Article

Quantum Hair on Colliding Black Holes

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Abstract: Black hole collision produce gravitational radiation which is generally thought in a quantum limit to be gravitons. The stretched horizon of a black hole contains quantum information, or a form of quantum hair, which in a coalescence of black holes participates in the generation of gravitons. This may be facilitated with a Bohr-like approach to black hole (BH) quantum physics with quasi-normal mode (QNM) approach to BH quantum mechanics. Quantum gravity and quantum hair on event horizons is excited to higher energy in BH coalescence. The near horizon condition for two BHs right before collision is a deformed AdS spacetime. These excited states of BH quantum hair then relax with the production of gravitons. This is then argued to define RT entropy given by quantum hair on the horizons. These qubits of information from a BH coalescence should then appear in gravitational wave (GW) data. This is a form of the standard AdS/CFT correspondence and the Ryu-Takayanagi (RT) formula[1]. The foundations of physics is proposed to be quantum information and a duality between spacetime observables and quantum fields.

Keywords: quantum information; quantum hair; black hole quantum physics; quantum levels; AdS/CF T correspondence; Ryu-Takayanagi formula

1. Introduction

Quantum gravitation suffers primarily from an experimental problem. It is common to read critiques that it has gone off into mathematical fantasies, but the real problem is the scale at which such putative physics holds. It is not hard to see that an accelerator with current technology would be a ring encompassing the Milky Way galaxy. Even if we were to use laser physics to accelerate particles the energy of the fields proportional to the frequency could potentially reduce this by a factor of about 10^6 so a Planck mass accelerator would be far smaller; it would encompass the solar system including the Oort cloud out to at least .1 light years. It is also easy to see that a proton-proton collision that produces a quantum BH of a few Planck masses would decay into around a mole of daughter particles. The detection and track finding work would be daunting. Such experiments are from a practical perspective nearly impossible. This is independent of whether one is working with string theory or loop variables and related models.

It is then best to let nature do the heavy lifting for us. Gravitation is a field with a coupling that scales with the square of mass-energy. Gravitation is only a strong field when lots of mass-energy is concentrated in a small region, such as a BH. The area of the horizon is a measure of maximum entropy any quantity of mass-energy may posses, and the change in horizon area with lower and upper bounds in BH thermodynamic a range for gravitational wave production. Gravitational waves produced in BH coalescence contains information concerning the BHs configuration, which it is argued includes quantum hair on the horizons. This information will then appear as gravitational memory, which is found when test masses are not restored to their initial configuration. This information may be used to find data on quantum gravitation.

There are three main systems in physics, quantum mechanics (QM), statistical mechanics and general relativity (GR) along with gauge theory. These three systems connect with each other in certain ways. There is quantum statistical mechanics in the theory of phase transitions, BH thermodynamics connects GR with statistical mechanics, and Hawking-Unruh radiation connects QM to GR as well. These are connections, but are incomplete and there has yet to be any general unification or reduction of degrees of freedom. Unification of QM with GR appeared to work well with holography, but now faces an obstruction called the firewall[2].

Hawking radiation is often thought of as positive and negative energy entangled states where positive energy escapes and negative energy enters the BH. The state which enters the BH effectively removes mass from the same BH and increases the entanglement entropy of the BH through its entanglement with the escaping state. This continues but this entanglement entropy is limited by the Bekenstein bound. In addition later emitted bosons are entangled with both the black hole and previously emitted bosons. This means a bipartite entanglement is transformed into a tripartite entangled state. This is not a unitary process. This will occur once the BH is at about half its mass at the Page time [3], and it appears the unitary principle (UP) is violated. In order to avoid a violation of UP the equivalence principle (EP) is assumed to be violated with the imposition of a firewall. The unification of QM and GR is still not complete. An elementary approach to unitarity of black holes prior to the Page time is with a Bohr-like approach to BH quantum physics, [4–6], which will be shortly discussed in next section.

Quantum gravity hair on BHs may be revealed in the collision of two BHs. This quantum gravity hair on horizons will present itself as gravitational memory in a GW. This is presented according to the near horizon condition on Reissnor-Nordstrom BHs, which is $AdS_2 \times \mathbb{S}^2$, which leads to conformal structures and complementarity principle between GR and QM.

2. Bohr-like approach to black hole quantum physics

At the present time, there is a large agreement, among researchers in quantum gravity, that BHs should be highly excited states representing the fundamental bricks of the yet unknown theory of quantum gravitation [4–6]. This is parallel to quantum mechanics of atoms. In the 1920s the founding fathers of quantum mechanics considered atoms as being the fundamental bricks of their new theory. The analogy permits one to argue that BHs could have a discrete mass spectrum [4–6]. In fact, by assuming the BH should be the nucleus the "gravitational atom", then, a quite natural question is: What are the electrons? In a recent approach, which involves various papers (see [4–6] and references within), this important question obtained an intriguing answer. The BH quasi-normal modes (QNMs) (i.e. the horizon's oscillations in a semi-classical approach) triggered by captures of external particles and by emissions of Hawking quanta, represent the electrons of the BH which is seen as being a gravitational hydrogen atom [4–6]. In [4–6] it has been indeed shown that, in the the semi-classical approximation, the evaporating Schwarzschild BH can be considered as the gravitational analogous of the historical, semi-classical hydrogen atom, introduced by Niels Bohr in 1913 [7,8] and awarded with the Nobel Prize in Physics. The analysis in [4–6] starts from the non-thermal spectrum of Hawking radiation in [9]. In fact, such a non-thermal spectrum implies the countable character of subsequent emissions of Hawking quanta [4–6]. Those emitted quanta generate a natural and important correspondence between Hawking radiation and the BH QNMs [4–6]. Thus, BH QNMs are interpreted as the BH electron-like states, which can jump from a quantum level to another one. One can also identify the energy shells of this gravitational hydrogen atom as the absolute values of the quasi-normal frequencies [4–6].

Within the approximation of this Bohr-like model unitarity holds in BH evaporation. This is because the time evolution of the Bohr-like BH is governed by a time-dependent Schrodinger equation [5,6]. In addition, subsequent emissions of Hawking quanta are entangled with the QNMs (the BH electron states) [5,6]. Various results of BH quantum physics are consistent with the results of [5,6], starting from the famous result of Bekenstein on the area quantization [10]. Recently, this Bohr-like approach to BH quantum physics has been also generalized to the Large AdS BHs, see [11]. For the sake of simplicity, in this Section we will use Planck units ($G = c = k_B = \hbar = \frac{1}{4\pi\epsilon_0} = 1$). For large values of the principal quantum number n (i.e. for excited BHs), the energy levels of the Schwarzschild BH are [4–6]

$$E_n \equiv |\omega_n| = M - \sqrt{M^2 - \frac{n}{2}}, \tag{1}$$

where M is the initial BH mass. E_n is the total energy emitted by the BH when the same BH is excited at the level n [4–6] in units of Planck mass, where $M_p = 1$. A discrete amount of energy is radiated by the BH in a quantum jump. A key point is that, for large values of n, the process does not depend on the other quantum numbers. This issue is surely consistent with Bohr's *Correspondence Principle* [12]. In fact, the Correspondence Principle states that transition frequencies at large quantum numbers should equal classical oscillation frequencies [12]. In the analysis of Bohr [7,8], electrons can only lose and gain energy during quantum jumps among various allowed energy shells. In each jump, the hydrogen atom can absorb or emit radiation and the energy difference between the two involved quantum levels is given by the Planck relation (in standard units) $E = h\nu$. In the BH case, the BH QNMs can gain or lose energy by quantum jumps from one allowed energy shell to another by absorbing or emitting radiation (Hawking quanta). The energy difference between the two quantum levels results [4–6]

$$\begin{aligned} \Delta E_{n_1 \to n_2} &\equiv E_{n_2} - E_{n_1} = M_{n_1} - M_{n_2} = \\ &= \sqrt{M^2 - \tfrac{n_1}{2}} - \sqrt{M^2 - \tfrac{n_2}{2}}, \end{aligned} \tag{2}$$

This equation governs the energy transition between two generic, allowed levels n_1 and $n_2 > n_1$ and consists in the emission of a particle with a frequency $\Delta E_{n_1 \to n_2}$ [4–6]. The quantity M_n in Eq. (2), represents the residual mass of the BH which is now excited at the level n. It is exactly the original BH mass minus the total energy emitted when the BH is excited at the level n [5,6]. Then, $M_n = M - E_n$, and one sees that the energy transition between the two generic allowed levels depends only on the two different values of the BH principal quantum number and on the initial BH mass [4–6]. If one considers the case of an absorption instead, one uses the equation [4–6]

$$\begin{aligned} \Delta E_{n_2 \to n_1} &\equiv E_{n_1} - E_{n_2} = M_{n_2} - M_{n_1} = \\ &= \sqrt{M^2 - \tfrac{n_2}{2}} - \sqrt{M^2 - \tfrac{n_1}{2}} = -\Delta E_{n_1 \to n_2}. \end{aligned} \tag{3}$$

We also recall the following intriguing remark which finalizes the analogy between the current BH analysis and Bohr's hydrogen atom. The interpretation of Eq. (1) is of a particle, that is, the electron of the gravitational atom, which is quantized on a circle of length [4–6]

$$L = 4\pi \left(M + \sqrt{M^2 - \frac{n}{2}} \right). \tag{4}$$

Hence, one really finds the analogous of the electron traveling in circular orbits around the nucleus in Bohr's hydrogen atom. One sees that it is also

$$M_n = \sqrt{M^2 - \tfrac{n}{2}} \;. \tag{5}$$

,

Thus the uncertainty in a clock measuring a time t becomes, with the Planck mass is equal to 1 in planck units,

$$\frac{\delta t}{t} = \frac{1}{2M_n} = \frac{1}{\sqrt{M^2 - \frac{n}{2}}}, \tag{6}$$

which means that the accuracy of the clock required to record physics at the horizon depends on the BH excited state, which corresponds to the number of Planck masses it has.

Finally, for the sake of completeness and clarity, we stress the following for what concerns the coordinates used in this Section. The main point concerns time. Recall time must be considered a parameter in quantum mechanics instead of a quantum mechanical operator. In particular, time is not a quantum observable. In other words, in quantum mechanics it is not possible to define a time operator in the same way that we do, for example for the position operator and for the momentum operator. In fact, within the framework of quantum mechanics there is no room for a symmetric analysis for both time and position, even if, from an historical point of view, quantum mechanics has been developed by De Broglie and Schroedinger following the idea of a covariant analogy between time and energy on one hand and position and momentum on the other hand. This discussion works from the quantum mechanical point of view of the analysis. But, in the current analysis, we have to discuss about time also from the point of view of general relativity. As we the analysis in this Section arises from Hawking radiation as tunnelling in the framework [9], we recall that in such work the Painlev and Gullstrand coordinates for the Schwarzschild geometry have been used. On the other hand, the radial an time coordinates are the same in both the Painlev and Gullstrand and Schwarzschild line elements.

3. Near Horizon Spacetime and Collision of Black Holes

The quantum basis of black holes may be detected in gravitational radiation. Signatures of quantum modes may exist in gravitational radiation. Gravitational memory or BMS symmetries are one way in which quantum hair associated with a black hole may be detected. Conservation of quantum information suggests that quantum states on the horizon may be emitted or entangled with gravitational radiation and its quantum numbers and information. In what follows a toy model is presented where a black hole coalescence excites quantum hair on the stretched horizon in the events leading up to the merger of the two horizons. The model is the Poincare disk for spatial surface in time. To motivate this we look at the near horizon condition for a near extremal black hole.

The Reissnor-Nordstrom (RN) metric is

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega^2.$$

Here Q is an electric or Yang-Mills charge and m is the BH mass. In previous section, considering the Schwarzschild BH, we labeled the BH mass as M instead. The accelerated observer near the horizon has a constant radial distance.

For the sake of completeness, we recall that the Bohr-like approach to BH quantum physics has been also partially developed for the Reissnor-Nordstrom black hole (RNBH) in [13]. In that case, the expression of the energy levels of the RNBH is a bit more complicated than the expression of the energy levels of the Schwarzschild BH, being given by (in Planck units and for small values of Q) [13]

$$E_n \simeq m - \sqrt{m^2 + \frac{q^2}{2} - Qq - \frac{n}{2}}, \tag{7}$$

where q is the total charge that has been loss by the BH excited at the level n.

Now consider

$$\rho = \int_{r_+}^{r} dr\sqrt{g_{rr}} = \int_{r^+}^{r} \frac{dr}{\sqrt{1 - 2m/r + Q^2/r^2}}$$

with lower integration limit r_+ is some small distance from the horizon and the upper limit r removed from the black hole. The result is

$$\rho = m\,log[\sqrt{r^2 - 2mr + Q^2} + r - m] + \sqrt{r^2 - 2mr + Q^2}\Big|_{r_+}^{r}$$

$$= m\, log[\sqrt{r^2 - 2mr + Q^2} + r - m] + r\sqrt{g_{tt}} - \Lambda.$$

Here Λ is a large number evaluated within an infinitesimal distance from the horizon. One writes the metric at this position as

$$ds^2 = \left(1 - \frac{2m}{r(\rho)} + \frac{Q^2}{r(\rho)^2}\right) dt^2 - d\rho^2 - r(\rho)^2 d\Omega^2.$$

With the near horizon condition set $r^2 - 2mr + Q^2 \simeq 0$, so that

$$\rho \simeq m\, log(r - m) + r\sqrt{g_{tt}} - \Lambda.$$

Since $m \tilde{>} Q$ the divergence of the log cancels the arbitrarily large Λ

$$\frac{\rho}{r_+} \simeq \sqrt{g_{tt}}.$$

Now, one writes the metric as

$$ds^2 = \frac{\rho^2}{r_+^2} dt^2 - d\rho^2 - m^2 d\Omega^2$$

We observe that $d\rho^2 = dr^2/g_{tt}^2$ and replace in ρ/m for g_{tt} for the extremal condition $r_+ = m + \epsilon$, obtaining

$$ds^2 = \left(\frac{\rho}{m}\right)^2 dt^2 - \left(\frac{m}{\rho}\right)^2 dr^2 - r^2 d\Omega^2 \; for\; \rho = r. \tag{8}$$

The divergence from the above integration and that due to deviation from the extremal condition means $\rho/r_+ = \sqrt{g_{tt}} + \omega$ for ω the subtraction between two divergent quantities. This means there is a term $\omega^2 dt^2$ that we can absorbed into ds^2. However since $r = (m/\rho)\rho$ the metric in $t,\ \rho,\ \theta,\ \phi$ coordinates is

$$ds^2 = \left(\frac{\rho}{m}\right)^2 dt^2 - \left(\frac{m}{\rho}\right)^2 d\rho^2 - m^2 d\Omega^2, \tag{9}$$

This is the metric for $AdS_2 \times \mathbb{S}^2$ for AdS_2 in the $(t,\ \rho)$ variables tensored with a two-sphere $\mathbb{S}^2$ of constant radius $= m$ in the angular variables at every point of AdS_2. This metric was derived by Carroll, Johnson and Randall[14]. Generalizing this for a nonconstant radius of $\mathbb{S}^2$ leads to the AdS_4 metric.

Then a near horizon condition for a near extremal black hole is $ds^2 = ds^2(AdS_4) + d\tau^2$. The $d\tau^2$ is then a residual term left over from $log(r - m)$ and $-\Lambda$. An accelerated frame may be chosen so these terms cancel each other or are not a significant contribution. This means the accelerated observer witnesses an AdS_4 spacetime that is relatively unstable with respect to tuning of the acceleration parameter $g = c^2/(r - m)$ and the nearness to extremal condition $\epsilon = \sqrt{m^2 - Q^2}$. If $g \simeq 1/\epsilon$ then the AdS_4 spacetime is relatively stable. Stability is insured then on the stretched horizon of the extremal black hole. The deviation from extremal condition means the black hole will emit Hawking radiation, which translates into meaning the accelerated observer will witness positive energy radiation emerge from the negative vacuum energy of the AdS_4 as the vacuum becomes more negative. The metric clearly has the mass of the black hole m serving as the radius of curvature for the AdS_2 or generally AdS_4 and the emission of Hawking radiation with $m \to m - \delta m$ means the radius of curvature decreases and the Ricci curvature $1/m^2$ increases.

The accelerated observer witnesses a spacetime with a negative average curvature, which emits an abundance of radiation. This is from the quantum hair on the horizon, which is designated in the RN metric by the charge Q. This information is held close to the event horizon if the Yang-Mills (YM) field is confined such as quantum chromodynamics (QCD). This is in the extreme UV limit accessed by the accelerated observe close enough to witness this as high temperature radiation. This GW information produced by BH collisions will reach the outside world highly red shifted by the tortoise coordinate $r^* = r' - r - 2m\, ln|1 - 2m/r|$. For a 30 solar mass BH, which is mass of some of the BHs which produce gravitational waves detected by LIGO, the frequency of this ripple, as measured from the horizon to $\delta r \sim \lambda$

$$\delta r' = \lambda - 2m\, ln\left(\frac{\lambda}{2m}\right) \simeq 2 \times 10^6 m.$$

A ripple in spacetime originating an atomic distance 10^{-10}m from the horizon gives a $\nu = 150 Hz$ signal, detectable by LIGO[15]. Similarly, a ripple 10^{-13} to 10^{-17} cm from the horizon will give a 10^{-1}Hz signal detectable by the eLISA interferometer system[18]. Thus, quantum hair associated with QCD and electroweak interactions that produce GWs could be detected. More exact calculations are obviously required.

Following [16], one can use Hawking's periodicity argument from the RN metric in order to obtain an "effective" RN metric which takes into account the BH dynamical geometry due to the subsequent emissions of Hawking quanta as

$$ds^2 = -\left(1 - \frac{2\,(m - E_n)}{r} + \frac{Q - q}{r^2}\right) dt^2 + \left(1 - \frac{2\,(m - E_n)}{r} + \frac{(Q - q)^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{10}$$

which permits to write a dynamical expression for the frequency of the ripple as

$$\delta r' = \lambda - 2m - E_n\, ln\left(\frac{\lambda}{2\,(m - E_n)}\right) \tag{11}$$

These weak gravitons produced by BH hair would manifest themselves in gravitational memory. The Bondi-Metzner-Sachs (BMS) symmetry of gravitational radiation results in the displacement of test masses[19]. This displacement requires an interferometer with free floating mirrors, such as what will be available with the eLISA system. The BMS symmetry is a record of YM charges or potentials on the horizon converted into gravitational information. The BMS metric provide phenomenology for YM gauge fields, entanglements of states on horizons and gravitational radiation. The physics is correspondence between YM gauge fields and gravitation. The BHs coalescence is a process which converts qubits on the BHs horizons into gravitons.

Two BHs close to coalescence define a region between their horizons with a vacuum similar to that in a Casimir experiment. The two horizons have quantum hair that forms a type of holographic "charge" that performs work on spacetime as the region contracts. The quantum hair on the stretched horizon is raised into excited states. The AdS_2 is mapped into the corresponding AdS_3. Thus, the spatial region is a Poincare disk with the same $SL(2,\mathbb{R})$ symmetry. The manifold with genus g for charges has Euler characteristic $\chi = 2g - 2$ and with the 3 dimensions of $SL(2, \mathbb{R})$ this is the index $6g - 6$ for Teichmuller space[19]. The $SL(2, \mathbb{R})$ is the symmetry of the spatial region with local charges modeled as a $U(1)$ field theory on an AdS_3. The Poincare disk is then transformed into $\mathbb{H}^2_p$ that is a strip. The $\mathbb{H}^2_p \subset AdS_3$ is simply a Poincare disk in complex variables then mapped into a strip with two boundaries that define the region between the two event horizons.

4. AdS geometry in BH Coalescence

The near horizon condition for a near extremal black hole approximates an anti-de Sitter spacetime. In general this is AdS_4. In [14] the extremal blackhole replaces the spacelike region in (r_+, r_-) with $AdS_2 \times \mathbb{S}^2$. The result above suggests an accelerated observer on the stretched horizon of a near extremal black hole with $r_+ - r_- \simeq \ell_p$ witnesses a near AdS_4 spacetime. This AdS_4 is however perturbed, as seen with the above $1/\epsilon - \Lambda$ cancellation, and so is not zero temperature and thus generates radiation. In a perfect balance the acceleration of the observers is by $T \sim g$ zero, the black hole emits no Hawking radiation and the spacetime observed is AdS_4.

Now reduce the dimensions and consider AdS_3 in 2 plus 1 spacetime. The near horizon condition for a near extremal black hole in 4 dimensions is considered for the BTZ black hole. This AdS_3 spacetime is then a foliations of hyperbolic spatial surfaces in time. These surfaces under conformal mapping are a Poincare disk. The motion of a particle on this disk are arcs that reach the conformal boundary as $t \rightarrow \infty$. This is then the spatial region we consider the dynamics of a quantum particle. This particle we start out treating as a Dirac particle, but the spinor field we then largely ignore by taking the square of the Dirac equation to get a Klein-Gordon wave.

Define the z and $\bar{z}$ of the Poincare disk with the metric

$$ds^2_{p-disk} = R^2 g_{z\bar{z}} dz d\bar{z} = R^2 \frac{dz d\bar{z}}{1 - z\bar{z}}$$

with constant negative Gaussian curvature $\mathcal{R} = -4/R^2$. This metric $g_{z\bar{x}} = R^2/(1 - \bar{z}z)$ is invariant under the $SL(2, \mathbb{R}) \sim SU(1, 1)$ group action, which, for $g \in SU(1, 1)$, takes the form

$$z \rightarrow gz = \frac{az + b}{\bar{b}z + \bar{a}},\ g = \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix}. \tag{12}$$

The Dirac equation $i\gamma^\mu D_\mu \psi + m\psi = 0$, $D_\mu = \partial_\mu + iA_\mu$ on the Poincare disk has the Hamiltonian matrix

$$\mathcal{H} = \begin{pmatrix} m & H_w \\ H_w^* & -m \end{pmatrix} \tag{13}$$

for the Weyl Hamiltonians

$$H_w = \frac{1}{\sqrt{g_{z\bar{z}}}} \alpha_z \left(2D_z + \frac{1}{2}\partial_z (ln\ g_{z\bar{z}})\right),$$

$$H_w^* = \frac{1}{\sqrt{g_{z\bar{z}}}} \alpha_{\bar{z}} \left(2D_{\bar{z}} + \frac{1}{2}\partial_{\bar{z}} (ln\ g_{z\bar{z}})\right),$$

with $D_z = \partial_z + iA_z$ and $D_{\bar{z}} = \partial_{\bar{z}} + iA_{\bar{z}}$. here α_z and $\bar{\alpha}_z$ are the 2×2 Weyl matrices.

Now consider gauge fields, in this case magnetic fields, in the disk. These magnetic fields are topological in the sense of the Dirac monopole with vanishing Ahranov-Bohm phase. The vector potential for this field is

$$A^\phi = -i\frac{\phi}{2}\left(\frac{dz}{z} - \frac{d\bar{z}}{\bar{z}}\right).$$

the magnetic field is evaluated as a line integral around the solenoid opening, which is zero, but the Stokes' rule indicates this field will be $\phi(\bar{z} - z)/r^2$, for $r^2 = \bar{z}z$. A constant magnetic field dependent upon the volume $\mathbf{V} = \frac{1}{2} dz \wedge d\bar{z}$ in the space with constant Gaussian curvature $\mathcal{R} = -4/R^2$

$$\mathbf{A}^v = i\frac{BR^2}{4}\left(\frac{zd\bar{z} - \bar{z}dz}{1 - \bar{z}z}\right).$$

The Weyl Hamiltonians are then

$$H_w = \frac{1 - r^2}{R} e^{-i\theta} \left(\alpha_z \left(\partial_r - \frac{i}{r}\partial_\theta - \frac{\sqrt{\ell(\ell + 1)} + \phi}{r} + i\frac{kr}{1 - r^2}\right)\right)$$

$$H_w^* = \frac{1 - r^2}{R} e^{i\theta} \left(\alpha_{\bar{z}} \left(\partial_r - \frac{i}{r}\partial_\theta + \frac{\sqrt{\ell(\ell + 1)} + \phi}{r} + i\frac{kr}{1 - r^2}\right)\right), \tag{14}$$

for $k = BR^2/4$. With the approximation that $r << 1$ or small orbits the product gives the Klein-Gordon equation

$$\partial_t^2\psi = R^{-2}\left(\partial_r^2 + \frac{\ell(\ell+1)+\phi^2}{r^2} + k^2r^2 + (\ell(\ell+1)+\phi^2)k\right).$$

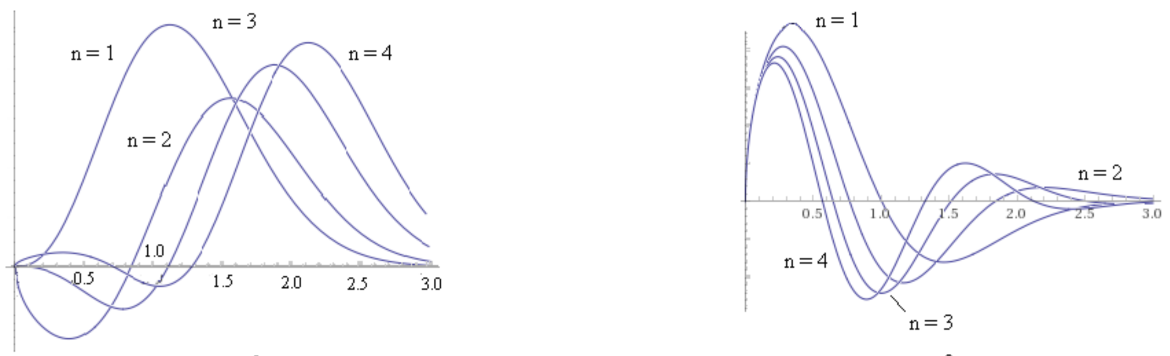
For $\ell(\ell+1)+\phi^2 = 0$ this gives the Weber equation with parabolic cylinder functions for solutions. The last term $(\ell(\ell+1)+\phi^2)k$ can be absorbed into the constant phase $\psi(r,t) = \psi(r)e^{-it\sqrt{E^2+\ell(\ell+1)+\phi^2}}$.

This dynamics for a particle in a Poincare disk is used to model the same dynamics for a particle in a region bounded by the event horizons of a black hole. With AdS black hole correspondence the field content of the AdS boundary is the same as the horizon of a black hole. An elementary way to accomplish this is to map the Poincare disk into a strip. The boundaries of the strip then play the role of the event horizons. The fields of interest between the horizons are assumed to have orbits or dynamics not close to the horizons. The maps is $z = tanh(\xi)$. The Klein-Gordon equation is then

$$\partial_t^2\psi = R^{-2}\left((1+2\xi^2)\partial_\xi\partial_{\bar\xi} + \frac{\ell(\ell+1)+\phi^2}{|\xi|^2} - k|\xi|^2\right)\psi, \tag{15}$$

where the ξ^2 is set to zero under this approximation. The Klein-Gordon equation is identical to the above.

hair image.PNG



Solution of the form $x^{1/4}e^{-x^2}H_n(x^2)$ given by parabolic cylinder function for n = 1, 2, ... 4 represented as a Hermite polynomial

Laguerre wave function $x^{1/4}e^{-x^2}L_n^0(x^2)$ for hydrogen atomic-like states for n = 1, 2, 3, 4.

These are the wave function components contributed by the parabolic cylinder functions, or Hermite polynomials and the Laguerre polynomials. These depend on $x^2 = k\xi^2$ so the wave function is radial. These are not normalized.

The solution to this differential equation for $\Phi = \ell(\ell+1)+\phi^2$ is

$$\psi = (2\xi)^{1/4(\sqrt{1-4\Phi}+1}e^{-\frac{1}{2}k\xi^2}\times$$

$$\left[c_1U\left(\frac{1}{4}\left(\frac{E^2R^2}{k} + \sqrt{1-4\Phi}+1\right), \frac{1}{2}(\sqrt{1-4\Phi}+1), k\xi^2\right) + c_2L^{\frac{1}{2}\sqrt{1-4\Phi}}_{\frac{E^2R^2}{k}+\sqrt{1-4\Phi}}(k\xi^2)\right].$$

The first of these is the confluent hypergeometric function of the second kind. For $\Phi = 0$ this reduces to the parabolic cylinder function. The second term is the associated Laguerre polynomial. The wave determined by the parabolic cylinder function and the radial hydrogen-like function have eigenmodes of the form in the diagram above.

The parabolic cylinder function $D_n = 2^{n/2}e^{-x^2/4}H_n(x/\sqrt{2})$ with integer n gives the Hermite polynomial. The recursion formula then gives the modes for the quantum harmonic oscillator. The generalized Laguerre polynomial $L^{2\ell+1}_{n-\ell-1}(r)$ of degree $n - \ell - 1$ gives the radial solutions to the hydrogen atom. The associated Laguerre polynomial with general non-integer indices has degree associated with angular momentum and the magnetic fields. This means a part of this function is similar to the quantum harmonic oscillator and the hydrogen atom. The two parts in a general solution have amplitudes c_1 and c_2 and quantum states in between the close horizons of coalescing black holes are then in some superposition of these types of quantum states.

To transition from the BTZ black hole to the 4-spacetime black hole we think of the $AdS_2 \times \mathbb{S}^2$ restricted as well to $AdS_2 \times \mathcal{P}$, for $\mathcal{P}$ a 1-dimensional principal bundle for a $U(1)$ gauge field. This gauge action is a lift on the AdS_2 a 3 dimensional total space. If we take the circle group $U(1)$ and map it to $\mathbb{R}^1$ with $SO(2) = \mathbb{R}^1/Z_2$ then the $\mathbb{R}^1$ as the internal space constructs a hyperbolic 2-space that is the Poincare disk foliated in time. This spacetime is AdS_3.

We introduce a gauge field into the AdS_2 with the introduction of an additional dimension. The total metric is

$$ds_3^2 = ds^2(AdS_2) - (dy + A_\mu dx^\mu)^2,$$

for $x^0 = t$ and $x^1 = x$ and the AdS_2 metric is

$$ds^2(AdS_2) = \left(\frac{r}{R}\right)^2 dt^2 - \left(\frac{R}{r}\right)^2 dr^2.$$

For a static monopole field we have $A_\mu dx^\mu = (R/r)dt$. The $dy = \nabla_\mu y dx^\mu$. This defines the gauge space of the field. We then have with the introduction of a gauge field extended this into one additional dimension. With the replacement $t' = rt/R$ and $y' = y + Rt/r$ this metric is

$$ds^2 = dt'^2 - dy'^2.$$

On this two dimensional space there are the generators of motion $i\partial_{t'}$ and $i\partial_{y'}$, which are defined according to their respective differential forms dt', dy'. This is extended into AdS_3 with $r' = r^2/R$.

This Hamiltonian

$$H = \frac{1}{2}|\pi|^2 - \frac{g}{r^2},\ \pi = -i\partial_r,$$

which contains the monopole field, describes the motion of a gauge particle in the hyperbolic space. In addition there is a contribution from the constant magnetic field $U = -kr^2/2$. Now convert this theory to a scalar field theory with $r \rightarrow \phi$ and $\pi = -i\partial_r\phi$. Finally introduce the dilaton operator D and the scalar theory consists of the operators

$$H_0 = \frac{1}{2}|\pi|^2 - \frac{g}{\phi^2},\ U = -k\frac{\phi^2}{2},\ D = \frac{1}{4}(\phi\pi + \pi\phi),$$

where $H_0 + U$ is the field theoretic form of the potential in equation16. These potentials then lead to the algebra

$$[H_0, U] = -2iD.\ [H_0, D] = -iH_0,\ [U, D] = iM.$$

This may be written in a more compact form with $L_0 = 2^{-1/2}(H_0 + U)$, which is the total Hamiltonian, and $L_\pm = 2^{-1/2}(U - H_0 \pm iD)$. This leaves the $SL(2, \mathbb{R})$ algebra

$$[L_0, L_\pm] = \pm iL_0,\ [L_+, L_-] = L_0. \tag{16}$$

This is the standard algebra $\sim \mathfrak{su}(2)$.

Given the presence of the dilaton operator this indicates conformal structure. The space and time scale as $(t, x) \rightarrow \lambda(t, x)$ and the field transforms as $\phi \rightarrow \lambda^\Delta\phi$. The measure of the integral $d^4x\sqrt{g}$ is invariant, where $\lambda = \partial x'/\partial x$ gives the Jacobian $J = det|\frac{\partial x'}{\partial x}|$ that cancels the $\sqrt{g}$ and the measure is independent of scale. In doing this we are anticipating this theory in four dimensions. We then simply have the scaling $\phi \rightarrow \lambda^{-1}\phi$ and $\pi \rightarrow \pi$, . For the potential term $-g/2\phi^2$ invariance of the action requires $g \rightarrow \lambda^{-2}g$ and for $U = -k\frac{\phi^2}{2}$ clearly $k \rightarrow \lambda^2 k$. This means we can consider this theory for 2 space plus 1 time and its gauge-like group $SL(2, \mathbb{R})$ as one part of an $SL(2, \mathbb{C}) \sim SL(2, \mathbb{R})^2$.

The differential equation number 15 is a modified form of the Weber equation $\psi_{xx} - (\frac{1}{4}x^2 + c)\psi = \; = 0$ The solution in Abramowit and Stegun are parabolic cylinder functions $D_{-a-1/2}(x)$, written according to hypergeometric functions. The ξ^{-1} part of the differential equation contributes the Laguerre polynomial solution. If we let $\xi = e^x$ and expand to quadratic powers we then have the potential in the variable x.

$$V(x) = (g + k)(1 + 2x^2) + 2(k - g)x$$

The Schrodinger equation for this potential with a a stationary phase in time has the parabolic cylinder function solution

$$\psi(x) = c_1 D_{\frac{\beta^2 - 4(\alpha + 2\sqrt{2}\alpha^{3/2})}{16\sqrt{2}\alpha^{3/2}}}\left(\frac{\beta + 4\alpha x}{\sqrt{2}(2\alpha)^{3/4}}\right) + c_2 D_{\frac{-\beta^2 - 4(\alpha - 2\sqrt{2}\alpha^{3/2})}{16\sqrt{2}\alpha^{3/2}}}\left(\frac{i(\beta + 4\alpha x)}{\sqrt{2}(2\alpha)^{3/4}}\right),$$

where $\alpha = g + k$ and $\beta = k - g$.

The field theory form of this will also have parabolic cylinder function solutions. The field theory with the field expanded as $\phi = e^{\chi}$ is expanded around unity so $\phi \simeq 1 + \chi + \frac{1}{2}\chi^2$. A constant C such that $C\chi$ is unitless is assumed or implied. The Lagrangian for this theory is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \alpha + \frac{1}{2}\mu^2\chi^2 + 2\beta\mu\chi.$$

The constant μ, standing for mass and absorbing α, is written for dimensional purposes. We then consider the path integral $Z = D[\chi]e^{-iS - i\chi J}$. Consider the functional differentials acting on the path integral

$$\left((p^2 + m^2)\frac{\delta}{\delta J} - 2i\beta\right)Z = -i\left\langle\frac{\delta S}{\delta\chi}\right\rangle,$$

where $\partial_\mu\chi = p_\mu\chi$. The Dyson-Schwinger theorem tells us that $\left\langle\frac{\delta S}{\delta\chi}\right\rangle = \langle J\rangle$ mean we have a polynomial expression $\langle\frac{1}{2}(p^2 + m^2)\chi - i\beta - J\rangle = 0$, where we can trivially let $J - i\beta \rightarrow J$. This does not lead to parabolic cylinder functions. There has been a disconnect between the ordinary quantum mechanical theory and the QFT. We may however, continue the expansion to quartic terms. This will also mean there is a cubic term, we may impose that only the real functional variation terms contribute and so only even power of the field define the Lagrangian

$$\mathcal{L} \rightarrow \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \alpha + \frac{1}{2}\mu^2\chi^2 + \frac{1}{4}\lambda\chi^4,$$

where $\frac{2}{3}\alpha \rightarrow \frac{1}{4}\lambda$. The functional derivatives are then

$$\left((p^2 + m^2)\frac{\delta}{\delta J} + \lambda\frac{\delta^3}{\delta J^3}\right)Z = -i\left\langle\frac{\delta S}{\delta\chi}\right\rangle,$$

This cubic form has three parabolic cylinder solutions. We may think of this as $ap + bp^3 = J$ and is a cubic equation for the source J that is annulled at three points. The correspond to distinct solutions with distinct paths. These three solutions correspond to three contours and define three distinct vacua. The over all action is a quartic function, which will have three distinct vacua, where one of these is the low energy physical vacua. It is worth noting this transformation of the problem has converted it into a system similar to the Higgs field.

This system with both harmonic oscillator and a Coulomb potentials is conformal and it maps into a system with parabolic cylinder functions solutions. In effect there is a transformation $harmonic~oscillator~states \leftrightarrow hydrogen - like~states$. The three solutions would correspond to the continuance of conformal symmetry, but where the low energy vacuum for one of these may not appear to be conformally invariant.

This scale transformation above is easily seen to be the conformal transformation with $\lambda = \Omega$. The scalar tensor theory of gravity for coupling constant $\kappa = 16\pi G$

$$S[g,\,\phi] = \int d^4x\sqrt{g}\left(\frac{1}{\kappa}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right). \tag{17}$$

This then has the conformal transformations

$$g'_{\mu\nu} = \Omega^2 g_{\mu\nu},\, \phi' = \Omega^{-1}\phi,\, \Omega^2 = 1 + \kappa\phi^2.$$

with the transformed action

$$S[g',\,\phi'] = \int d^4x\sqrt{g'}\left(\frac{1}{\kappa}R' + \frac{1}{2}g'^{\mu\nu}\partial_\mu\phi'\partial_\nu\phi' + V(\phi') + \frac{1}{12}R\phi'^2\right). \tag{18}$$

There is then a hidden $SO(3,\,1) \simeq SL(2,\,\mathbb{C})$ symmetry. Given an internal index on the scalar field ϕ^i there is a linear $SO(n)$ transformation $\delta\phi^i = C^{ijk}\phi_j\delta\tau_k$ for τ_k a parameter. There is also a nonlinear transformation from equation 19 as $\delta\phi^i = (1 + \kappa\phi^2)^{1/2}\kappa\delta\chi^i$ for χ^i a parameterization. In the primed coordinates the scalar field and metric transform as

$$\delta\phi^i = \delta\tau^i - \kappa\phi'^i\phi^j\delta\chi^j$$

$$\delta g_{\mu\nu} = \frac{2g'_{\mu\nu}\kappa\phi'^i\delta\chi^i}{1 - \kappa\phi'^2}. \tag{19}$$

The gauge-like dynamics have been buried into the scalar field. With this semi-classical model the scalar field adds some renormalizability. Further this model is conformal. The conformal transformation mixes the scalar field, which is by itself renormalizable, with the spacetime metric. Quantum gravitation is however difficult to renormalize. Yet we see the linear group theoretic transformation of the scalar field in $SO(n)$ is nonlinear in $SO(n,\,1)$.

Conformal symmetry is manifested in sourceless spacetime, or spatial regions without matter or fields. The two dimensional spatial surface in AdS_3 is the Poincare disk that with complexified coordinates has metric with $SL(2,\,\mathbb{R})$ algebraic structure. This may of course be easily extended into $SL(2,\,\mathbb{C})$ as $SL(2,\,\mathbb{R}) \times SL(2,\,\mathbb{R})$. In this conformal setting quantum states share features similar to the emission of photons by a harmonic oscillator or an atom. The orbits of these paths are contained in regions bounded by hyperbolic surfaces, or arcs for the two dimensional Poincare disk. The entropy associated with these arcs is a measure of the area contained within these curves. This is in a nutshell the Mirzakhani result on entropy for hyperbolic curves.

This development is meant to illustrate how radiation from black holes is produced by quantum mechanical means not that different from bosons produced by a harmonic oscillator or atom. Hawking radiation in principle is detected with a wavelength not different from the size of the black hole. The wavelength approximately equal to the Schwarzschild radius has energy $E = h\nu$ corresponding to a unit mass emitted. The mass of the black hole is n of these units and it is easy to find $m_p = \sqrt{\hbar c/G}$. These modes emitted are Planck units of mass-energy that reach $\mathcal{I}^\infty$. In the case of gravitons, these carry gravitational memory. For the coalescence of black holes gravitational waves are ultimately gravitons. For Hawking radiation there is the metric back reaction, which in a quantum mechanical setting is an adjustment of the black hole with the emission of gravitons. The emission of Hawking radiation might then be compared to a black hole quantum emitting a Planck unit of black hole that

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then decays into bosons. The quantum induced change in the metric is a mechanism for producing gravitons.

In the coalescence of black holes the quantum hair on the stretched horizons interacts with the vacuum and generates quanta. In general these are gravitons. We might see this as not that different from a scattering experiment with two Planck mass black holes. These will coalesce, form a larger black hole, produce gravitons, and then quantum states excited by this process will decay. The production of gravitons by this mechanism is affiliated with normal modes in the production of gravitons, which in principle is not different from the production of photons and other particles by other quantum mechanical processes. I fact quantum mechanical processes underlying black hole coalescence might well be compared to nuclear fusion.

The 2 LIGOs plus now the VIRGO detector are recording and triangulating the positions of distant black hole collisions almost weekly. This information may contain quantum mechanical information associated with quantum gravitation. This information is argued below to contain BMS symmetries or information. This will be most easily detected with a space based system such as eLISA, where the shift in metric positions of test masses is most readily detectable. However, preliminary data with the gross displacement of the LIGO mass may give preliminary information as well.

5. Foundation Issues

Quantum hair is a set of quantum fields that build up quantum gravitation, in the manner of gauge-gravity duality and BMS symmetry. This is holography, with the fields on the horizons of two BHs that determine the graviton/GW content of the BH coalescence. A detailed analysis of this may reveal BMS charges that reach $\mathcal{I}^+$ are entangled with Hawking radiation by a form of entanglement swap. In this way Hawking radiation may not be entangled with the black hole and thus not with previously emitted Hawking radiation. This will be addressed later, but a preliminary to this idea is seen in[22], for disentanglement between Hawking radiation and a black hole. The authors are working on current calculations where this is an entanglement swap with gravitons. The black hole production of gravitons in general is then a manifestation of quantum hair entanglement.

Small deviations from the classical result may occur at the peak of the received signal. It will also manifest itself in the gravitational memory observed in the post detection position of test masses. The construction of spacetime from entangled quantum fields will manifest itself in the adjusted position of test masses as a signature of how quantum entangled fields have restitched the fabric of spacetime according to vacuum states within it. These quantum states are holographic projections from quantum hair on stretched horizons. Gravitons $g \sim b^\dagger_+ b^\dagger_\times e^{ikx}$ are produced with interactions from black hole hair, say from two gluons that form a colorless entanglement or bound state, with maybe some STU transformation of their opposite color charges, which are signatures of metric back reaction. These gravitons will have properties, as yet not fully understood, which may be measured by a gravitational wave antenna capable of detecting memory or metric change.

It is illustrative for physical understanding to consider a linearized form of gravitational memory. Gravitational memory from a physical perspective is the change in the spatial metric of a surface according to[23]

$$\Delta h_{+.\times} = \lim_{t\to\infty} h_{+,\times}(t) - \lim_{t\to-\infty} h_{+,\times}(t).$$

Here $+$ and x refer to the two polarization directions of the GW. See [24] for more on this. Consider the GW as being a linear form of diphotons or colorless state of two gluons, where each photon (or gluon) has a generic state,

$$|\Psi_{+,\times}\rangle = |h_+(t) + ih_\times(t)\rangle = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} Y_{lm}[(\theta,\phi)\,(|\uparrow\rangle_+ + |\downarrow\rangle_+) - i(\theta,\phi)\,(|\uparrow\rangle_\times + |\downarrow\rangle_\times)],$$

where the arrows indicate the polarization directions according to their respective axes. The matrix element $H_{+,\times} = |\Psi_{+,\times}\rangle\langle\Psi_{+,\times}|$ describes the interaction of the GW with a quantum particle. This expanded out is

$$H_{+,\times} = \sum_{l,l'=2}^{\infty}\sum_{m=-l}^{l}\sum_{m'=-l'}^{l'} Y_{lm}(\theta,\phi)Y_{l'm'}(\theta,\phi)\left[(|\uparrow\uparrow\rangle_{+\times} + |\downarrow\downarrow\rangle_{+\times}) + i(|\uparrow\downarrow\rangle_{+\times} - |\downarrow\uparrow\rangle_{+\times})\right]$$

This tensor operation sets to zero terms like $|\rangle_{++}$ and $|\rangle_{\times\times}$ as unphysical states. This matrix contains a GW term $|\uparrow\uparrow\rangle_{+\times} + |\downarrow\downarrow\rangle_{+\times}$ plus a scalar term $|\uparrow\downarrow\rangle_{+\times} - |\downarrow\uparrow\rangle_{+\times}$ that again we set to zero. This is a linearized model for how a gravitational wave can change the isotropy of a distribution of test masses. This change defines entropy $S = -H_{+,\times}log(H_{+,\times})$ that with correspondence between black holes and AdS is related to Mirzakhani's entropy measure in hyperbolic spaces.

The entanglement entropy of CFT_2 entropy with AdS_3 lattice spacing a is

$$S \simeq \frac{R}{4G}ln(|\gamma|) = \frac{R}{4G}ln\left[\frac{\ell}{L} + e^{2\rho_c}sin\left(\frac{\pi\ell}{L}\right)\right].$$

where the small lattice cut off avoids the singular condition for $\ell = 0$ or L for $\rho_c = 0$. For the metric in the form $ds^2 = (R/r)^2(-dt^2 + dr^2 + dz^2)$ the geodesic line determines the entropy as the Ryu-Takayanagi (RT) result[1]

$$S = \frac{R}{2G}\int_{2\ell/L}^{\pi/2}\frac{ds}{sin\ s} = -\frac{R}{2G}ln[cot(s) + csc(s)]\Big|_{2\ell/L}^{\pi/2}$$
$$\simeq \frac{R}{2G}ln\left(\frac{\ell}{L}\right),$$

which is the small ℓ limit of the above entropy.

The RT result specifies entropy, which is connected to action $S_a \leftrightarrow S_e$[25]. Complexity, a form of Kolmogoroff entropy [26] is $S_a/\pi\hbar$ which can also assume the form of the entropy of a system $S \sim k\,log(dim\,\mathcal{H})$ for $\mathcal{H}$ the Hilbert space and the dimension over the number of states occupied in the Hilbert space. We may also see complexity as the volume of the Einstein-Rosen bridge [27] vol/GR_{ads} or equivalently the RT area $\sim vol/R_{AdS}$. We have an equivalency of such entropy or complexity according to the geodesic paths in hyperbolic $\mathbb{H}^2$ by geometric means [19]. The near horizon condition for coalescing black holes, in particular with the final 10^{-20}sec to the Planck time unit, is similar to the AdS spacetime, and the physics of field therein is determined by entropy defined in hyperbolic arcs or sections, and this is for quantum hair a holographic realization of quantum entropy. The interaction of this hair in the merging of horizons then produces gravitons.

The detection of quantum hair in gravitational radiation may then put quantum gravity theories to a test. The coalescence of black holes in the universe as potential Planck scale colliders means there is some prospect for testing aspect of quantum gravitation. BH coalescence is a way of probing the event horizon or holographic screen. Such measurements might give information on the firewall problem. It may be the occurrence of BMS charges at $\mathcal{I}^+$ is a nonlocal physics that removes the firewall.

6. Conclusion

The generation of gravitational waves should have an underlying quantum mechanical basis. It is sometimes argued that spacetime physics may not be at all quantum mechanical. This is probably a good approximation for energy sufficient orders of magnitude lower than the Planck scale. However, if we have a scalar field that define the metric $g' = g'(g,\ \phi)$ with action $S[g,\ \phi]$ then a quantum field ϕ and a purely classical g means the transformation of g by this field has no quantum physics. In particular for a conformal theory $\Omega = 1 + \kappa\phi^a\phi^a$, here a an internal index, the conformal transformation $g'_{\mu\nu} = \Omega^2 g_{\mu\nu}$ has no quantum content. This is an apparent inconsistency. For the inflationary universe the line element

$$ds'^2 = g'_{\mu\nu}dx^\mu dx^\nu = \Omega^2(du^2 - d\Sigma^{(3)})$$

with $dt/du = \Omega^2$ gives a de Sitter-like line element that expands space with $\Omega^2 = e^{t\sqrt{\Lambda/3}}$. The current slow accelerated universe we observe is approximately of this nature. The inflaton scalars are then fields that stretch space as a time dependent conformal transformation and are quantum mechanical.

The generation of gravitational waves will then ultimately be the generation of gravitons. Signatures of these quantum effects in black hole coalescence will enatail the measurement of quantum information. Gravitons carry BMS charges and these may be detected with a gravitational wave interferometer capable of measuring the net displacement of a test mass. The black hole hair on the stretched horizon is excited by the merger and these results in the generation of gravitons. The Weyl Hamiltonians in equation 14 depend on the curvature as $\propto \sqrt{\mathcal{R}}$. For the curvature extreme during the merging of black holes this means many modes are excited. The two black holes are pumped with energy by the collision, this generates or excites more modes on the horizons, where this results in a black hole with a net larger horizon area. This results in a metric response, or equivalently the generation of gravitons.

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