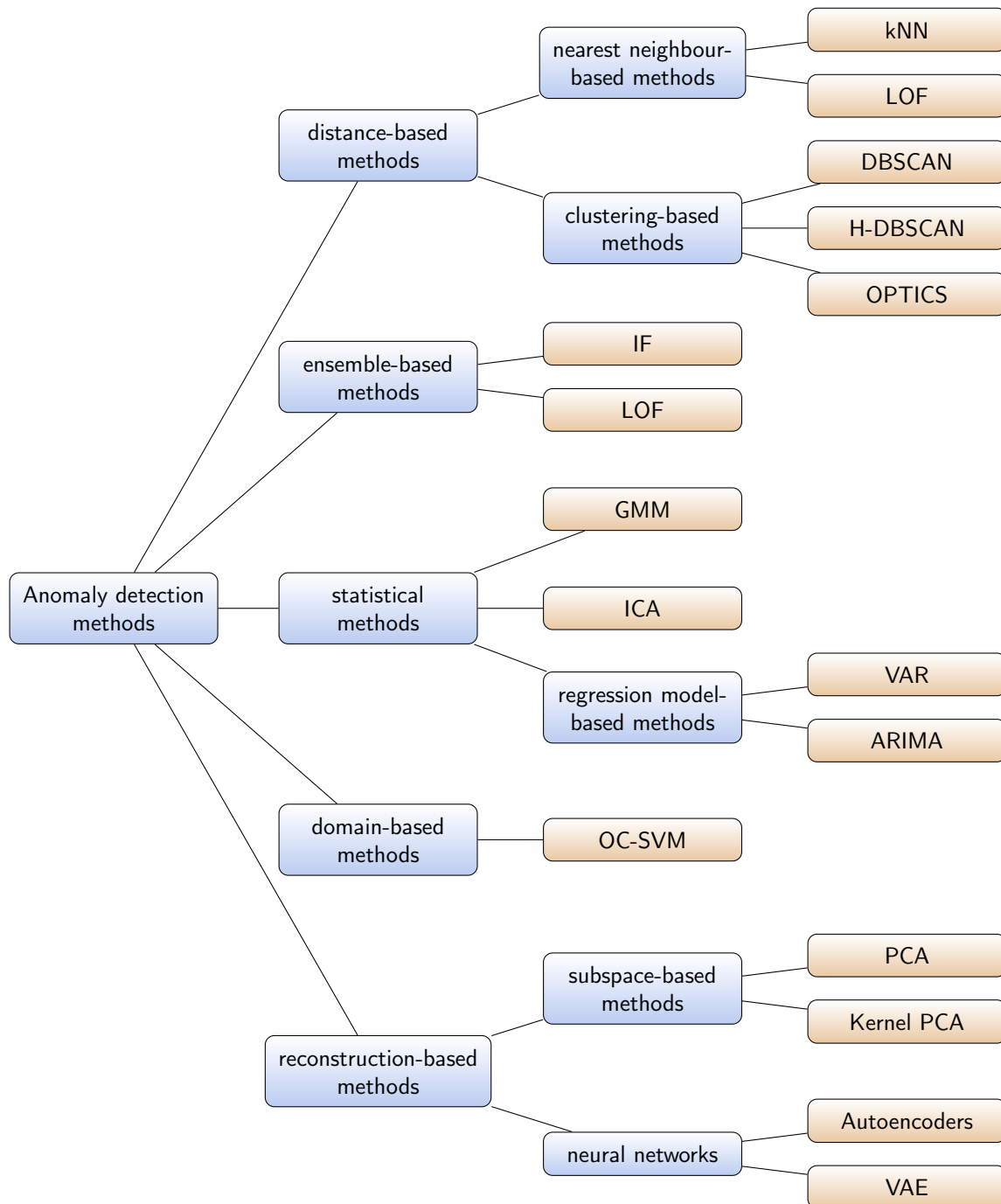


Figure 1. Taxonomy of classical anomaly detection methods

171 One of the basic distance-based techniques is the k-Nearest Neighbours (kNN) method in which an
172 anomaly score is computed for each data instance defined as the distance to its k-Nearest Neighbours.
173 Then, a threshold is used to determine whether a data point is anomalous or not. Several variants
174 of this technique exist to deal with different data types (continuous and discrete) by using different
175 distance/similarity functions, to compute the scores differently or to improve the complexity of the
176 basic algorithm which is $O(n^2)$ (where n is the data size).

177 Density-based approaches assume that density around an outlier or anomalous point should be
178 significantly lower than the density around a normal data point. For instance, the Local Outlier Factor
179 (LOF) [13] method computes the densities of the (k-nearest) neighbours and the anomaly score of an
180 instance is the ratio between the local density of the instance and the average of the densities of its
181 neighbours. LOF performs relatively well to detect very sparse anomalies among a large volume of
182 normal data such in the case of network intrusion attacks [14]. This makes it also potentially practical
183 for some of the aviation applications.

184 In fact, one improved variant of LOF, called Local Outlier Probability (LoOP) algorithm [15], is
185 used by Oehling et al. [16] to look for rare safety events in large amounts of sensor-generated flight
186 data. The main advantage of LoOP is that it provides a score which can be directly understood as the
187 probability for a data instance to be an outlier. This standardized outlier score allows for comparisons
188 over one dataset and even over different datasets. The probabilistic approach of LoOP is also more
189 reliable and tolerant to noise than LOF, where an inappropriate choice of parameter k can cause
190 unstable results.

191 Although LOF works better than kNN with datasets of varying density, both techniques do not
192 scale well with large and high-dimensional data as they need to compute pairwise distances between
193 data points and determine the nearest-neighbours. In fact, the computational complexity not only can
194 be high during the training but also during the testing phase. See [1,2] for a more exhaustive survey
195 on the multiple improved variants reducing the quadratic complexity of the basic techniques. For
196 instance, the Orca program [17] is an example of an improved variant often cited in aviation papers as
197 part of the literature review in anomaly detection.

198 2.1.2. Clustering-based methods

199 Clustering is a well-known unsupervised and semi-supervised technique to group similar
200 data instances into clusters based on the definition of a pairwise distance or similarity function.
201 Clustering-based anomaly detection will be introduced in slightly more detail because it is widely
202 used to identify relevant flight operational events [18–22] (see Section 4.1.2).

203 Chandola et al. [2] distinguish three different categories of clustering-based anomaly detection
204 techniques:

- 205 • In the first category, the techniques assume that normal data instances belong to a cluster whereas
206 anomalies do not: anomalies correspond instead to the so called clustering *outliers* or noise. Thus,
207 any clustering algorithm that does not force all data instances to belong to a cluster can be
208 used. The most popular ones are density-based clustering algorithms such as DBSCAN [23],
209 HDBSCAN [24] or OPTICS [25].
- 210 • In the second category, the assumption is that normal instances are near their closest cluster
211 centroid whereas anomalies lie far away from them. In this case, two steps are required for
212 anomaly detection: run an algorithm to cluster the data, and then compute an anomaly score
213 for each data instance based on the distance to its closest cluster centroid. An example of
214 technique in this category often cited in aerospace papers is the Inductive Monitoring System
215 (IMS) algorithm [26].
- 216 • The third category addresses the issue with the methods in the two previous categories when
217 clusters of anomalies are formed. This is because the assumption is now that normal instances
218 belong to large and dense clusters and anomalies to sparse or small clusters. A threshold is thus
219 defined on the cluster size or density to determine the anomaly cases.

220 Some clustering methods are designed to support outlier detection algorithms. For instance,
221 HDBSCAN can compute outlier scores between 0 and 1 through the density-based GLOSH outlier
222 detection algorithm [27]. GLOSH is computed from a hierarchy a clusters by comparing the density of a
223 point to the densities of the points in the associated current and child clusters. Points with substantially
224 lower density than the cluster density are considered outliers and are given a relatively high outlier
225 score.

226 Clustering-based anomaly detection methods also suffer from the lack of scalability and the curse
227 of dimensionality, but the test phase is faster as only requires comparison with a few clusters. Thus,
228 more computationally efficient variants are proposed in the literature (see [1,2]) based on heuristic
229 techniques such as in k-means [28], approximate clustering or advanced-indexing techniques to
230 partition the data. A possible way to tackle problems of dimensionality is to first project samples into a
231 smaller dimension space (dimension reduction) before applying clustering techniques on this space.
232 Dimension reduction techniques include PCA [29], t-SNE [30] or autoencoders (see Section 3.3).

233 2.2. Ensemble-based methods

234 Aggarwal [7] provides a broad review of the ensemble-based algorithms for outlier detection and
235 identifies several sub-categories based on a set of principles underlying them. Inside the category of
236 ensemble-based methods, Aggarwal includes classical models for anomaly detection such as local
237 outlier factor (LOF) [13] when they are used with several sets of hyper parameters to combine the
238 resulting scores.

239 Isolation Forest (IF) [31,32] is another method cited in [7] among the ensemble-based algorithms.
240 This algorithm is specifically designed for anomaly detection with performances challenging sometimes
241 those of the more sophisticated and recent neural network approaches [33]. The method starts building
242 an ensemble of decision trees to classify the data instances. Then, the average of path lengths from the
243 root to the sample location in the trees is computed to determine an anomaly score. The assumption is
244 that anomalies are easier to isolate and so they should have shorter path lengths than normal instances.
245 The method is computationally efficient and can be adapted for application to detect anomalies in
246 streaming data by using a sliding window [34]. More recently, Hariri et al. [35] present a variant of
247 the IF improving the quality of the anomaly scores by correcting the bias induced by the way the
248 branching is done in the classical IF.

249 2.3. Statistical methods

250 Statistical anomaly techniques is one of the main categories identified by Chandola et al. [2] in
251 their survey. A similar group of methods is included in the review by Pimentel et al. [1] under the
252 name of *probabilistic novelty detection*. In any case, the authors agree that anomaly detection methods
253 in these categories are based on the estimation of the probability densities of the data and on the
254 assumption that normal data will fall in high probability regions whereas anomalies will fall in low
255 probability ones. The underlying probability distribution is estimated from the training data (assuming
256 it is populated mostly with normal data) and a threshold is set to discriminate anomalous from normal
257 instances.

258 Both Pimentel [1] and Chandola [2] further classify the methods under the subcategories of
259 parametric and non-parametric techniques and enumerate a multitude of methods which will not
260 be reviewed here again. Instead we will focus on introducing some of the techniques applicable to
261 time series or widely used by more recent approaches, e.g. Gaussian Mixture Models which are often
262 chosen as prior distributions by the neural network generative models.

263 2.3.1. Gaussian mixture models

264 Gaussian Mixture Models (GMM) are probabilistic models based on the assumption that the
265 instances were generated from a weighted mixture of Gaussian distributions. GMMs can be used for
266 anomaly detection as the distance of a data instance to the estimated mean can be used as an anomaly

267 score. Instances with a score beyond a given threshold are marked as anomalies. GMMs present
268 two main limitations: first they try to fit all the data including the potential outliers in the training
269 set. If the set contains too many outliers, it may hence be useful to remove some of them after a first
270 application of the model. Secondly, in simplest GMM models, the number of Gaussian distributions
271 must be known in advance. Bayesian Gaussian Mixture Model can be used to simplify this process as
272 they eliminate automatically unnecessary clusters.

273 2.3.2. Independent component analysis

274 Independent Component Analysis (ICA) is a statistical technique for data analysis allowing for
275 the identification of latent variables in observed multivariate data. ICA assumes the observed data to
276 be an unknown linear mixture of non-Gaussian and mutually independent latent variables, which are
277 also called independent components, sources or factors.

278 As an example of application of ICA to anomaly detection, Pimentel et al. [1] refers to Pontoppidan
279 and Larsen [36] who describe a probabilistic framework based on ICA to detect changes in the condition
280 of diesel engines from acoustic emission signals. In aviation, this method have been applied by Jiang
281 et al. [37] to identify air traffic congestion problems (see Section 4).

282 2.3.3. Regression model-based

283 The regression model-based anomaly detection is a subcategory of the parametric techniques
284 identified by Chandola et al. [2] including a number of methods widely applied to time series data.
285 These methods are based on a two-step approach. A regression model is first fitted on the training
286 data. Then the resulting model is used on test sequences to compute the residuals, e.g. the difference
287 between the predicted value and the real value. The anomaly scores are finally determined based on
288 the residuals. Inside this category, we can include anomaly detection techniques based on traditional
289 time series forecasting models such as Vector Auto-Regressive (VAR) [38,39] and Autoregressive
290 Integrated Moving Average (ARIMA) [40,41]. Also, RNN have been used as regression models and
291 will be covered later on in a specific section of the survey.

292 2.4. Domain-based methods

293 This category identified by Pimentel et al. [1] include the methods which define a boundary
294 or domain to separate normal data from anomalies based on the training data. The most widely
295 applied technique in this category and the only one cover in this survey is the Support Vector Machines
296 (SVM) [42] and more precisely the variant known as one-class SVM (OC-SVM) [43].

297 This method assumes that training data is mostly representative of *normal data* so that the learned
298 boundary properly defines the normal region or class. The boundary is not defined directly on the
299 training data, but in the feature space obtained after applying the kernel trick, i.e. after projecting the
300 data in a space where it is linearly separable. A test instance is then considered as anomalous if falling
301 outside of the defined normal domain.

302 OC-SVM is part of the Multiple Kernel Anomaly Detection (MKAD) [44] algorithm developed by
303 NASA and considered as one of the first methods proven successful in the detection of anomalies in
304 heterogeneous flight data (see Section 4.1.1).

305 2.5. Reconstruction-based methods

306 Chandola et al. [2] identify a category called spectral-based anomaly detection in which it is
307 assumed that data embedding in a lower dimension helps separate normal instances from anomalous
308 ones. In the taxonomy by Pimentel et al. [1], the reconstruction-based approach encompasses
309 the spectral-based approach (called subspace-based) as well as a neural network-based approach.
310 Reconstruction-based methods assume that anomalies lose information when they are projected to a
311 lower dimension space, hence cannot be effectively reconstructed.

312 2.5.1. Subspace-based methods

313 In this subcategory, most of the anomaly detection methods use Principal Component Analysis
314 (PCA) [29]. For instance, the surveys [1,2] mention a simple anomaly detection algorithm based on
315 PCA and applied by Dutta et al. [45] to astronomy data. The assumption in this algorithm is that
316 samples with large values for the last principal components (the ones with the lowest variance) are
317 anomalies since this is indicative of a deviation from the correlation structure of data.

318 Several variants exist to address the different limitations of the basic PCA technique: Kernel
319 PCA [46] introduce specific kernels for non linear projections; Robust PCA [47] aims at making PCA
320 less sensitive to noise by enforcing sparse structures; Functional PCA [48] is a PCA extension [49–51]
321 to the case where data has a functional nature (sample of curves) such as flight trajectories.

322 PCA models are designed to be trained on a training set: then the fitted linear transformation
323 can be efficiently applied to any volume of further samples. Although the fitted transformations can
324 be applied on datasets of any size, their high computational complexity make them unsuitable to be
325 trained on very large datasets.

326 The application of subspace-methods as dimensionality reduction techniques are particularly
327 useful when applied to high-dimensional data. Several applications of subspace-methods to aviation
328 exist (see Section 4), including an improved faster method based on Kernel PCA [46] and Functional
329 PCA [52].

330 2.5.2. Neural network methods

331 Pimentel et al. [1] refer to a variety of NN techniques that can be applied for anomaly detection.
332 We focus here on Autoencoders (AE), one the most widely applied anomaly detection techniques
333 nowadays, which includes variants such as Deep Autoencoder (DAE) or Variational Autoencoder
334 (VAE) (See Section 3).

335 Autencoders have the same number of input and output neurons, and one or several hidden
336 layers with a smaller number of neurons acting up as a compression or dimensionality reduction
337 mechanism. The assumption behind reconstruction-based anomaly detection is that anomalies are
338 incompressible and cannot be properly reconstructed from the lower dimensional representation of
339 the latent variables.

340 Extreme Learning Machines (ELM) [53,54] are feed-forward neural networks much faster to
341 train than SVM or back-propagation neural networks and able to produce good results on many
342 classification and regression problems [55]. They are specially used for scalable anomaly detection
343 in very large datasets: Janakiraman and Nielsen [56] have applied unsupervised ELM models such
344 as autoencoders and embedding models to identify operationally significant events in aviation (see
345 Section 4.1.3).

346 3. Recent advances in anomaly detection

347 This section reviews some recent techniques applicable to anomaly detection which have been
348 developed in the fields of neural networks or deep learning as well as temporal-logic learning. Table 1
349 presents an overview of the recent techniques covered in this section.

350 3.1. Recent advances in recurrent neural networks

351 Recurrent Neural Network (RNN) is a special kind of neural network considered as well suited
352 for time series processing. The main issue with the standard RNN is its inability to learn long
353 term patterns in sequential data due to the gradient vanishing/exploding problem when applying
354 backpropagation-through-time (BPTT) algorithm during the training phase. For this reason, a standard
355 RNN is rarely used in real world applications which are usually based instead on two improved RNN
356 variants: the Long Short-Term Memory (LSTM) [74] and the Gated Recurrent Unit (GRU) [75].

Recurrent Neural Networks	3.1	<i>Stacked LSTM</i> : [57] (2015) <i>LSTM and GRU</i> : [58] (2016) <i>Hybrid LSTM with OC-SVM or SVDD</i> : [59] (2017)
Convolutional Neural Networks	3.2	<i>Intrusion detection</i> : [60] (2017), [61] (2017) <i>Comparative study with other NN</i> : [62] (2018)
Advanced Autoencoders	3.3	<i>LSTM-ED</i> : [63] (2016) <i>MSCRED</i> : [64] (2018) <i>Multi-modal DAE</i> : [65] (2016) <i>ConvLSTM-AE</i> : [66] (2017)
Generative Models	3.4	<i>GAN</i> : [67], [33], [68] (2018) <i>Variational Inference</i> : [69] (2016), [70] (2018)
Temporal-logic Learning Models	3.5	<i>Supervised model</i> : [71] (2014) <i>Unsupervised model</i> : [72] (2014) <i>Online model</i> : [73] (2016)

Table 1. Recent Bibliography in Anomaly Detection

RNN can be used as a regression model for anomaly detection and as such it can be classified as a method belonging to the regression model-based subcategory identified by Chandola et al. [2] inside the parametric techniques for statistical anomaly detection approaches. Compared to other classical anomaly detection techniques such as the ones based on clustering or OC-SVM, RNN are more convenient to capture temporal and non-linear dependencies in multivariate time series, especially when multiple layers of RNN are stacked together in deep architectures.

Goel et al. [76] perform a comparative study of the performance between two types of LSTM and the traditional Vector Auto-Regressive (VAR) as a regression model for multivariate time series from aviation. Surprisingly, the results of their research show that VAR significantly outperforms LSTM, which according to the authors could be explained by the fact that the LSTM capability of capturing long term dependencies may not be necessary.

Malhotra et al. [57] present a RNN model with several layers of stacked LSTM which is trained with normal data. The trained model is then used as a predictor over a number of multiple time steps and the residuals computed over the training set are modeled as a multivariate Gaussian distribution. The probabilities on the residuals can thus be computed and a threshold to discriminate the anomalies is determined by maximising the F_{β} score over a validation dataset.

More recently, Ergen et al. [59] present a hybrid framework for variable length sequences based on LSTM and the use of either OC-SVM [43] or SVDD [77] as anomaly detectors. The novelty in the approach comes from the fact they jointly optimise the parameters of both LSTM and the anomaly detector by developing specific gradient based training methods. The experiments on several real and simulated datasets show significant performance improvements over the traditional OC-SVM and SVDD methods.

It is also interesting to note that some approaches use RNN as part of an autoencoder (e.g. LSTM encoder-decoder (LSTM-ED) [63,78]) or generative architecture (e.g. GAN-AD [68]). See Sections 3.3 and 3.4 for further details.

3.2. Recent advances in convolutional neural networks

RNN have been traditionally considered as the best technique for sequence and time series modelling, recent research [79] seems to suggest that CNN can outperform canonical RNN such as LSTM in the task.

CNN are especially well-known for image feature extraction, but they can also be applied to extract complex hidden features in sequential data [80]. In this case, CNN are sometimes combined with some variant of RNN, such as the Convolutional LSTM (ConvLSTM) [81] to better capture spatio-temporal features.

390 In order to be processed by a CNN, a sliding window is the most used preprocessing technique
391 on time series data. In the case of multivariate time series, some studies [82,83] also suggest computing
392 the pair-wise correlations between the time series to model the system status. The resulting signature
393 matrices can then be fed to a CNN for pattern extraction.

394 CNN-based anomaly detection methods have been mainly applied to intrusion detection [60,61]
395 by preprocessing data samples with float and integer attributes into an image form convenient for
396 CNN processing. In a more recent study, Kwon et al. [62] assess several CNN architectures for anomaly
397 detection using different network traffic datasets by comparing their performance to other techniques
398 including Variational Autoencoders (VAE), Fully Connected Networks (FCN) [84] and LSTM. Their
399 results indicate that CNN perform better than VAE, but worse than FCN and LSTM.

400 While the use CNN for anomaly detection is an active area of research, several hybrid architectures
401 integrating CNN exist to perform anomaly detection. This is especially the case when CNN are used
402 as part of more complex AE architectures, like in [64,85,86] which have been applied in the aviation
403 domain and will be further addressed in the next section.

404 3.3. Recent advances in autoencoders

405 Autoencoders (AE) are powerful non linear dimensionality reduction tools commonly used for
406 anomaly detection, with many references in Pimentel's review [1] and more [87]. Autoencoders fall
407 in the unsupervised learning category: they learn to reconstruct, i.e. maximize a similarity measure,
408 samples that go through a lower dimension bottleneck. They *encode*, or *project*, samples into a low
409 dimension representation (the latent space), then *decode*, or *reconstruct*, it back into the original space.
410 The network is trained to minimize the global reconstruction error, e.g. a mean squared error. Once
411 the optimisation converged, anomalies are samples with the higher reconstruction error.

412 In this section, we focus on recent AE-based anomaly detection approaches including hybrid AE,
413 using RNN or CNN cells in the encoding and decoding parts of the neural networks, as well as Deep
414 Autoencoders (DAE), e.g. Stacked Denoising Autoencoders.

415 In the case of AE using RNN, it is worth mentioning the research by Malhotra et al. [63]. The
416 authors propose a reconstruction-based approach for anomaly detection in time series based on LSTM
417 Encoder-Decoder (LSTM-ED) models, which has been previously used for machine translation [75].
418 A LSTM-ED is trained only with normal time series data so that higher reconstruction errors should
419 be obtained for anomalous sequences. A normal distribution is fitted on the reconstruction errors
420 computed over a subset of the validation data. The estimated mean and covariance is then used to
421 compute an anomaly score for each point in the time series. The threshold to determine whether a
422 point is anomalous or not is computed so that it maximises F_{β} score over a validation dataset.

423 A ConvLSTM based autoencoder (ConvLSTM-AE) is proposed by [66] to encode appearance
424 and change of appearance (motion) for anomaly detection in videos. More recently, Zhang et al. [64]
425 propose a framework called Multi-Scale Convolutional Recurrent Encoder-Decoder (MSCRED) to
426 perform anomaly detection and diagnosis in multivariate time series data. The architecture combines
427 a convolutional encoder to capture the spatial patterns in the signature matrices, an attention based
428 ConvLSTM to capture the temporal patterns on the previously generated feature maps and finally a
429 convolutional decoder to decode the feature maps obtained in the previous steps in order to get the
430 reconstructed feature matrices.

431 In [65] a multi-modal Deep Autoencoder (DAE) framework is proposed for anomaly detection
432 and fault disambiguation on multivariate time-series corresponding to flight data generated from
433 multiple sensors. A DAE is a multi-hidden layer autoencoder capable of capturing several levels of
434 data abstraction [88]. In the framework, an overlapping sliding window technique is used over each
435 time series and the resulting sliced time series are concatenated into a large vector and fed into the
436 DAE. The average reconstruction error of a time window over all the sensors is used as an anomaly
437 score. The characterisation of the different fault signatures is based on the analysis of the distribution
438 of the anomaly scores.

439 3.4. Recent advances in generative models

440 Generative modelling is an area of machine learning which deals with models of distribution
441 defined in some potentially high-dimensional space. Generative models aim at capturing dependencies
442 between dimensions: they are trained to produce realistic data samples looking alike what is in the
443 original data set based on their representation in a lower dimension projected space, commonly
444 referred to as a *latent space*.

445 Generative Adversarial Networks (GAN) [89] are a well known framework for producing
446 generative models. They consist of two competing networks, a generator and a discriminator. The
447 generator models the data by learning how to transform samples taken from a prior distribution while
448 the discriminator learns to distinguish between real data and samples generated by the generator.
449 GANs have recently been used for anomaly detection [33,67,68,90], also in more advanced variants [91].

450 Variational Autoencoders (VAE) [92] have also been extensively used for anomaly detection [93].
451 The neural network representation of VAE is based on traditional autoencoders, although the
452 mathematical foundations have few in common. VAE model high-dimensional distributions by
453 casting learning representations as a Variational Inference [94] problem. VAE aim at learning a
454 mechanism to draw new samples from random variables taking values in the latent space following a
455 fixed prior distribution, classically Gaussian. The optimisation process takes into account the quality
456 of autoencoded samples with respect to their reconstruction probability and the Kullback-Leibler (KL)
457 divergence between the prior distribution and the transformed posterior distribution through the
458 encoding process. An anomaly reconstruct poorly through the generative process, and its encoding
459 fall outside the prior distribution.

460 VAE based anomaly detection has been generalised to time series by applying RNN with hidden
461 layers as encoder and decoder [95] under the name of Stochastic Recurrent Network (STORN). Similar
462 approach has also been mentioned in [69], or combined with Gaussian Mixtures with Gated Recurrent
463 Unit (GRU) cells [70]. The Gaussian assumption on the prior may be a limitation. Recent approaches
464 attempted to model more complex distributions in the latent space with energy-based models [96] or
465 Gaussian Mixtures Models [97]; and tried to free themselves from the variational framework [98].

466 More recently, the relevance of VAE over deterministic AE has been discussed [99]: reconstructed
467 examples are often blurry in case of images, the Gaussian assumption on the prior may be too restrictive
468 and the measure of the KL divergence in the optimization problem may lead to over-regularization.
469 The impact of such allegation on anomaly detection has, to our knowledge, not been addressed.

470 3.5. Recent advances in temporal logic-based learning

471 In the previous sections, we have covered the recent advances in anomaly detection in the field of
472 neural networks and deep learning. However, a well known drawback of neural networks is the lack
473 of interpretability of the results. Also, the output of classical methods represented by hyper-planes or
474 surfaces embedded in high-dimensional feature spaces to separate normal from anomalous behaviour
475 is hard to interpret by domain experts.

476 Most of the research effort in the last two decades in the field of machine learning and statistics has
477 primarily been focused on designing scalable and accurate black-box models. The interpretability of
478 the results has mostly been neglected because of the general belief it necessarily reduces accuracy [100].
479 In this section, we introduce a recent anomaly detection approach that can learn from data temporal
480 logic properties of a system in the form of a more human-readable formalism based on temporal
481 logic expressions. This approach can be better accepted by domain experts who intrinsically dislike
482 black-box models and ultimately reject them because of their lack of transparency.

483 Thus, Jones et al. [72] and Kong et al. [71,73] present an approach capable of inferring signal
484 temporal logic (STL) [101,102] formulae from data resembling natural language. STL is a specification
485 language used in the field of formal methods to specify system properties including time bounds and
486 bounds on physical system parameters, which can be used to describe the normal system behaviour.

487 For instance, we can express invariant properties such as “If x is greater than x_r , then within T_1 seconds,
488 it will drop below x_r and remain below x_r for at least T_2 seconds” [71].

489 The original supervised method [71] and unsupervised method [72] have been recently extended
490 to allow for on-line anomaly detection [73]. This new algorithm has reduced the computational cost
491 compared to the supervised version [72] and can be now applicable to high dimensional systems
492 producing large amounts of data. A further advantage is that the output is expressed in STL, which
493 can be directly processed by a computer system for automatic monitoring of anomalous behaviour.

494 The approach has been applied to several domains including naval surveillance and train braking
495 system [73]. Concerning the aviation domain, Deshmukh et al. [103] has recently used the approach
496 to detect anomalies in the terminal airspace operations (more details in Section 4.1).

497 4. Applications

498 This survey reviews the use of some of the previously introduced anomaly detection methods in
499 two important areas of the aviation: air traffic operations and predictive maintenance. Because of the
500 significant number of the techniques covered in the first application area, air traffic operations, we
501 have created a classification of the methods based on the category of the anomaly detection approach
502 (see Table 2).

4.1.1 Domain-based	<i>Abnormal approaches with MKAD</i> : [104] (2011) <i>GA approach and landing anomalies with OC-SVM</i> : [105] (2017)
4.1.2 Distance-based	<i>Anomalous pilot switching with SequenceMiner</i> : [18] (2009) <i>Anomalous take-off and approach operations</i> : [19] (2011), [20](2015) <i>Anomalous safety events with LoOP</i> : [16] (2019) <i>Anomalous taxi paths with hierarchical clustering</i> : [22] (2019) <i>Anomalous radiotelephony readbacks with kNN</i> : [106] (2018)
4.1.3 Reconstruction-based	<i>Atypical aviation safety data with KPCA</i> : [107] (2017) <i>Atypical approaches and landings with FPCA</i> : [52] (2018) <i>Anomalous trajectories in TMA and en-route</i> : [108] (2018), [109] (2019) <i>Anomalous transitions between sector configurations</i> : [110] (2018) <i>Anomalous ADS-B messages with ConvLSTM-AE</i> : [86] (2019)
4.1.4 Statistical-based	<i>Anomalous flights with VARX</i> : [38] (2016), <i>Anomalous flight switches with VAR</i> : [39] (2016) <i>Abnormal flight data with GMM</i> : [21] (2016) <i>Anomalous air traffic congestion with ICA</i> : [37] (2019)
4.1.5 Temporal-logic based	<i>Anomalous trajectories in terminal airspace with TempAD</i> : [103], [111] (2019)

Table 2. Application of anomaly detection models to aviation use cases

503 4.1. Anomaly detection for air traffic operations

504 One application area in aviation where anomaly detection techniques have particularly been
505 applied to is in the identification of significant operational events in flight data. In this context,
506 significant events mean patterns or behaviours that can be worth detecting in flight data because
507 of their potential impact on the performance (usually safety) of flight operations. For instance, the
508 identification of events such as runway excursions, go-around operations, trajectory deviation due
509 to conflict resolution actions. Other significant events occur in the broader context of Air Traffic
510 Management (ATM) operations. These are also covered in this section and include anomalous
511 ATC-pilot communications and anomalies in the sequences of airspace sector configurations. Table 2
512 presents an overview of the different applications classified by the category of the anomaly detection
513 approach applied.

514 In the US, NASA established in 2007 a program to store Flight Operations Quality Assurance
515 (FOQA) data from most of the major airlines which is also used by the FAA to monitor and address

operational risk issues. The FOQA database currently contains millions of flights and each entry represents hundreds of parameters from the avionics and other on-board systems. Likewise, in Europe, Flight Data Monitoring (FDM) programs promoted by EASA requires airlines to gather, monitor and analyse data to improve the performance and safety of flight operations.

The objective of FOQA and FDM programs is to switch from a purely reactive mode based on reports or interviews to a more proactive mode where data analytics can be used to assess trends, risks and undesired events in order to help implement mitigation measures. The applications reviewed here support this proactive approach by automatically detecting statistically anomalous events in vast amounts of historical on-board generated data.

However, the process is not fully automatic as the flagged events need further consideration from operational experts to determine whether the identified anomalies are only low occurrence events or true significant events with potential safety or performance implications.

For decades, the only approach to automatically detect anomalies from generated data has been based on exceedance detection algorithms, which check flight data against predetermined thresholds set by subject matter experts. When one or a combination of thresholds are exceeded, the corresponding flight is flagged as anomalous. Even though this approach has been improved and is nowadays largely trusted by the industry, it still presents significant shortcomings such as the difficulty to properly set the thresholds to avoid false-positives and false-negatives as well as the impossibility to anticipate all possible events.

The availability of extensive amounts of generated flight data along with the significant advances in the machine learning community offer new opportunities for approaches capable of a better detection of unknown (not pre-programmed) events, which should improve the current rate of false-negatives in exceedance-based approaches and be able to cope with a large volume of high-dimensional data. In the following subsections, we present the application of some of the previously reviewed data-driven anomaly detection methods (see Section 2 and Section 3) to support the identification of significant flight operational events.

4.1.1. Domain-based approaches

If SequenceMiner [18] is one of the few anomaly detection methods specifically designed for the processing of discrete sequences, MKAD developed by Das et al. [44] is one of the first methods designed to effectively detect operationally significant anomalies with heterogeneous sequences of both discrete and continuous variables. Based on kernel functions and OC-SVM, MKAD can identify operational situations in FOQA data such as go-around operations, unusually high airspeed flights, flights impacted by gusty winds and abnormal approaches. More recently, Das et al. [104] applied MKAD to detect anomalies in the approach phase but this time with a much larger set of flights of the same fleet and aircraft type. In the paper, the authors report exclusively on two anomalous situations correctly identified by MKAD corresponding to two significant operational events: high energy approaches and turbulent approaches.

With the aim of improving the safety of General Aviation operations, Puranik et al. [105] propose a framework to identify anomalies based on a OC-SVM model. After a classical preprocessing phase to clean the raw multivariate time series data, a set of feature vectors corresponding to the energy metrics detailed in [112] are computed, such as the Specific Total Energy (STE) or the Specific Potential Energy (SPE). The DBSCAN algorithm is first applied to the feature vector in order to determine the number of clusters. Based on the identified clusters, the OC-SVM algorithm is used to compute the anomaly scores of each flight. The methodology is evaluated with both simulated data with anomalies and real data from a Cessna 172S during the approach and landing phase. The results show a good performance in terms of anomalous flight identification even when only a limited number of parameters are recorded.

4.1.2. Distance-based approaches

One of the first attempts in the field was the research by Budalakoti et al. [18] who address the problem of anomaly detection in a set of sequences of switches used by the pilot and co-pilot to maneuver an aircraft. Their method (SequenceMiner), based on a clustering approach, is able to detect anomalous switching behaviour linked to the loss of autopilot mode awareness by the flight crew.

Li et al. [19] apply a cluster-based anomaly detection (ClusterAD) method based on DBSCAN to detect anomalies in a FOQA dataset of an airline for 365 B777 take-off and approach operations. The anomalous operational situations correctly identified by ClusterAD include high/low energy approaches, unusual pitch excursions, abnormal flap settings and high wind conditions. One of the advantages of ClusterAD compared to MKAD is that it can automatically identify multiple types of flight operation patterns (different nominal operations) corresponding to the identified clusters.

Following up this research, Li et al. [20] present a method based on DBSCAN called ClusterAD – Flight, which is able to detect abnormal flights during take-off or approach as whole. In this work, more extensive tests are conducted with an additional dataset of 25,519 A320 flights. Results show that both ClusterAD – Flight and MKAD are able to identify more operationally significant anomalies than exceedance-based methods. ClusterAD – Flight performs better with continuous parameters, whereas MKAD is more sensitive toward discrete parameters. The latest research by Li et al. [21] is on an improved ClusterAD approach called ClusterAD – DataSample. However, as this method is based on a GMM, we cover it as part of the statistical approaches in Section 4.1.4.

Compared to MKAD and ClusterAD which are able to process hundreds to tens-of-thousands of flights, Oehling et al. [16] propose an approach able to scale to very large datasets as the ones used in the production environments of big airlines. The approach, based on the Local Outlier Probability (LoOP) method, is applied to an airline dataset of 1,2 million flights in order to detect anomalies related to safety events. The top outliers identified by their approach are reviewed by the airline pilots in order to assess their safety-relevance. The results of the research show that their method is able to reduce the number of undetected safety-relevant events compared to the current exceedance based approaches implemented in FDM systems.

Churchill et al. [22] present a hierarchical clustering method to group in space and time aircraft trajectories in the airport surface. The goal is the identification of statistically anomalous taxi paths, which may be unplanned and unexpected by the controllers and thus could represent a safety risk.

In [106,113], semantic checking models based on LSTM and kNN are introduced to identify read-back errors in ATC radio-telephony communications. Civil aviation radio-telephony recordings are converted to textual format, and similarity functions are defined to verify whether the semantics is the same between controller instructions and pilot read-backs.

4.1.3. Reconstruction-based approaches

Zhang et al. [107] point out two known issues when the classical Kernel PCA algorithm [46] is applied to a large dataset for anomaly detection: it is computationally expensive ($O(n^3)$ where n is the size of the dataset) as well as hard to adapt as parameters such as the number of principal components and the confidence for the confidence limit needs to be set before anomaly detection. Thus, the authors develop an optimized GPU implementation where the previous parameters are computed automatically. The improved algorithm is applied to synthetic datasets [44] and compared to the OC-SVM [43] technique. The results show significant speed increases and a detection efficacy close to the OC-SVM one.

Jarry et al. [52] propose a method based on FPCA to identify atypical approaches and landings both in post-operational analysis and on-line. The method was tested with track radar data (20,756 records) of landing operations at Paris Charles-De-Gaulle (CDG) airport. The goal is to improve the detection rate of Non Compliant Approaches (NCA), i.e. an approach in which the intercepting conditions of the intermediate and final legs are not compliant with the operational prescriptions. NCA is a precursor of Non Stabilised Approaches (NSA) which may lead to fatal events like Control

611 Flight Into Terrain (CFIT). The authors propose to extend current tools capabilities based on geometric
612 criteria by taking into account additional features such the specific total energy of the aircraft. The
613 method uses a sliding window over the trajectories in order to apply FPCA first and then HDBSCAN
614 with GLOSH [27]. From the set of computed outlier scores, it is determined whether a trajectory is
615 anomalous. The results show the method can effectively identify atypical flights although the results
616 can be very sensitive to the size of the sliding window.

617 Janakiraman and Nielsen [56] propose an unsupervised anomaly detection approach based on
618 ELM. This approach developed by NASA is an alternative to MKAD for the identification of safety
619 risks in very large aviation datasets. The performance of the three ELM variants are evaluated and
620 compared to MKAD on a dataset of over 40,000 flights corresponding to landing operations at Denver
621 airport. While the results of the ELM-based approach are comparable to MKAD in terms of detection
622 accuracy, the training of ELM models are faster by two orders of magnitude.

623 Olive et al. [108] present a method based on autoencoders to analyse flight trajectories, detect
624 unusual flight behaviours and infer ATC actions from past Mode S data. The method is evaluated with
625 three different city-pairs and one year of traffic within a bounding box defined just before the entry
626 to the Terminal Manoeuvring Area. The identified anomalous situations are analysed based on the
627 distribution of reconstruction errors (anomaly scores). It is shown that the highest anomaly scores
628 correspond to weather impact or traffic regulations whereas the lowest ones to relatively more usual
629 ATC deconfliction or sequencing actions.

630 Following up on the previous research on air traffic anomaly detection [108] and on identification
631 of traffic flows [114] in en-route ATC sectors, Olive and Basora [109] propose a method to detect
632 anomalous flight trajectories in the flows of a en-route ATC sector. A clustering approach is used first
633 to automatically identify from ADS-B traffic a set of clusters corresponding to sector flows. Then, an
634 autoencoder is applied to each cluster in order to detect anomalous trajectories. The analysis of the
635 distribution of reconstruction errors confirms the conclusions reached in [108].

636 In [110], autoencoders are used to detect anomalous transitions between sector configurations in
637 Area Control Centres (ACC). The model is trained with transitions performed in the past and then
638 applied to transitions never realized. Transitions with highest autoencoder reconstruction error are
639 considered as anomalies, unlikely to be realized.

640 Based on the ConvLSTM method by Shi et al. [81], Akerman et al. [86] present a convolutional
641 LSTM based autoencoder (ConvLSTM-AE) framework to detect anomalous ADS-B messages. In this
642 framework, aircraft flying in the same airspace are represented as images and the ConvLSTM-AE
643 model is used to detect anomalies in the sequences of images leading to anomalous ADS-B location
644 reports.

645 4.1.4. Statistical-based approaches

646 Melnyk et al. [38] propose an unsupervised model-based framework adapted to online anomaly
647 detection where each flight is represented as a Vector AutoRegressive eXogenous model (VARX)
648 model [115]. The key step in the approach is to compute a distance matrix between flights defined in
649 terms of residuals of modeling one flight's data using another flight's VARX model. Once the distance
650 matrix is built, a LOF method [13] is applied to identify the anomalous flights. The evaluation results
651 on a large FOQA dataset (over a million flights) show a good performance into detecting already
652 known safety events as well as previously undetected ones compared to state-of-the-art algorithms
653 like MKAD.

654 In another framework also based on VAR modelling and adapted to online anomaly, Melnyk et
655 al. [39] represent each flight with a semi-Markov switching vector autoregressive (SMS-VAR) model.
656 With this approach, each phase of a flight determined by the set of pilot switches is represented by
657 a different VAR process [115] and a semi-Markov model (SMM) [116] is used for the dynamics of
658 flight switches. Anomaly detection is based on the dissimilarities between the one-step ahead model's

659 predictions and observed data. The framework is extensively evaluated on both a synthetic and an
660 airline FOQA dataset and the achieved performance is similar or slightly better than the MKAD one.

661 Nanduri and Sherry [58] present a regression-based approach applied to simulated FOQA-like
662 data [117] corresponding to 500 approaches into San Francisco airport. Several different kinds
663 of RNN architectures (GRU and LSTM) are tested and compared with MKAD. In all cases, the
664 RNN-based models are able to detect more anomalies than MKAD. The authors explain the superiority
665 of RNN-based approaches by the fact that they do not have the limitations of MKAD: the need for
666 dimensionality reduction which results in loss of information and poor sensitivity to short duration
667 anomalies, and its inability to detect anomalies in latent features. Unfortunately, the authors give few
668 details on how the anomaly threshold based on the residuals (prediction errors) was chosen.

669 ClusterAD – DataSample by Li et al. [21] is a method based on a GMM which is capable of
670 instantaneously detecting abnormal data samples during a flight rather than abnormal flights as a
671 whole during a specific flight phase. The method is tested with a real dataset of 10,528 A320 flights and
672 compared with exceedance-based methods. Then, it is compared with MKAD and ClusterAD – Flight
673 with a second dataset of 25,519 A320 flights (already used in [20]). The results indicate ClusterAD
674 – DataSample performs better in detecting known unsafe events (detected with exceedance-based
675 methods), but the authors point out the need for further evaluation of the performance with detecting
676 unknown issues.

677 Jiang et al [37] propose a method based on independent component analysis (ICA) for online
678 monitoring of air traffic congestion. Based on the complex networks topology, a model is trained
679 with a dataset of smooth situations. Any new situation is then compared to the reference ‘normal’
680 representation by analyzing the change of statistics. As the confidence limits cannot be determined
681 directly from a particular approximate distribution, a kernel density estimation (KDE) is used to set
682 the control limits.

683 4.1.5. Temporal-logic learning based approaches

684 Deshmukh et al. [103] propose a temporal logic based anomaly detection algorithm (TempAD)
685 applicable to trajectories in the terminal airspace. The algorithm, based on a temporal-logic learning
686 approach [71–73], can learn human-readable mathematical expressions from data which facilitates the
687 feedback and interaction with operational experts. The method uses DBSCAN as a preprocessing step
688 to identify the clusters with similar trajectories on which the detection of anomalies with TempAD
689 becomes more effective. TempAD is able to generate for each cluster STL predicates defining the
690 bounds of normal flights as a function of time, distance to touchdown or aircraft state vectors (including
691 latitude, longitude, altitude, ground speed). The representative features to find anomalies include
692 some of the energy features used in [105]. The method is evaluated on real surveillance data from the
693 terminal airspace at New York La Guardia airport, covering several thousands of arrival flights. The
694 algorithm is able to effectively identify anomalous situations such go-around operations as well as
695 arrivals with excessive total energy, above or below the recommended glideslope.

696 Following up this research, Deshmukh et al. [111] develop a supervised precursor detection
697 algorithm called reactive TempAD by correlating surveillance data to specific anomalies identified by
698 the TempAD algorithm [103]. Thus, the prediction of an anomaly is performed by identifying events
699 that precede the occurrence of an anomaly, which are called precursors.

700 4.2. Anomaly detection for predictive maintenance operations in aviation

701 Flight data recorders generate large volumes of heterogeneous time-series data from arrays of
702 sensors. This massive amount of sensor-generated data can be exploited to perform fault diagnosis
703 and estimate the remaining useful life (RUL). The long-term objective is to reduce and ultimately
704 avoid unscheduled maintenance by optimising the scheduling of maintenance operations based on the
705 RUL prediction, i.e. condition based maintenance (CBM). The ability to predict the RUL of a system
706 component after the occurrence of a fault corresponds to the widely accepted definition of prognostics.

707 The field of prognostics and health management (PHM) has drawn significant interest from industrial
708 and academic research in the last few years as system availability and reliability becomes a serious
709 concern, especially in safety-critical systems such the ones found in aviation.

710 In the emerging field of data-driven prognosis, predictive models are learned from flight and
711 maintenance data. These models can then be integrated into PHM systems for health monitoring
712 and incipient system failure prediction. There exists a number of data-driven methods for prognosis,
713 but it is usually difficult to compare them based on a common reference baseline due to the use of
714 sensitive commercial data. Fortunately, the following open datasets related to aviation are widely
715 acknowledged as reference for comparison:

- 716 • NASA DASHlink open database originally designed and collected by Balaban [118] and available
717 at <https://c3.nasa.gov/dashlink/projects/85/>;
- 718 • a turbofan engine degradation simulation dataset based on thermo-dynamical simulation models,
719 introduced in [119];
- 720 • other datasets, also shared on the Prognostic Data Repository of NASA refer to bearing systems
721 or milling machines. These do not necessarily refer to aviation problems but are still worth
722 mentioning as they are commonly used as reference.

723 Although prognostics and RUL estimation is a core function of PHM, it falls out of the strict
724 anomaly detection scope and hence it is not covered in this survey (two recent reviews by Elattar
725 (2016) [120] or Lei (2018) [121] already address this topic). Instead, we focus our review on the
726 application of data-driven techniques aimed at detecting anomalous behaviour in aviation systems
727 with the goal of identifying faults after their occurrence or anticipating potential failures as part of the
728 condition monitoring process in PHM [122].

729 Effective anomaly detection techniques to predict incipient failures from historical data is
730 important to estimate time-to-failure and help schedule maintenance activities. Some of the reviewed
731 techniques can also support fault diagnostics, which is a PHM process encompassing fault detection,
732 isolation (i.e. which component has failed), failure mode identification (i.e. what is the cause of
733 failure or fault) and quantification of the failure severity. Fault detection is typically based on the
734 quantification of the inconsistencies between the actual and the expected behavior of the system in
735 nominal conditions [122].

736 For instance, Rabatel et al. [123] present an anomaly detection framework for preventive
737 maintenance based on anomalous pattern detection in data. The data is based on closed railway
738 data; the approach, from pattern extraction to anomaly detection methods to apply on sequences,
739 could be extended to aircraft data which are subject to common characteristics.

740 More recently, Nicchiotti et al. [124] (2018) leverage closed commercial aircraft maintenance
741 operational data and apply SVM based methods and PCA as a tool to reduce dimensionality in order
742 to predict such unscheduled maintenance operations.

743 Deep autoencoders [65] and convolutional denoising autoencoders [85] (2019) have been used for
744 fault detection and anomaly detection, both on the NASA open database and on a dataset of Customer
745 Notification Reports sent over ACARS to airlines to help them detect engine faults.

746 Recurrent Neural Networks autoencoders have also been used on time series [125] in order to find
747 a proper embedding or representation of time series that is in turn used for predicting a RUL estimation
748 on the turbofan dataset. More recently, Zhao et al. compare in [126] different approaches of feature
749 selection mechanism based on dimensionality reduction. Autoencoders, Riemann Boltzmann Machines
750 (RBM) and Deep Belief Networks (DBN), CNN and RNN based methods are compared on a traditional
751 milling machine health monitoring application with similar results, the older RBM/DBN-based
752 techniques being slightly behind.

753 5. Conclusions

754 In this survey, we have reviewed the state-of-the-art in data-driven anomaly detection and its
755 application to the aviation domain. Thus, we have introduced a large number of classical and more

756 recent approaches and described how some of them have been applied to areas such as air traffic
757 operations and predictive maintenance. Machine learning models can work with offline or online
758 data to detect significant events for further analysis by aviation experts as part of decision support
759 or condition monitoring tools. The ultimate goal of the presented applications is to help improve the
760 performance of ATM and maintenance operations, in particular safety.

761 In general, as stated by Janakiraman and Nielsen [56], the data-driven detection of anomalies in
762 aviation data is particularly challenging because of its large volume, high-dimensionality, heterogeneity
763 (mixed categorical and continuous attributes), multi-modality (multiple modes of nominal and
764 non-nominal operations with different types of aircraft, airports and airspaces) and temporality
765 (long time-series). The challenge is expected to be even bigger in the future because of the forecast
766 world-wide growth of air traffic and the ever higher number of sensor-equipped aviation systems and
767 operational complexity.

768 Classical nearest-neighbour and clustering-based approaches do not scale well with such
769 massive amounts of high-dimensional data. In the case of high-dimensional data, the use of a
770 dimensionality reduction technique as a preprocessing step (e.g. to clustering) or the application of a
771 reconstruction-based method is often a better solution. On the other hand, distance-based methods are
772 computationally expensive when applied to large volumes of data, even during the test phase, which
773 makes them unsuitable for real-time applications. However, in the case of probabilistic, domain-based
774 and reconstruction-based methods, even though the training phase can be time-consuming, the test
775 phase is very efficient. This is not an issue for applications where models can be trained offline, but
776 some real-time safety monitoring applications may require some kind of incremental or very fast
777 online training.

778 In aviation, among the traditional approaches, the domain-based MKAD [44] developed by NASA
779 is still one of the state-of-the-art methods for the detection of operationally significant events in flight
780 data. However, its computational complexity is quadratic with respect to the number of training
781 examples, which makes it unsuitable for very large datasets and certain applications. ClusterAD
782 methods are also among the most widely applied, in spite of having the same performance issues than
783 MKAD. For faster learning with large datasets, ELM [56] or LoOP [16] based anomaly detection seem
784 to be two good alternatives to MKAD.

785 The recent advances in anomaly detection we have covered in this review are mainly based on
786 techniques developed in the field of neural networks and deep learning. In principle, deep-learning
787 approaches should be better adapted than traditional machine learning methods [11] when it comes to
788 find anomalies in large-scale high-complex data as the one generally available in aviation. We have
789 also reviewed the advances in temporal-logic based learning as an alternative approach that should
790 help the user more naturally understand and trust the results expressed in terms of logical formulae.

791 The application area related to the identification of significant events in air traffic operations is
792 particularly rich in terms of the number and variety of the anomaly methods applied. While traditional
793 techniques are widely used, there exists also some attempts to apply recent advances in temporal-logic
794 learning [103,111], RNN [58] and advanced autoencoders (e.g. ConvLSTM-AE [86]). The vast majority
795 of the research in this application area concern the detection of anomalies relevant to safety, although
796 we provided also a few examples of anomalies related to potential cyberattacks or air traffic congestion.
797 Another observation is that most of the introduced applications work in an offline configuration
798 with post-operational data for analysis purposes rather than with online data to support real-time
799 monitoring tasks.

800 As for the other application area concerning predictive maintenance in aviation, we have reviewed
801 a few anomaly detection methods aimed at identifying incipient failures in aviation system components
802 from flight and maintenance data. These data-driven methods play an increasingly important role in
803 PHM which is necessary to achieve true condition-based maintenance. In spite of that, the number of
804 reviewed research is relatively limited compared to the air traffic operations application area. This
805 is because a lot of the literature on data-driven methods for PHM is more focus on RUL prediction

806 which is out of the scope of this review. Also, some of the work on anomaly detection for predictive
807 maintenance is not specific to aviation. Nevertheless, we have covered the application of both classical
808 approaches such as SVM [124] and more recent approaches based on deep learning such as deep
809 autoencoders [65].

810 Finally, the operational usability of anomaly detection methods as part of a decision support
811 tool is an aspect marginally addressed and which would probably deserve further attention and
812 research. A first consideration about the usability of anomaly detection methods is how to provide
813 the user with a proper uncertainty measure associated to the model output (e.g. confidence intervals)
814 as a better way to deal with false alarms. A second consideration is that for an expert to trust and
815 understand the prediction of an anomaly detection model, the model and its outputs should be
816 explainable in some degree. Although this issue is more generally addressed in an emerging research
817 field called explainable artificial intelligence [127], its main focus has been on supervised machine
818 learning approaches, which is not the main approach in anomaly detection.

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821 T.D.; data curation, n/a; writing—original draft preparation, L.B., X.O. and T.D.; writing—review and editing,
822 L.B., X.O. and T.D.; visualization, n/a; supervision, L.B.; project administration, X.O.; funding acquisition, X.O.

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826 Abbreviations

827 The following abbreviations are used in this manuscript:

828

829 Abbreviations related to machine learning methods

AE	Autoencoder
ARIMA	Auto Regressive Integrated Moving Averages
CNN	Convolutional Neural Network
DAE	Deep Autoencoder
DBN	Deep Belief Network
DBSCAN	Density-Based Spatial Clustering of Applications with Noise
ELM	Extreme Learning Machines
GAN	Generative Adversarial Network
GLOSH	Global-Local Outlier Score from Hierarchies
GMM	Gaussian Mixture Model
GRU	Gated Recurrent Unit
ICA	Independent Component Analysis
IF	Isolation Forest
KDE	Kernel Density Estimation
kNN	K-Nearest Neighbours
830 IMF	Inductive Monitoring System
LOF	Local Outlier Factor
LoOP	Local Outlier Probability
LSTM	Long Short-Term Memory
MKAD	Multiple Kernel Anomaly Detection
NN	Neural Network
OC-SVM	One-Class Support Vector Machine
OPTICS	Ordering Points To Identify the Clustering Structure
PCA	Principal Component Analysis
RBM	Riemann Boltzmann Machine
RNN	Recurrent Neural Network
STORN	Stochastic Recurrent Network
SVM	Support Vector Machine
VAE	Variational Autoencoder
VAR	Vector Auto-Regressive

831 Abbreviations related to aviation

ACARS	Aircraft Communication Addressing and Reporting System
ADS-B	Automatic Dependent Surveillance–Broadcast
ATC	Air Traffic Control
ATM	Air Traffic Management
832 CBM	Condition Based Maintenance
FDM	Flight Data Monitoring
FOQA	Flight Operations Quality Assurance
PHM	Prognostics and Health Management
RUL	Remained Useful Life

833

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