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Linearized Stability of Bardeen de-Sitter Thin-Shell Wormholes

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ABSTRACT

A thin-shell wormhole is crafted by the cut-and-paste method of two Bardeen de-Sitter black holes using Darmois-Israel formalism. Energy conditions are considered for different values of magnetic charge while both mass and cosmological constant are fixed. The attractive and repulsive characteristics of the throat of the thin-shell wormhole are also examined through the radial acceleration. Dynamics and stability of the wormhole are studied around the static solutions of the linearized radial perturbations at the throat of the wormhole. The regions of stability are determined by checking out the condition of concavity of the potential as a function in the throat radius for different values of magnetic charges.

PACS numbers: 04.90.+e, 04.20.-q, 04.20.Gz

I. INTRODUCTION

The first wormhole solution was discovered by Ludwig Flamm [1]. It was rediscovered as the *Einstein-Rosen bridge* [2] while Einstein and Rosen were trying to develop a non-Boscovichian, i.e., singularity-free, atomic model of gravity and electromagnetism. Later, Wheeler developed a theory about *geons* [3], topologically unstable gravitoelectromagnetic quasi-solitons that can connect widely separated spacetime regions. Misner and Wheeler tried to develop the theory of geons into a geometrical unified classical theory [4]. In Misner and Wheeler project the *wormhole* term was coined.

Between the development of geons and the rejuvenation of Morris and Thorne *traversable* wormholes [5], Ellis studied the flow of "substantial ether" through a *drainhole* [6]. Also Bronnikov analyzed tunnel-like solutions [7], which are considered the precursors to the studies of wormholes in modified theories of gravity [8]. Geons reappear again in *galileon* theory as a scalar-tensor theory [9]. Even in Euclidean space, Ellis variant *p-norm* drainholes can be used as *pedagogical* examples to study electrostatics [10–13]. There is also a recent letter with interesting pedagogical examples, in both flat space and curved spacetime, in which the behaviors of objects around wormholes are discussed in terms of scalar, electromagnetic, and gravitational fields [14]. More on wormholes can be found in Ref. [15].

To find a wormhole solution to field equation, one can choose some equations of state such as phantom energy [16, 17], Chaplygin gas [18], and/or quintessence [19]. Then, rotating spacetimes [20], evolving wormholes [21], thin-shell spacetimes [22], and/or dust shell wormholes [23] can be implemented to the field equations to "ameliorate" the violation of energy conditions associated with the *flaring-out* condition, which is necessary for the field equations to have wormhole solutions. There are numerous studies that consider different black holes creating thin-shell wormholes in de-Sitter and anti-de-Sitter spacetimes [24–29]. Stability of these thin-shell wormholes are examined too [30–46]. Also thin-shell wormholes can be obtained from regular black holes [47, 48], which is the general theme of this letter.

In this letter we construct Bardeen de-Sitter thin-shell wormholes. The Bardeen black hole [49] is an interesting *regular* black hole, i.e., with no *geometric* singularity. Bardeen black hole can be discerned as a quantum-corrected Schwarzschild black hole [50] by applying the *generalized uncertainty principle* [51–58] to the black hole and studying the consequent effects on the corresponding thermodynamics. Also Bardeen black hole can be implemented in de-Sitter background (BdS) [59]. The BdS solution in arbitrary dimensions and the corresponding thermodynamics for each dimension are also considered [60].

In section (**II**), we use Visser's technique of cut-and-paste [61, 62], together with Darmois-Israel formalism [63], to connect two BdS regions of spacetime through a thin shell. We also study the components of the stress-energy-momentum surface tensor using the extrinsic curvature. We comment on the violation of energy conditions in terms of stress components. We also calculate the attraction and repulsion nature of the wormhole throat in terms of the acceleration.

In section (**III**), we analyze the linear stability of BdS thin-shell wormhole by studying the concavity condition on the "speed of sound" as a function in BdS parameters: the mass, the magnetic monopoles and the cosmological constant. And we see the change in stability regions upon varying the amount of magnetic monopoles while both mass and cosmological constant are fixed.

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In section (**IV**) we summarize and comment on the results of the previous two sections.

II. VISSER'S CUT-AND-PASTE TECHNIQUE AND THE DARMOIS-ISRAEL FORMALISM

The BdS black hole is constructed [59] starting with the metric

$$ds^2_{\text{BdS}} = -\left(1-\frac{2m(r)}{r}\right)dt^2 + \left(1-\frac{2m(r)}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2\ . \tag{1}$$

The field equations are derived from the action [64]:

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi} - \frac{1}{4\pi}\frac{3M}{|\mu|^3}\left(\frac{\sqrt{2\mu^2F}}{1+\sqrt{2\mu^2F}}\right)^{\frac{5}{2}}\right], \tag{2}$$

where μ is the magnetic monopole charge, M is the mass of the black hole, and $F = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ is the trace of electromagnetic field tensor $F_{\mu\nu}$.
Therefore, eq.(1) becomes:

$$ds^2_{\text{BdS}} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2, \tag{3}$$

where

$$f(r) = 1 - \frac{2Mr^2}{(r^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}r^2\ . \tag{4}$$

And by finding the roots of $f(r)=0$, or the roots of the *decic* polynomial

$$r^{10}\Lambda^2 + r^8\left(3\Lambda^2\mu^2 - 6\Lambda\right) + r^6\left(3\Lambda^2\mu^4 - 18\Lambda\mu^2 + 9\right) + r^4\left(\Lambda^2\mu^6 - 18\Lambda\mu^4 + 27\mu^2 - 36M^2\right) + r^2\left(27q^4 - 6\Lambda\mu^6\right) + 9\mu^6 = 0, \tag{5}$$

one can determine the location of the inner, event (r_h) and cosmological (r_c) horizons of the BdS. However, we must avoid the combinations of M, μ, and Λ that lead to formation *extreme* BdS [27]—at which the event and cosmological horizons coincide by setting $f(r) = f'(r) = 0$ —so the throat radius a of the wormhole still exists as $r_h < a < r_c$.

Following the Cut-and-Paste technique [61, 65], one can easily construct a geodesically complete manifold $\Gamma = \Gamma_+ \bigcup \Gamma_-$ by *pasting* the region of timelike hypersurfaces, named a *thin shell* $\partial\Gamma = \partial\Gamma_+ \bigcup \partial\Gamma_-$, where $\partial\Gamma_\pm := \{r_\pm = a \mid a > r_h\}$, that bounds the bulk of two BdS. This follows after *cutting* spacetime regions $\Gamma_\pm := \{r_\pm \leq a \mid a > r_h\}$ inside the throat radius a.
Now we follow Darmois-Israel formalism [66, 67] by defining the coordinates of Γ as $x^\mu := (t, r, \theta, \phi)$ and the coordinates of the shell $\partial\Gamma$ as $\zeta^i := (\tau, \theta, \phi)$, where τ is the proper time that a comoving frame measures

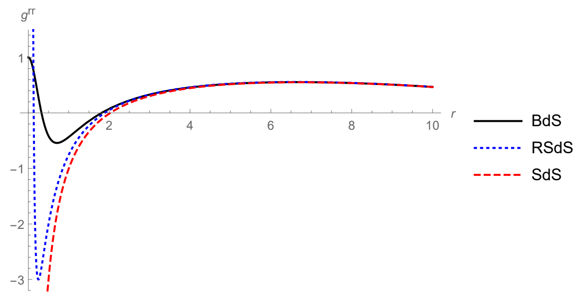

Figure 1: The behavior of g^{rr} metric component for Bardeen de-Sitter (BdS: $M=1, \Lambda=0.01, \mu=0.5$), Reissner-Nordstrom de-Sitter
(RNdS: $M=1, \Lambda=0.01, Q=0.5$),
and Schwarzschild de-Sitter (SdS: $M=1, \Lambda=0.01$).

on the throat of the wormhole. The induced metric of the shell is:

$$ds^2_{\partial\Gamma} = -d\tau^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2, \tag{6}$$

where the parametric equation that relates Γ to $\partial\Gamma$ is $r = a(\tau)$.
We use the Gauss-Kodazzi decomposition of spacetime such that it yields Israel's junction condition on Γ. The condition is described by the energy momentum tensor on the shell $\mathcal{S}^i_j = \text{diag}\left(-\sigma, p_\theta, p_\phi\right)$ as

$$\mathcal{S}^i_j = -\frac{1}{8\pi}\left(\left[\mathcal{K}^i_j\right] - \delta^i_j\mathcal{K}\right), \tag{7}$$

where the $\mathcal{K}^i_j$ is the extrinsic curvature, $\left[\mathcal{K}^i_j\right] = \mathcal{K}^{i\ +}_j - \mathcal{K}^{i\ -}_j$, and $\mathcal{K} = \left[\mathcal{K}^i_i\right]$.
We define the unit vectors $n^\pm_\mu$ normal to $\partial\Gamma$ as

$$n^\pm_\mu = \pm\left(\left|g^{\alpha\beta}\frac{\partial f}{\partial x^\alpha}\frac{\partial f}{\partial x^\beta}\right|^{-1/2}\frac{\partial f}{\partial x^\mu}\right)\ . \tag{8}$$

And the extrinsic curvature, or the second fundamental form, is defined in terms of the unit vectors as

$$\mathcal{K}^\pm_{ij} = -n_\mu\left(\frac{\partial^2x^\mu}{\partial\zeta^i\zeta^j} + \Gamma^{\mu\pm}_{\nu\rho}\frac{\partial x^\nu}{\partial\zeta^i}\frac{\partial x^\rho}{\partial\zeta^j}\right) \tag{9}$$

We substitute eq.(4) in eq.(8) to get:

$$n^\pm_\mu = \left(\mp\dot{a}, \pm\frac{\sqrt{\dot{a}^2 + f(a)}}{f(a)}, 0, 0\right)\ . \tag{10}$$

Then, we substitute eq.(10) in eq.(9) to get the components of the extrinsic curvature as

$$\mathcal{K}^{\theta\ \pm}_\theta = \mathcal{K}^{\phi\ \pm}_\phi = \pm\frac{1}{a}\sqrt{1 - \frac{2Ma^2}{(a^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a^2 + \dot{a}^2}\ ,$$
$$\mathcal{K}^{\tau\ \pm}_\tau = \pm\frac{\frac{6Ma^3}{(\mu^2+a^2)^{5/2}} - \frac{4Ma}{(\mu^2+a^2)^{3/2}} - \frac{2\Lambda a}{3} + \ddot{a}}{\sqrt{1 - \frac{2Ma^2}{(a^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a^2 + \dot{a}^2}}\ . \tag{11}$$

We use the the last results to define the surface stresses as

$$\sigma = -\frac{1}{\pi}\mathcal{K}^\theta_\theta = -\frac{1}{2\pi a}\sqrt{1 - \frac{2Ma^2}{(a^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a^2 + \dot{a}^2}\ , \tag{12}$$

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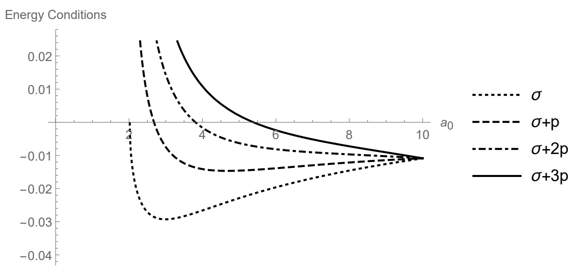
$$
\begin{aligned}
p = p_\theta = p_\phi &= \frac{1}{8\pi}\left(\mathcal{K}^\tau_\tau + \mathcal{K}^\theta_\theta\right) \\
&= \frac{3}{8\pi a}\,\frac{\frac{1}{3} - \frac{2Ma^2}{(a^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a^2 + \frac{2Ma^4}{(a^2+\mu^2)^{5/2}} + a\ddot{a} + \dot{a}^2}{\sqrt{1 - \frac{2Ma^2}{(a^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a^2 + \dot{a}^2}} .
\end{aligned} \tag{13}
$$

And for the static configuration, i.e., $\dot{a} = \ddot{a} = 0$, the surface stress become

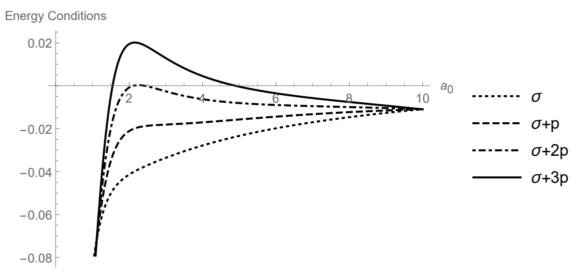
$$
\sigma_0 = -\frac{1}{2\pi a_0}\sqrt{1 - \frac{2Ma_0^2}{(a_0^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a_0^2}\ , \tag{14}
$$

$$
\begin{aligned}
p_0 &= \frac{1}{8\pi}\left(\mathcal{K}^\tau_\tau + \mathcal{K}^\theta_\theta\right) \\
&= \frac{3}{8\pi a_0}\,\frac{1 - \frac{2Ma_0^2}{(a_0^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a_0^2 + \frac{2Ma_0^4}{(a_0^2+\mu^2)^{5/2}}}{\sqrt{1 - \frac{2Ma_0^2}{(a_0^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a_0^2}} \\
&\quad - \frac{1}{4\pi a_0}\,\frac{1}{\sqrt{1 - \frac{2Ma_0^2}{(a_0^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a_0^2}} .
\end{aligned} \tag{15}
$$

From the last two equations, surface density σ_0 imposes the violation of the weak energy condition (WEC). Meanwhile, the null energy condition (NEC), $\sigma_0 + p_0 > 0$, can be maintained with no need to any *exotic* effect from the combined mass and pressure of the matter as long as $f(a_0) < \dfrac{6Ma_0^4}{(a_0^2+\mu^2)^{5/2}}$. And for the strong energy condition (SEC), $\sigma_0 + 3p_0 > 0$, it is also maintained with $f(a_0) < \dfrac{9Ma_0^4}{(\mu^2+a_0^2)^{5/2}} - \dfrac{6Ma_0^2}{(\mu^2+a_0^2)^{3/2}} - \Lambda a_0^2$.



(a) $\mu = 0.1$.



(b) $\mu = 1$.

Figure 2: The energy conditions expressed in terms of σ and p vs. the throat radius a_0 with fixed $M = 1$ and $\Lambda = 0.01$, and different values of magnetic monopole: $\mu = 0.1$ for figure.2.(a) and $\mu = 1$ for figure.2.(b).

For a BdS black hole with no radial pressure, $p_r = 0$, and a mass density that it localized at the throat $\rho = \sigma_0\ \delta(r - a_0)$, the total amount of exotic matter necessary to keep the wormhole open is

$$
\begin{aligned}
\Omega_\sigma &= \int_0^{2\pi}\int_0^{\pi}\int_{-\infty}^{+\infty}\sqrt{-g}\ \sigma_0\ \delta(r-a_0)\ dr\ d\theta\ d\phi \\
&= -2a_0\sqrt{1 - \frac{2Ma_0^2}{(a_0^2+\mu^2)^{3/2}} - \frac{\Lambda}{3}a_0^2}\ .
\end{aligned} \tag{16}
$$

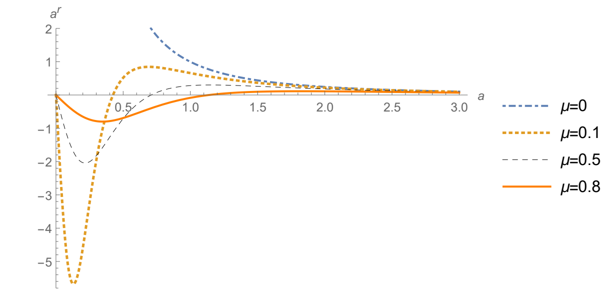
We can examine the attractive and repulsive characters of the constructed thin-shell wormhole by studying the four-acceleration $a^\mu = u^\nu \nabla_\nu u^\mu$, where $u^\mu = (1/\sqrt{f(r)}, 0, 0, 0)$. The geodesic equation of a test particle is

$$
\frac{d^2 r}{d\tau^2} = -a^r\ , \tag{17}
$$

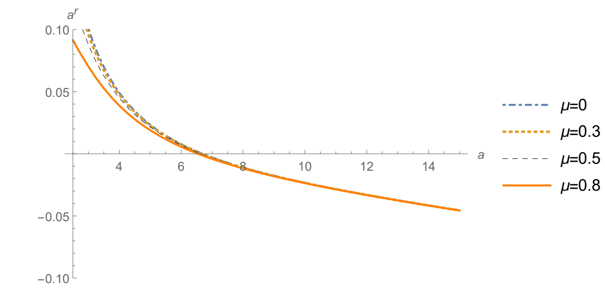
where the radial acceleration is given by

$$
a^r = \Gamma^r_{tt}\left(\frac{dt}{d\tau}\right)^2 = \frac{Mr^3}{(r^2+\mu^2)^{5/2}} - \frac{\Lambda}{3}r - \frac{2M\mu^2 r}{(r^2+\mu^2)^{5/2}}\ . \tag{18}
$$

We notice that the wormhole has attractive or repulsive nature if $a^r > 0$ or $a^r < 0$ respectively.



(a) Different attractive and repulsive behaviors of a^r at small a values for different μ values.



(b) Convergent behavior of a^r at large a values for different μ values. a^r becomes repulsive at almost the same value of a regardless the value of μ.

Figure 3: Attraction and repulsion in terms of acceleration a^r vs. the throat radius a with fixed $M = 1$ and $\Lambda = 0.01$, and different values of magnetic monopole μ.

III. LINEARIZED STABILITY ANALYSIS

The stability of the wormhole can be checked [65] by performing linear perturbation about the static configuration ($a = a_0$) for eq.(14) and eq.(15). One can easily notice that differentiating eq.(12) with respect to τ yields the continuity equation

$$\frac{d(\sigma A)}{d\tau} + p\frac{dA}{d\tau} = 0 \ , \tag{19}$$

which directly leads to

$$\sigma' = -\frac{2}{a}(\sigma + p) \ , \tag{20}$$

where $A = 4\pi a^2$ is the area of the wormhole throat, $\sigma' = \dot{\sigma}/\dot{a}$, the dot means $d/d\tau$, and the prime means d/da.
If we rearrange eq.(12), we define a potential function

$$V(a) = f(a) - 4\pi^2 a^2 \sigma^2 = -\dot{a} \ . \tag{21}$$

Then we substitute with eq.(20) in the first derivative of eq.(21) to get

$$\begin{aligned} V'(a) =& \frac{6Ma^3}{\left(\mu^2 + a^2\right)^{5/2}} - \frac{4Ma}{\left(\mu^2 + a^2\right)^{3/2}} \\ &- \frac{2\Lambda a}{3} + 8\pi^2 a\sigma(\sigma + 2p) \ . \end{aligned} \tag{22}$$

And for the second derivative of (21), we parameterize the pressure to be a function in the density $p := p(\sigma)$ [22]. Then we introduce a new parameter $\vartheta(\sigma) = dp/d\sigma$, which can be seen as the "speed of sound". And the second derivative of (21) becomes

$$\begin{aligned} V''(a) &= f''(a) - 8\pi^2 \left[2\sigma(\sigma + p)(1 + 2\vartheta) + (\sigma + 2p)^2\right] \\ &= f''(a) + \Bigg[\frac{1}{a^2}\Big(af'(a) - 2f(a)\Big)\Big(1 + 2\vartheta\Big) \\ &\qquad - \frac{1}{2}\left(\frac{f'(a)}{f(a)}\right)^2 \Bigg] \ . \end{aligned} \tag{23}$$

To linearize the model, we apply Taylor expansion to the potential function around the static point $a = a_0$ such that eq.(21) becomes

$$\begin{aligned} V(a) =& V(a_0) + (a - a_0)V'(a_0) \\ &+ \frac{1}{2}(a - a_0)^2 V''(a_0) + \mathcal{O}\left[(a - a_0)^3\right] \ . \end{aligned} \tag{24}$$

We use eq.(14) and eq.(15) to evaluate eq.(21) and eq.(22) at $a = a_0$. Therefore, we get $V(a_0) = V'(a_0) = 0$. Meanwhile eq.(23) becomes

$$\begin{aligned} V''(a_0) =& \frac{30a_0^2 M}{\left(a_0^2 + \mu^2\right)^{5/2}} - \frac{4M}{\left(a_0^2 + \mu^2\right)^{3/2}} - \frac{30a_0^4 M}{\left(a_0^2 + \mu^2\right)^{7/2}} \\ &- \frac{2\Lambda}{3} - \frac{1}{a_0^2}(1 + 2\vartheta)\left(\frac{6Ma_0^2}{\left(\mu^2 + a_0^2\right)^{5/2}} - \frac{2}{a_0^2}\right) \\ &- \frac{1}{2}\left(\frac{-\frac{4Ma_0}{\left(\mu^2+a_0^2\right)^{3/2}} + \frac{6Ma_0^3}{\left(\mu^2+a_0^2\right)^{5/2}} - \frac{2\Lambda a_0}{3}}{1 - \frac{2Ma_0^2}{\left(\mu^2+a_0^2\right)^{3/2}} - \frac{\Lambda a_0^2}{3}}\right)^2 \ . \end{aligned} \tag{25}$$

Of course we can use $(1 + 2\vartheta) = (\sigma' + 2p')/\sigma'$ to express ϑ in terms of the metric parameters M, μ, and a . But we will not as we need to study the behavior of ϑ when the throat is stable.
The concave down condition $V''(a_0) < 0$ results in provoking either expansion or contraction of the throat when any small perturbation occurs. While the convex, or the concave up, condition $V(a_0)'' > 0$ stabilizes the throat with a local minimum of $V(a_0)$ at a_0. Therefore, we solve for ϑ_0 at that local minimum to get

$$\begin{aligned} \vartheta_0 < \frac{1}{2}\Bigg\{1 - \frac{a_0^2}{\left(\frac{6Ma_0^2}{\left(\mu^2+a_0^2\right)^{5/2}} - \frac{2}{a_0^2}\right)}\Bigg[&\frac{30a_0^2 M}{\left(a_0^2 + \mu^2\right)^{5/2}} - \frac{4M}{\left(a_0^2 + \mu^2\right)^{3/2}} - \frac{30a_0^4 M}{\left(a_0^2 + \mu^2\right)^{7/2}} - \frac{2\Lambda}{3} \\ &- \frac{1}{2}\left(\frac{-\frac{4Ma_0}{\left(\mu^2+a_0^2\right)^{3/2}} + \frac{6Ma_0^3}{\left(\mu^2+a_0^2\right)^{5/2}} - \frac{2\Lambda a_0}{3}}{1 - \frac{2Ma_0^2}{\left(\mu^2+a_0^2\right)^{3/2}} - \frac{\Lambda a_0^2}{3}}\right)^2 \Bigg]\Bigg\} \ . \end{aligned} \tag{26}$$

Or

$$\vartheta_0 < \frac{1}{2}\left(1 - \frac{-\frac{4\Lambda a_0^2\left(6\mu^2 M - 3Ma_0^2 + \Lambda\left(\mu^2+a_0^2\right)^{5/2}\right)^2}{3\left(\mu^2+a_0^2\right)^2\left(6Ma_0^2 + \left(\Lambda a_0^2 - 3\right)\left(\mu^2+a_0^2\right)^{3/2}\right)^2} + \frac{30Ma_0^2}{\left(\mu^2+a_0^2\right)^{5/2}} - \frac{4M}{\left(\mu^2+a_0^2\right)^{3/2}} - \frac{30Ma_0^4}{\left(\mu^2+a_0^2\right)^{7/2}}}{\frac{6M}{\left(\mu^2+a_0^2\right)^{5/2}} - \frac{2}{a_0^4}}\right) \ . \tag{27}$$

IV. DISCUSSION

In this letter we construct Bardeen de-Sitter thin-shell wormhole. We use Visser's technique of cut-and-paste with Darmois-Israel formalism to connect two BdS regions of spacetime through a thin shell. We compare the asymptotic behavior of the metric with

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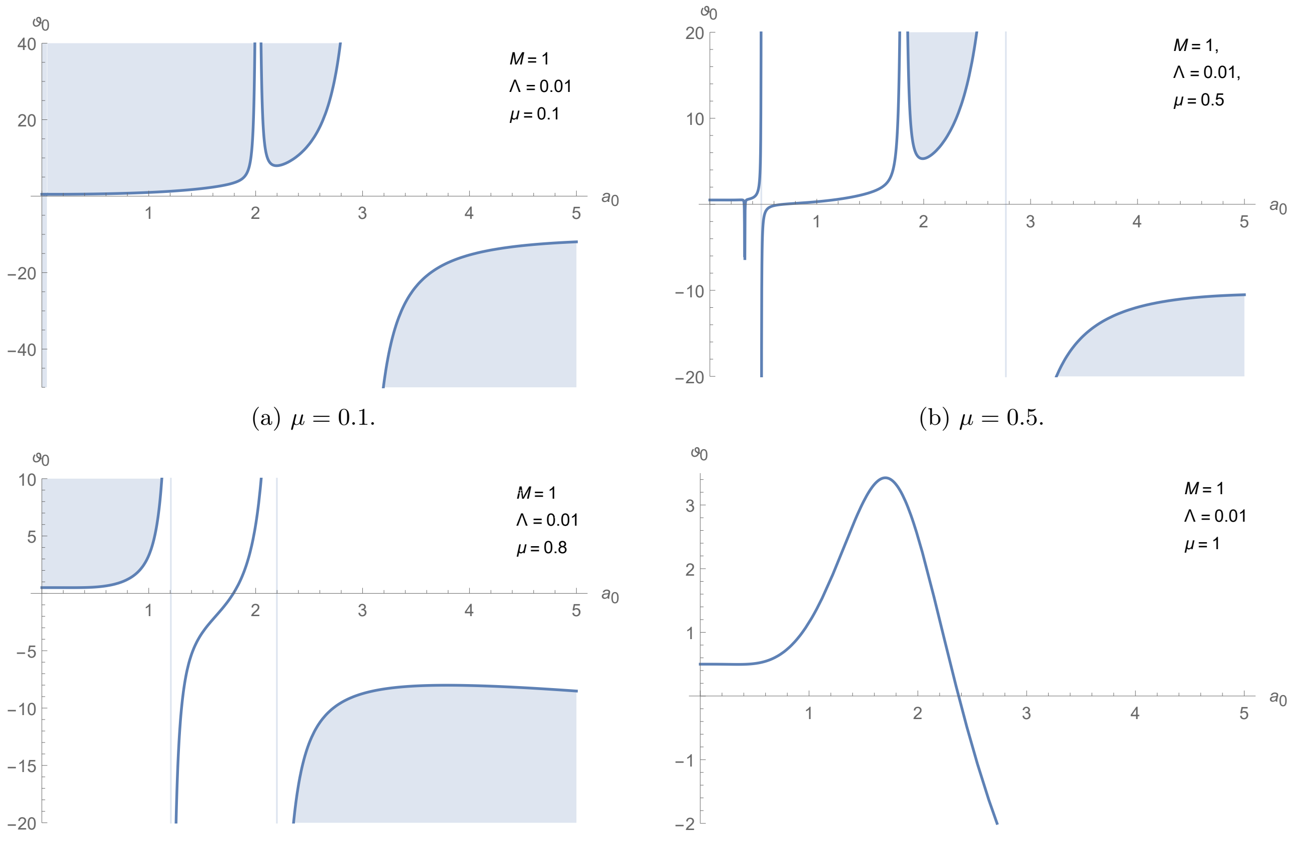
that of SdS and RNdS as in fig.(1). We also study the components of the stress-energy-momentum surface tensor using the extrinsic curvature. We find that WEC is always violated. However, both NEC and SEC can be maintained upon imposing the inequalities that relate $f(r)$ to $f'(r)$. The energy conditions are shown in fig.(2). Then, we calculate the radial acceleration to express the attractive and repulsive nature of the wormhole throat. The results are plotted in fig.(3).



(a) $\mu = 0.1$. (b) $\mu = 0.5$. (c) $\mu = 0.8$. (d) $\mu = 1$.

Figure 4: Regions of stability of the thin-shell wormhole for the Bardeen de-Sitter solution for fixed values of $M = 1$ and $\Lambda = 0.01$, and different values of μ. Stable regions are the blue shaded domains.

Also we analyze the linear stability of BdS thin-shell wormhole by studying the concavity behavior on the "speed of sound" as a function in BdS parameters: the mass, the magnetic monopoles and the cosmological constant. And we see the change in stability regions upon varying the charge of magnetic monopoles while both mass and cosmological constant are fixed. The analysis is demonstrated in fig.(4). We conclude that for a diminutive value of cosmological constant and small value of magnetic charge, relative to the amount of mass, we find different regions of stability. Once the mass is equal to the magnetic charge, we no longer have stability regions. So to keep the Bardeen de-Sitter thin shell wormhole, and for a minute value of the cosmological constant, we suggest choosing the value of magnetic charge to be always less than the value of mass.

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