Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 August 2019 doi:10.20944/preprints201908.0091.v1

Peer-reviewed version available at *Galaxies* **2019**, *7*, 77; doi:10.3390/galaxies7030077

The flatness problem and the varying physical constants

Rajendra P. Gupta

Department of Physics, University of Ottawa, Ottawa, Canada K1N 6N5

Abstract: We have used the varying physical constant approach to resolve the flatness problem in cosmology. Friedmann equations are modified to include variability of speed of light, gravitational constant, cosmological constant and the curvature constant. The continuity equation obtained with such modifications includes scale factor dependent cosmological term as well as the curvature term along with the standard energy-momentum term. The result is that as the scale factor tends to zero (i.e. as the big-bang is approached) the universe becomes strongly curved rather than flatter and flatter in the standard cosmology. We have used the supernovae 1a redshift versus distance modulus data to determine the curvature variation parameter of the new model, which yields a better fit to the data than the standard ΛCDM model. The universe is found to be open type with radius of curvature $R_c = 1.64\,(1+z)^{-3.3} c_0/H_0$, where z is the redshift, c_0 is the current speed of light and H_0 is the Hubble constant.

Keywords: flatness problem; variable physical constants; inflation; cosmology theory; dark energy

PACS: 98.80.-k; 98.80.Es; 98.62.Py

1. Introduction

While the Big-Bang cosmology has been successful in explaining most observables of the universe, and has become arguably the most accepted theory since the discovery of microwave background in 1964 by Penzias and Wilson [1], the inflation phenomenon proposed by Guth in 1981 [2-4] as explanation for the flatness, horizon, and magnetic monopole problems remains rather contentious. Within the purview of Big-Bang, several alternatives have been suggested to explain one or more of these problems. We will mention only a few here to give a flavour of the alternatives proposed as our intent is not to review the field but to explore if the recently proposed variable constants approach [5] can resolve the flatness problem, especially since the approach was able to resolve three astrometric anomalies and fit the SNe Ia redshift data better than the standard ΛCDM model.

An early review of the inflationary universe was written by Olive in 1990 [6] and an easy textbook description was provided in chapter 10 by Ryden in 2017 [7]. Levine and Freese in 1993 [8] attempted a possible solution to the horizon problem using the so called MAD (massively aged and detained) approach in massless scalar theory of gravity. Their approach is based on the time dependent Planck mass without the dominance of dark energy that is pivotal in the inflationary cosmology. Hu, Turner and Weinberg [9] studied the dynamical solutions to the horizon and flatness problems and showed that in the context of scalar-tensor theories, the time-varying Planck mass cannot lead to a solution of the horizon problem. They also pointed out that there is an apparent paradox in discussing the horizon problem in the context of Friedmann-Robertson-Walker (FRW) model, which has been developed assuming the universe to be isotropic and homogeneous. While modeling of generally inhomogeneous and anisotropic universe is very difficult, and interpretation of solutions therefrom is even more difficult [10, 11], an easier approach is to consider perturbation of FRW model and explore if the inhomogeneity created by such perturbations is contained to the level of observations. One could further question that even if the universe was causally connected, and thus homogeneous and isotropic during the inflationary epoch, what kept the inhomogeneities from developing from the time inflation ended ($t \approx 10^{-32}s$) to the time when cosmic microwave background (CMB) radiation was emitted ($t \approx 380{,}000$ yrs $\approx 10^{13}s$) when the universe was no longer causally connected. More recently, Singal has asserted that based on theories developed using FRW metric, which *a priori* assume the universe to be homogeneous and isotropic, one cannot use homogeneity and flatness problems in support of inflation [12].

While the horizon problem can be stated as 'why there is high level of isotropy and homogeneity observed in CMB data when most of the universe could not possibly be causally connected', the flatness problem seeks the answer to 'why energy density of the universe is so close to the critical energy density (the latter being the energy density assuming curvature term to be absent in FRW model or the universe to be flat) today, meaning that universe is almost flat today, and why it was even flatter in the past' [7]. This means that if we write the ratio of the energy density to its critical energy density as Ω then what we get in FRW model is that if we have, say $|1-\Omega_0| \approx 0.005$ now at time $t=t_0$, then at Planck time $t=t_P$, $|1-\Omega_P| \approx 10^{-62}$!

Barrow and Magueijo have addressed the flatness and cosmological constant (Λ) problem with an evolutionary speed of light theory in which speed of light falls-off with increasing cosmic time [13, 14]. Berera, Gleiser and Ramos presented a quantum field theory warm inflation model for the solution of horizon and flatness problem wherein in the realm of the elementary dynamics of particle physics, cosmological scale factor trajectories that originate in the radiation dominated epoch, enter an inflationary epoch, and finally exit back to the radiation dominated epoch, with significant radiation throughout the evolution [15]. Lake has shown through complete integration of Friedmann equations that for $\Lambda > 0$ there exist nonflat FRW models for which Ω remains ~ 1 throughout the entire history of the universe [16]. Fathi, Jalalzadeh and Moniz used quantum cosmology, based on the application of de Broglie-Bohm formulation in quantum mechanics to spatially closed universe comprising radiation and matter perfect fluids, to show that expanding classical universe can emerge from an oscillating quantum universe without singularity and without the horizon or flatness problems [17]. Using anisotropic scaling that leads to a novel mechanism of generating scale-invariant cosmological perturbations and resolution of horizon problem without inflation, Bramberger *et al.* propose a possible solution of the flatness problem by assuming that the initial condition of the universe is set by a small instanton respecting the same scaling [18].

In this paper we show that the flatness problem is easily resolved by incorporating variable physical constants in deriving the Friedmann equation from Einstein equations and Robertson-Walker metric. The continuity equation obtained with such modifications includes scale factor dependent cosmological term as well as the curvature term along with the standard energy-momentum term. The result is that as the scale factor tends to zero (i.e. as the big-bang is approached) the universe becomes strongly curved rather than flatter and flatter in the standard cosmology. The approach used here defers from that in reference [5] as follows: a) we do not assume the curvature term to be zero; b) the continuity equation is not artificially split into two continuity equations.

Section 2 develops the theoretical background of the approach used in this paper. Section 3 delineates the flatness problem. Section 4 discusses the resolution of the flatness problem using the new approach. In Section 5 the curvature parameter is estimated by fitting the SNe Ia data. Section 6 is dedicated to results and discussions, and finally Section 7 narrates the conclusions reached herein.

2. Theory

Let us start with the Robertson-Walker metric with the usual coordinates x^μ (ct, r, θ, ϕ):

$$ds^2 = c^2dt^2 - a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \qquad (1)$$

where $a(t)$ is the scale factor; $K \equiv k/R_c^2$ with k determining the spatial geometry of the universe ($k=-1$, 0 +1 for open, flat, and closed respectively) and R_c representing the spatial curvature of the universe; and c is

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 August 2019 doi:10.20944/preprints201908.0091.v1

Peer-reviewed version available at *Galaxies* **2019**, *7*, 77; doi:10.3390/galaxies7030077

the speed of light. The Einstein field equations may be written in terms of the Einstein tensor $G^{\mu\nu}$, metric tensor $g^{\mu\nu}$, energy-momentum tensor $T^{\mu\nu}$, cosmological constant Λ and gravitational constant G, as [19]:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu} \tag{2}$$

When solved for the Robertson-Walker metric, we get the following non-trivial equations:

$$\left(\frac{\dot a}{a}\right)^2 + \frac{kc^2}{R_c^2 a^2} = \frac{8\pi G\varepsilon}{3c^2} + \frac{\Lambda}{3} \quad , \tag{3}$$

$$\left(\frac{\ddot a}{a}\right) + a\frac{1}{2}\left(\frac{\dot a}{a}\right)^2 + \frac{1}{2}\frac{kc^2}{R_c^2 a^2} = -\frac{4\pi G p}{c^2} + \frac{1}{2}\Lambda. \tag{4}$$

Here G is the Newton's gravitational constant, ε is the energy density, Λ is the Einstein's cosmological constant, and $p \equiv w\varepsilon$ with w as the equation of state parameter (0 for matter, 1/3 for radiation and -1 for Λ). It is implicitly assumed in the above equations that the curvature of the universe scales as $R_c a$. If we relax this constraint then R_c could evolve differently. Also, it is assumed that c, G and Λ too are constants. We would like to superficially relax these constancy constraints. Why superficially, because the constraints should ideally be relaxed in the derivation of the Equation (2) from Einstein-Hilbert action, which is non-trivial and no one has been able to do it to our knowledge. Our formulation here should therefore be considered phenomenological. It would be interesting to know to what extent it differs from relativistically correct equations if and when they are developed.

Let us define two composite constants[1] $J \equiv G/c^2$ and $U \equiv c^2/R_c^2$ and relax the constancy constraint on them as well as on Λ. We may now write Eq. (3) as follows:

$$\dot a^2 = \frac{8\pi J}{3}\varepsilon a^2 + \frac{\Lambda}{3}a^2 - kU \tag{5}$$

Differentiating it with respect to time t, we get

$$2\dot a\ddot a = 2a\dot a\left(\frac{8\pi J}{3}\varepsilon + \frac{\Lambda}{3}\right) + a^2\left(\frac{8\pi}{3}\dot J\varepsilon + \frac{8\pi}{3}J\dot\varepsilon + \frac{\dot\Lambda}{3}\right) - k\dot U. \tag{6}$$

Dividing the above equation by $2a\dot a$, we may write

$$\left(\frac{\ddot a}{a}\right) = \frac{8\pi J}{3}\varepsilon + \frac{\Lambda}{3} + \frac{4\pi}{3}\left(\frac{a}{\dot a}\right)\dot J\varepsilon + \frac{4\pi}{3}\left(\frac{a}{\dot a}\right)J\dot\varepsilon + \frac{1}{6}\left(\frac{a}{\dot a}\right)\dot\Lambda - k\dot U\left(\frac{1}{2a\dot a}\right). \tag{7}$$

Equating it with $\ddot a/a$ in Eq. (4) and rearranging we get

$$4\pi J\varepsilon(1+w) + \frac{4\pi}{3}J\dot\varepsilon\left(\frac{a}{\dot a}\right) + \frac{4\pi}{3}\dot J\varepsilon\left(\frac{a}{\dot a}\right) + \frac{\dot\Lambda}{6}\left(\frac{a}{\dot a}\right) - \frac{k\dot U}{2a\dot a} = 0. \tag{8}$$

Multiplying by $\frac{3}{4\pi J}\left(\frac{\dot a}{a}\right)$, this equation becomes

$$\dot\varepsilon + \left(\frac{\dot a}{a}\right)(3+3w)\varepsilon + \frac{\dot J}{J}\varepsilon + \frac{1}{8\pi}\left(\frac{\dot\Lambda}{J}\right) - \frac{3k}{8\pi a^2}\left(\frac{\dot U}{J}\right) = 0. \tag{9}$$

This is the new continuity equation. If we assume the time dependence of ε and all constants is proportional to $\dot a/a$, we may write following Barrow [20]

$$\frac{\dot\varepsilon}{\varepsilon} = s\left(\frac{\dot a}{a}\right),\ \frac{\dot J}{J} = j\left(\frac{\dot a}{a}\right),\ \frac{\dot\Lambda}{\Lambda} = l\left(\frac{\dot a}{a}\right),\ \text{and}\ \frac{\dot U}{U} = u\left(\frac{\dot a}{a}\right),\ \text{i.e.} \tag{10}$$

$$\varepsilon = \varepsilon_0 a^s,\ J = J_0 a^j,\ \Lambda = \Lambda_0 a^l,\ \text{and}\ U = U_0 a^u\ . \tag{11}$$

The continuity equation, Equation (9), may now be written

$$\dot\varepsilon + \varepsilon(3+3w)\left(\frac{\dot a}{a}\right) + \left(\frac{\dot J}{J}\right)\varepsilon + \frac{1}{8\pi}\left(\frac{\dot\Lambda}{\Lambda}\right)\left(\frac{\Lambda}{J}\right) - \frac{3k}{8\pi a^2}\left(\frac{\dot U}{U}\right)\left(\frac{U}{J}\right) = 0. \tag{12}$$

[1] In reference [5] symbol K was used instead of J for G/c^2. Here we are using K for curvature parameter.

Peer-reviewed version available at Galaxies 2019, 7, 77; doi:10.3390/galaxies7030077

In the standard ΛCDM model, J, Λ and U are constants, and thus the last three terms in the above equation are zero. Thus, we get the usual continuity equation

$$\dot{\varepsilon} + \varepsilon(3+3w)\left(\frac{\dot{a}}{a}\right) = 0. \tag{13}$$

The solution of the this equation is

$$\varepsilon = \varepsilon_0 a^{-3-3w}\,. \tag{14}$$

Here ε_0 is the energy density at the current time $t = t_0$. Substitution of Eq. (13) in Eq. (12), and using Eq. (10) we get the second continuity equation superimposed on the first

$$j\left(\frac{\dot{a}}{a}\right)\varepsilon + \frac{l}{8\pi}\left(\frac{\dot{a}}{a}\right)\left(\frac{\Lambda}{J}\right) - \frac{3ku}{8\pi a^2}\left(\frac{\dot{a}}{a}\right)\left(\frac{U}{J}\right) = 0, \tag{15}$$

which, simplifies to (using Eq. (11))

$$8\pi\varepsilon jJ + l\Lambda - \frac{3ku}{a^2}U = 0, \text{ or } 8\pi\varepsilon_0 jJ_0 a^{j-3-3w} + l\Lambda_0 a^l = 3kuU_0 a^{u-2}. \tag{16}$$

It yields, assuming exponents of a, the only time varying parameter, to be equal, the following:

$$j - 3 - 3w = l = u - 2 \text{ and } 8\pi\varepsilon_0 jJ_0 + l\Lambda_0 = 3kuU_0. \tag{17}$$

The separation of the continuity equation, Eq. (12), is tantamount to no direct sharing of the energy between the components represented by Eq. (13) and the components represented by Eq. (15). This is not acceptable if we wish to properly take into account the effect of the variation of c, G, and Λ on the evolution of the universe. We therefore rewrite Eq. (12) as

$$\frac{\dot{\varepsilon}}{\varepsilon}\varepsilon + (3+3w)\left(\frac{\dot{a}}{a}\right)\varepsilon + \left(\frac{\dot{J}}{J}\right)\varepsilon + \frac{1}{8\pi}\left(\frac{\dot{\Lambda}}{\Lambda}\right)\left(\frac{\Lambda}{J}\right) - \frac{3k}{8\pi a^2}\left(\frac{\dot{U}}{U}\right)\left(\frac{U}{J}\right) = 0, \text{ or}$$

$$s\left(\frac{\dot{a}}{a}\right)\varepsilon_0 a^s + (3+3w)\left(\frac{\dot{a}}{a}\right)\varepsilon_0 a^s + j\left(\frac{\dot{a}}{a}\right)\varepsilon_0 a^s + \frac{l}{8\pi}\left(\frac{\dot{a}}{a}\right)\left(\frac{\Lambda_0}{J_0}\right)a^{l-j} - \frac{3ku}{8\pi}\left(\frac{\dot{a}}{a}\right)\left(\frac{U_0}{J_0}\right)a^{u-j-2} = 0\,. \tag{18}$$

Dividing by $\frac{\dot{a}}{a}$ and rearranging

$$(s+3+3w+j)\varepsilon_0 a^s + \frac{l}{8\pi}\left(\frac{\Lambda_0}{J_0}\right)a^{l-j} - \frac{3ku}{8\pi}\left(\frac{U_0}{J_0}\right)a^{u-j-2} = 0. \tag{19}$$

This is therefore is the undivided continuity equation. Since $a_0 = 1$, we get the following constraining condition

$$(s+3+3w+j)\varepsilon_0 + \frac{l}{8\pi}\left(\frac{\Lambda_0}{J_0}\right) = \frac{3ku}{8\pi}\left(\frac{U_0}{J_0}\right). \tag{20}$$

From Eq. (19), if we would like the Λ term to be dominant in the past (over the radiation term), i.e. when $a \to 0$, then $l - j < s$, and if the curvature term is to be dominant then $u - j - 2 < s$. One simple solution is obtained when we consider $s = l - j = u - j - 2$. However, we will not be limited to such a constraint.

3. Flatness Problem

We may rewrite Eq. (3), with $\varepsilon_\Lambda \equiv \Lambda/8\pi J$, as

$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi J}{3}\varepsilon + \frac{\Lambda}{3} - \frac{kU}{a^2} = \frac{8\pi J}{3}\varepsilon + \frac{8\pi J}{3}\varepsilon_\Lambda - \frac{kU}{a^2}$$
$$= \frac{8\pi}{3}J\varepsilon_T - \frac{kU}{a^2}\,. \tag{21}$$

Here $H(t)$ is the Hubble parameter, and $\varepsilon_T = \varepsilon + \varepsilon_\Lambda$ is the total energy density as it includes the same due to Λ. From the definition of critical density, i.e. the density in a spatially flat universe ($k = 0$)

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 August 2019 doi:10.20944/preprints201908.0091.v1

Peer-reviewed version available at Galaxies 2019, 7, 77; doi:10.3390/galaxies7030077

$$\varepsilon_{Tc} = \frac{3}{8\pi J} H(t)^2 \text{ , or } \frac{8\pi J}{3} = \frac{H(t)^2}{\varepsilon_{Tc}} \text{ , } \therefore H_0^2 = \frac{8\pi J_0}{3} \varepsilon_{Tc,0} \text{ ,} \tag{22}$$

we may define the relative energy density $\Omega(t)$ at time t as

$$\Omega(t) \equiv \frac{\varepsilon_T(t)}{\varepsilon_{Tc}(t)} . \tag{23}$$

We may rewrite Eq. (21) using Eq. (22) as

$$H(t)^2 = \frac{8\pi}{3} J\varepsilon_T - \frac{kU}{a^2} = H(t)^2 \left(\frac{\varepsilon_T(t)}{\varepsilon_{Tc}(t)}\right) - k\left(\frac{U}{a^2}\right) . \tag{24}$$

Dividing by $H(t)^2$ and rearranging, we get (recall that $a_0 \equiv 1$)

$$1 - \Omega(t) = -\frac{kU}{a^2 H(t)^2} \text{, and} \tag{25}$$

$$1 - \Omega_0 = -\frac{kU_0}{H_0^2} , \tag{26}$$

Using Eq. (21) and Eq. (22) we may write

$$\begin{aligned} \frac{H(t)^2}{H_0^2} &= \frac{8\pi}{3} J\varepsilon_T / (\frac{8\pi}{3} J_0 \varepsilon_{Tc,0}) \ - \frac{kU}{a^2}\left(\frac{3}{8\pi J_0 \varepsilon_{Tc,0}}\right) , \\ &= \frac{a^j}{\varepsilon_{Tc,0}}\left(\varepsilon_0 a^s + \frac{\Lambda_0}{8\pi J_0} a^{l-j}\right) - kU_0 a^{u-2}\left(\frac{3}{8\pi J_0 \varepsilon_{Tc,0}}\right) , \\ &= \frac{1}{8\pi J_0 \varepsilon_{Tc,0}} (8\pi J_0 \varepsilon_0 a^{s+j} + \Lambda_0 a^l - 3kU_0 a^{u-2}) . \end{aligned} \tag{27}$$

Dividing Eq. (25) by Eq. (26) and rearranging, then using Eq. (27), we have

$$\begin{aligned} 1 - \Omega(t) &= (1 - \Omega_0)\left(\frac{UH_0^2}{U_0 a^2 H(t)^2}\right) , \\ &= (1 - \Omega_0) a^{u-2}\left(\frac{H_0^2}{H(t)^2}\right) , \\ &= (1 - \Omega_0) a^{u-2} 8\pi J_0 \varepsilon_{Tc,0} / (8\pi J_0 \varepsilon_0 a^{s+j} + \Lambda_0 a^l - 3kU_0 a^{u-2}) \text{ , or} \end{aligned}$$

$$1 - \Omega(a) = (1 - \Omega_0) 8\pi J_0 \varepsilon_{Tc,0} / (8\pi J_0 \varepsilon_0 a^{s+j-u+2} + \Lambda_0 a^{l-u+2} - 3kU_0). \tag{28}$$

It may also be written in terms of the relative energy densities $\Omega_{\varepsilon,0} = \varepsilon_0/\varepsilon_{Tc,0}$, $\Omega_{\Lambda,0} = \Lambda_0/(8\pi J_0 \varepsilon_{Tc,0}) = \varepsilon_{\Lambda,0}/\varepsilon_{\mathrm{Tc},0}$

$$1 - \Omega(a) = (1 - \Omega_0)/(\Omega_{\varepsilon,0} a^{s+j-u+2} + \Omega_{\Lambda,0} a^{l-u+2} + (1 - \Omega_0)). \tag{29}$$

As a check it can be seen referring to Eq. (27) that the denominator on the right hand side of the above equation reduces to 1 at $t = t_0$ because $a(t_0) \equiv 1$ and $\Omega_0 = \Omega_{\varepsilon,0} + \Omega_{\Lambda,0}$.

When none of the constants are varying, so $j = u = l = 0$, and $s = -3 - 3w$ (from Eqs. 11 and 14), and Eq. 29 becomes

$$\begin{aligned} 1 - \Omega(a) &= (1 - \Omega_0)/(\Omega_{\varepsilon,0} a^{-1-3w} + \Omega_{\Lambda,0} a^2 + (1 - \Omega_0)), \\ &= (1 - \Omega_0) a^2/(\Omega_{\varepsilon,0} a^{+1-3w} + \Omega_{\Lambda,0} a^4 + (1 - \Omega_0) a^2). \end{aligned} \tag{30}$$

As $a \to 0$, we have $1 - \Omega(a) \to (1 - \Omega_0)a$ in the matter dominated universe, i.e. $w = 0$, and $1 - \Omega(a) \to (1 - \Omega_0)a^2$ in radiation dominated universe. In both the cases $1 - \Omega(t)$ is becoming smaller and smaller as we approach the big-bang, i.e the universe is becoming flatter and flatter linearly with decreasing scale factor in matter dominated universe and quadratically in the radiation dominated universe. This indeed is the flatness problem.

4. Resolution of Flatness Problem

Let us focus on Eq. (29). If we have $s+j-u+2=0$ and $l-u+2=0$ then the denominator reduces to 1 and thus, irrespective of the value of a, $1-\Omega(a)=(1-\Omega_0)$. If $s+j-u+2>0$ and $l-u+2>0$ then the denominator keeps decreasing in absolute value with decreasing scale factor (assuming all relative densities have the same sign). Ultimately, at $a=0$, the first two terms in the denominator on the right hand side vanish and $(1-\Omega(t))=1$; i.e. the universe become strongly curved as the scale factor a approaches zero. However, if $s+j-u+2<0$ or $l-u+2<0$, then denominator keep increasing with decreasing scale factor and the flatness problem remains. Thus, flatness problem is resolved provided $s+j-u+2\geq 0$ and $l-u+2\geq 0$. It should be noted that the flatness problem is resolved even if $\Omega_{\Lambda,0}=0$.

One may ask if it is possible to determine the parameters s, j, u and l. If we take $s=-3-3w$ (Eq. 14), based on the split continuity equation, and $j=1.8$ [5], we get (a) for radiation dominated epochs (i.e. $w=1/3$), $u\leq -0.2$ and $l\geq u-2$; and (b) for matter dominated epochs, (i.e. $w=0$), $u\leq 0.8$ and $l\geq u-2$.

Now, $s\neq -3-3w$ if we wish to keep the undivided continuity equation, i.e. Eq. (19). However, if we assume $\Omega_{\Lambda,0}=0$ (Einstein-de Sitter type model) and consider the curvature term is observationally very small at present and thus ignore it, we get from Eq. (20), $s=-3-3w-j$ rather than $s=-3-3w$ obtained from the split continuity equation. This yields $u\leq -2$ for radiation dominated epochs and $u\leq -1$ for matter dominated epoch. And since $\Omega_{\Lambda,0}$ term is absent, we don't need to worry about l. Nevertheless, the limiting flatness conclusions remain unchanged.

The above reasoning only gives the high limits of u under various scenarios and not actual value of the parameter. Let us see if u can be determined from the supernovae 1a redshift z versus distance modulus μ observational data [21] for 1048 extragalactic sources up to $z\leq 2.26$.

5. Estimation of the Curvature Parameter u

Eq. (27) may be written

$$\frac{H(t)^2}{H_0^2}=\Omega_{m,0}a^{-1.2}+\Omega_{r,0}a^{-2.2}+\Omega_{\Lambda,0}a^{l}+(1-\Omega_0)a^{u-2}\text{ , or using Peebles convention [22]}$$

$$E^2(z)\equiv\frac{H(z)^2}{H_0^2}=\Omega_{m,0}(1+z)^{1.2}+\Omega_{r,0}(1+z)^{2.2}+\Omega_{\Lambda,0}(1+z)^{-l}+(1-\Omega_0)(1+z)^{2-u}\text{ , or} \tag{31}$$

$$H(z)\equiv\left(\frac{\dot a}{a}\right)=H_0E(z). \tag{32}$$

Now the proper distance $d_P(t_0)$ of a galaxy emitting the light is given by [7]

$$d_P(t_0)=\int_{t_e}^{t_0}c\left(\frac{dt}{a(t)}\right)=\int_{a_e}^{a_0}c\left(\frac{da}{a\dot a}\right)=\int_{a_e}^{a_0}c\left(\frac{da}{a^2\left(\frac{\dot a}{a}\right)}\right). \tag{33}$$

Since $a=1/(1+z)$, $da=-dz/(1+z)^2=\ -a^2dz$, and $c=c_0a^j=c_0/(1+z)^{1.8}$ [5] we may write Eq. (33) as

$$d_P(z)=\int_0^z\left(\frac{c_0}{(1+z')^{1.8}}\right)\left(\frac{dz'}{H_0E(z')}\right)=\left(\frac{c_0}{H_0}\right)\int_0^z\left(\frac{dz'}{(1+z')^{1.8}E(z')}\right). \tag{34}$$

The luminosity distance $d_L(z)=(1+z)d_P(z)$, and the distance modulus $\mu=5\log_{10}\big(d_L(z)\big)+25$ when distance is expressed in Mpc, i.e.

$$\mu=5\log_{10}\left(\left(\frac{c_0}{H_0}\right)\int_0^z\left(\frac{dz'}{(1+z')^{1.8}E(z')}\right)\right)+5\log_{10}(1+z)+25. \tag{35}$$

If we assume that the cosmological constant is an artifact that compensates the inadequacy of a model, then we can try to fit data assuming $\Omega_{\Lambda,0}=0$. In addition, we know that $\Omega_{r,0}\ll\Omega_{m,0}$. We may therefore write

$$E^2(z)=\Omega_{m,0}(1+z)^{1.2}+\big(1-\Omega_{m,0}\big)(1+z)^{2-u}. \tag{36}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 August 2019 doi:10.20944/preprints201908.0091.v1

Peer-reviewed version available at Galaxies 2019, 7, 77; doi:10.3390/galaxies7030077

Substituting this in Eq. (35) and fitting the SNe Ia data, we can obtain H_0, $\Omega_{m,0}$ and u.

6. Results and Discussion

We have used the gold standard data of redshift versus distance modulus, so-called Pantheon Sample comprising 1048 supernovae 1a in the range of $0.01 < z \leq 2.26$, compiled by Scolnic et al. [20]. The data is in terms of the apparent magnitude and we added 19.35 to it to obtain normal luminosity distance numbers as suggested by Scolnic [23].

The Matlab curve fitting tool was used to fit the data by minimizing χ^2 and the latter was used for determining the corresponding χ^2 probability [24] P. Here χ^2 is the weighted summed square of residual of μ:

$$\chi^2 = \sum_{i=1}^{N} w_i\left[\mu(z_i; R_0, p_1, p_2 \dots) - \mu_{obs,i}\right]^2 \tag{37}$$

where N is the number of data points, w_i is the weight of the ith data point $\mu_{obs,i}$ determined from the measurement error $\sigma_{\mu_{Obs,i}}$ in the observed distance modulus $\mu_{obs,i}$ using the relation $w_i = 1/\sigma^2_{\mu_{Obs,i}}$, and $\mu(z_i; R_0, p_1, p_2 ..)$ is the model calculated distance modulus dependent on parameters R_0 and all other model dependent parameter p_1, p_2, etc. As an example, for the ΛCDM models considered here, $p_1 \equiv \Omega_{m,0}$ and there is no other unknown parameter.

We then quantified the goodness-of-fit of a model by calculating the χ^2 probability for a model whose χ^2 has been determined by fitting the observed data with known measurement error as above. This probability P for a χ^2 distribution with n degrees of freedom (DOF), the latter being the number of data points less the number of fitted parameters, is given by:

$$P(\chi^2, n) = \left(\frac{1}{\Gamma\left(\frac{n}{2}\right)}\right)\int_{\frac{\chi^2}{2}}^{\infty} e^{-u}u^{\frac{n}{2}-1}du, \tag{38}$$

where Γ is the well know gamma function that is generalization of the factorial function to complex and non-integer numbers. The lower the value of χ^2, the better the fit, but the real test of the goodness-of-fit is the χ^2 probability P; the higher the value of P for a model, the better the model's fit to the data. We used an online calculator to determine P from the input of χ^2 and DOF [25].

Our primary findings are presented in Table 1. The unit of the Hubble constant H_0 is km s^{-1} Mpc^{-1}. The table includes the data fit results for the standard ΛCDM model for comparison with the varying constant model, the latter being identified as VcGU model (varying c, G and U model). The table shows goodness of fit slightly in favour of the varying constant model. The results are also compared graphically in Figure 1.

We get $u \approx -3$ from the table. We may therefore rewrite Eq. (29) in the VcGU model as

$$1 - \Omega(a) = (1 - \Omega_0)/(\Omega_{m,0}a^{3.8} + (1 - \Omega_0)), \tag{39}$$

in the matter dominated universe, and in the radiation dominated universe as

$$1 - \Omega(a) = (1 - \Omega_0)/(\Omega_{r,0}a^{2.8} + (1 - \Omega_0)). \tag{40}$$

Table 1. Model parameters and goodness-of-fit parameters for the ΛCDM model the VcGU model. The unit of H_0 is km s^{-1} Mpc^{-1}. P% is the χ2 probability in percent that is used to assess the best model for each category; the higher the χ2 probability *P*, the better the model fits to the data. R^2 is the square of the correlation between the response values and the predicted response values. RMSE is the root mean square error.

Parameter/Model	ΛCDM	VcGU
H_0	70.18±0.43	70.65±0.60
$\Omega_{m,0}$	0.2845±0.0245	0.6309±0.1103
u	NA	-2.938±0.549
χ2	1036	1032
DOF	1046	1045
P%	58	61
R^2	0.9970	0.9970
RMSE	0.995	0.994

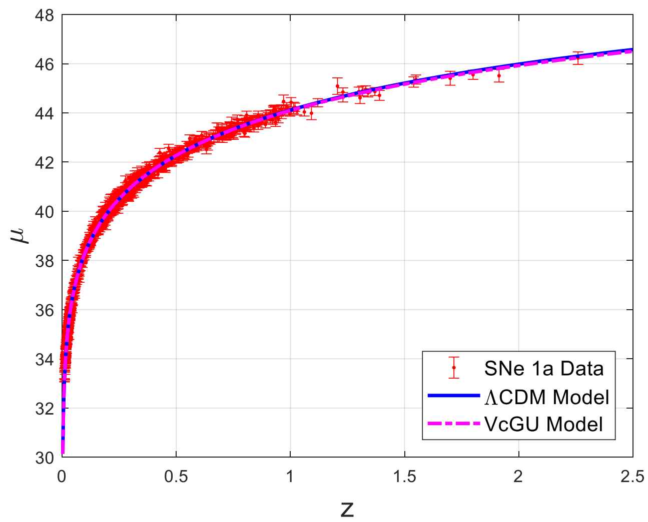

Figure 1. Supernovae Ia redshift z vs. distance modulus μ data fit with ΛCDM model and the fit with the new model using the varying speed of light c, varying gravitational constant G and variable curvature constant U (VcGU) model.

As $a \to 0$, both the above expressions, Eqs. (39 and 40) tend to unity, i.e. the universe was strongly curved in the past. By taking $\Omega_0 = \Omega_{m,0} = 0.63$ from Table 1 for Eq. (39), and $\Omega_0 = \Omega_{r,0} = 9.0 \times 10^{-5}$ from Table 5.2 in reference [7] we get radiation density equal to matter density at $z \approx 7{,}000$ determined by the relation

$$\Omega_{r,0} a^{-4} = \Omega_{m,0} a^{-3}, \text{ or } \Omega_{r,0}(1+z)^4 = \Omega_{m,0}(1+z)^3. \tag{41}$$

However, we can easily calculate from Eq. (39), and see from Figure 2 displaying the plot of $1 - \Omega$ against a, that $1 - \Omega$ was almost unity within three decimal places at $z = 15$, i.e. well below the redshift when radiation density equaled matter density. The flatness of the universe problem has now become the universe being curved now and strongly curved in the past.

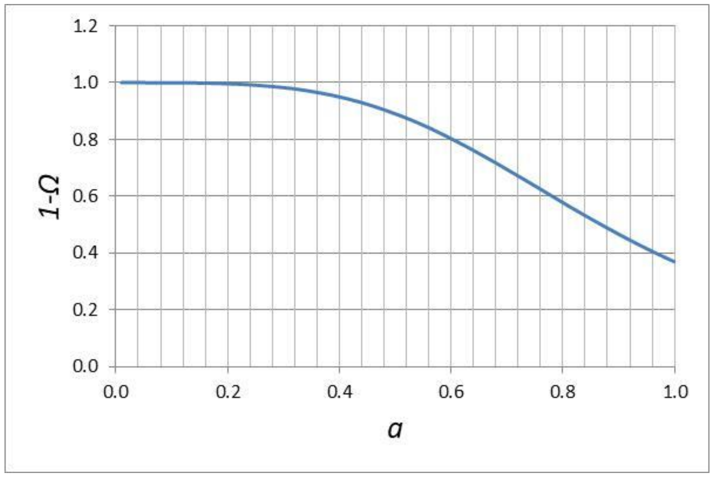

Figure 2. $1 - \Omega$ plotted against $a \equiv (1+z)^{-1}$ using Eq. 39.

We can also determine $R_{c,0}$ using Eq. (26):

$$1 - \Omega_0 = -\frac{kU_0}{H_0^2} = -\frac{k}{H_0^2}\left(\frac{c_0^2}{R_{c,0}^2}\right), \text{ or} \tag{42}$$

$$R_{c,0}^2 = -\frac{kc_0^2}{H_0^2}\left(\frac{1}{1-\Omega_0}\right) = -\frac{k}{1-0.63}\left(\frac{c_0^2}{H_0^2}\right) = -2.7k\left(\frac{c_0^2}{H_0^2}\right). \tag{43}$$

This expression shows that k must be –ve ($= -1$) in order for $R_{c,0}$ to be real, i.e. the universe is open with $R_{c,0} = 1.64 c_0/H_0$.

We will now determine how the radius of curvature R_c in Eqs. (3) and (4), that is embedded in the parameter U, varies with the scale factor a. Now $U \equiv c^2/R_c^2$, $U = U_0 a^{-3} = (c_0^2/R_{c,0}^2)a^{-3}$ and $c = c_0 a^{1.8}$, and $R_{c,0} = 1.64 c_0/H_0$. Therefore, $R_c = R_{c,0} a^{3.3} = 1.64(1+z)^{-3.3} c_0/H_0)$. This is presented in Figure 3.

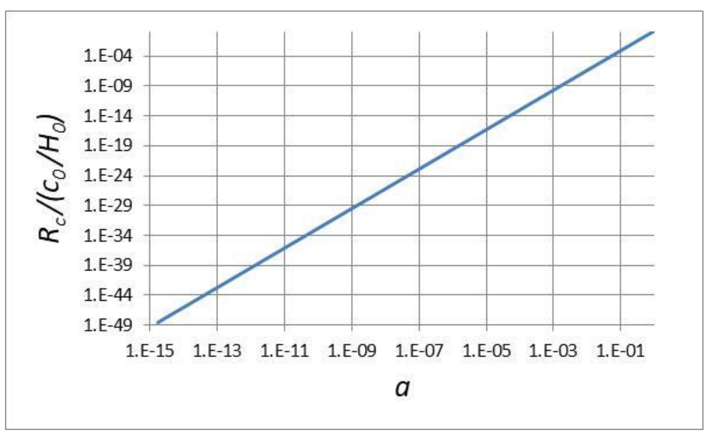

Figure 3. Curvature in units of c_0/H_0 plotted against $a \equiv (1+z)^{-1}$.

Einstein introduced his most undesirable cosmological constant Λ to prevent the universe from collapsing. Currently, it represents the all elusive dark energy in most cosmological models need to explain observables. The inflation theories require Λ that is 107 orders of magnitude larger during inflation then in the current epoch. Additionally, it has to be turned on and off at appropriate time: inflation started at $t \approx 10^{-36} s$ and lasted for a period of about $10^{-34} s$ [7]. The variable physical

Peer-reviewed version available at Galaxies 2019, 7, 77; doi:10.3390/galaxies7030077

constants model, and its extension by relaxing the usual constraint on the curvature of the universe to evolve exactly as the scale factor, makes it possible to refrain using cosmological constant in cosmological modeling.

7. Conclusions

The following are the conclusions:

1. The variable physical constant approach can naturally eliminate the flatness problem that has been pervasive in most cosmological models.
2. The universe is open type, and was strongly curved in the past and substantial curved at present with a curvature $1.64c_0/H_0$.
3. The scaling of the curvature can be reliably determined by fitting the most recently available supernovae 1a data; the new model fits the data as well or better than the standard ΛCDM model.
4. The radius of curvature of the universe evolves differently than assumed in the standard model; it evolves as proportional to the $a^{3.3}$.
5. The cosmological constant, and consequently the dark energy, is no longer required to save the cosmos from collapsing.

Funding: This research received no external funding.

Acknowledgement: The author is grateful to Dan Scolnic for providing the SNe Ia Pantheon Sample data used in this work.

Conflict of Interest: The author declares no conflict of interest.

References

[1] Penzias, A. A.; Wilson R. W. A Measurement of Excess Antenna Temperature at 4080 Mc/s, *Ap. J.* **1965**, *142*, 419-421.

[2] Guth, A. H. Inflationary universe: A possible solution for the horizon and flatness problem, *Phys. Rev. D* **1981**, *23*, 347-356.

[3] Albrecht, A; Steinhardt, P. J. Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking, *Phys. Rev. Lett.* **1982**, *48*, 1220-1223.

[4] Linde, A. A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Phys. Lett. B* **1982**, *108*, 389-393.

[5] Gupta, R. P. Varying physical constants, astrometric anomalies, redshift and Hubble units, *Galaxies* (2019) **7**, 55; https://doi.org/10.3390/galaxies7020055.

[6] Olive, K. A. Inflation. *Phys. Rep.* **1990**, *190*, 307-403; https://doi.org/10.1016/0370-1573(90)90144-Q.

[7] Ryden, B. *Introduction to Cosmology*; Cambridge University Press: Cambridge, UK (2017), Ch. 10.

[8] Levine, J; Freese, K. Possible solution to the horizon problem: Modified aging in massive scalar theories of gravity. *Phys. Rev. D* **1993**, *47*, 4282-4291. arXiv: 9211011.

[9] Hu, Y; Turner, M. S.; Weinberg, E. J. Dynamical solutions to the horizon and flatness problems, *Phys. Rev. D* **1994**, *49*, 383-3836; arXiv: 9302002.

[10] Barrow, J. D.; Tipler, F. Analysis of the generic singularity studies by Belinskii, Khalatnikov, and Lifschitz, *Phys. Rep.* **1979**, *56*, 371-402.

[11] Belinski, V. A.; Khalatnikov, I. M.; Lifshitz, E. M. A general solution of the Einstein equations with a time singularity, *Adv. Phys.* **1982**, *31*, 639-667.

[12] Singal, A. K. Horizon, homogeneity and flatness problems – do their resolutions really depend upon inflation? **2016** arXiv: 1603.01539.

[13] Barrow, J. D.; Magueijo, J. Solution to the quasi-flatness and quasi-lambda problems. *Phys. Lett. B* **1999**, *447*, 246-250; arXiv: 9811073.

[14] Barrow, J. D.; Magueijo, J. Solving the flatness and quasi-flatness problems in Brans-Dicke cosmologies with varying light speed. *Class. Quantum Grav.* **1999**, *16*, 1435-1454; arXiv: 9901049.

[15] Berera, A.; Gleiser, M.; Ramos, R. O. A first principle warm inflation model that solves cosmological horizon and flatness problems. *Phys. Rev. Lett.* **1999**, *83*, 264-267.

[16] Lake, K. The flatness problem and Λ. *Phys. Rev. Lett.* **2005**, *94*, 201102.
[17] Fathi, M; Jalalzadeh, S; Moniz, P. V. Classical universe emerging from quantum cosmology without horizon and flatness problems. *Eur. Phys. J. C* **2016**, *76*, 527.
[18] Bramberger, S. F.; Coates, A.; Magueijo, J; Mukohyama, S.; Namba, R.; Watanabe, Y. Solving the flatness problem with an anisotropic instanton in Horava-Lifshitz gravity. *Phys. Rev. D* **2018**, *97*, 043512; arXiv: 1709.07084.
[19] Narlikar, J. V. *An Introduction to Cosmology*, 3rd Ed., Cambridge University Press, U.K. (2002).
[20] Barrow, J. D. Cosmologies with varying light speed, *Phys. Rev. D* **1999**, ***59***, 043515.
[21] Scolnic, D.M.; Jones, D.O.; Rest, A.; Pan, Y.C.; Chornock, R.; Foley, R. J.; Huber, M. E.; Kessler, R.; Narayan, G.; Riess, A. G.; et al. The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample. *Astrophys. J.* **2018**, *859*, 101. Available online: https://archive.stsci.edu/hlsps/ps1cosmo/scolnic/ https://github.com/dscolnic/Pantheon (both accessed on May 10, 2019).
[22] Peebles, P. J. E. *Principles of Physical Cosmology*, Princeton University Press, N. J. (1993).
[23] Scolnic, D.M. - email communication dated November 22, 2018.
[24] Press, W.H.; Teukolsky, S.A.; Vetterling, W.T.; Flannery, B.P. *Numerical Recipes in C—The Art of Scientific Computing*, 2nd ed.; Cambridge University Press: Cambridge, UK, 1992.
[25] Walker, J. Chi-Square Calculator. Available online: https://www.fourmilab.ch.rpkp/experiments/analysis/chiCalc.html (accessed on 3rd August, 2019).