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Spatial Molecular AlO Temperature Distributions in Laser-Induced Plasma

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Abstract: Spatially resolved, line-of-sight measurements of aluminum monoxide emission spectra in laser ablation plasma are used with Abel inversion techniques to extract radial plasma temperatures. Contour mapping of the radially deconvolved signal intensity shows a ring of AlO formation near the plasma boundary with the ambient atmosphere. Simulations of the molecular spectra were coupled with the line profile fitting routines. Temperature results are presented with simultaneous inferences from lateral, asymmetric radial, and symmetric radial AlO spectral intensity profiles. This analysis indicates that we measured shockwave phenomena in the radial profiles, including a temperature drop behind the blast wave created during plasma initiation.

Keywords: Molecular Spectroscopy; Diatomic Spectroscopy; Abel Transform; Plasma Spectroscopy; Laser-Induced Breakdown Spectroscopy; Laser-Induced Plasma; Plasma Dynamics

1. Introduction

The act of tightly focusing laser radiation on to a target volume initiates plasma coupled with the propagation of a shockwave. Optical spectroscopy measurements of the dynamics are made in an effort to understand the chemistry kinetics of laser ablation plasma. Development of line-of-sight diagnostics for such systems are advantageous due to the numerous applications that are of interest, including scaling laws between high explosive events and laser-induced plasma shock, temperature, and electron density phenomena [1,2], molecular formation of nanocluster formation in laser plasma plumes [3], pulsed laser vapor deposition [4,5], and analytical applications of laser-induced breakdown spectroscopy (LIBS) [6,7]. In each of these applications the distribution of atomic and molecular emissions and the associated plasma state quantities, i.e., electron density and temperature, are of interest. In particular, correlations between spatial distributions molecular emission intensities and subsequent temperature inferences and the shock phenomena associated with laser-induced plasma are desirable. This information is most desirable for laser produced plasma at atmospheric conditions, as opposed to vacuum conditions, where the plasma and shock dynamics are more complex due to plume splitting, and confinement.

We consider asymmetric Abel inverted, diatomic molecular aluminum monoxide (AlO) emission spectra measured from laser produced plasma on the surface of an Al target sample in ambient laboratory air as a method for extracting further spatial distribution information about laser ablation plasma. Abel transformations and the more generalized Radon transformation relate lateral, line of sight measurements of the plasma emission intensity to the radial intensity distribution through well-known integral transformations [8,9]. This is done in an effort to extract information about the radial distributions of the molecular emission. Radon transformation techniques are commonly applied to laser ablation scenarios [10,11], however due to the more general nature of the Radon transformation, measurements made along multiple lines-of-sight are typically required. The Abel inversion assumes some type of symmetry, which alleviates the need for multiple line-of sight measurements. For a cylindrically symmetric emission source, the Abel transformation is given as

$$I(z, \lambda) = 2 \int_r^R I(r, \lambda) \frac{r dr}{\sqrt{r^2 - z^2}}. \quad (1)$$

37 Here, $I(z, \lambda)$ is the lateral spectral measurement and $I(r, \lambda)$ is the radial reconstruction. The upper
38 limit of the integral, R , is often carried out to be much greater than the extent of the plasma. The
39 integral pre-factor of 2 indicates the plasma symmetry along the radial and lateral axes.

40 The desire in attempting to apply Abel reconstructions over the Radon deconvolutional methods
41 is to adopt a procedure that is more widely applicable given the ease with which a single lateral,
42 spectral measurement can be made along a single axis of the plasma plume. Further, numeric methods
43 for implementing Abel transformation are plenty and can be applied with little computational expense.
44 Asymmetries are included by using a procedure developed by Blades [12], originally applied to
45 inductively coupled plasma with slightly asymmetric spectral emissions. This method focusses on
46 the creation of a symmetric profile from two plasma emission halves that are symmetric, yet show
47 different symmetries. Particular to laser-induced plasma, algorithmic implementations of asymmetric
48 Abel inversions exist [13–15], in which the emission intensity is decomposed into symmetric and
49 asymmetric components. The method of asymmetric inclusion for this work is selected for the relative
50 ease with which the method can be implemented. Consequences of applying the asymmetric Abel
51 inversions are considered for application of inferring the plasma temperature from the measured,
52 lateral AIO emission profiles.

53 2. Experiment

54 Spectra were collected following laser-induced breakdown initiated in ambient laboratory air
55 using a Quantel Ultra Light Q-switched Nd:YAG laser with a pulse width of 8.5 ns, as measured by a
56 fast silicon photodiode. The laser operated at the fundamental wavelength of 1064 nm with an average
57 energy per pulse of 38 mJ at the ablation site. The ablation target was the narrow edge of 5 cm square
58 aluminum alloy 6061 sheet that was approximately 1 mm thick. The AIO spectra are imaged onto the
59 slit of a Princeton instruments Isoplane SCT 320 spectrometer installed with an 1800 grooves/mm
60 grating and a focal length of 0.33 meters. Spatially and temporally resolved spectral images of the
61 ablation event were recorded using a Princeton Instruments PiMax2 ICCD. The ICCD had a pixel
62 arrangement of 256x10245 vertical by horizontal pixels. The horizontal axis was used to achieve the
63 desired spectral resolution and the vertical axis was used to achieve spatial resolution along the plasma
64 plume expansion height. Groups of 2 vertical pixels were binned together in order to improve the
65 signal quality of the AIO emissions with respect to the measurement noise. The detector pixel size is
66 $25 \mu\text{m}$ square such that in the binned configuration the effective pixel size was $50 \times 25 \mu\text{m}$. The imaging
67 of the spectrometer-detector arrangement is such that the breakdown plasma is magnified at a ratio
68 of 1:1.8. This was determined using a negative of the 1951 USAF target. The spectral resolution of
69 the spectrometer-detector arrangement was approximately 0.1 nm in the spectral region between 480
70 to 500 nm. The spectra used for analysis were the result of 100 accumulations on a single ablation
71 site. The spectrometer-detector arrangement was calibrated for the system wavelength response using
72 a Hg/Ne-Ar lamp and the system intensity response using a tungsten lamp with a known spectral
73 intensity response curve. Time synchronization of the system was achieved by synchronizing the
74 ICCD gate opening to the Q-switch of the laser.

75 3. Results and Discussion

76 Figure 1 shows contour mappings of spatially resolved measurements of AIO spectra between
77 time delays of 20 and 35 μs following optical breakdown along the axis of laser incidence. The ablation
78 surface is located at a slit height of 1.0 mm. Here we note that slit height units refer to the image plane
79 measurement and not the exact positions of the spectral emissions in the plasma. Detailed ray tracings
80 of the spectral imaging system (non trivial) would be required to correlate image plane units to the
81 plasma emission source. Initially the AIO signal intensity is relatively isotropic between slit heights of

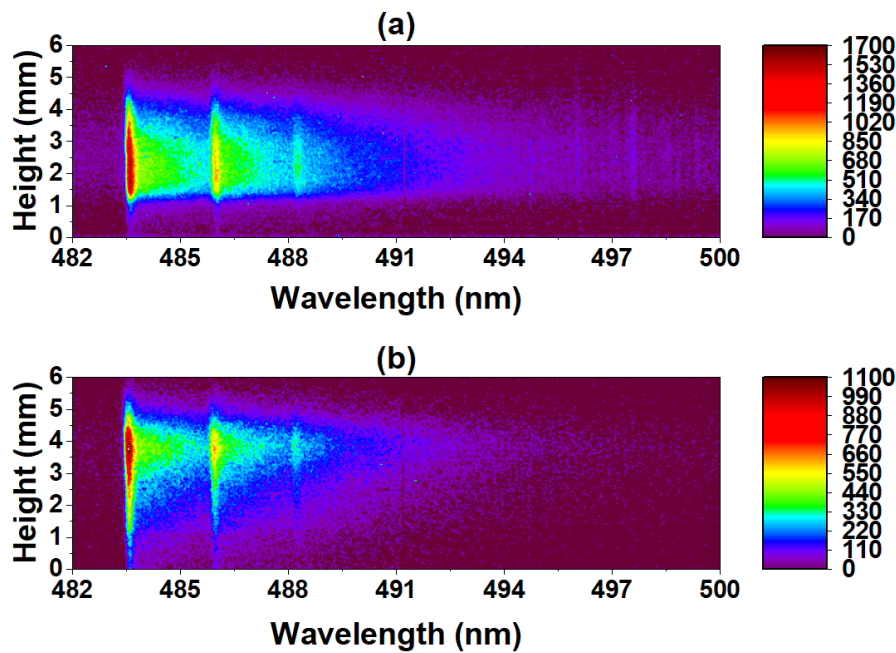


Figure 1. Contour mappings of spatially resolved AIO spectral measurements along the plasma plume height measured at (a) 20 μ s and (b) 35 μ s time delays.

82 1.5 and 3.5 mm (Fig.1a). As the time delay increases, the spectroscopic intensity is seen to diminish,
 83 which is roughly correlated with the decay of the laser-induced plasma. As the time delay increases,
 84 areas of larger AIO signal intensity farther from the ablation surface, see 3.5 to 4 mm slit heights in
 85 Fig1(b). These areas of increased intensity also appear to be asymmetric, with stronger emissions
 86 occurring in the higher half of the plume, which is also the direction of the plasma plume and plasma
 87 formation blast wave propagation. The asymmetry of the axial, spectroscopic lines-of-sight likely
 88 indicates asymmetries in the radial profiles of the AIO emissions and the need for an asymmetric Abel
 89 deconvolution.

The radial intensity distribution of plasma emissions are extracted through application of the Abel inversion under the assumption of cylindrical symmetry. Under such a scenario the Abel inverse transform is given by

$$I(r, \lambda) = \frac{-1}{\pi} \int_r^R \frac{dI(z, \lambda)}{dz} \frac{dz}{\sqrt{z^2 - r^2}}, \quad (2)$$

90 where z is the lateral coordinate along the light of sight, r is the radial coordinate, $I(z, \lambda)$ is the line of
 91 sight measurement, and $I(r, \lambda)$ is the deconvolved radial intensity profile.

92 In order to obtain an Abel inversion that accounts for minor asymmetries in the plasma spectral
 93 emissions, we consider the procedure introduced by Blades, in which Abel inversion symmetry is
 94 assigned to upper and lower plasma spectral regions [12]. These two regions are used to generate a
 95 symmetric intensity profile,

$$I_0(z, \lambda) = \frac{I(+z, \lambda) + I(-z, \lambda)}{2} \quad (3)$$

96 by averaging the upper and lower plasma regions. The total inverted profile is given by

$$I(\pm r, \lambda) = G(\pm z, \lambda) I_0(r, \lambda) \quad (4)$$

97 where $I_0(r, \lambda)$ is the Abel inversion of $I_0(z, \lambda)$ and $G(\pm z, \lambda)$ is given by

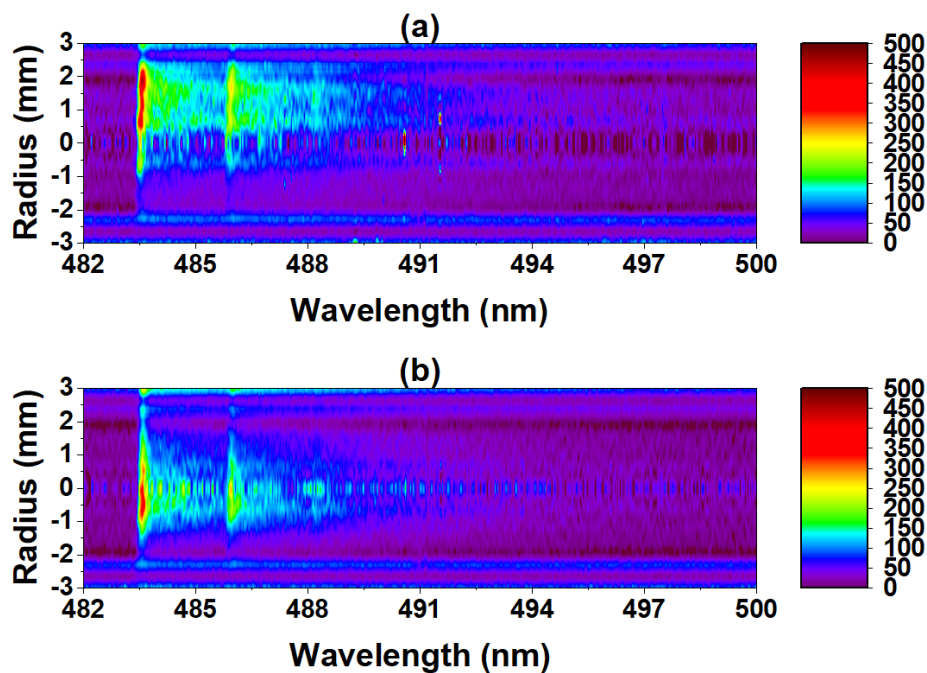


Figure 2. Contour mappings of extracted radial deconvolutions of measured AIO spectra at (a) 20 μ s and (b) 35 μ s time delays.

$$G(\pm z, \lambda) = \frac{I(\pm z, \lambda)}{I_0(r, \lambda)}. \quad (5)$$

98 This factor accounts for difference between the upper and lower half plasma regions and the
 99 symmetrized axial intensity distribution.

100 Application of the above procedure was carried out on the emission spectra depicted in the
 101 spectral contour mappings presented in Figure 1. The results of the asymmetric Abel inversion are
 102 presented in spatial-spectral contour maps in Figure 2. The method by which the Abel inversion of the
 103 symmetrized axial profile represented by Equation 2 uses a series expansion of orthogonal polynomials
 104 in which series expansion coefficients are determined through minimization techniques [16]. The
 105 orthogonal set of polynomials is taken to be Chebyshev polynomials of the first kind. The number of
 106 terms to be used in the procedure is 10. This maintains a decent fidelity of the deconvolved profile
 107 while avoiding potential oscillatory behavior and noise addition by considering significantly more
 108 series terms. The advantage of such a method is it alleviates the need to numerically compute the
 109 derivative presented in Equation 1 by way of the series expansion approximation. Such advantages
 110 exist in other Abel inversion implementations as well [17]. The Abel inversion is performed through
 111 use of a formalized MatLab script [18].

112 The largest source of error in the deconvolution procedure results from considering too small
 113 of a radius or applying the deconvolution to too small of an axial region. Attempting to apply the
 114 deconvolution procedure with a smaller axial region than the plasma radius will result in an inflated
 115 deconvolved signal intensity. By considering a smaller region of the plasma, oscillatory behavior from
 116 the Abel inversion algorithm becomes prevalent in the radial profile. Also, edge effects tend to occur,
 117 whereby the radial deconvolution tends toward arbitrarily large values for the largest radius values in
 118 the deconvolution. Given the limitations in the size of an ICCD detector vs. the plasma image size,
 119 some of these effects may not be completely overcome in the outlined procedure. These effects were
 120 reduced in magnitude by applying the deconvolution procedure over most of the detector height,

121 using slit heights from 0 to 6 mm. This was done since the AIO emission intensities are centered about
 122 a slit height of 3 mm. The calculated radial profiles correspond with this axial position.

123 The noisy nature of the deconvolution procedure is evident in the contour images of Figure 2,
 124 which is depicted by the fuzziness of the images. Potential edge effects are also apparent in each of
 125 the spectroscopic contour mappings and each favor a particular radial direction. Namely, the positive
 126 radial direction are favored. The positive direction corresponds to contributions from the lower plasma
 127 region. Reductions in quality of the deconvolution in this direction occur due to the presence of the
 128 aluminum surface.

129 Temperatures are determined from the molecular AIO emissions by fitting simulated AIO spectra
 130 to the line of sight measurements and radially deconvolved spectra. The theory spectra are simulated
 131 by making use of accurately compiled tables of line strengths for the diatomic molecule of interest [19].
 132 In short, the intensity of a measured molecular line from upper state, u , to lower state, l , is given by

$$I_{ul} = \frac{16\pi^3 c (a_0 e)^2 C_{abs} N_0}{3\epsilon_0 Q} C_v \nu_{ul}^4 S_{ul} e^{hF_u/kT} \quad (6)$$

133 where a_0 is the Bohr radius, e , is the electronic charge, c is the speed of light, Q is the partition function,
 134 N_0 is the total population of the species, C_{abs} and C_v are the absolute and relative calibration factors, h
 135 is Planck's constant, F_u is the upper term value, k is Boltzmann's constant, and T is the temperature.
 136 The term S_{ul} is the diatomic line strength and is calculated in a factorized form to account for electronic,
 137 vibrational, and rotational molecular structure. Detailed procedures for calculating a spectrum from
 138 the diatomic line strength are outlined in References [20–22].

139 Given the temperature dependence of the molecular spectrum, a fitting routine was used to extract
 140 the temperature of each temporally and spatially resolved AIO spectrum. Fitting was performed
 141 using a Trust-Region fitting routine with fitting parameters of temperature, line amplitude, and
 142 linear offset [23,24]. Additionally, the spectral resolution is a variable parameter that is used to
 143 establish the uncertainty of the inferred temperature from diatomic line profile fitting. Following
 144 initial fitting of each spectrum, the linear offset and spectral resolution parameters were randomly
 145 varied 1000 times and subsequently re-fit to generate a distribution of possible temperatures for each
 146 AIO spectrum. An additional fit was performed in which the line amplitude, temperature, linear
 147 offset, and spectral resolution were fit parameters. A total 1002 inferred temperatures were used to
 148 establish $1-\sigma$ uncertainties according to the so-called Three Sigma Rule (68-95-99 Rule) for Gaussian
 149 distributions[25].

150 Figures 3 and 4 show calculated temperatures both in the lateral direction on the plasma plume
 151 height and in the deconvolved radial directions for 20 μ s and 35 μ s time delay following plasma
 152 initiation, respectively. These times were chosen to illustrate the different behaviors of the plasma
 153 expansions at two time delays that were as distinct as possible from each other. For the sake of validity
 154 of application of the asymmetric Abel inversion method to molecular emissions in laser ablation
 155 plasma, temperature inferences in which a symmetric Abel inversion was calculated are also presented.
 156 The shaded region surrounding the symmetric Abel deconvolution (red dashed line) in Figures 3 and
 157 4 represents the uncertainty of this temperature inference.

158 The axial temperatures at both 20, Fig. 3(b) and 35, Fig. 4(b) μ s do not show much structure
 159 beyond the calculated $1-\sigma$ uncertainties and show relatively constant temperatures of approximately
 160 3400 and 3100 K, respectively, when averaging along the radial and axial directions. The displayed
 161 spectra and spatial temperature expansions of central part of the plume. The spatial temperature
 162 distributions along the axial coordinates in Figs. 3(b) and (4(b) indicate slightly higher temperatures
 163 near the boundaries, especially away from the target indicating the remnants of the laser-driven plasma
 164 excitation. Similarly, Figs. 3(a) and 3(b) reveal slightly higher temperature near the edges, and with
 165 larger variation away from the target as indicated by the asymmetric results. The larger uncertainties
 166 in the negative radial directions correspond to weaker AIO signals in the upper most regions of the
 167 spatially resolved measurement (see Fig. 1). The axial temperature expansion of the AIO molecules is

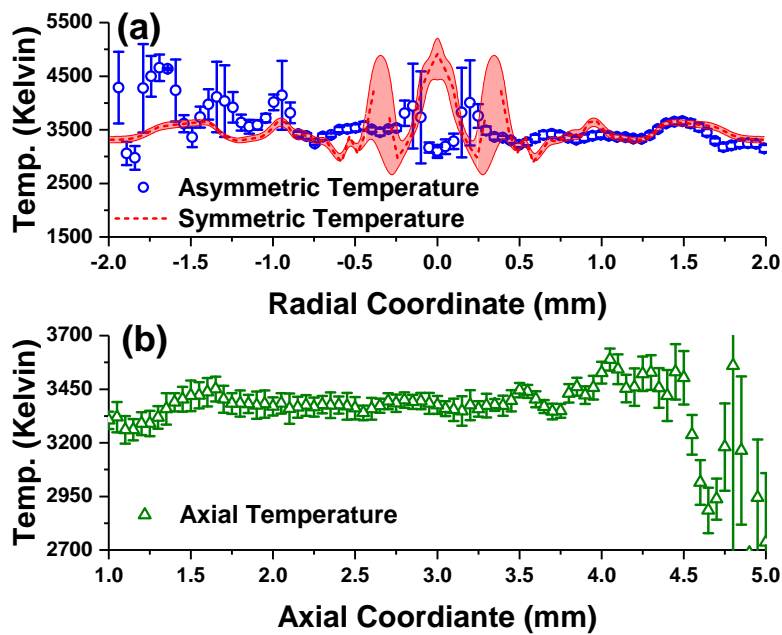


Figure 3. Temperatures inferred from AIO spectra at a time delay of $20 \mu\text{s}$ following ablation (a) along the plasma radius using asymmetric and symmetric Abel inverted spectra and (b) along the plasma using spatially resolved line-of-sight spectral measurements.

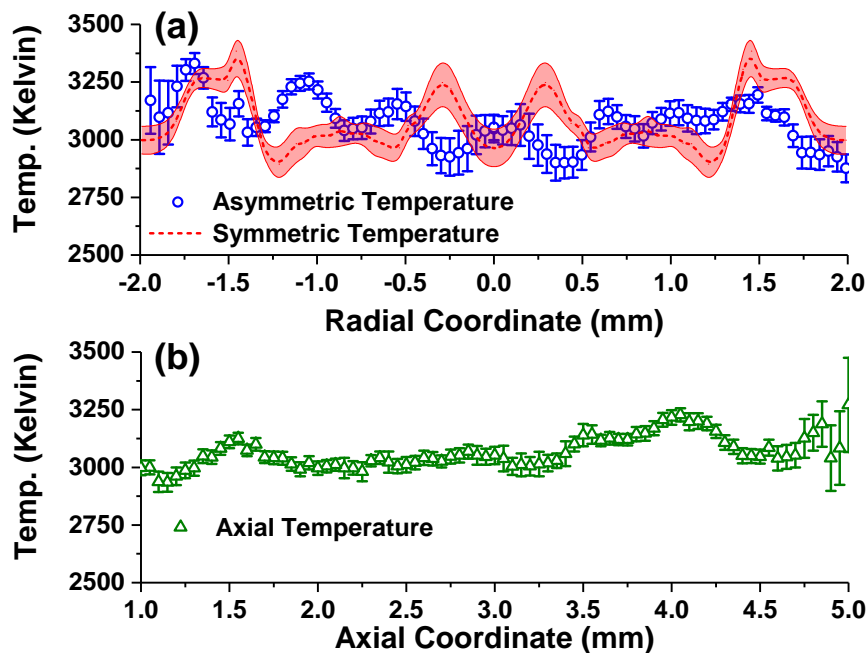


Figure 4. Temperatures inferred from AIO spectra at a time delay of $35 \mu\text{s}$ following ablation (a) along the plasma radius using asymmetric and symmetric Abel inverted spectra and (b) along the plasma using spatially resolved line-of-sight spectral measurements.

168 of the order of 80 to 100 m/s, but the radial expansion appears to occur at lower speeds towards the
 169 plasma center.

170 For nominal 100 mJ, 10 ns pulsed laser radiation, the shock wave expands at speeds of the order
 171 of $1 \text{ mm}/\mu\text{s}$ or 1000 m/s ($\text{Ma}=3$) at $1 \mu\text{s}$ time delay. For 38 mJ pulses the shock wave radius will be
 172 smaller according to the Taylor-Sedov blast wave model [26,27]. Moreover, for the investigated time
 173 delays of 20 and $35 \mu\text{s}$, the Taylor-Sedov blast wave model predicts a shock wave radius of 2.7 and

174 3.4 mm, respectively, moving at speeds approaching the speed of sound ($Ma=1$). Therefore, the axial
175 temperature profiles show in part, the effects of the shock wave, indicated near 2.5 mm from center (or
176 at the abscissa of 5 mm) in Fig. 3(b), but the shock wave appears to be just outside the investigated
177 axial position in Fig 4(b). Consequently, a temperature drop near the shock wave from about 3300 K to
178 2800 K (see Fig. 3 (b)) is quite reasonable. The radial profiles also indicate, in part, the shock wave
179 effects near 2 mm. Moreover, it would not be unexpected to see an elevated temperature of AIO just
180 inside the shock wave, with a slight indication of smaller temperatures at center.

181 4. Conclusions

182 In summary, we have shown that asymmetric Abel deconvolutional methods can be applied
183 to obtain spatial information of molecular spectra in laser ablation plasma plumes. When coupled
184 with diatomic line profile fitting of AIO emissions, we were able to demonstrate the impact of the
185 propagating shockwave has on the plasma state in both radial and axial directions. Namely, multiple
186 temperature gradients exist which correspond to the leading/following edges of the plasma blast wave.
187 Further insight into the shockwave interaction with the plasma material may be obtained by coupling the
188 radial deconvolution analysis with shockwave imaging studies, such as shadowgraph or schlieren
189 techniques, or interferometric methods [28,29]. The purpose of such a study would be to elucidate the
190 affect of the plasma shockwave on the chemical kinetics of the plasma plume expansion into ambient
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