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Article

Fixed-time Stabilization for uncertain chained systems with sliding mode and RBF neural network

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Abstract: In this paper, the fixed-time stabilization problem for a class of uncertain chained system is addressed by using a novel nonsingular recursive terminal sliding mode control approach. A fixed-time controller and an adaptive law are designed to guarantee the uncertain chained form system both Lyapunov stable and fixed-time convergent within the settling time. The advantage of the controller based on the sliding mode is that the settling time does not depend on the system initial state. Furthermore, we use RBF neural network to estimate the uncertainty of the system. Finally, the simulation results demonstrate the performance of the control laws.

Keywords: Fixed-time stabilization; Sliding mode control; Adaptive control; Neural network

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1. Introduction

Over the last years, the problems of nonholonomic system have been widely studied[1]. Many methods are proposed for the stabilization problems of nonholonomic systems, like adaptive control process [2], discontinuous feedback controller design method [3], and continuous time-varying state feedback controller design method [4]. With these control methods, the systems can be stabilized with well performance, but they all have some problems like the time of stabilization control is long and so on.

In recent years, with the development of automatic control technology and the increasing demand for stability, rapidity, and accuracy of control systems, controllers based on finite time have become one of the frontier topics in the field of control theory. Finite-time control is a kind of time optimal control. It has the characteristics of fast response, steady state accuracy, and good robust performance. The main design methods of finite-time controllers are homogeneous system method [5], finite-time Lyapunov function backsteping method [6], sliding mode control [7], and so on. Sliding mode control is one of the most important control methodology in particular. A novel optimal guaranteed cost sliding mode surface for constrained-input nonlinear systems with matched and unmatched disturbances was presented in [8]. J. Ernesto Solanes et al. presented a human-robot closely collaborative solution to cooperatively perform surface treatment tasks by using adaptive sliding-mode control methodology [9]. Aghababa, Mohammad Pourmahmood et al. proposed a robust nonsingular terminal sliding mode controller for synchronizing two different uncertain chaotic systems with nonlinear inputs [10]. For a class of uncertain chained form systems, Hong Y et al. designed a nonsmooth state feedback law to ensure the controlled chained form system both Lyapunov stable and finite-time convergent within any given settling time [11].

However, the explicit expressions for the bound of the finite settling time depend upon initial states of the system, which prohibit their practical applications since the knowledge of initial conditions is unavailable in advance. Therefore, Ployakov put forward an algorithm called fixed-time, which can ensure convergent time of system does not rely on system initial state but on the controller parameters [12]. This soon attracted a lot of scholars' attention. In recent years, the literature [13] considered a novel class of nonlinear consensus protocols for single-integrator multi-agent network. Zuo Z and Parsegov et al. independently provided a fixed-time stable first-order system with application to the network consensus problem [14,15]. Then, Zuo Z employed a fixed-time controller based on

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terminal sliding mode surfaces for nonlinear systems [16]. In order to design an uniform robust exact differentiator based on a second-order super-twisting algorithm with modification [17], another important application of the fixed-time stability was considered [18]. Huang W discussed a fixed-time tracking control problem for a class of nonholonomic mobile robots with integral terminal sliding mode surface [18]. The fixed-time synchronization problem of memristive fuzzy bidirectional associative memory cellular neural networks with time-varying delays was also discussed in the work [20]. For a class of nonholonomic uncertain chained form system, the work [12] put forward a feedback controller design process by using block control principles and finite-time attractivity properties of polynomial feedbacks. However, few articles discussed the fixed-time stabilization problem of nonholonomic uncertain chained form system with sliding mode methodology. Inspired by this, we consider a class of uncertain chained form system by using a new recursive sliding mode surface in this paper.

The main contributions of this paper are as follows.
(1) A new controller and an adaptive law based on fixed time are proposed for a chained system with unknown parameter and system mode uncertain function. And the uncertainty of the system is estimated by RBF neural network. (2) The convergence time does not depend on the system initial state but only on the controller parameters. (3) A recursive sliding surface mode is designed to guarantee that the uncertain chained system converges to zero within fixed time.

This paper is organized as follows. In Section 2, the problem statement is presented. The fixed-time controller and adaptive law are designed by using sliding mode methodology in Section 3. Numerical simulations are provided to illustrate the effectiveness of the proposed control strategy in Section 4. Finally, the major conclusions of the paper are summarized in Section 5.

2. Problem Statement

Consider a class of uncertain chained system as follows

$$\begin{cases} \dot{x}_0 = u_0 \\ \dot{x}_1 = x_2 u_0 \\ \dot{x}_2 = x_3 u_0 \\ \dot{x}_3 = f(x(t)) + d((x), t) + q u_1 \end{cases} \tag{1}$$

where $x_i(i = 0, 1, 2, 3) \in R$ and $u_i(i = 0, 1)$ are the states and input variables, respectively. q is unknown parameter. $f(x(t))$ and $d((x), t)$ represent the system mode uncertain function and the external disturbance respectively.

The objective of our paper is to design the control laws $u_j(j = 0, 1)$ to ensure the state variables $x_i(i = 0, 1, 2, 3)$ of system (1) reach the fixed-time equilibrium point completely with settling time T, which does not depend upon system initial state $x(0)$ but only on the controller parameters. And the controller parameters can be selected in advance. We will give the explicit function expression of settling time T in subsequent analysis.

3. Fixed-time controller design

3.1. Fixed-time control

In this section, we will design controllers to solve the fixed-time problem for system (1). In order to discuss the methodology of control laws designed conveniently, we give some basic knowledge as follows.

Definition 1 [12]. Consider the following system

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0 \tag{2}$$

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where $x \in R^n$ is the system states, $f \in R_+ \times R^n \to R^n$ is a nonlinear function. If the any solution $x(t, x_0)$ of system (2) reaches the equilibrium at a finite time, then the system (2) is said to be globally finite-time stable, i.e. $x(t, x_0) = 0, \forall t \geq T(x_0)$, where $T : R^n \to R_+ \cup 0$ is the settling-time function.

Definition 2 [12]. The origin of the system (2) is said to be globally fixed-time stable if it is globally finite-time stable and the settling-time function $T(x_0)$ is bounded, i.e. $T(x_0) \leq T_{max}$ for $\forall x_0 \in R^n$.

Lemma 1 [14]. Consider a scalar system

$$\dot{y} = -\alpha y^{\frac{m}{n}} - \beta y^{\frac{p}{h}}, \quad y(0) = y_0 \tag{3}$$

where $\alpha > 0$, $\beta > 0$, and m, n, p, h are positive odd integers satisfying $m > n$, and $p < h$. Then the equilibrium point of (3) is fixed-time stable and the settling time T is bounded by

$$T < T_{max} = \frac{1}{\alpha}\frac{n}{m-n} + \frac{1}{\beta}\frac{h}{h-p} \tag{4}$$

Moreover, if $\varepsilon \leq [h(m-n)/n(h-p)] \leq 1$, then a less conservative upper-bound estimation for the settling time can be rewritten as

$$T < T_{max} = \frac{h}{h-p}\left(\frac{1}{\sqrt{\alpha\beta}}\tan^{-1}\sqrt{\frac{\alpha}{\beta}} + \frac{1}{\alpha\varepsilon}\right) \tag{5}$$

Lemma 2[12]. Consider the system of differential equations (2). If there exists a continuous, positive-definite function V satisfies the following two conditions:

(1) $V(x) = 0$ when $x = 0$,

(2) Any solution $x(t)$ of system (2) satisfies $\dot{V}(x) \leq -\alpha V^{\frac{m}{n}}(x) - \beta V^{\frac{p}{h}}(x)$, where $\alpha > 0$, $\beta > 0$, and m, n, p, h are positive odd integers satisfying $m > n$, and $p < h$. Then the system (2) is globally fixed-time stabilization, where the settling time was defined as (4)-(5).

Remark 1: Definition 1 shows the definition of finite-time stabilization. As we have mentioned in the previous, the settling-time function $T(x_0)$ of finite-time stabilization depends on the initial states of system (2). However, The settling-time function (4) of fixed-time stabilization does not rely on the system initial states x_0 but only on the design parameters α, β, m, n, p, and h. Therefore, the convergence time can be guaranteed by any initial state.

3.2. Controller Design and Stability Analysis

For system (1), consider the x_0-subsystem

$$\dot{x}_0 = u_0 \tag{6}$$

Then, we design the fixed-time controller as follows

$$u_0 = -\alpha_0 x_0^{\frac{m_0}{n_0}} - \beta_0 x_0^{\frac{p_0}{h_0}} \tag{7}$$

where $\alpha_0 > 0$ and $\beta_0 > 0$. m_0, n_0, p_0, and h_0 are positive odd integers satisfying $m_0 > n_0$ and $p_0 < h_0$. We have the following Theorem.

Theorem 1. With the action of control law (7), the equilibrium of x_0-subsystem (6) is fixed-time stable. The settling time T_0 is bounded by

$$T_0 = \frac{1}{\alpha_0 2^{\frac{m_0+n_0}{2n_0}}}\frac{2n_0}{m_0-n_0} + \frac{1}{\beta_0 2^{\frac{p_0+h_0}{2h_0}}}\frac{2h_0}{h_0-p_0} \tag{8}$$

Proof: Choose the Lyapunov candidate

$$V(x_0) = \frac{1}{2}x_0^2 \tag{9}$$

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The derivative of Eq.(9) is

$$\dot{V}(x_0) = x_0\dot{x}_0 = x_0u_0 \tag{10}$$

By substituting the fixed time controller (7) into (10), we get

$$\begin{aligned}\dot{V}(x_0) &= x_0(-\alpha_0 x_0^{\frac{m_0}{n_0}} - \beta_0 x_0^{\frac{p_0}{h_0}}) \\ &= -\alpha_0(x_0^2)^{\frac{m_0+n_0}{2n_0}} - \beta_0(x_0^2)^{\frac{p_0+h_0}{2h_0}}\end{aligned} \tag{11}$$

then

$$\dot{V}(x_0) = -\alpha_0 2^{\frac{m_0+n_0}{2n_0}} V_0(x)^{\frac{m_0+n_0}{2n_0}} - \beta_0 2^{\frac{p_0+h_0}{2h_0}} V_0(x)^{\frac{p_0+h_0}{2h_0}} \tag{12}$$

According to the Lemma 2, x_0 converges to zero in fixed time T_0. And the settling time T_0 is only determined by designed parameters in advance. The proof is completed.

After the above proof, we can know that $x_0 = 0$ can be always hold when the subsystem (6) reaches the settling time T_0.

Now, we consider the following x_i-subsystem$(i = 1, 2, 3)$ of system (1)

$$\begin{cases}\dot{x}_1 = x_2u_0 \\ \dot{x}_2 = x_3u_0 \\ \dot{x}_3 = f(x(t)) + d((x), t) + qu_1\end{cases} \tag{13}$$

Now, we introduce RBF neural network to estimate system uncertain function $f(x)$.

$$f(x) = W^{*\top}h + \varepsilon(x) \tag{14}$$

where $W^* = [W_1, W_2, \cdots W_n]^\top$ is the optimal weight vector of Neural Network. $n(n > 1)$ is the number of the hidden layer neuron. $h = [h_1, h_2, \cdots h_n]^\top$ is the radial basis vector function, and $h_i \quad (i = 1, 2, \cdots n)$ are Gauss function defined as follows:

$$h_i = \exp(-\frac{\|X - c_i\|^2}{2b_i}), \quad i = 1, 2, \cdots n \tag{15}$$

where, $X \in \Omega_\infty \subset R_q$ is the input variable. c_i and b_i are the central positions and base width parameters of Gaussian function respectively. $\varepsilon(x)$ is the network approximation error which satisfies $|d((x), t) + \varepsilon(x)| \le \varphi$, φ is a any positive constant.

The system (13) can be rewritten as

$$\begin{cases}\dot{x}_1 = x_2u_0 \\ \dot{x}_2 = x_3u_0 \\ \dot{x}_3 = W^{*\top}h + \varepsilon(x) + d((x), t) + qu_1\end{cases} \tag{16}$$

For the problem of fixed-time stabilization of x_i-subsystem (16)$(i = 1, 2, 3)$, a new nonsingular recursive terminal sliding mode surface is proposed

$$\begin{cases}s_1 = x_1 \\ s_2 = \dot{s}_1 + \alpha_1 s_1^{\frac{m_1}{n_1}} + \beta_1 s_1^{\frac{p_1}{h_1}} \\ s_3 = \dot{s}_2 + \alpha_2 s_2^{\frac{m_2}{n_2}} + \beta_1 s_2^{\frac{p_2}{h_2}}\end{cases} \tag{17}$$

where s_1, s_2, and s_3 are the terminal sliding mode surfaces. $\alpha_i > 0$, $\beta_i > 0$ are constants. m_i, n_i, p_i, and h_i $(i = 1, 2)$ are positive odd integers satisfying $m_1 > n_1$, $m_2 > n_2$, $p_1 < h_1$, and $p_2 < h_2$. $\tilde{q}$ is the system parameter error defined as $\tilde{q} = q - \hat{q}$, where $\hat{q}$ is the estimation of q. Hence, the feedback control law u_1 and the adaptive law $\dot{\hat{q}}$ can be designed as follows

$$u_1 = \frac{1}{\hat{q}} \frac{-\Gamma - \alpha_3 s_3^{\frac{m_3}{n_3}} - \beta_3 s_3^{\frac{p_3}{h_3}}}{u_0} - \frac{\hat{W}^\top h(x) + \hat{\varphi}}{\hat{q}} \tag{18}$$

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$$\dot{\hat{q}} \;=\; \tfrac{s_3}{\hat{q}}[-\Gamma-\alpha_3\dot{s}_3^{\frac{m_3}{n_3}}-\beta_3\dot{s}_3^{\frac{p_3}{h_3}}+u_0(\hat{W}^\top h(x)+\hat{\varphi})] \tag{19}$$

where

$$\begin{aligned}\Gamma \;=\;& 2x_3\dot{u}_0+x_2\ddot{u}_0+\alpha_1\tfrac{m_1}{n_1}(\tfrac{m_1}{n_1}-1)s_1^{(\frac{m_1}{n_1}-2)}\ddot{s}_1\\ &+\beta_1\tfrac{p_1}{h_1}(\tfrac{p_1}{h_1}-1)s_1^{(\frac{p_1}{h_1}-2)}\ddot{s}_1+\beta_1\tfrac{p_1}{h_1}(\tfrac{p_1}{h_1}-1)\dot{s}_1^2\\ &+\dot{s}_2\alpha_2\tfrac{m_2}{n_2}s_2^{\frac{m_2}{n_2}-1}+\dot{s}_2\beta_b\tfrac{p_b}{h_2}s_2^{\frac{p_2}{h_2}-1}+\alpha_1\tfrac{m_1}{n_1}(\tfrac{m_1}{n_1}-1)\dot{s}_1^2\end{aligned} \tag{20}$$

Then, we design neural network adaptive law as follows

$$\dot{\hat{W}} = s_3\Lambda h u_0 \tag{21}$$

where Λ is adaptive gain matrix. $\hat{W}$ is the estimate of W^*, and $\tilde{W}=W^*-\hat{W}$. $\hat{\varphi}$ is the unknown external disturbance and the estimation of upper bound of neural network error, and $\tilde{\varphi}=\varphi-\hat{\varphi}$. The parameter adaptive law is designed as

$$\dot{\hat{\varphi}} = s_3 u_0 \tag{22}$$

Then, we have the following Theorem.

Theorem 2: With the methodology of nonsingular recursive terminal sliding mode surface, system (16) is stabilized to the equilibrium position at fixed time under the action of controllers (18), (19), (21), and (22). Then, the settling time is

$$T_{11} < T_{max} = T_{12} + T_{13} \tag{23}$$

where

$$T_{12} = \frac{1}{\alpha_3 2^{\frac{m_3+n_3}{2n_3}}}\frac{2n_3}{m_3-n_3}+\frac{1}{\beta_3 2^{\frac{p_3+h_3}{2h_3}}}\frac{2h_3}{h_3-p_3}$$

and

$$T_{13} = \tfrac{1}{\alpha_1}\tfrac{n_1}{m_1-n_1}+\tfrac{1}{\beta_1}\tfrac{h_1}{h_1-p_1}$$

Proof: For the sliding mode (17), taking the derivatives of s_1, s_2, and s_3, we have

$$\dot{s}_1 = \dot{x}_1 \tag{24}$$

By substituting (24) into the terminal sliding mode surface s_2, we can get the derivative of s_2 as follows

$$\dot{s}_2 \;=\; x_3u_0+x_2\dot{u}_0+\alpha_1\tfrac{m_1}{n_1}s_1^{\frac{m_1}{n_1}-1}x_2u_0+\beta_1\tfrac{p_1}{h_1}s_1^{\frac{p_1}{h_1}-1}x_2u_0 \tag{25}$$

Then, substituting (25) into s_3, we have the derivative of s_3 as follows

$$\dot{s}_3 \;=\; u_0(W^{*\top}h(x)+\varepsilon(x)+d((x),t)+qu_1)+\Gamma \tag{26}$$

Choose the Lyapunov candidate as

$$V_1(s) = \tfrac{1}{2}s_3^2+\tfrac{1}{2}\tilde{W}^\top\Lambda^{-1}\tilde{W}+\tfrac{1}{2}\tilde{q}^2+\tfrac{1}{2}\tilde{\varphi}^2 \tag{27}$$

The derivative of above equation is

$$\begin{aligned}\dot{V}_1(s) \;=\;& s_3\dot{s}_3-\tilde{W}^\top\Lambda^{-1}\dot{\hat{W}}+\tilde{\varphi}\dot{\hat{\varphi}}+\tilde{q}\dot{\hat{q}}\\ =\;& s_3[u_0(W^{*\top}h(x)+\varepsilon(x)+d((x),t)+qu_1)+\Gamma]\\ &-\tilde{W}^\top\Lambda^{-1}\dot{\hat{W}}-\tilde{\varphi}\dot{\hat{\varphi}}-\tilde{q}\dot{\hat{q}}\end{aligned} \tag{28}$$

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By substituting the fixed-time state feedback controller u_1 into the Eq. (28), we can obtain

$$\begin{aligned}
\dot{V}_1(s) &= s_3[u_0(W^{*\top}h+\varepsilon(x)+d((x),t)+q(\tfrac{1}{\hat{q}}\tfrac{-\Gamma-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}}}{u_0} \\
&\quad -\tfrac{\hat{W}^\top h(x)+\hat{\varphi}}{\hat{q}})+\Gamma]-\tilde{W}^\top\Lambda^{-1}\dot{\hat{W}}-\tilde{\varphi}\dot{\hat{\varphi}}-\tilde{q}\dot{\hat{q}} \\
&= s_3[(u_0(W^{*\top}h+\varepsilon(x)+d((x),t)+\tfrac{(\tilde{q}+\hat{q})}{\hat{q}}(\tfrac{-\Gamma-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}}}{u_0} \\
&\quad -\hat{W}^\top h-\hat{\varphi}))+\Gamma]-\tilde{W}^\top\Lambda^{-1}\dot{\hat{W}}-\tilde{\varphi}\dot{\hat{\varphi}}-\tilde{q}\dot{\hat{q}} \\
&= s_3(-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}})+\tilde{q}[\tfrac{s_3}{\hat{q}}(-\Gamma-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}} \\
&\quad +\tfrac{s_3}{\hat{q}}u_0(\hat{W}^\top h+\hat{\varphi}))-\dot{\hat{q}}]+s_3u_0\tilde{W}h \\
&\quad -\tilde{W}\Lambda^{-1}\dot{\hat{W}}+s_3u_0(\varepsilon(x)+d((x),t))-s_3u_0\hat{\varphi}-\tilde{\varphi}\dot{\hat{\varphi}} \\
&= s_3(-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}})+\tilde{q}[\tfrac{s_3}{\hat{q}}(-\Gamma-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}} \\
&\quad +\tfrac{s_3}{\hat{q}}u_0(\hat{W}^\top h+\hat{\varphi}))-\dot{\hat{q}}]+\tilde{W}(s_3u_0h-\Lambda^{-1}\dot{\hat{W}}) \\
&\quad +s_3u_0(\varepsilon(x)+d((x),t))-s_3u_0\hat{\varphi}-\tilde{\varphi}\dot{\hat{\varphi}}
\end{aligned} \tag{29}$$

By using the designed adaptive laws $\dot{\hat{q}}$ and $\dot{\hat{W}}$, we have

$$\dot{V}_1(s) = -\alpha_3 s_3^{\frac{m_3+n_3}{n_3}}-\beta_3 s_3^{\frac{p_3+h_3}{h_3}}+s_3u_0(\varepsilon(x)+d((x),t))-s_3u_0\hat{\varphi}-\tilde{\varphi}\dot{\hat{\varphi}} \tag{30}$$

According to $|d((x),t)+\varepsilon(x)|\leq\varphi$ and the Eq.(22), we can get

$$\begin{aligned}
\dot{V}_1(s) &\leq -\alpha_3 s_3^{\frac{m_3+n_3}{n_3}}-\beta_3 s_3^{\frac{p_3+h_3}{h_3}}+s_3u_0\varphi-s_3u_0\hat{\varphi}-\tilde{\varphi}\dot{\hat{\varphi}} \\
&= -\alpha_3 s_3^{\frac{m_3+n_3}{n_3}}-\beta_3 s_3^{\frac{p_3+h_3}{h_3}}+s_3u_0\tilde{\varphi}-\tilde{\varphi}\dot{\hat{\varphi}} \\
&= -\alpha_3 s_3^{\frac{m_3+n_3}{n_3}}-\beta_3 s_3^{\frac{p_3+h_3}{h_3}}+\tilde{\varphi}(s_3u_0-\dot{\hat{\varphi}}) \\
&= -\alpha_3 s_3^{\frac{m_3+n_3}{n_3}}-\beta_3 s_3^{\frac{p_3+h_3}{h_3}}
\end{aligned} \tag{31}$$

It is easy to see that the terminal sliding mode surface s_3 can be stabilized to zero since $V(s)\geq 0$ and $\dot{V}_1(s)\leq 0$ as $t\to+\infty$. Next, we prove the sliding mode surface s_3 is fixed-time stable within given settling time T_{12}.

Based on the above analysis, we can rewrite $\dot{s}_3$ as follows

$$\dot{s}_3 \leq u_0(\hat{W}^\top h(x)+\hat{\varphi}+\hat{q}u_1)+\Gamma \tag{32}$$

Then we choose the following Lyapunov candidate

$$V_2(s)=\tfrac{1}{2}s_3^2 \tag{33}$$

The derivative of above equation is

$$\begin{aligned}
\dot{V}_2(s) &= s_3\dot{s_3} \\
&\leq s_3[u_0(\hat{W}^\top h+\hat{\varphi}+\hat{q}(\tfrac{1}{\hat{q}}\tfrac{-\Gamma-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}}}{u_0} \\
&\quad -\tfrac{\hat{W}^\top h+\hat{\varphi}}{\hat{q}}))+\Gamma+\alpha_3 s_3^{\frac{m_3}{n_3}}+\beta_3 s_3^{\frac{p_3}{h_3}}] \\
&= s_3(-\alpha_3 s_3^{\frac{m_3}{n_3}}-\beta_3 s_3^{\frac{p_3}{h_3}}) \\
&= -\alpha_3 2^{\frac{m_3+n_3}{2n_3}}V_2^{\frac{m_3+n_3}{2n_3}}-\beta_3 2^{\frac{p_3+h_3}{2h_3}}V_2^{\frac{p_3+h_3}{2h_3}}
\end{aligned} \tag{34}$$

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Using Lemma 2, we can obtain that s_3 converge to zero in fixed time T_{12}. In other words, the sliding surface $s_3 = 0$ can always be kept when the system reaches the settling time T_{12}. Substituting $s_3 = 0$ into the nonsingular recursive terminal sliding mode surface (17), we have

$$\dot{s}_2 = -\alpha_2 s_2^{\frac{m_2}{n_2}} - \beta_2 s_2^{\frac{p_2}{h_2}} \tag{35}$$

According to Lemma 1, s_2 stays at fixed-time equilibrium position as $t \to T_{12}$. By substituting $s_2 = 0$ into the nonsingular recursive terminal sliding mode surface (17), we have

$$\dot{s}_1 = -\alpha_1 s_1^{\frac{m_1}{n_1}} - \beta_1 s_1^{\frac{p_1}{h_1}} \tag{36}$$

By Lemma 1, we have the conclusion that $s_1 = 0$ in fixed time T_{13} given in (23). It is obvious that $s_1 = x_1$ from Eq.(24). Therefore, we can obtain the following state easily

$$x_1 = \alpha_1 x_1^{\frac{m_1}{n_1}} - \beta_1 x_1^{\frac{p_1}{h_1}}$$

By Lemma 1, x_1 is fixed-time stabilization within settling T_{13} distinctly. Based on the above analysis, we can know that $s_1 = 0$, $s_2 = 0$, and $s_3 = 0$ as $t \geq T_{12}$. Thus, according to the relationship between sliding surfaces and system states, we can get

$$x_2 = \frac{1}{u_0}(s_2 - \alpha_1 x_1^{\frac{m_1}{n_1}} - \beta_1 x_1^{\frac{p_1}{h_1}}) \tag{37}$$

and

$$\begin{aligned} x_3 \;=\; & \tfrac{1}{u_0}(s_3 - \alpha_2 s_2^{\frac{m_2}{n_2}} - \beta_2 x_2^{\frac{p_2}{h_2}} - x_2 \dot{u}_0 + \alpha_1 \tfrac{m_1}{n_1} s_1^{\frac{m_1}{n_1}-1} x_2 u_0 \\ & + \beta_1 \tfrac{p_1}{h_1} s_1^{\frac{p_1}{h_1}-1} x_2 u_0) \end{aligned} \tag{38}$$

Taking $x_1 = 0$ and $s_2 = 0$ to Eq.(37), we find that x_2 converges to zero within settling time T_{13}. Moreover, taking $s_1 = 0$, $s_2 = 0$, $s_3 = 0$, $x_1 = 0$, and $x_2 = 0$ to Eq.(38), x_3 will be converged to zero within settling T_{13} either. Thus, we have the conclusion that x_1, x_2, and x_3 converge to zero. This implies that system (13) is stabilized to the origin equilibrium by fixed time under the action of controller (18). Hence, the proof of Theorem 2 is completed.

Remark 2: According to Theorems 2, the terminal sliding mode surfaces (17) are stabilized with settling time T_{12}. Once the terminal sliding mode surfaces reach to the zero, the states of system (13) converge to zero within the settling time T_{13}. Hence, the system can be fixed-time stabilization within settling time $T_{11} \leq T_{12} + T_{13}$.

Remark 3: It is worth mentioning that, the form of system (1) is derived from [11]. In paper [11], Yiguang Hong discussed the finite-time stabilization of a class of uncertain chained form system. A novel switching control methodology with help of homogeneity, time-rescaling, and Lyapunov function method were proposed for it. Then, with the action of the feedback controller, system can be finite-time stabilization within settling time with well performance. And the settling time were bounded by: $T_s \leq \frac{|x_0(0)|^{1-\alpha}}{q_0^{min} K_0(1-\alpha)}$ and $T_x \leq -\frac{2^{\frac{2+k+2r_2}{1+r2}}(1+r_2)}{\gamma l k} V_n(x(0))^{\frac{-k}{1+r_2}}$ where $x_0(0)$ is the initial state of subsystem $x_0 = u_0$, and $x(0)$ is the initial state of another subsystem $x = [x_1, x_2, \dots x_n]$. It is obvious that the finite settling-time T_s and T_x depend upon initial states $x_0(0)$ and $x(0)$, which prohibit their practical applications since the knowledge of initial conditions is very large or unavailable in advance. Compared with it, the fixed-time controller designed by us can guarantee that the four-order chained system converges to zero within settling time T_0 and T_{11} which only determined by controller parameters.

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4. Simulation

In this section, the simulation results are presented to demonstrate the effectiveness of the fixed-time controller designed for the system (1). Here, we define the system unknown parameter $q = 1$ and system mode uncertain function $f(x(t)) = \sin(2x) + \cos(x)$ respectively. We take the initial states of system (1) as $[x_0(0), x_1(0), x_3(0), x_4(0)]^\top = [0.2, -0.3, 0.4, 0.6]^\top$. Choose the parameters of controller and nonsingular recursive integral terminal sliding mode surface as $\alpha_0 = 1$, $\beta_0 = 1$, $\alpha_1 = 1.1$, $\beta_1 = 1.1$, $\alpha_2 = 1.2$, $\beta_2 = 1.2$, $\alpha_3 = 1.3$, $\beta_3 = 1.3$, $\alpha_4 = 1.4$, $\beta_4 = 1.4$, $m_0 = 5$, $n_0 = 3$, $p_0 = 7$, $h_0 = 9$, $m_1 = 5$, $n_1 = 3$, $p_1 = 7$, $h_1 = 9$, $m_2 = 13$, $n_2 = 11$, $p_2 = 15$, $h_2 = 17$, $m_3 = 21$, $n_3 = 19$, $p_3 = 23$, and $h_3 = 25$. The simulation results are plotted as Figs.1-4.

As shown in Fig.1, the curve represents the simulation result of state variable x_0 of subsystem (6). It is obvious that the state x_0 can converge to zero at fixed time with the action of u_0, which is represented in Fig.2. Then, the states variables x_1, x_2, and x_3 of the system (13) with respect to time are reveled in Fig.3. The Fig.3 can illustrate that the states variables x_1, x_2, and x_3 stabilized to zero in fixed-time with the action of controller u_1 obviously. And the controller u_1 with respect to time is shown in Fig.4. Thus, based on Figs.1-4, we can know that the controllers u_0 and u_1 can guarantee the states of system (1) stabilization within fixed time.

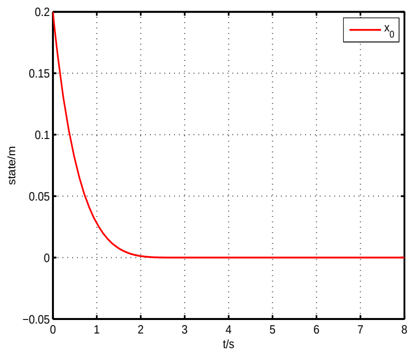

Figure 1. System state x_0 with respect to time.

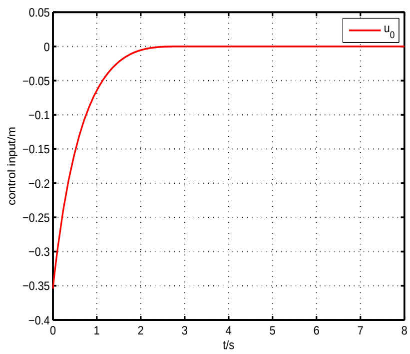

Figure 2. Controller u_0 with respect to time.

5. Conclusion

In this paper, the fixed-time problem of a class of four-order chined system with unknown parameter, system mode uncertainty, and external disturbance is solved. By proposing a new nonsingular recursive terminal sliding mode surface, a fixed-time feedback controller is designed to guarantee the system states converging to zero within fixed-time. The settling time is only determined by controller parameters which can obtained in advance. Finally, the simulation results demonstrate the performance of the control laws.

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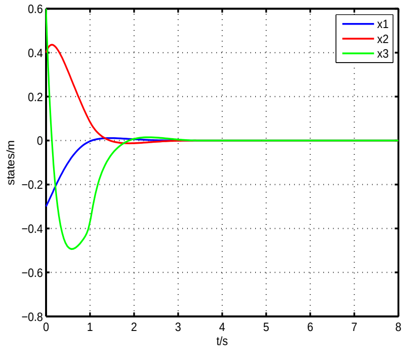

Figure 3. System state x_1, x_2, and x_3 with respect to time.

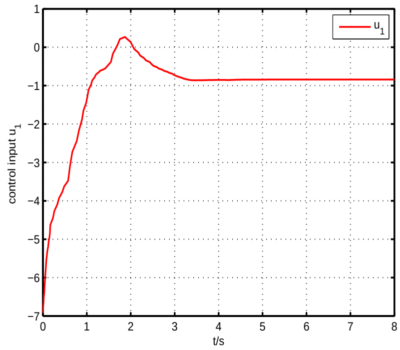

Figure 4. Controller u_1 with respect to time.

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Guo Pengfei contributed significantly to analysis, manuscript preparation and wrote the manuscript;
Wang Xi and Jin Zengke helped perform the analysis with constructive discussions

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	linear dichroism

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Sample Availability: Samples of the compounds ...... are available from the authors.