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A New Unified Electro-Gravity Theory for the Electron

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A rigorous model for an electron is presented by generalizing the Coulomb's Law or Gauss's Law of electrostatics, using a unified theory of electricity and gravity. The permittivity of the free-space is allowed to be variable, dependent on the energy density associated with the electric field at a given location, employing generalized concepts of gravity and mass/energy density. The electric field becomes a non-linear function of the source charge, where concept of the energy density needs to be properly defined. Stable solutions are derived for a spherically symmetric, surface-charge distribution of an elementary charge. This is implemented by assuming that the gravitational field and its equivalent permittivity function is proportional to the energy density, as a simple first-order approximation, with the constant of proportionality referred to as the Unifield Electro-Gravity (UEG) constant. The stable solution with the lowest mass/energy is assumed to represent a "static" electron without any spin. Further, assuming that the mass/energy of a static electron is half of the total mass/energy of an electron including its spin contribution, the required UEG constant is estimated. More fundamentally, the lowest stable mass of a static elementary charged particle, its associated classical radius, and the UEG constant are related to each other by a dimensionless constant, independent of any specific value of the charge or mass of the particle. This dimensionless constant is numerologically suspected to be closely related to the the fine structure constant. This finding may carry greater fundamental significance, with scope of the UEG theory covering other elementary particles in the standard model of particle physics.

Keywords: Electron, Fine-Structure Constant, Unified Electro-Gravity, Non Linear Free-Space Model, Elementary Particles

I. INTRODUCTION

The electron is the most fundamental charged particle of nature [1], carrying the smallest mass among all known charged particles, and is classified as a lepton in the standard model of particle physics [2, 3]. It plays a fundamental role in our everyday nature as a basic building block of all chemical elements, which consist of one or more electrons orbiting in different spatial forms around an oppositely charged, massive central nucleus [4, 5]. Different physical parameters of the electron - its charge, mass, as well as the spin angular momentum and the magnetic moment [6–8]- have been measured in great precision. The electron's characteristics in an electromagnetic field have also been successfully modeled using quantum mechanical wave functions [9–11] and quantum electro-dynamics [12]. However, any internal structure of the electron, and the origin of its mass, remain mysterious. It is sometimes considered to be a "point particle" with no particular internal structure [13]. However, the electromagnetic energy, or its equivalent mass, for the point-particle would be infinite [14], which is unphysical and inconsistent with the finite measured mass of the electron [6]. Further, the question of how the electronic charge could withstand the repulsive force due to its own electric field [14], which is infinite for the point-structure with a zero radius (or even a finite value if the electron had a non-zero radius), can not be properly answered.

In this paper we model an electron using a new Unified Electro-Gravity (UEG) theory. The theory attempts to unify the concept of the electric field surrounding a source charge, as defined by the Coulomb's Law or Gauss' Law of electrostatics [15–17], together with a generalized concept of gravity produced due to energy density associated with the electric field, that would be consistent with the Newton's Law of Gravity [18, 19]. The permittivity of the "free-space" around a charge, which is conventionally assumed to be a fixed constant in the Coulomb's Law or Gauss' Law, is now modeled as a functional distribution, dependent on the distribution of the electric field or its associated energy density. The permittivity function needs to be consistent with the Newton's Law of gravity, where a gravitational field is recognized to be directly proportional to the gradient of the inverse-permittivity function. Accordingly, such an "unified electo-gravitational (UEG)" field may be modeled as a non-linear field, where the permittivity distribution is a general function of the source charge, or equivalently the electric field is a non-linear function of the source charge. Under this non-linear condition, the definition of energy density and its expression in terms of the source charge or the electric field, used in conventional electromagnetic theory, may have to be properly modified.

With a proper definition of the energy density associated with the non-linear UEG field, and a suitable relationship between the gravitational field and the energy density, the permittivity function surrounding a spherically symmetric surface-charge distribution may be solved, either analytically or numerically. Consequently, the total energy, or its equivalent mass as per special relativity, may be derived as a function of the charge ra-

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dius. It is discovered that stable solutions, where the first derivative of the total energy with respect to the charge radius is zero, and the second derivative positive, are possible for certain discrete values of the charge radius. The derivation assumes a simple proportional relationship between the energy density and the UEG field, with the constant of proportionality referred to as the UEG constant. It maybe reasonable to assume that the stable solution having the smallest possible mass/energy is associated with the mass/energy of an ideal "static electron" that does not spin around itself. Further, the mass of the static electron may be ideally assumed to be half of the total mass of an electron that includes its spin contribution. Accordingly, by reverse deduction, the UEG constant can be calculated, and is recognized as a new fundamental constant of nature. This is a significant fundamental development.

The new UEG constant is defined as the gravitational acceleration per unit energy density, carrying a dimension of $(m/s^2)/(J/m^3)$. More significantly, a dimensionless constant relating the UEG constant, the stable static mass, and its associated classical radius, is identified which would apply to any basic charge particle, independent of the specific charge or mass of the particle. The value of this dimensionless constant is numerologically recognized to be closely related to the fine structure constant [20]. This general finding may suggest a much broader scope of application of the UEG theory to other known elementary particles in the standard model of particle physics [2, 3, 21, 22], which might be associated with different effective values of the UEG constant, resulting in different mass and classical radii of the particles, while they carry the same value of the elementary charge as the electron. Considering the broad reach of the fine structure constant in quantum mechanics and electro-dynamics [20, 23, 24], the recognition that the fine-structure constant may have its fundamental origin in the UEG theory may carry profound theoretical and fundamental implications.

II. GRAVITY AS GRADIENT OF FREE-SPACE PERMITTIVITY

A massive body in a gravitational field $\overline{E}_g$ experiences a force $\overline{F}$ in a certain direction in space. In the theory of general relativity this force is seen as a result of curvature of the surrounding "free-space" [25]. The force may be alternatively modeled by considering the permittivity ϵ of the surrounding "free-space" to be a non-uniform function $\epsilon(\bar{r})$ of the location $\bar{r}$ (unlike a constant value $\epsilon = \epsilon_0$ normally used), and assuming that the mass of a given body at a particular location is a function of the local permittivity (see Fig.1). As the mass is displaced from one location over an incremental distance along a given direction, its mass or equivalent energy is also incrementally changed due to the incremental change in the permittivity associated with the displacement. This change

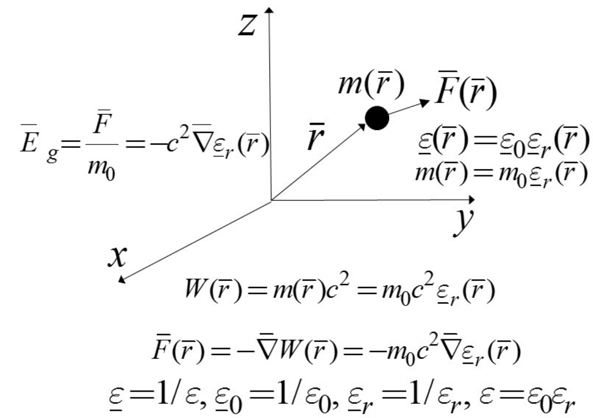

FIG. 1.

in energy per unit displacement in the given direction would be equal to the force component in the particular direction. Accordingly, the gravitational field is modeled in terms of gradient of the permittivity function of the "free-space" medium.

We assume that the mass m or the equivalent energy $W = mc^2$, where c is the speed of light in an isolated free-space, is inversely proportional to the permittivity ϵ, or directly proportional to $\underline{\epsilon} = 1/\epsilon$. This is in consistency with the energy $W = \frac{q^2}{8\pi\epsilon r_q}$ of a spherical surface charge q of radius r_q, placed in a medium with permittivity ϵ.

$$\underline{\epsilon}(\bar{r}) = \underline{\epsilon}_0\underline{\epsilon}_r(\bar{r}),\ \epsilon = \epsilon_0\epsilon_r,\ \underline{\epsilon} = \frac{1}{\epsilon} = \frac{1}{\epsilon_0\epsilon_r} = \underline{\epsilon}_0\underline{\epsilon}_r,$$
$$\underline{\epsilon}_0 = \frac{1}{\epsilon_0},\ \underline{\epsilon}_r = \frac{1}{\epsilon_r};$$
$$m(\bar{r})\ \alpha\ \underline{\epsilon}(\bar{r}),\ m(\bar{r}) = m_0\underline{\epsilon}_r(\bar{r}),$$
$$m_0 = m(\underline{\epsilon} \to \underline{\epsilon}_0,\ \underline{\epsilon}_r \to 1),\ \underline{\epsilon}_r = \frac{1}{\epsilon_r} = \frac{\underline{\epsilon}}{\underline{\epsilon}_0} = \frac{m}{m_0};$$
$$\overline{E}_g = \frac{\overline{F}}{m_0} = \frac{-\overline{\nabla}W(\bar{r})}{m_0} = \frac{-\overline{\nabla}[m(\bar{r})c^2]}{m_0}$$
$$= \frac{-\overline{\nabla}[m_0\underline{\epsilon}_r(\bar{r})c^2]}{m_0} = -c^2\overline{\nabla}\underline{\epsilon}_r(\bar{r}). \qquad (1)$$

A. Gravitational Field and Permittivity Function in a Region with Energy/Mass Distribution

Consider the gravitational field produced by a body of mass of m_0, as per the Newton's Law of Gravitation, exerting a force on an external mass δm_0. The permittivity function around the mass m_0 may be expressed using the model (1) developed above.

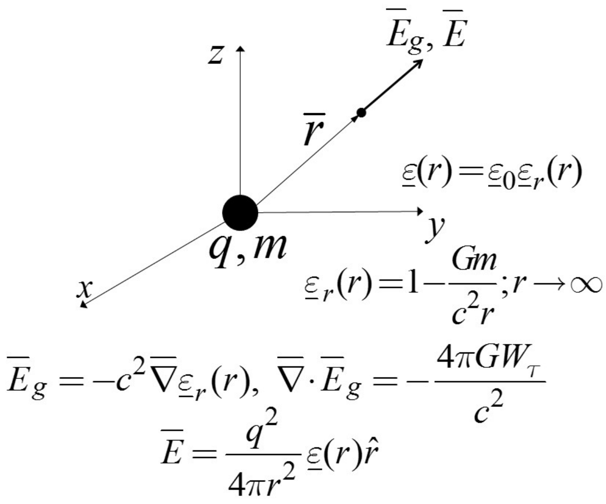

FIG. 2.

$$\overline{F} = -\frac{Gm_0\delta m_0}{r^2}\hat{r},$$
$$\overline{E}_g = \frac{\overline{F}}{\delta m_0} = -\frac{Gm_0}{r^2}\hat{r} = -c^2\overline{\nabla}\underline{\epsilon}_r(\overline{r}) = -c^2\frac{\partial\underline{\epsilon}_r(r)}{\partial r}\hat{r},$$
$$\frac{\partial\underline{\epsilon}_r(r)}{\partial r} = \frac{Gm_0}{c^2r^2},\ \underline{\epsilon}_r = 1-\frac{Gm_0}{c^2r},\ \underline{\epsilon}_r(r\to\infty)=1\ . \qquad (2)$$

The above result would be applicable for all distances $r > 0$ for an ideal point-body, and would apply only outside the body for a body of non-zero radius.

The permittivity function for a body with distributed mass/energy, such as an electric charge, may be similarly developed (see Fig.2), by relating the divergence of the gravitational field $\overline{E}_g$ in (2) to the mass-density $m_{\tau 0}$. The mass-density $m_{\tau 0}$ of a distributed body at a particular location is defined as the mass per a unit elemental volume $d\tau = 1$ at the given location. The equivalent energy-density $W_{\tau 0} = c^2 m_{\tau 0}$.

$$\overline{\nabla}\cdot\overline{E}_g = -4\pi Gm_{\tau 0} = \overline{\nabla}\cdot(-c^2\overline{\nabla}\underline{\epsilon}_r),$$
$$\overline{\nabla}\cdot\overline{\nabla}\underline{\epsilon}_r = \frac{4\pi Gm_{\tau 0}}{c^2} = \frac{4\pi GW_{\tau 0}}{c^4}\ . \qquad (3)$$

III. MODELING ENERGY DENSITY IN A NON-LINEAR MEDIUM AROUND AN ELECTRIC CHARGE

In the unified electro-gravity (UEG) model, the permittivity distribution of the free-space is dependent upon the energy density distribution, which is dependent upon the source charge. This is unlike a linear dielectric medium where the permittivity function is independent of the field strength or the source charge. Having the permittivity distribution to be a function of the source charge, is equivalent to having the electric field distribution to be a non-linear function of the source charge. The energy density in such a non-linear medium needs to be properly modeled, starting from the fundamentals. This would result in a general expression for the energy density for a non-linear medium, which may be verified with a standard expression of the energy density for a linear medium, as a special case when the permittivity is a constant independent of the charge.

The electric field $\overline{E}$ and the electric flux density $\overline{D}$ produced due to a charge q, at a distance r from the center of the charge, in the presence of a permittivity distribution $\epsilon(r) = 1/\underline{\epsilon}(r)$ may be expressed using the Coulomb's Law.

$$\overline{E} = \frac{q}{4\pi r^2\epsilon(r)}\hat{r},\ \overline{D} = \frac{q}{4\pi r^2}\hat{r},\ \overline{E} = \frac{\overline{D}}{\epsilon(r)} = \underline{\epsilon}(r)\overline{D}. \qquad (4)$$

Let us calculate an incremental energy dW required in moving an incremental charge dq from infinity to a radius $r = r_q$, using the above electric field. This is equivalent to having $dW = V(q)dq$ using a potential concept, where $V(q)$ is the potential (function of q) at the radius $r = r_q$. Integrating the dW over the total charge q would give the total energy W.

$$dW = dq\int_{r_q}^{\infty}\overline{E}(q)\cdot\overline{dr}$$
$$= V(q; r = r_q)dq = \int\int\int_{\tau;r>r_q} dW_\tau d\tau,$$
$$W = \int_{q=0}^{q} dW = \int\int\int_{\tau;r>r_q}\left(\int_{q=0}^{q} dW_\tau\right)d\tau. \qquad (5)$$

The incremental charge dq may be expressed in terms of an incremental change in the electric flux density $d\overline{D}$ using Gauss Law. The incremental energy dW can then be expressed as an integral over the external volume $\tau; r > r_q$ using the divergence theorem.

$$dW = V(q; r = r_q)dq = \int\int_{S,r=r_q+\delta} V(q)d\overline{D}\cdot\overline{ds}$$
$$= \int\int\int_{\tau;r>r_q}\overline{\nabla}\cdot(-V(q)d\overline{D})d\tau$$
$$= \left[\int\int\int_{\tau;r>r_q}(-\overline{\nabla}V(q)\cdot d\overline{D})d\tau\right.$$
$$\left.+ \int\int\int_{\tau;r>r_q} -V(q)(\overline{\nabla}\cdot d\overline{D})d\tau\right]$$
$$= \int\int\int_{\tau;r>r_q}\overline{E}(q)\cdot d\overline{D}d\tau$$
$$= \int\int\int_{\tau;r>r_q}\underline{\epsilon}(q)\overline{D}(q)\cdot d\overline{D}d\tau$$
$$= \int\int\int_{\tau;r>r_q}\underline{\epsilon}(q)\frac{1}{2}\frac{\partial}{\partial q}(\overline{D}(q)\cdot\overline{D}(q))dqd\tau$$
$$= \int\int\int_{\tau;r>r_q}\frac{1}{2}\underline{\epsilon}(q)\frac{\partial|\overline{D}|^2}{\partial q}dqd\tau = \int\int\int_{\tau;r>r_q} dW_\tau d\tau;$$
$$\overline{\nabla}\cdot d\overline{D} = 0 \text{ in } \tau. \qquad (6)$$

We have now established an expression for an incremental energy density dW_τ, which may be integrated over the total charge q to obtain the required expression of the energy density W_τ. The general expression may be

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verified to be the conventional energy density for a linear medium, when the permittivity is a constant independent of the charge q. The total energy W can then be calculated as the volume integral of the energy density W_τ.

$$dW_\tau = \frac{1}{2}\underline{\epsilon}(q)\frac{\partial|\overline{D}|^2}{\partial q}dq,$$
$$W = \int\int\int_\tau W_\tau d\tau = \int\int\int_\tau (\int_{q=0}^{q} dW_\tau)d\tau = m_0c^2 \ . \quad (7)$$

In equivalency to a conventional definition of the energy density for a linear medium, it may be useful to define a new variable $\underline{\epsilon}'$ for a non-linear medium. The conventional expression of the energy density for a linear medium, with the inverse-permittivity $\underline{\epsilon}$ for the linear medium simply substituted by the new equivalent variable $\underline{\epsilon}'$, would be valid as well for the non-linear medium.

$$W_\tau = \int_{q=0}^{q} dW_\tau = \int_{q=0}^{q} \frac{1}{2}\underline{\epsilon}(q)\frac{\partial|\overline{D}|^2}{\partial q}dq = \frac{1}{2}\underline{\epsilon}'|\overline{D}|^2 = \frac{1}{2}\underline{\epsilon}_0\underline{\epsilon}'_r|\overline{D}|^2,$$
$$\underline{\epsilon}' = \frac{1}{|\overline{D}|^2}\int_{q=0}^{q}\underline{\epsilon}(q)\frac{\partial|\overline{D}|^2}{\partial q}dq = \frac{1}{q^2}\int_{q=0}^{q}\underline{\epsilon}(q)\frac{\partial q^2}{\partial q}dq$$
$$= \frac{2}{q^2}\int_{q=0}^{q}\underline{\epsilon}(q)qdq = \frac{2\underline{\epsilon}_0}{q^2}\int_{q=0}^{q}\underline{\epsilon}_r(q)qdq = \underline{\epsilon}_0\underline{\epsilon}'_r. \quad (8)$$

IV. A UNIFIED ELECTRO-GRAVITY MODEL FOR AN ELEMENTARY CHARGE, WITH A NEW DEFINITION OF THE ENERGY DENSITY

For a given total energy W, the energy density W_τ we derived may not be unique. An alternate expression of the energy density W'_τ may be defined by adding a distribution f to the original energy density W_τ, such that the W'_τ would result in the same total energy W when integrated over the total volume τ as that due to the original energy density W_τ. Accordingly, a fixed total energy W is redistributed into the different energy densities W_τ and W'_τ inside the volume τ. This can be accomplished by having the additional distribution f expressed as divergence of a suitable vector distribution $\overline{U}$, which is identically zero everywhere outside the volume τ.

$$W = \int\int\int_\tau W_\tau d\tau = m_0c^2,$$
$$W_\tau = \frac{\triangle W}{\triangle\tau} = \frac{1}{2}\underline{\epsilon}'|\overline{D}|^2 = \frac{1}{16\pi^2r^4\epsilon_0}\int_0^q q\underline{\epsilon}_r(q)dq,$$
$$W'_\tau = W_\tau + f,$$
$$f = \overline{\nabla}\cdot\overline{U} = \overline{\nabla}\cdot(U\hat{u});\ \overline{U} = 0 \text{ outside of } \tau, \quad (9)$$

$$W = \int\int\int_\tau W'_\tau d\tau$$
$$= \int\int\int_\tau (W_\tau + f)d\tau = \int\int\int_\tau (W_\tau + \overline{\nabla}\cdot\overline{U})d\tau$$
$$= \int\int\int_\tau W_\tau d\tau + \int\int\int_\tau \overline{\nabla}\cdot\overline{U}d\tau$$
$$= \int\int\int_\tau W_\tau d\tau + \int\int_S \overline{U}\cdot\overline{ds} = \int\int\int_\tau W_\tau d\tau, \quad (10)$$

$$U(W_\tau):\ U(W_\tau = 0) = 0,\ \hat{u} = \hat{r};$$
$$W_\tau = 0 \text{ outside of } \tau,\ \overline{U} = \zeta W_\tau\hat{r}. \quad (11)$$

An alternate expression of the energy density W'_τ, as in (9), would require revision of the Poynting theorem of the electromagnetic theory [26, 27], in order to re-establish proper relationship between different energy and power associated with an electromagnetic field.

Theoretically, there are many possible expressions for the vector function $\overline{U}$. A simple, physically meaningful proposition is to express the function $\overline{U}$ (11), referred to as the UEG function, proportional to the original energy density W_τ, and directed toward the center of mass/gravity of the particle.

Consider the external free-space region of a "neutral" material body, that appears to be charge-less to an external observer, with the electromagnetic field and its associated energy density in the external region equal to zero. With the above choice of the UEG function $\overline{U}$ (11), no new, special treatment would be required to model the gravitational field in the external region, because the original as well as the revised energy densities of (9), W_τ and W'_τ respectively, would be zero in this region. Further, with the choice of the UEG function (11), the total energy W, or its equivalent mass $m = W/c^2$ of the neutral body, as seen by an external observer, would remain the same whether the W is calculated by integrating the original or the revised energy density in the internal region, as per the deduction in (10). Accordingly, Newtonian gravitational field in the external region of such neutral material bodies would remain unaffected by the new UEG theory, which would be consistent with observation.

The selected UEG function $\overline{U}$ (11) could be non-zero in the internal region of a neutral body discussed above, due to non-zero electromagnetic fields associated with any charged sub-structure internal to the body. This would lead to having the revised energy density W'_τ in (9) to be different from the original energy density W_τ in the internal region. Accordingly, it would require a revised treatment for modeling the gravitational field, in the internal charged region of such a neutral material body, or for that matter in any general region in the presence of a non-zero electromagnetic field.

The new alternate expression for the energy density W'_τ of (9), using the new UEG function $\overline{U}$ of (11), may now be substituted for the original energy density $W_{\tau 0} = W_\tau$ in the UEG modeling of the gravitational field in (3).

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$$
\begin{aligned}
\overline{\nabla}\cdot\overline{E}_g &= -c^2\overline{\nabla}\cdot\overline{\nabla}\underline{\epsilon}_r = -4\pi G m'_\tau = -\frac{4\pi G W'_\tau}{c^2} \\
&= -\frac{4\pi G}{c^2}(W_\tau + \overline{\nabla}\cdot\overline{U}) = -\frac{4\pi G}{c^2}W_\tau - \overline{\nabla}\cdot(\gamma W_\tau \hat{r}), \\
\frac{4\pi G}{c^2}\overline{U} &= \frac{4\pi G}{c^2}\zeta W_\tau \hat{r} = \gamma W_\tau \hat{r}\ . \qquad (12)
\end{aligned}
$$

It may be observed from the above expression of the gravitational field $\overline{E}_g$, that the new UEG function $\overline{U}$, which was introduced for an alternate definition of the energy density W'_τ in (11), would be equivalent to having an additional gravitational field equal to $-\gamma\overline{U}$, referred to as the UEG field. The parameter γ in (12) is a new scalar constant, referred to as the UEG constant, which is related to the constant ζ used in (11).

A. Series Solution for $\underline{\epsilon}_r$, with a Strong UEG Force Assumption

We will solve for the inverse-relative permittivity function $\underline{\epsilon}_r(r)$, by expanding it as power-series of r^{-i} with unknown coefficients b_i, and then solve for the coefficients in order to satisfy the above UEG relation (12). In the limit of large distance r, the $\underline{\epsilon}_r(r)$ needs to satisfy the Newtonian gravitational field (2) due to the particle mass m_0, approaching unity at infinite distance $r \to \infty$. The limiting conditions would fix the first two coefficients b_0 and b_1.

$$
\underline{\epsilon}_r(r,q) = \sum_{i=0}^{\infty} b_i r^{-i},\ b_0 = 1,\ b_1 = -\frac{Gm_0}{c^2}. \qquad (13)
$$

This assumes that the surrounding medium at infinite distance from the particle is a free-space with $\epsilon = \epsilon_0$, $\epsilon_r = 1 = 1/\epsilon_r = \underline{\epsilon}_r$, and the $m = m_0$ is the mass of the particle when measured in the free-space medium. If the surrounding medium is different from the free-space, with $\epsilon = \epsilon_r\epsilon_0$, $1/\epsilon_r = \underline{\epsilon}_r \neq 1$, then the above solution (13) needs to be scaled with $b_0 = \underline{\epsilon}_r$ and $b_1 = \frac{Gm_0\underline{\epsilon}_r}{c^2}$. It may be shown from the following iterative solution for the $\underline{\epsilon}_r(r)$, that each term in the series expression of (13), and therefore the entire expression of (13), would be multiplied by the $\underline{\epsilon}_r(r \to \infty)$ of the surrounding medium, in order to obtain the $\underline{\epsilon}_r(r)$ for the particle in the given surrounding medium. Further, the mass function $m(r)$ for the particle measured in the given surrounding medium, as derived in section IV B using the above scaled $\underline{\epsilon}_r$, may be shown to be equal to $m = m_0\underline{\epsilon}_r(r \to \infty)$, as expected in section II. For simplicity, in the following derivations we will assume the surrounding medium to be free-space, the results from which may be properly scaled as needed for any other surrounding medium.

We may assume that the new UEG field $\gamma\overline{U}$ is much stronger than the conventional Newtonian gravitational field of the charge particle, contributed due to the original energy density W_τ. This is because the conventional Newtonian gravitational field of an elementary charge is known to be very week, having a negligible (essentially no) effect on the permittivity function. It may be shown, that this assumption would be valid given the radius r of the charge particle is much larger than the radius r_0 of a black-hole produced by an elementary charge q, with a mass equal to the classical mass $q^2/(8\pi\epsilon_0 r_0 c^2)$ of the charge with the radius r_0.

$$
\begin{aligned}
\overline{\nabla}\cdot\overline{E}_g &= -c^2\overline{\nabla}\cdot\overline{\nabla}\underline{\epsilon}_r \simeq -\overline{\nabla}\cdot(\gamma W_\tau \hat{r}) \\
&= -\bar{\nabla}\cdot\Big(\frac{\gamma\hat{r}}{16\pi^2 r^4\epsilon_0}\int_0^q q\underline{\epsilon}_r(q,r)dq\Big) \\
&= -\bar{\nabla}\cdot\Big(\frac{\gamma q^2\underline{\epsilon}'_r(r)\hat{r}}{32\pi^2 r^4\epsilon_0}\Big),\ r >> r_0 = \sqrt{\frac{Gq^2}{8\pi\epsilon_0 c^4}}, \\
\bar{E}_g &\simeq -\gamma W_\tau \hat{r} = -\frac{\gamma\hat{r}}{16\pi^2 r^4\epsilon_0}\int_0^q q\underline{\epsilon}_r(q,r)dq \\
&= -\frac{\gamma q^2\underline{\epsilon}'_r(r)\hat{r}}{32\pi^2 r^4\epsilon_0},\ r >> r_0 = \sqrt{\frac{Gq^2}{8\pi\epsilon_0 c^4}}. \qquad (14)
\end{aligned}
$$

The expression (8) for the energy density W_τ in a non-linear medium is used in the above derivation. Substituting the series expression of (13) in (14) we get

$$
\begin{aligned}
&\underline{\epsilon}_r(r,q) = \sum_{i=0}^{\infty} b_i r^{-i},\ \underline{\epsilon}_r(r\to\infty) = 1,\ b_0 = 1, \\
&\overline{\nabla}\cdot\overline{\nabla}\underline{\epsilon}_r \simeq \frac{1}{c^2}\overline{\nabla}\cdot(\gamma W_\tau\hat{r}) = \frac{1}{c^2}\overline{\nabla}\cdot\Big(\frac{\gamma\hat{r}}{16\pi^2 r^4\epsilon_0}\int_0^q q\underline{\epsilon}_r(r,q)dq\Big) \\
&= \overline{\nabla}\cdot\Big(\frac{3r_\mu^3\hat{r}}{2q^2r^4}\int_0^q q\underline{\epsilon}_r(r,q)dq\Big),\ r_\mu^3 = \frac{\gamma q^2}{24\pi^2c^2\epsilon_0} \propto q^2, \\
&r_\mu = \Big(\frac{\gamma q^2}{24\pi^2 c^2\epsilon_0}\Big)^{1/3} = 5.14\times 10^{-16}\gamma^{1/3}\ . \qquad (15)
\end{aligned}
$$

Assuming that the charge distribution and the UEG solution are spherically symmetric, the differential operators in the above expression can be expressed in terms of derivatives with respect to the radius.

$$
\begin{aligned}
&\frac{\partial}{\partial r}\Big(r^2\frac{\partial}{\partial r}\underline{\epsilon}_r\Big) = \sum_{i=0}^{\infty} b_i(i)(i-1)r^{-i} \simeq \frac{\partial}{\partial r}\Big(\frac{3r_\mu^3}{2q^2r^2}\int_0^q q\underline{\epsilon}_r(r,q)dq\Big) \\
&= -\frac{3r_\mu^3}{2q^2}\sum_{i=0}^{\infty}\Big(\int_0^q qb_i(q)dq\Big)(i+2)r^{-i-3}\ , \\
&\sum_{i=0}^{\infty} b_i(i)(i-1)r^{-i} \\
&\simeq -\frac{3r_\mu^3}{2q^2}\sum_{i=3}^{\infty}\Big(\int_0^q qb_{i-3}(q)dq\Big)(i-1)r^{-i}\ . \qquad (16)
\end{aligned}
$$

The above relation provides an iterative solution for the coefficients b_i.

$$
\begin{aligned}
&b_i \simeq -\frac{3r_\mu^3}{2q^2(i)}\Big(\int_0^q qb_{i-3}(q)dq\Big), \\
&b_0 = 1,\ b_i = 0;\ i \neq 3k = 0, 3, 6, 9\cdots\ . \qquad (17)
\end{aligned}
$$

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The series may be re-sequenced with $a_k = b_{3k}$, because all coefficients b_i for i other than $i = 3k = 0, 3, 6, 9\cdots$ are zero.

$$\begin{aligned} a_k = b_{3k} &\simeq -\frac{r_\mu^3}{q^2(2k)}\Big(\int_0^q q b_{3k-3}(q)dq\Big) \\ &= -\frac{r_\mu^3}{q^2(2k)}\Big(\int_0^q q a_{k-1}(q)dq\Big) \ . \end{aligned} \tag{18}$$

From the above iterative relation it maybe recognized that a_k would be proportional to q^{2k}. This condition may be used to simplify the iterative relation for a_k and then solve for all the coefficients a_k starting with the known coefficient $a_0 = 1$.

$$\begin{aligned} &a_k(q) \propto q^{2k},\ a_0 = 1,\ a_1 = -\frac{r_\mu^3}{2{\times}2}, \\ &a_2 = \frac{r_\mu^6}{2{\times}2{\times}4{\times}4};\ a_k = -a_{k-1}\frac{r_\mu^3}{(2k)(2k)} \ . \end{aligned} \tag{19}$$

The series expression for the inverse-relative-permittivity function $\underline{\epsilon}_r(r)$ may be re-formatted as a power series of t^{2k}, where t is a normalized variable $t = (r_\mu/r)^{1.5}$, with corresponding normalized coefficients a'_k.

$$\begin{aligned} \underline{\epsilon}_r(r) &= \sum_{i=0}^{\infty} b_i r^{-i} \simeq \sum_{k=0}^{\infty} a_k r^{-3k} \\ &= \sum_{k=0}^{\infty} a'_k\Big(\frac{r_\mu}{r}\Big)^{3k} = \sum_{k=0}^{\infty} a'_k t^{2k}, \\ a'_k r_\mu^{3k} &= a_k,\ t = \Big(\frac{r_\mu}{r}\Big)^{1.5},\ a'_k = -a'_{k-1}\frac{1}{(2k)(2k)}, \end{aligned} \tag{20}$$

$$a'_k = \frac{(-1)^k}{2^{2k}(k!)^2},\ k! = (k)(k-1)(k-2)\cdots(1), \tag{21}$$

$$\underline{\epsilon}_r(r) \simeq 1 - \frac{t^2}{2^2[1!]^2} + \frac{t^4}{2^4[2!]^2} - \frac{t^6}{2^6[3!]^2} + \cdots . \tag{22}$$

The inverse-relative permittivity function $\underline{\epsilon}_r = 1/\epsilon_r$ of (22), as well as the corresponding effective function $\underline{\epsilon}'_r = 1/\epsilon'_r$ of (23) deduced from (22) using the definition (8), are plotted in Fig.3 as a function of the normalized radius $r_\mu/r = t^{2/3}$.

$$\begin{aligned} \underline{\epsilon}'_r(r) &= \frac{2}{q^2}\int_0^q q\underline{\epsilon}_r(q,r)dq \\ &\simeq 1 - \frac{t^2}{2^2[1!]^2{\times}2} + \frac{t^4}{2^4[2!]^2{\times}3} - \frac{t^6}{2^6[3!]^2{\times}4} + \cdots . \end{aligned} \tag{23}$$

The function $\underline{\epsilon}_r = 1/\epsilon_r$ that would have resulted if a conventional energy density for a linear medium were used (incorrectly) in the above derivation of section IV A (equations (14,8)), where the effective function $\underline{\epsilon}'_r = 1/\epsilon'_r$ from (8) that defines the energy density would be equal to the function $\underline{\epsilon}_r = 1/\epsilon_r$, is expressed in (24), and is also plotted in Fig.3 for reference. Notice in the Fig.3 that the function $\underline{\epsilon}_r$ of (22) (and the corresponding effective function $\underline{\epsilon}'_r$ of (23)), derived using the rigorous definition of the energy density (8) for a non-linear medium, exhibits an oscillatory behavior changing its sign from positive to negative values and vice versa. This is in contrast with the result for $\underline{\epsilon}_r = 1/\epsilon_r = \underline{\epsilon}'_r = 1/\epsilon'_r$ from (24) (using a simplistic (incorrect) UEG model), which monotonically approaches zero with no oscillatory behavior. The rigorously derived, oscillatory behavior of the $\underline{\epsilon}_r = 1/\epsilon_r$ and $\underline{\epsilon}'_r = 1/\epsilon'_r$ functions is a key development, which would lead also to an oscillatory behavior of the total energy/mass of the charge particle as a function of radius, to be established in the following section. This would allow the charge particle to maintain a stable structure at discrete values of radius, where the total energy/mass of the particle would be locally minimum.

$$\underline{\epsilon}_r(r) = \underline{\epsilon}'_r(r) \simeq 1 - \frac{t^2}{2^2 1!} + \frac{t^4}{2^4 2!} - \frac{t^6}{2^6 3!} + \cdots \ . \tag{24}$$

From Fig.3, it maybe noted that at discrete locations where $\underline{\epsilon}_r$ of (22) is zero, the corresponding $\underline{\epsilon}'_r$ of (23) is non-zero, and vice versa. Accordingly, the energy density $W_\mathcal{T}$ of (8) would be non-zero, at the discrete locations where the field $\overline{E}$ of (4) is zero, and vice versa. This is unlike a conventional field in a "free-space" medium having a fixed relative permittivity $\epsilon_r = 1$, in which case a non-zero or zero electric field is respectively associated with a non-zero or zero energy density. The above non-conventional behavior is a result of the non-conventional nature of the "free-space" medium, as per the UEG theory, which is no longer a fixed but is a "flexible" or variable medium with a non-linear behavior. The electric field in such a flexible medium would be a non-linear function of the source charge, and the equivalent permittivity is a function of the source charge and location. The energy density in such a non-linear medium needs to be properly re-defined as in (8), resulting in the effective inverse relative-permittivity $\underline{\epsilon}'_r$ of (23), which leads to the non-conventional disconnect between the energy density and the electric field, discussed above.

Further, the relativity permittivity ϵ_r from (22) in Fig.3 is allowed to be negative, which maybe theoretically associated with a negative speed of light. The effective permittivity ϵ'_r from (23) in Fig.3 is also allowed to be negative, which as per its definition in (8) would allow the energy density to be negative. These possibilities of negative light speed and negative energy density are remarkable new developments, not encountered in conventional physical problems, which may carry far-reaching physical and philosophical implications.

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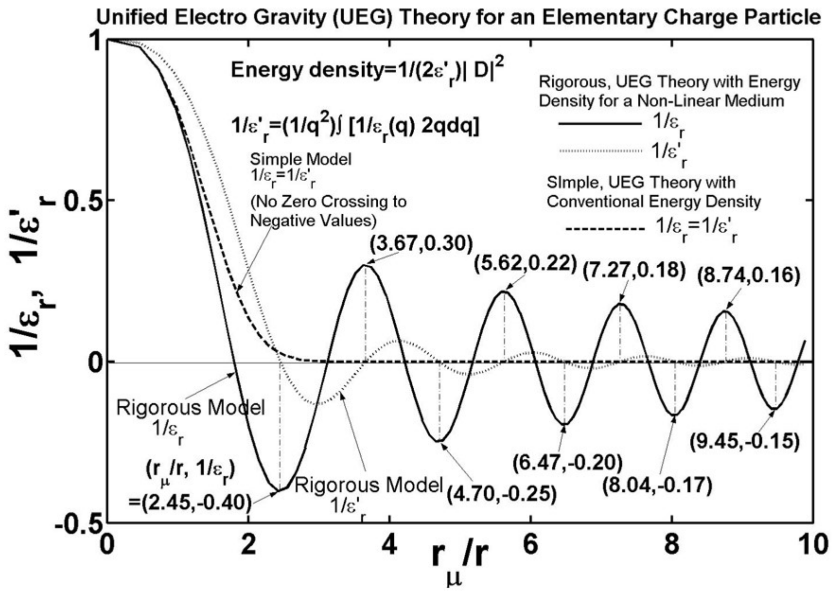

FIG. 3.

B. Particle Energy and Mass, as a Function of the Charge Radius

Once the inverse-relative permittivity function $\underline{\epsilon}_r(r)$ is solved, the energy density can be expressed in terms of the $\underline{\epsilon}_r(r)$ using (8), which can then be integrated over the total volume outside the charge radius (there is no field inside the charge radius) to obtain the total energy or the equivalent mass m $(=m_0$ in (13)) of the particle.

$$W = \int\int\int_\tau W_\tau d\tau = m_0 c^2$$
$$= \int\int\int_\tau [\frac{1}{16\pi^2 r^4 \epsilon_0} \int_0^q q\underline{\epsilon}_r(q,r)dq]d\tau,$$
$$m = m_0 = \frac{W}{c^2} = \frac{1}{4\pi c^2 \epsilon_0} \int_r^\infty \frac{1}{r^2} \int_0^q q\underline{\epsilon}_r(q,r)dqdr$$
$$= m_\mu \sum_{k=0}^{\infty} \frac{(-1)^k t^{(2k+\frac{2}{3})}}{2^{2k}(k!)^2(k+1)(3k+1)},\ t = (\frac{r_\mu}{r})^{1.5},$$
$$m_\mu = \frac{q^2}{8\pi c^2 \epsilon_0 r_\mu} = 2.49 \times 10^{-30} \gamma^{-1/3},$$
$$r_\mu = (\frac{\gamma q^2}{24\pi^2 c^2 \epsilon_0})^{1/3} = 5.14 \times 10^{-16} \gamma^{1/3}\ . \qquad (25)$$

The charge radius in (25) is maintained as a general variable $(=r)$. The general mass function $m(r)$ in (25) would also represent the equivalent energy $(=c^2 m(r))$ contained in the field external to a sphere of radius r, produced due to the charge placed at any radius less than r.

Fig.4 and Fig.5 (with different mass scales/resolutions) plot the normalized mass m/m_μ of (25) as a function of the normalized radius r_μ/r, showing the oscillatory behavior of the mass function, as we anticipated earlier. Any of the minimum points of the mass function would correspond to a possible stable particle with the particular charge radius, as we also anticipated. The mass $m = m_0$ that would have resulted, if the inverse-relative permittivity function of (24) were used in the derivation of (25,8), based on a simplistic (incorrect) UEG model assuming a linear medium, is expressed in (26). This mass (26) normalized with respect to m_μ is also plotted in Figs.4,5 for reference, showing no stable radius. Also plotted in Figs.4,5 for reference is the normalized mass $(m/m_\mu) = (r_\mu/r)$, based on a simple Coulomb's field, which asymptotically approaches the normalized masses of (25) and (26) for $r \to \infty$, as should be expected. Clearly, the Coulomb mass does not allow any stable radius.

$$m = m_0 = \frac{W}{c^2} = \frac{q^2}{8\pi c^2 \epsilon_0} \int_r^\infty \frac{\underline{\epsilon}_r(r)}{r^2} dr$$
$$= m_\mu \sum_{k=0}^{\infty} \frac{(-1)^k t^{(2k+\frac{2}{3})}}{2^{2k} k!(3k+1)}. \qquad (26)$$

The smallest possible stable mass deduced from the oscillatory mass of (25) (Figs.4,5) is expected to be the mass of an electron (or a positron) without any spin. This is referred to as the static UEG mass m_e' of an electron. We will assume that the static UEG mass m_e' of an electron is about half of the total electron mass m_e, that includes additional mass/energy due to the electron's spin. This factor of about 2 between the m_e' and m_e is suggested by recognizing that the electron's spin g-factor, as

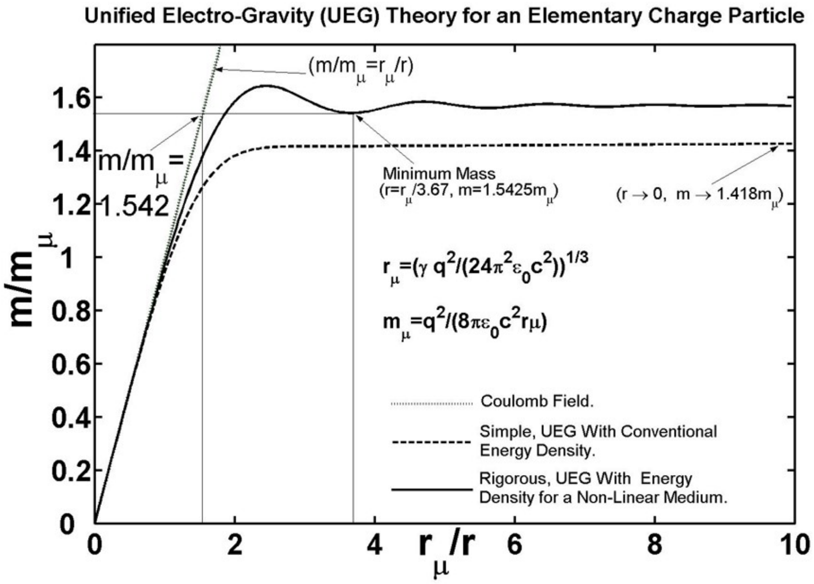

FIG. 4.

defined below in (27), is approximately equal to 2. The bare static UEG mass m'_e of an electron spins effectively at the same speed and at the same radial distance as the electron's charge q. This would result in having the ratio of the spin magnetic moment M and the spin angular momentum p equal to $q/(2m'_e)$. This is equivalent to having a total electron mass $m_e = gm'_e \simeq 2m'_e$ spinning at about half of a given speed or about half of a given radius (or at about half of a given speed-radius product), in order to produce the same given angular momentum p. This factor of about 2 is represented by the electron's spin g-factor.

$$\begin{aligned} \frac{M}{p} &= \frac{q}{2m'_e} = \frac{gq}{2m_e}, \\ g \simeq 2, \quad m'_e &= \frac{m_e}{g} \simeq \frac{m_e}{2}. \end{aligned} \tag{27}$$

The same conclusion may also be suggested by observing that the orbital magnetic moment of an atomic electron with an orbital angular momentum $\hbar$ is approximately equal to the magnetic moment of a spinning electron with spin angular momentum $\hbar/2$. The approximately same magnetic moments in the two cases means the velocity-radius product of the orbital and the spinning electrons are about the same. With about the same speed-radius product, having the spin angular moment ($= \hbar/2$) half of the orbital angular moment ($= \hbar$) suggests that the bare UEG static mass m'_e of the spinning electron is about half of the total mass $m_e = 9.109 \times 10^{-31} kg$ of the orbiting electron.

$$\begin{aligned} m'_e \times (vr)_{\text{spin}} &= \frac{\hbar}{2}, \; m_e \times (vr)_{\text{orbital}} = \hbar, \\ (vr)_{\text{spin}} &= \frac{g}{2} \times (vr)_{\text{orbital}} \simeq (vr)_{\text{orbital}}, \\ m'_e &= \frac{m_e}{g} \simeq \frac{m_e}{2}. \end{aligned} \tag{28}$$

With the assumption of $m'_e = m_e/2$ for the minimum stable mass in Figs.4,5, the value of the normalization constant m_μ can be calculated, from which the value of the UEG constant γ is estimated.

$$\begin{aligned} \frac{m'_e}{m_\mu} &= \frac{m_e}{2m_\mu} = 1.5425, \\ m_\mu &= \frac{m_e}{3.085} = 2.49 \times 10^{-30}\gamma^{-1/3}, \\ \gamma^{1/3} &= 3.085 \times 2.49 \times 10^{-30}/m_e, \\ \gamma &= 5.997 \times 10^2 (m/s^2)/(J/m^3). \end{aligned} \tag{29}$$

As per the UEG theory of the electron, the constant γ is declared to be a new natural constant, which is equal to a new gravitational acceleration in m/s^2 toward the center of gravity, produced due to one J/m^3 of energy density.

C. General Relationship Between the UEG Constant γ, the Particle Mass and Classical Radius.

The above estimate of the value of the UEG constant requires the actual UEG static mass m'_e of the electron. However, a general relationship between the smallest stable UEG static mass m'_e of an elementary particle, the

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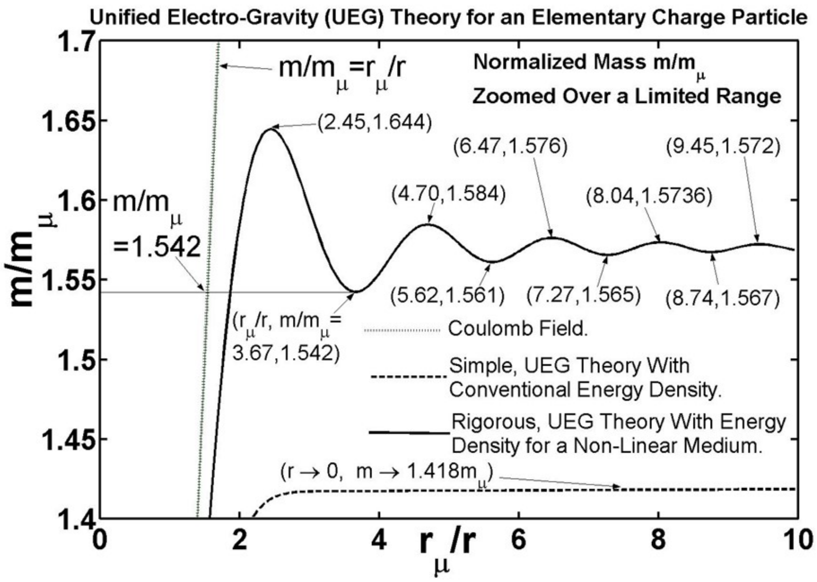

FIG. 5.

corresponding classical radius r'_e, and the UEG constant γ required to produce the mass m'_e, can be derived based on the expressions for the reference mass m_μ (25) and reference radius r_μ (15) used in the above analysis.

$$(\frac{m_\mu}{m'_e})^3 = \frac{3q^4}{64\pi c^4 \epsilon_0^2 \gamma {m'_e}^3} = \frac{3{r'_e}^2\pi}{\gamma m'_e},$$
$$\frac{\gamma m'_e}{{r'_e}^2} = 3\pi(\frac{m'_e}{m_\mu})^3, \quad m'_e = \frac{q^2}{8\pi\epsilon_0 r'_e c^2}. \qquad (30)$$

The value of the ratio $m/m_\mu = 1.5425$ from the Figs.4,5, for the smallest possible stable mass $m = m'_e$. Using this value, the γ, m'_e and r'_e maybe related in term of a dimensionless constant.

$$\frac{\gamma m'_e}{{r'_e}^2} = 3\pi(\frac{m'_e}{m_\mu})^3 = 34.590 \quad . \qquad (31)$$

If we simply assume the total mass m_e of the elementary particle with spin to be twice the UEG mass m'_e, and the classical radius r_e associated with m_e half of that ($= r'_e$) with m'_e, the γ, m_e and r_e may be related using a new dimensionless constant, which would be eight times the above constant.

$$\frac{\gamma m_e}{r_e^2} = 24\pi(\frac{m'_e}{m_\mu})^3 = 8 \times 34.590 = 276.720 \quad . \qquad (32)$$

Notice that the above constant is close to twice the inverse-fine structure constant $1/\alpha = 137.036$, and the earlier constant in (31) is one fourth of the $1/\alpha$, with less than one percent of difference. It may be possible that the normalized stable mass in Figs.4,5 is not accurate. This may reflect possible inaccuracy in computation due to poor convergence of the power series in (25), when the normalized parameter t is sufficiently greater than unity (t is close to 4 at the smallest stable mass of Figs.4,5). More significantly, the small difference may also be due to lack of generality or rigor of the basic UEG static theory for the particle, presented in this paper with assumption of a simple UEG function in (11), and without including the particle's spin. The small difference may perhaps be related to the small difference between the actual value of the g-factor and its ideal value of 2 suggested in (27). This may point to possible physical origin of the g-factor associated with the spin, governed by a more rigorous version of the new UEG theory.

Leaving aside any small computational inaccuracy, or any small difference due to lack of generality or rigor of the basic UEG model, the close relations of the above dimensionless constant (31 or 32) to the fine-structure constant is intriguing. First, the very existence of a dimensionless constant based on the UEG theory, and its intriguing close numerological relationship with the known fine-structure constant α, may strongly suggest certain fundamental basis and significance of the new UEG theory. The close numerological relationship may also strongly suggest an explicit close relationship between the UEG constant γ associated with the dimensionless constant (31 or 32) from the UEG theory, and the particle's quantum-theoretical spin angular momentum $\hbar/2$ (consequently, the Plank's constant $\hbar$) associated with the fine-structure constant α. However, any modeling of a physically spinning particle is beyond the scope of the present UEG theory, which is valid only for a static charge. A more advanced modeling, extending the static UEG theory to model an electrodynamic problem of a physically spinning charge, would be needed in order to study any direct physical relationship between the UEG theory and the quantum spin theory (and quantum theory in general), and consequently between the associated dimensionless constant (31 or 32) and the fine-structure constant α, respectively.

V. SUMMARY AND FUTURE SCOPE.

A new unified electro-gravity (UEG) theory is presented to model an elementary charge particle, based on a non-linear permittivity function of the empty space around the charge, which is dependent on distribution of the energy density. A new fundamental physical constant γ, referred to as the UEG constant, is introduced in order to redefine the energy density around the charge, leading to a new gravitational field. The value of the constant γ is estimated to be about 600 $(m/s^2)/(J/m^3)$, by recognizing that the lightest possible elementary charge particle is an electron (or a positron). A fundamental dimensionless constant exists, relating the mass of an elementary charge particle, its classical radius, and the UEG constant γ required to produce the particle as the lightest possible stable particle based on the UEG theory. This dimensionless constant is shown to be closely related to the fine-structure constant α used in quantum electrodynamics [20, 23], with less than one percent of difference. This would strongly suggest a deeper fundamental basis of the UEG theory, with fundamental relationship with the quantum-mechanical concepts, that could possibly be extended to model any other elementary particles.

The basic UEG theory models only a static elementary charge without spin. Further, the energy density associated with the electric field around a charge, which is revised in this paper in terms of a new UEG function, is still not a uniquely-defined concept. The simple UEG theory used in this work may need to to be extended to model the electrodynamic problem of a spinning electron [11]. The theory may be further refined and extended using higher-order UEG functions to model other elementary charge particles [3, 21, 22], such as a proton, in the standard model of particle physics [2, 28]. The basic theory for a charged particle could also be extended for neutral particles composed of concentric layers of opposite charges, and similarly for other possible composite charged or neutral particles consisting of many layers of charge particles in definite concentric patterns. Accordingly, the fundamental basis of the new UEG theory may open research avenues, providing an alternate paradigm to the existing standard model of the particle physics. This could succeed in achieving the long-pending unification of the electromagnetism and gravity into one complete theory, which would allow modeling of all charged and neutral particles of the standard model without need for any other additional force, possibly making the weak and strong forces currently used in the standard model redundant.

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