

1 Article

2 Characterization and Correction of the Geometric 3 Errors in Using Confocal Microscope for Extended 4 Topography Measurement. Part II: Experimental 5 Study and Uncertainty Evaluation

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13 **Abstract:** This paper presents the experimental implementations of the mathematical models and
14 algorithms developed in Part I. Two experiments are carried out. The first experiment aims at the
15 determinations of the correction coefficients of the mathematical model. The dot grid target is
16 measured and the measurement data are processed by our developed and validated algorithms
17 introduced in Part I. The values of the coefficients are indicated and analysed. Uncertainties are
18 evaluated with implementation of the Monte Carlo method. The second experiment measures a
19 different area of the dot grid target. The measurement results are corrected according to the
20 coefficients determined in the first experiment. The mean residual between the measured points
21 and their corresponding certified values reduced 29.6% after the correction. The sum of squared
22 errors reduced 47.7%. The methods and the algorithms for raw data processing, such as data
23 partition, fittings of dots' centres, K-means clustering, etc., are the same for both two experiments.
24 The experimental results demonstrate that our method for the correction of the errors produced by
25 the movement of lateral stage of confocal microscope is meaningful and practicable.

26 **Keywords:** Geometric errors; rigid body kinematics; lateral stage errors; Imaging Confocal
27 Microscope; MCM uncertainty evaluation; dot grid target.

28 1. Introduction

29 The increasing demands for manufacturing accuracies and quality control due to the rapid
30 development of nanotechnology, ultraprecision machining, micro- and nano-fabrications, etc. [1, 2]
31 and the requirements for precision in surface finishing in different technologies such as additive
32 manufacturing [3], mechanical parts with structured surfaces [4], etc., require the use of
33 increasingly sophisticated measurement systems and measurement traceability from a metrological
34 point of view.

35 Calibration provides wide range information of the microscope performances. The ever
36 increasing demand for improved surface quality and tighter geometric tolerances leads to the
37 augmentations in the investigations of manufacturing technologies [5]. Measurements using optical
38 microscopes are often affected by common path noise, disturbance in light source and ambient
39 lighting, etc., causing measurement defects and outliers [6, 7] as well as attracting investigations on
40 noisy data processing [8]. The need for standardization is becoming ever greater as the range of
41 capturing 3D information of microscope techniques continues to increase [5, 9]. For optical confocal
42 microscopes, the Z-calibrations at nm-levels are typically good while the X- and Y- accuracies are
43 often left without further notice than resolution limits of the optics [10, 11]. Among the

44 investigations of the lateral calibrations, lots of studies focus on its optical system [**Error! Reference**
45 **source not found.**]. For example, H. Ni, *et al.* proposed a new method to achieve structured
46 detection through using spatial light modulator, which modulates the Airy disk amplitude
47 distribution according to the detection function in collection arm [13]; B. Wang, *et al.* presented
48 confocal microscopy with structured detection in a coherent imaging process to achieve a higher
49 resolution with a comparably large pinhole [14]. However, the systematic geometric errors which
50 adversely affect the relative position and orientation between measuring probes and measurands
51 are usually neglected [15, 16]. B. Daemi, *et al.* designed a comprehensive verification test by using a
52 high precision metrology method based on subpixel resolution image analysis [17]. Calibration of
53 confocal microscopes usually relies on traceable standard artefacts, which are commonly made up
54 of regular patterns [10].

55 This paper carries out the experimental studies based on the kinematic modelling and
56 algorithms for the correction of the geometric errors developed in Part I [18]. Sections 2 and 3
57 introduce the methodologies for experiments and uncertainty evaluation individually. Section 4
58 presents the experiment on a dot grid target for correction coefficients determination and their
59 corresponding uncertainty evaluation. Section 5 implements an experiment and corrects the
60 measured data with determined coefficients, comparing the residuals with respect to certified
61 values before and after corrections. Section 6 presents the conclusions. Acknowledgements and
62 references are indicated in the final section.

63 2. Methodology for the experimental study

64 The experimental study aims at: First, determining the error correction coefficients, i.e.
65 a_1, a_2, \dots, f_3 , defined in the kinematic geometric error correction model developed in Part I; Second,
66 applying the determined parameters of coefficients and the correction mathematical model for new
67 measurement data calibration. The dot grid target standard artefact is implemented as measurand
68 of the experiments. By comparing the residuals of measured points and corrected points with
69 respect to the certified values, the practicality and significance of our developed models and
70 algorithms for lateral stage error calibration can be observed.

71 Two experiments are carried out with our Imaging Confocal Microscope, which type is Leica
72 Confocal Dual Core 3D Measuring Microscope (Leica DCM-3D), at the "*Laboratorio de Investigación*
73 *de Materiales de Interés Tecnológico*" (LIMIT) of the Technical University of Madrid. The first
74 experiment is to measure the dot grid target standard for the determination of the correction
75 coefficients. The second experiment measures another area of the dot grid standard, processing the
76 measurement data with the same developed and validated algorithms implemented in the first
77 experiment. The purpose of this experiment is to observe whether the corrected data improved
78 comparing with the raw measurement data. As the second experiment uses the same measurement
79 parameters, data processing algorithms and procedures, and measure different area of the same dot
80 grid standard, this comparison means a lot as other factors which might influence the results could
81 be excluded majorly, such as the uncertainties or inaccuracies generated by algorithms of cylinder
82 separation, centre fitting, movement scope of the lateral stage, etc.

83 3. Methodology and procedures for uncertainty estimation

84 A statement of measurement is complete only if it provides an estimate of the quantity
85 concerned as well as a quantitative evaluation of the estimate's reliability, i.e., the associated
86 uncertainty [**Error! Reference source not found.**]. Accompany measurement results by quantitative
87 statements about their accuracy is very important particularly when the result is part of a
88 measurement chain tracking back to national standards or when decisions about product
89 specifications are taken [**Error! Reference source not found.**].

90 The document issued by BIPM, *Guide to the Expression of Uncertainty in Measurement* (GUM)
91 [21] provides a method and procedure for the evaluation and expression of measurement

92 uncertainties [22]. This method is termed as GUM uncertainty framework in supplement 1 and
 93 supplement 2 (GUM-S1 and GUM-S2) [23, 24] and other bibliographies [19, 20]. The GUM
 94 uncertainty framework has two main limitations [**Error! Reference source not found.**, 25]. The first
 95 limitation is lacking generality of the procedure to obtain an interval to contain the values of the
 96 measurand with a stipulated coverage probability [25]. In the GUM uncertainty framework the way
 97 a coverage interval is constructed to contain values of the measurand with a stipulated coverage
 98 probability is approximate [22]. The second limitation is that insufficient guidance is given for the
 99 multivariate case in which there is more than one measurand, namely, more than one output
 100 quantity [22, 25]. In order to address these limitations, Working Group 1 of the Joint Committee for
 101 Guides in Metrology (JCGM) has produced two specific guidance documents, namely GUM-S1 and
 102 GUM-S2 [23, 24] , on Monte Carlo method (MCM) respectively for uncertainty evaluation and
 103 extensions to any number of measurand (output quantity) [22].

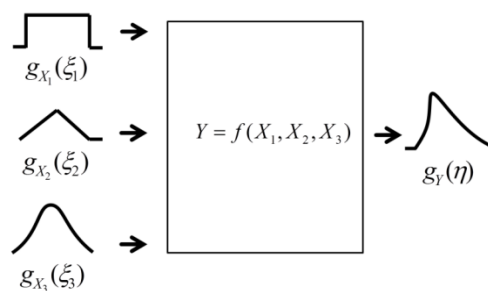
104 MCM provides a general approach to obtain a numerical representation \mathbf{G} of the distribution
 105 function $G_Y(\boldsymbol{\eta})$ for \mathbf{Y} . The heart of the approach is making repeated draws from the Probability
 106 Density Functions (PDFs) for the input variables X_i (or joint PDF for \mathbf{X}) and the evaluation of the
 107 output quantity. Assignment of the PDFs for the input variables will be dependent on each
 108 experiment and be explained in the part of experimental studies. The same case applies to the
 109 evaluation of the output quantity. The distribution function $G_Y(\boldsymbol{\eta})$ encodes all information known
 110 about the output quantity \mathbf{Y} . Properties of \mathbf{Y} can be approximated using $G_Y(\boldsymbol{\eta})$. The quality of
 111 \mathbf{G} depends on the number of draws made. The symbol \mathbf{y} represent the output measurement
 112 results. It is determined by the input measurement results \mathbf{x}_i :

$$113 \quad \mathbf{y} = f(\mathbf{x}_1, \dots, \mathbf{x}_N) \quad (1)$$

114 The relationship between the PDF of output measurement results and input measurement
 115 results is:

$$116 \quad \mathbf{g}_Y(\boldsymbol{\eta}) = \int \mathbf{g}_{X_1, \dots, X_N}(\xi_1, \dots, \xi_N) \times \delta[\boldsymbol{\eta} - f(\xi_1, \dots, \xi_N)] d\xi_1 \dots d\xi_N \quad (2)$$

117 where $\boldsymbol{\eta}$ denotes possible values that can be distributed to \mathbf{Y} , $\delta[\dots]$ denotes the Dirac delta
 118 function.



119
 120 **Figure 1.** Illustration of the propagation of distributions for input quantities and the obtained output
 121 quantities [**Error! Reference source not found.**].

122 Figure 1 makes an illustration of the propagation of distributions for input and output
 123 quantities. The expectation of the output quantities can be obtained by its PDF $\mathbf{g}_Y(\boldsymbol{\eta})$ as:

$$124 \quad E(Y_i) = \int_{-\infty}^{\infty} \eta_i g_{Y_i}(\eta_i) d\eta_i \quad (3)$$

125 The variance of the output quantities can be obtained by its PDF $\mathbf{g}_Y(\boldsymbol{\eta})$ as:

$$126 \quad V(Y_i) = \int_{-\infty}^{\infty} [\eta_i - E(Y_i)]^2 g_{Y_i}(\eta_i) d\eta_i \quad (4)$$

127 The covariance of the output quantities can be obtained by its PDF $\mathbf{g}_Y(\boldsymbol{\eta})$ as:

$$128 \quad Cov(Y_i, Y_j) = Cov(Y_j, Y_i) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\eta_i - E(Y_i)][\eta_j - E(Y_j)] g_{Y_i, Y_j}(\eta_i, \eta_j) d\eta_i d\eta_j \quad (5)$$

129 where $g_{Y_i, Y_j}(\eta_i, \eta_j)$ is the joint PDF for the two random variables Y_i, Y_j .

130 The correlation of the output quantities can be obtained by its PDF $\mathbf{g}_Y(\boldsymbol{\eta})$ as:

$$131 \quad Corr(Y_i, Y_j) = Corr(Y_j, Y_i) = \frac{Cov(Y_i, Y_j)}{\sqrt{V(Y_i)V(Y_j)}} \quad (6)$$

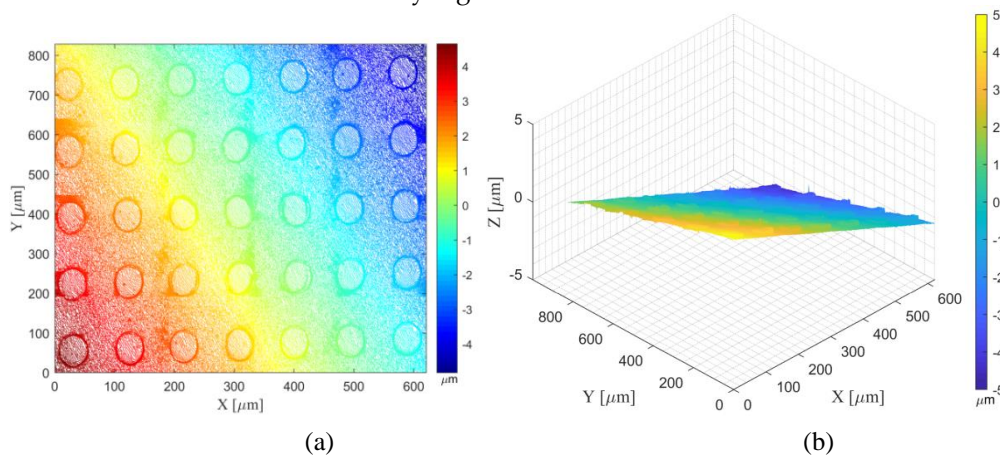
132 4. Experiment for determination and uncertainty evaluation of the error correction coefficients

133 The first experiment is the measurement of the standard artefact of the dot grid target. This
 134 experiment aims at obtaining the parameters of the coefficients of the kinematic rigid body errors.
 135 Uncertainties of the obtained correction coefficients are also evaluated.

136 4.1. Determination of error coefficients

137 The standard artefact of the dot grid target is introduced in Part I of this paper. This artefact is
 138 measured in an environment under controlled temperature of 20 ± 1 °C, using the Leica DCM-3D
 139 Confocal Microscopy at the Metrology Laboratory of the Technical University of Madrid. A
 140 magnification objective of 50X was used, with a numerical aperture of 0.90. The acquisition
 141 parameter of measurement area was defined as the topography stitching measurement, with a 4×4
 142 extended topographies, covering an area of 0.828×0.621 mm². The parameter of overlapping area is
 143 25% and the correlation takes XYZ option. The level of resolution is 1, and the measured extended
 144 topographies contain 2496×1872 pixels.

145 After the measurement, the confocal system generated a file with suffix name '.dat', containing
 146 three vectors, which are values of X-, Y-, and Z-coordinates. Data of this file is imported and
 147 analysed by our developed algorithms. The raw measurement data is shown in Figure 2, which is
 148 an inclined surface with some outliers. This surface is aligned to be parallel with the X- and Y-
 149 coordinate plane using our developed surface rotation methods introduced in Part I. The aligned
 150 surface is shown in Figure 3. The distribution of the values of the X-, Y-, and Z-coordinates of the
 151 surface after rotation is indicated by Figure 4.

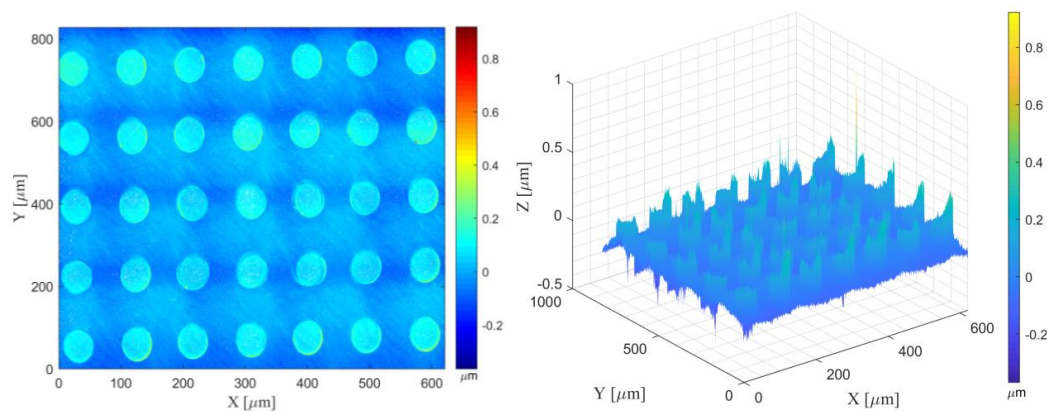


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Figure 2. Surface reconstruction by the raw measurement data: (a) 2D contour; (b) 3D surf.



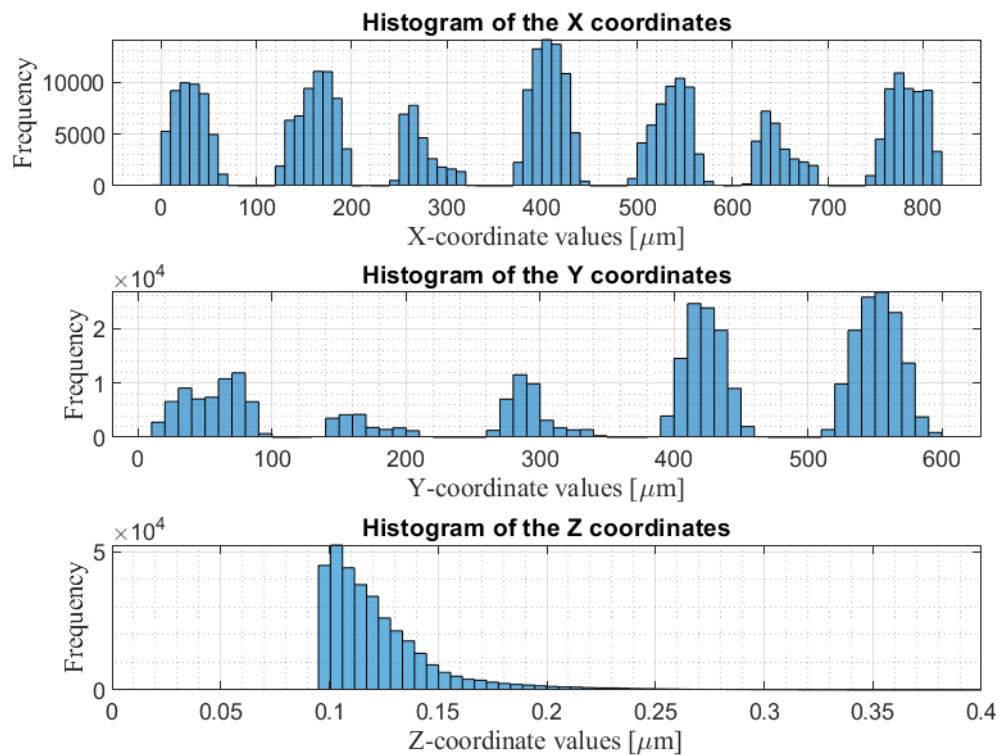
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Figure 3. Surface reconstruction after rotation of the raw measured surface: (a) 2D contour; (b) 3D surf.



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Figure 4. Histograms of the X-, Y-, and Z-coordinates of the aligned surface.

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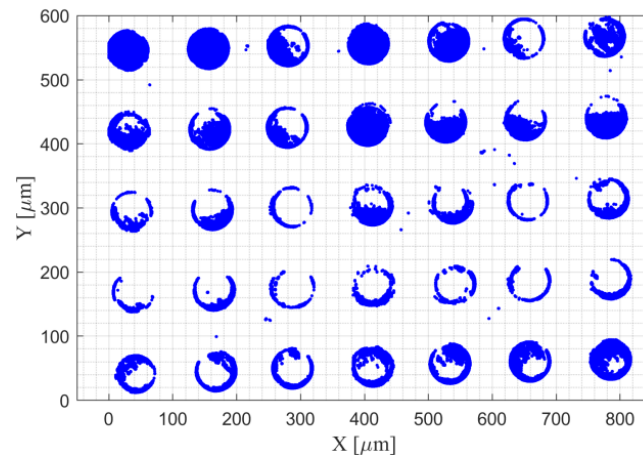
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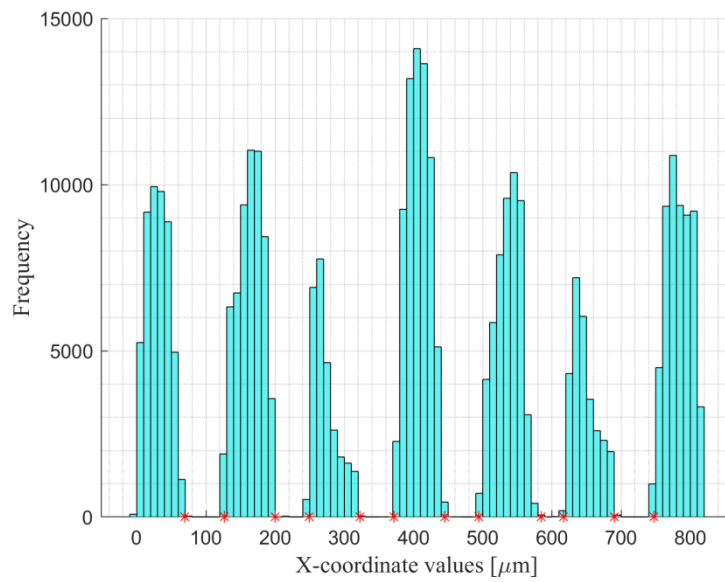
After rotation, the data is separated as surface plane and cylinders. The surface reconstruction of the data of cylinders is shown in Figure 5. It is obvious that this data has many outliers. Those outliers are detected and deleted by the method introduced in our previous work [Error! Reference source not found.]. The distribution of the measurement values of the X- and Y- coordinates as well as the threshold for outlier detections are shown in Figure 6 and Figure 7 individually. The surface reconstruction by the data of cylinders with outliers removed is shown in Figure 8.



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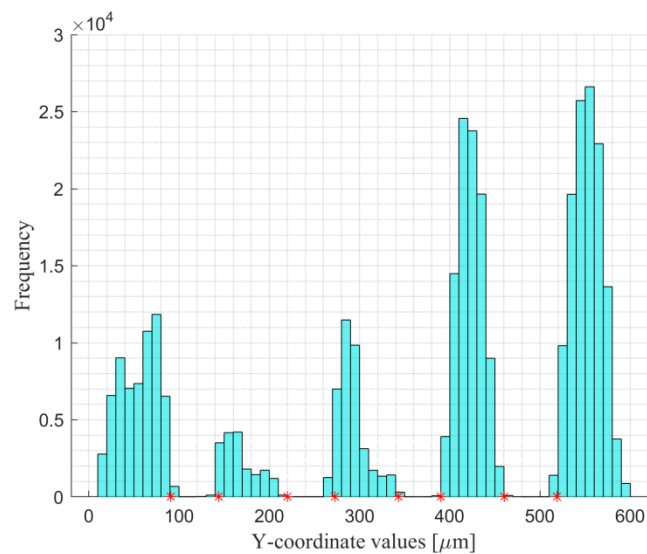
Figure 5. Surface reconstruction of the data of cylinders.



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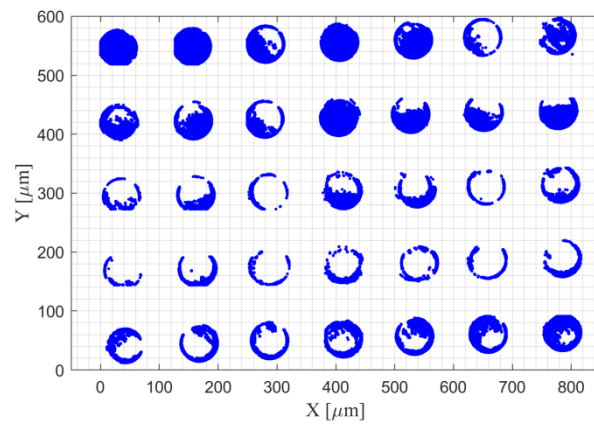
Figure 6. Distribution of the X-coordinate values and the thresholds for outlier detection.



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Figure 7. Distribution of the Y-coordinates values and the thresholds for outlier detection.



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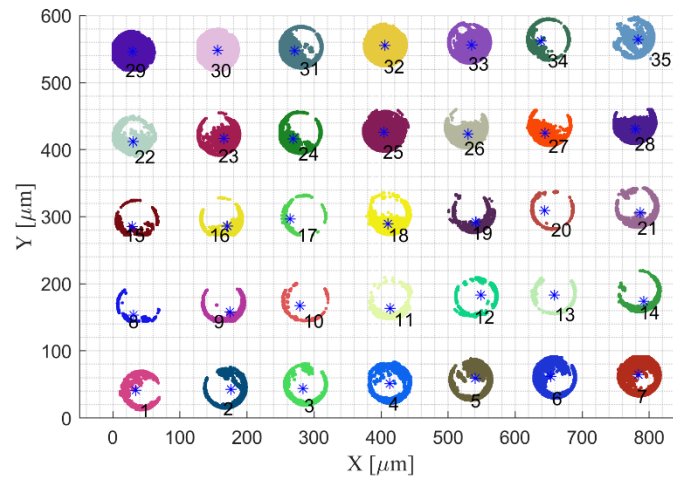
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Figure 8. Surface reconstruction of the data of cylinders with outliers removed.

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The separated cylinders are shown in Figure 9. They are denoted by different colours and numbers.

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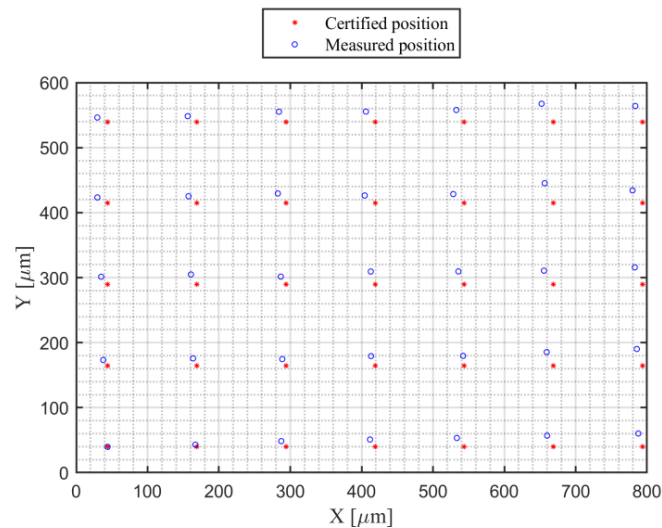
Figure 9. Separation of the cylinders.

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Those individual point cloud of cylinders are processed by our introduced algorithms. Their centres are fitted and shown in Figure 10, in comparison with their corresponding certified positions.

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Figure 10. Plot of the cylinders' centres and their corresponding certified positions.

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It can be found that the measured cylinder centres shown in Figure 10 are all distorted in one direction. This might be caused by the location of the measurand, as it is impossible to locate the measurand parallel with the X-axis. Thereof, those measured centres are first aligned to be parallel with the X-axis. Then they are calibrated to the certified points using our developed mathematical models and algorithms. From this, the results of the coefficients defined in the mathematical model, i.e. Equations (4) and (5) in Part I, can be obtained. The results are indicated in Table 1.

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Table 1. Parameters of the coefficients of the mathematical models for lateral geometric error correction.

Parameter	Value	Parameter	Value	Parameter	Value
a_1	7.38×10^{-3}	c_1	4.36×10^{-6}	e_1	6.00×10^{-6}
a_2	-1.46×10^{-6}	c_2	1.05×10^{-8}	e_2	-6.24×10^{-9}
a_3	3.19×10^{-11}	c_3	-1.20×10^{-12}	e_3	9.69×10^{-14}
b_1	1.05×10^{-3}	d_1	-7.22×10^{-3}	f_1	-2.01×10^{-2}
b_2	-1.11×10^{-5}	d_2	4.00×10^{-6}	f_2	1.25×10^{-5}
b_3	-3.15×10^{-11}	d_3	-1.01×10^{-10}	f_3	-3.10×10^{-9}

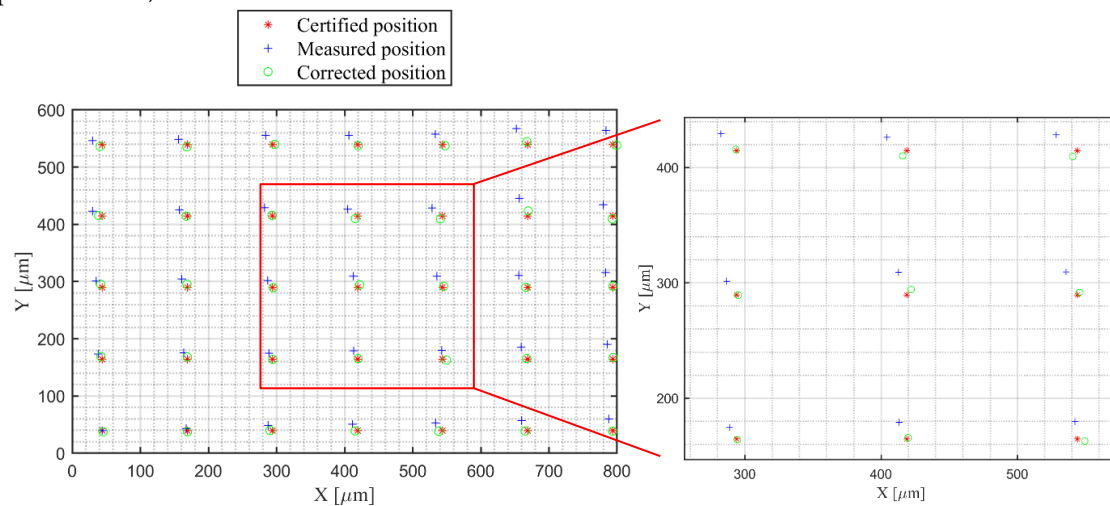
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With those obtained parameters of the coefficients, the corrected points are calculated according to Equations (4) and (5) in Part I. Figure 11 shows the corresponding positions of certified, measured, and corrected points. The results of the mean errors, maximum errors, sum of squared errors, and standard deviations of the errors are indicated in Table 2.



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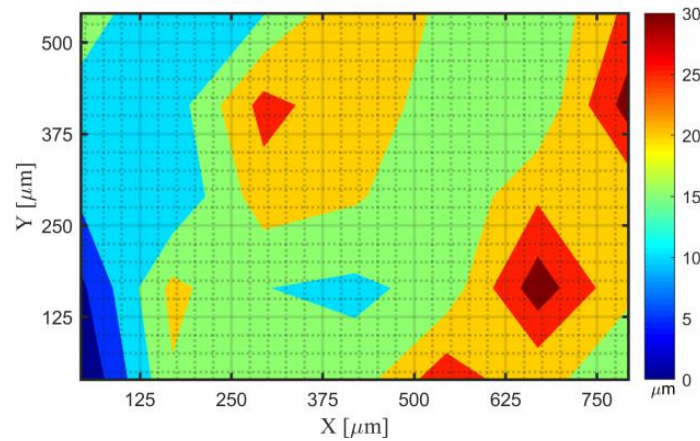
Figure 11. Comparison of the positions of certified, measured and corrected points.

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Table 2. Errors with respect to the certified positions before correction and after correction.

Data types	Mean Error [μm]	Maximum error [μm]	Sum of squared errors [μm^2]	Standard deviations of the errors [μm]
Measured points	18.3	33.1	1.4×10^4	7.0
Corrected points	3.8	8.9	628.1	1.9

199 The Euclidean residuals of each point are plotted by contours, as shown in Figure 12 and 13,
 200 which is the contour of the Euclidean residuals of the measured and corrected points with respect
 201 to the certified values.

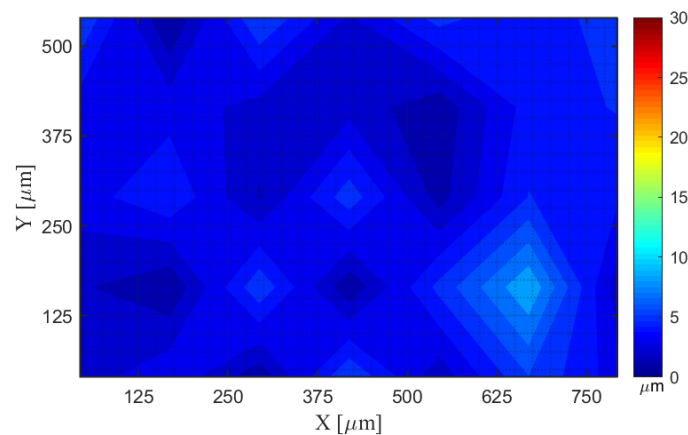


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Figure 12. Contour of the Euclidean residuals between each certified and measured points.

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Figure 13. Contour of the Euclidean residuals between each certified and corrected points.

207 It can be concluded that the determined coefficients for kinematic geometric error correction
 208 works very well in this measurement.

209 4.2. Uncertainty evaluation

210 Uncertainty evaluation of the geometric error coefficients is based on the algorithms
 211 introduced in Section 3. The heart of the approach is making repeated draws from the PDFs for the
 212 input variables X_i (or joint PDF for \mathbf{X}) and the evaluation of the output quantity. Here defines the
 213 determinations of the number of repeated draws, namely, the number of simulation trials, the PDFs
 214 for the input variables, and the evaluation of the output quantity.

215 According to GUM-S2 [**Error! Reference source not found.**] the main stages of uncertainty
 216 evaluation constitute formulation, propagation, and summarizing.

217 I) The first stage of formulation includes:

- 218 a) Define the output quantity, namely, the geometric error correction coefficients
 219 $\mathbf{C}\mathbf{c} = (a_1, \dots, g_3)$;
 220 b) Determine the input quantity upon which $\mathbf{C}\mathbf{c}$ depends, namely, the measurement results
 221 (\mathbf{x}, \mathbf{y}) and their corresponded certified values $(\mathbf{P}_x, \mathbf{P}_y)$;
 222 c) Develop a measurement function f or measurement model relating the input and output
 223 quantities, namely, Equations (4) and (5);
 224 d) On the basis of available knowledge assign PDFs to the components of the input quantities.
 225 As indicated by Table 2 in Part I, the certified values $(\mathbf{P}_x, \mathbf{P}_y)$ follow a rectangular
 226 distribution $U(-1,1)$ μm . As there are no more information about the sources of
 227 uncertainties for $(\mathbf{P}_x, \mathbf{P}_y)$ neither information for the measured values of X- and Y-
 228 coordinates (\mathbf{x}, \mathbf{y}) , here does not assign more uncertainties to the input quantities, for not
 229 introducing unnecessary uncertainties.

230 II) The second stage is propagation:

231 Propagate the PDFs for the components of input quantities through the model to obtain the
 232 (joint) PDF for the output quantity.

233 III) The final step is summarizing:

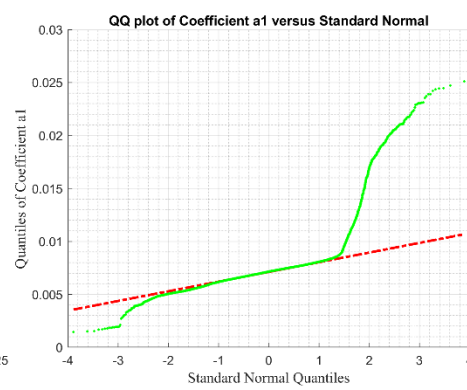
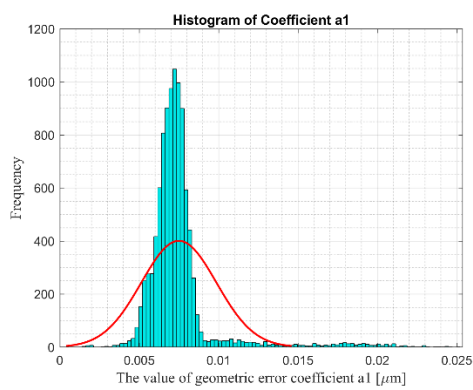
234 Use the PDF for the output quantity to obtain the expectation of the output quantity, the
 235 uncertainty matrix, also named covariance matrix, associated with the expectation of the output
 236 quantity, and a coverage region containing the output quantity with a specified probability p
 237 (the coverage probability).

238 The simulation was repeated for 1×10^4 times. The mean values, expanded uncertainties ($k = 2$
 239), lower and upper boundaries for a 95% coverage interval are indicated in Table 3. Distributions for
 240 the output quantities are shown in Figure 14. As it can be observed, non-symmetric distributions
 241 and not assimilable to normal distributions are obtained.

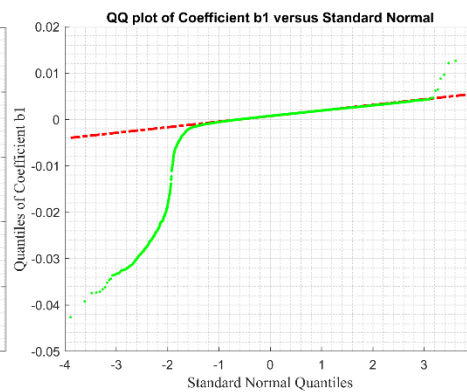
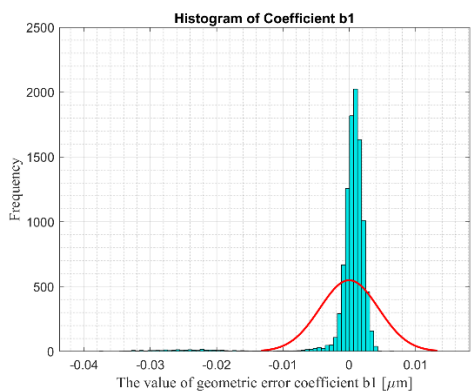
242 **Table 3.** Parameters of the coefficients of the mathematical models for lateral geometric error
 243 correction.

Parameter	Mean Value	Expanded Uncertainty	95% coverage Interval	
			Lower Boundary	Upper Boundary
a_1	7.50×10^{-3}	4.74×10^{-3}	5.07×10^{-3}	1.61×10^{-2}
a_2	-2.13×10^{-6}	5.41×10^{-6}	-1.28×10^{-5}	-6.12×10^{-8}
a_3	2.00×10^{-10}	7.14×10^{-10}	-1.35×10^{-10}	1.21×10^{-9}
b_1	-5.91×10^{-6}	8.85×10^{-3}	-1.62×10^{-2}	2.99×10^{-3}
b_2	-9.91×10^{-6}	9.19×10^{-6}	-1.50×10^{-5}	2.08×10^{-6}
b_3	-6.86×10^{-12}	2.36×10^{-9}	-8.94×10^{-10}	2.67×10^{-9}

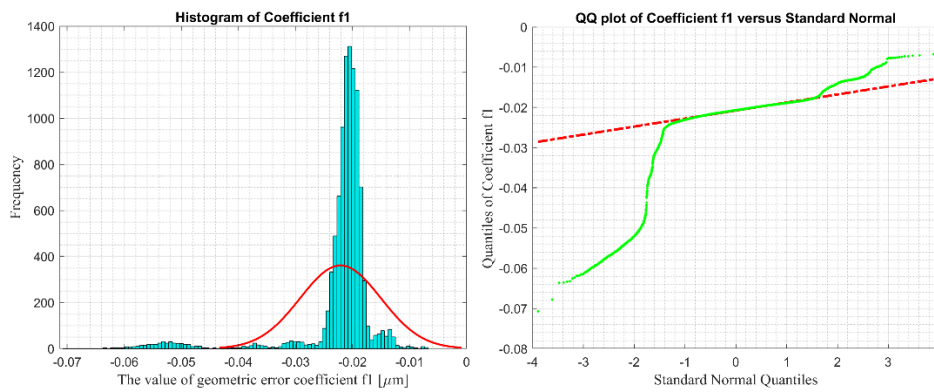
c_1	3.41×10^{-6}	7.89×10^{-6}	-6.22×10^{-6}	7.41×10^{-6}
c_2	1.26×10^{-8}	1.86×10^{-8}	2.21×10^{-10}	3.86×10^{-8}
c_3	-8.16×10^{-13}	3.44×10^{-12}	-5.74×10^{-12}	1.56×10^{-13}
d_1	-6.57×10^{-3}	4.97×10^{-3}	-8.53×10^{-3}	3.52×10^{-3}
d_2	3.93×10^{-6}	1.94×10^{-6}	1.39×10^{-6}	5.37×10^{-6}
d_3	-5.88×10^{-11}	3.78×10^{-10}	-4.59×10^{-10}	1.53×10^{-10}
e_1	5.89×10^{-6}	2.95×10^{-6}	2.00×10^{-6}	8.04×10^{-6}
e_2	-6.57×10^{-9}	6.71×10^{-9}	-1.44×10^{-8}	-4.88×10^{-11}
e_3	-2.34×10^{-15}	3.80×10^{-13}	-2.36×10^{-13}	3.30×10^{-13}
f_1	-2.21×10^{-2}	1.42×10^{-2}	-5.15×10^{-2}	-1.43×10^{-2}
f_2	1.40×10^{-5}	1.73×10^{-5}	6.71×10^{-6}	4.98×10^{-5}
f_3	-2.94×10^{-9}	2.92×10^{-9}	-5.37×10^{-9}	-1.14×10^{-11}



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Figure 14. Distributions of the uncertainties of some geometric error correction coefficients.

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5. Experimental study of the applications of determined coefficients

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This section aims at verifying the applicability of the determined error correction coefficients. The residuals of measured and corrected points with respect to certified points are compared.

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This experiment measures the dot grid target standard used in Section 4. The measured area of this standard is different from that in Section 4. All the environmental and operational parameters are the same with that in Section 4. The measurement data are processed using the same algorithms and procedures until the fitted cylinder centres are approximated parallel with the X-axis. Then the fitted centres are corrected by the error correction coefficients determined. By comparing the mean residuals, the sum of all squared residuals, and the standard deviation of residuals of measured points and corrected points with respect to certified positions, the effectiveness of the calculated coefficients and our model can be observed.

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Measurement data are processed using our developed and validated algorithms. The raw measurement data with form removed is shown in Figure 15, implementing the form removal method presented in the pair publication Part I [18] Section 4.1. The cylinders are separated from the base with our developed algorithms, with results demonstrated in Figure 16. Figure 16 (a) shows the initial separated cylinders. Figure 16 (b) manifests the cylinders with outliers removed and with clusters classified. Figure 17 indicates the histograms of the three coordinates of the cylinders.

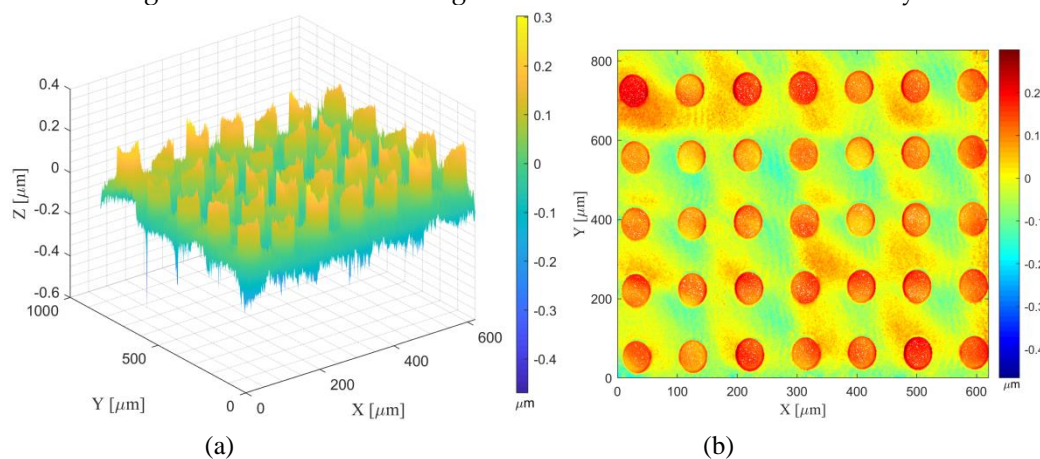
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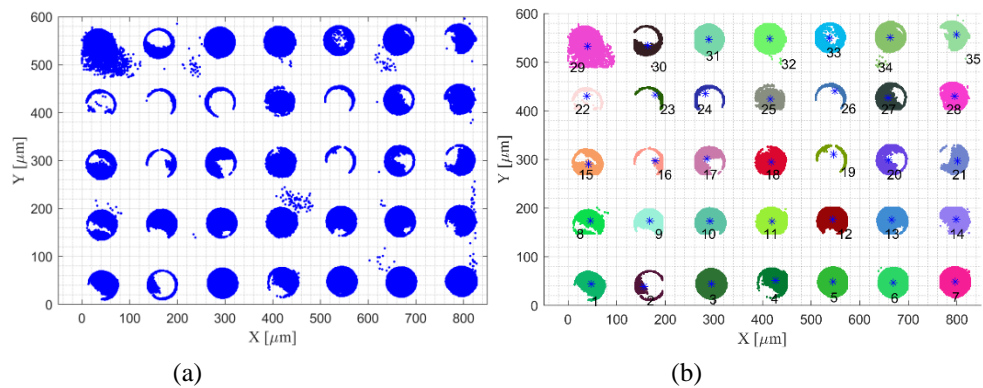
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Figure 15. The measured surface with form removed: (a) 3D reconstruction; (b) 2D reconstruction.

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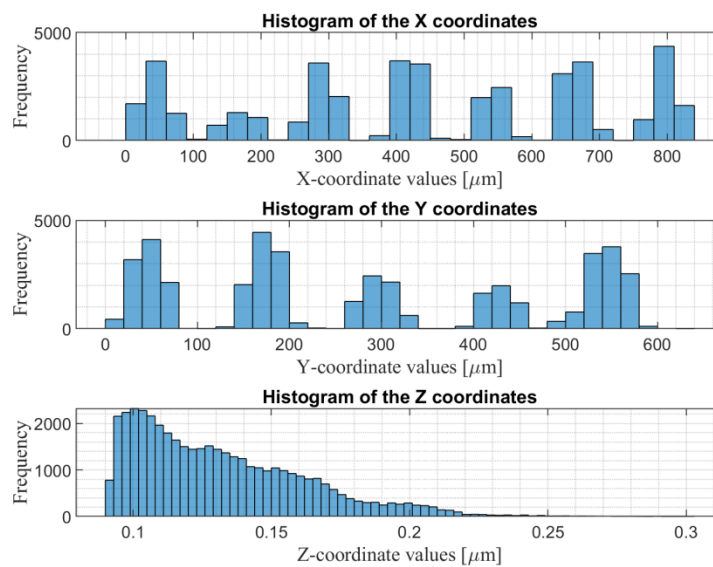
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Figure 16. Separation of the cylinders: (a) The separated cylinders from the base; (b) Separate cylinders into individual clusters.



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Figure 17. Histogram of the three coordinates of the separated cylinders.

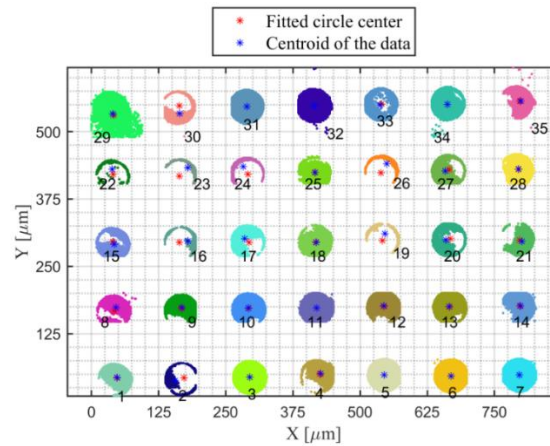
277 The separated clusters are fitted for their centres. The coordinate values of the fitted cylinder
278 centres are shown in Table 4. Figure 18 manifests the fitted cylinder centres as well as the centroids
279 of each cluster data.

280

Table 4. The raw coordinate values of the fitted cylinder centres.

Cluster N°	X-Coordinate [μm]	Y-Coordinate [μm]	Cluster N°	X-Coordinate [μm]	Y-Coordinate [μm]
1	47.3	42.6	19	541.8	298.1
2	172.3	42.9	20	667.7	301.3
3	294.8	44.0	21	796.8	298.4
4	425.1	49.5	22	40.2	420.6
5	544.9	48.1	23	163.3	417.8
6	669.2	46.6	24	291.1	421.0
7	796.4	48.3	25	415.9	424.6
8	42.4	166.5	26	538.9	423.9
9	167.7	171.9	27	666.1	430.0
10	292.6	172.3	28	794.6	430.3
11	419.3	172.6	29	41.2	530.7
12	544.1	175.2	30	163.3	548.3

13	666.6	174.6	31	289.5	546.6
14	797.0	176.0	32	415.0	548.2
15	40.1	295.9	33	539.9	551.7
16	163.4	294.6	34	662.6	550.7
17	293.0	294.5	35	796.9	550.8
18	417.9	295.3			



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Figure 18. Comparison of the fitted circle centre and the centroid of the data.

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As it can be observed in Figure 18, the cluster numbered 29 has too many outliers. Here only choose the first 28 clusters for kinematic geometric error correction. The correction employs the mathematical model Equations (4) and (5) as well as our calculated error coefficients. Before correction, the data are aligned to be parallel with the X-coordinate as much as possible. Table 5 manifests both the aligned measurement data and the corrected data.

288

Table 5. The aligned and corrected measurement data.

Cluster N°	Rotated measurement data		Corrected data by error coefficients	
	X-coordinate [μm]	Y-coordinate [μm]	X-coordinate [μm]	Y-coordinate [μm]
1	47.2	42.8	47.5	41.6
2	172.2	43.4	173.4	41.7
3	294.7	44.8	296.7	42.7
4	425.1	50.6	427.7	48.2
5	544.8	49.5	548.2	46.8
6	669.0	48.3	673.1	45.2
7	796.2	50.3	800.9	46.4
8	42.0	166.6	42.1	163.2
9	167.2	172.3	168.2	168.3
10	292.1	173.1	293.9	168.9
11	418.9	173.6	421.3	169.3
12	543.7	176.6	546.8	172.1
13	666.2	176.3	669.9	171.5
14	796.6	178.1	801.0	172.6
15	39.3	296.0	39.1	290.6
16	162.6	295.0	163.2	289.3
17	292.3	296.4	293.7	289.6
18	417.2	299.4	419.3	290.7
19	541.0	303.0	543.8	293.7
20	666.9	300.4	670.4	297.2
21	796.1	420.7	800.2	294.2

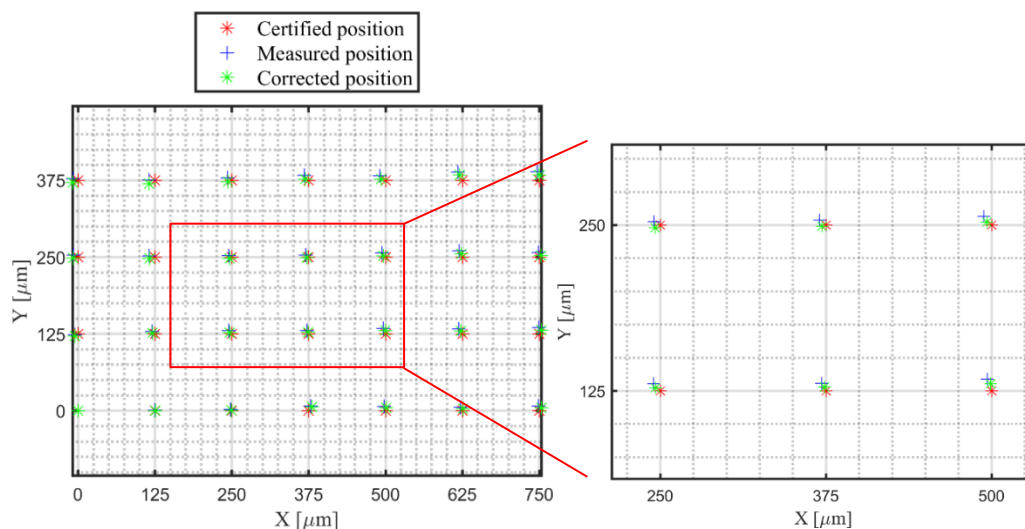
22	39.1	418.2	38.6	413.4
23	162.2	418.2	162.5	410.9
24	290.0	421.7	291.1	414.6
25	414.8	425.7	416.7	418.8
26	537.9	425.3	540.4	418.6
27	665.1	431.7	668.3	425.1
28	793.5	432.2	797.4	425.5

289 Both the aligned and the corrected data are adjusted to a begging of (0,0). The results of the
 290 measured and corrected positions are compared with the certified positions in Table 6. Figure 19
 291 illustrates the measured, corrected, and the certified positions. The mean error, the maximum error,
 292 the sum of the squared errors, and the standard deviations of the errors are indicated in Table 7.
 293 The mean error/residual between the measured positions and the certified positions is 8.1 μm , while
 294 the mean error/residual between corrected positions and the certified positions is 5.7 μm , improved
 295 29.6%. The maximum error between the measured positions and the certified positions is 15.6 μm ,
 296 while the maximum error between corrected positions and the certified positions is 11.5 μm ,
 297 reduced 26.3%. The sum of squared errors reduced from 2173.3 μm^2 to 1136.2 μm^2 , which is 47.7%. It
 298 can be observed that all four types of errors are much smaller after correction with the error
 299 coefficients.

300

Table 6. Comparison of the certified, measured, and corrected positions.

Cluster N°	Certified position		Measured position (alignment rotated)		Corrected position	
	X-coordinate [μm]	Y-coordinate [μm]	X-coordinate [μm]	Y-coordinate [μm]	X-coordinate [μm]	Y-coordinate [μm]
1	0.0	0.0	0.0	0.0	0.0	0.0
2	125.0	0.0	125.1	0.6	125.9	0.1
3	250.0	0.0	247.6	2.0	249.2	1.1
4	375.0	0.0	377.8	7.8	380.2	6.6
5	500.0	0.0	497.7	6.7	500.7	5.2
6	625.0	0.0	621.9	5.5	625.6	3.6
7	750.0	0.0	749.1	7.5	753.4	4.9
8	0.0	125.0	-5.1	123.8	-5.4	121.6
9	125.0	125.0	120.1	129.5	120.7	126.8
10	250.0	125.0	245.0	130.3	246.4	127.3
11	375.0	125.0	371.7	130.9	373.8	127.8
12	500.0	125.0	496.5	133.8	499.3	130.5
13	625.0	125.0	619.0	133.5	622.4	129.9
14	750.0	125.0	749.4	135.2	753.5	131.0
15	0.0	250.0	-7.8	253.2	-8.4	249.0
16	125.0	250.0	115.5	252.2	115.7	247.8
17	250.0	250.0	245.1	252.5	246.2	248.0
18	375.0	250.0	370.1	253.6	371.8	249.1
19	500.0	250.0	493.9	256.6	496.3	252.2
20	625.0	250.0	619.8	260.2	622.9	255.6
21	750.0	250.0	748.9	257.7	752.7	252.6
22	0.0	375.0	-8.0	377.9	-8.9	371.8
23	125.0	375.0	115.1	375.4	115.1	369.3
24	250.0	375.0	242.9	378.9	243.7	373.0
25	375.0	375.0	367.7	382.9	369.2	377.2
26	500.0	375.0	490.7	382.5	492.9	377.0
27	625.0	375.0	617.9	388.9	620.8	383.5
28	750.0	375.0	746.4	389.5	749.9	383.9



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Figure 19. Comparison of the certified, measured, and corrected positions.

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Table 7. Errors with respect to the certified positions before correction and after correction.

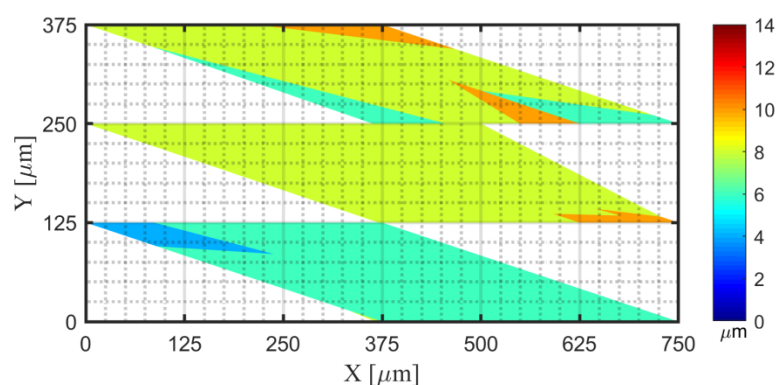
Data types	Mean Error [μm]	Maximum error [μm]	Sum of squared errors [μm^2]	Standard deviations of the errors [μm]
Measured points	8.1	15.6	2173.3	3.5
Corrected points	5.7	11.5	1136.2	2.8

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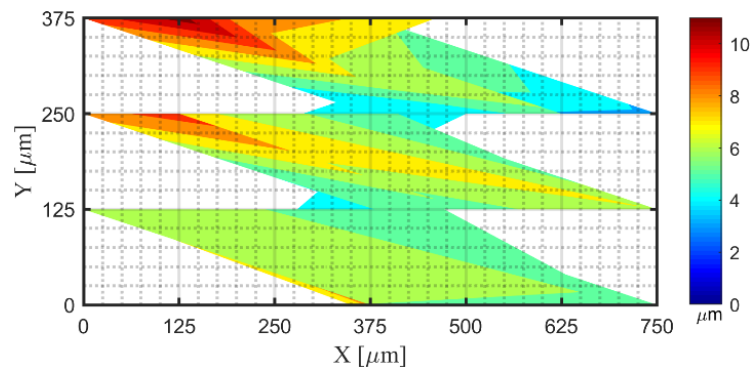
Figure 20 and Figure 21 manifests the contour of the mean errors of the measured data and the corrected data individually. Figure 22 compares the error vectors from the certified positions to the measured positions and the vectors from the certified positions to the corrected positions.



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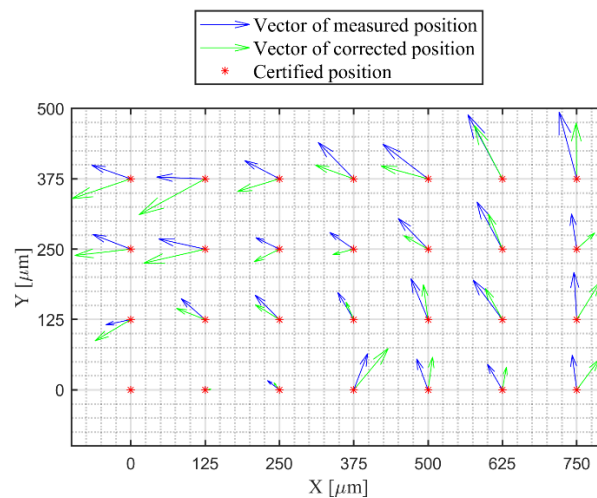
Figure 20. Contour of the Euclidean residuals between each certified and measured points.



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Figure 21. Contour of the Euclidean residuals between each certified and corrected points.



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Figure 22. Comparison of the error vectors between the measured and corrected points.

313 According to the above results, it can be found that the errors/residuals between the corrected
 314 positions and the certified positions are much smaller than the errors/residuals between the
 315 measured positions and certified positions. This indicates that our method for the X- and Y-
 316 coordinate calibration/correction is effective and useful.

317 6. Conclusions and future work

318 This paper implemented two experiments for the illustration and the verification of our
 319 proposed method for the correction of the kinematic geometric errors produced by the movement
 320 of the lateral stage of confocal microscopes. The experimental results indicate that the mean
 321 residual reduced 29.6%, the maximum error reduced 26.3%, and the sum of squared errors reduced
 322 47.7%.

323 The first experiment measured the dot grid targets with extended topography. After
 324 processing the measurement data, the error correction coefficients defined in the mathematical
 325 model, i.e. Equations (4) and (5) presented in Part I [18], were determined. The uncertainties of the
 326 values of those coefficients were also evaluated with implementation of the Monte Carlo method.
 327 The simulation number was 1×10^4 . Distributions of the uncertainties of each coefficient as well as
 328 their lower and upper boundaries of a 95% interval were indicated.

329 The second experiment measured a different area of the same standard artefact. By correcting
 330 the measurement data using our mathematical model and the determined coefficients, the corrected
 331 results were obtained. The residuals between the raw measured points and their corresponding
 332 certified values were compared to that between the corrected points and the certified values.

333 The data processing algorithms and procedures, such as separations of the flats and cylinders,
334 data partitions, outlier eliminations, K-means clustering, cylinder centres fittings, etc. were the
335 same for both two experimental studies. The difference between the data processing for the two
336 experiments lied in the final procedures. The first experiment fitted the values of the coefficients
337 used non-linear Least Squared method. The second method applied the mathematical models and
338 the determined values of the coefficients to the measured data for obtaining the corrected
339 coordinate values of the points.

340 Results of the experiments demonstrated that our proposed method for lateral stage kinematic
341 geometric error correction is efficient and useful.

342 Among the next practical steps for improving the proposed method is aimed at the stitching
343 algorithm of the optical element [26], which highly needs a calibration/correction of the stitching
344 result.

345

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349

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