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is*-Open Sets, is*- Mappings and is*-separation Axioms in Topological Spaces

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ABSTRACT

In this paper, we introduce a new class of open sets that is called is*-open set . Also, we present the notion of is*-continuous, is*-open, is*-irresolute, is*-totally continuous, and is-contra-continuous mappings, and we investigate some properties of these mappings. Furthermore, we introduce some is*-separation axioms, and is*-mappings are related with is*-separation axioms.

1 Introduction and Preliminaries

A Generalization of the concept of open sets is now well-known important notions in topology and its applications. Levine [7] introduced semi-open set and semi-continuous function, Njastad [8] introduced α-open set, Askander [15] introduced i-open set, i-irresolute mapping and i-homeomorphism, Biswas [6] introduced semi-open functions, Mashhour, Hasanein, and El-Deeb [1] introduced α-continuous and α-open mappings, Noiri [16] introduced totally (perfectly) continuous function, Crossley [11] introduced irresolute function, Maheshwari [14] introduced α-irresolute mapping, Beceren [17] introduced semi α-irresolute functions, Donchev [4] introduced contra continuous functions, Donchev and Noiri [5] introduced contra semi continuous functions, Jafari and Noiri [12] introduced Contra-α-continuous functions, Ekici and Caldas [3] introduced clopen-T_1, Staum [10] introduced, ultra hausdorff, ultra normal, clopen regular and clopen normal , Ellis [9] introduced ultra regular, Maheshwari [13] introduced s-normal space, Arhangel [2] introduced α-normal space, Mohammed and khattab[18]introduced On iα - Open Sets, Khattab [19] introduced On iα- Open Sets in Topological Spaces and Their Application. The main aim of this paper is to introduce and study a new class of open sets which is called iα-open set and we present the notion of iα-continuous mapping, iα-totally continuity mapping and some weak separation axioms for iα-open sets. Furthermore, we investigate some properties of these mappings. In section 2, we define iα-open set, and we investigate the relationship with, open set, semi-open set, α-open set and i-open set. In section 3, we present the notion of iα-continuous mapping, iα-open mapping, iα-irresolute mapping and iα-homeomorphism mapping, and we investigate the relationship between iα-continuous mapping with some types of continuous mappings, the relationship between iα-open mapping, with some types of open mappings and the relationship between iα-irresolute mapping with some types of irresolute mappings. Further, we compare iα-homeomorphism with i-homeomorphism. In section 4, we introduce new class of mappings called iα-totally continuous mapping and we introduce i-contra-continuous mapping and iα-contra-continuous mapping. Further, we study some of their basic properties. Finally in section 5, we introduce a new weak of separation axioms for iα-open set and we conclude iα-continuous mappings related with iα-separation axioms. Throughout this paper, we denote the topology spaces (X,τ) and (Y,σ) simply by X and Y respectively. We recall the following definitions, notations and characterizations. The closure (resp. interior) of a subset A of a topological space X is denoted by $Cl(A)$ (resp. $Int(A)$).

Definition 1.1 A subset A of a topological space X is said to be

(i) pre-open set, if $A \subseteq Int(Cl(A))$ [7]
(ii) semi-open set, if $A \subseteq Cl(Int(A))$ [7]
(iii) α–open set, if $A \subseteq Int(Cl(Int(A)))$ [8]
(iv) β-open set, if $A \subseteq Cl(Int(Cl(A)))$ [1]
(v)(b [2], sp [13], γ[23]) –open set, if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$
(vi) i-open set, if $A \subseteq Cl(A \cap O)$, where $\exists O \in \tau$ and $O \neq X, \phi$ [15]
(vii) iα-open set $A \subseteq Cl(A \cap O)$, where $\exists O \in$ α - open sets and $O \neq X, \phi$ [18]
(viii) clopen set, if A is open and closed

The family of all semi-open (resp. pre-open, b-open, β-open, α-open , i-open, iα-open, clopen) sets of a topological space is denoted by $SO(X)$ (resp. PO(X),BO(X), βO(X),α$O(X)$, $iO(X)$, iα$O(X)$, $CO(X)$). The complement of open (resp. semi-open, α-open, i-open, iα-open) sets of a topological space X is called closed (resp. semi-closed, α-closed, i-closed, iα-open) sets.

Definition 1.2 Let X and Y be a topological spaces, a mapping $f: X \to Y$ is said to be

(i) semi-continuous [7] if the inverse image of every open subset of Y is semi-open set in X.
(ii) α-continuous [1] if the inverse image of every open subset of Y is an α-open set in X.
(iii) i-continuous [15] if the inverse image of every open subset of Y is an i-open set in X.
(iv) iα-continuous [15] if the inverse image of every open subset of Y is an iα-open set in X.

(v) totally (perfectly) continuous [16], (i , iα [ ])-tollay continuous, if the inverse image of every open, (i , iα –open) subset of *Y* is clopen set in *X*.
(vi) Strongly continuous [19], if the inverse image of every open, (i , iα –open) subset of *Y* is clopen set in *X*.
(vii)
(viii) irresolute [11] if the inverse image of every semi-open subset of *Y* is semi-open subset in *X*.
(ix) α-irresolute [14] if the inverse image of every α-open subset of *Y* is an α-open subset in *X* .
(x) semi α-irresolute [17] if the inverse image of every α-open subset of *Y* is semi-open subset in *X*.
(xi) i-irresolute [15] if the inverse image of every i-open subset of *Y* is an i-open subset in *X*.
(xii) iα-irresolute [15] if the inverse image of every iα-open subset of *Y* is an i-open subset in *X*.
(xiii)contra-continuous [4] if the inverse image of every open subset of *Y* is closed set in *X*.
(xiv) contra semi continuous [5] if the inverse image of every open subset of *Y* is semi-closed set in *X*.
(xv) contra α-continuous [12] if the inverse image of every of open subset of *Y* is an α-closed set in *X*.
(xvi) semi-open [6] if the image of every open set in *X* is semi-open set in *Y*.
(xvii) α-open [1] if the image of every open set in *X* is an α-open set in *Y*.
(xviii) i-open [15] if the image of every open set in *X* is an i-open set in *Y*.

Definition 1.3 Let *X* and *Y* be a topology space, a bijective mapping $f: X \to Y$ is said to be iα [](resp. α-open [] , i-open[]) homeomorphism, if f is an iα (resp. α-open , i-open) continuous mappings, and iα (resp. α-open , i-open) open mappings .

Lemma 1.4 Every open (resp. semi–open, α-open , i-open) set in a topological space is an iα-open set [18].

Lemma1.5 A topological *X* is said to be extermally Disconnected [ ], if evey pre – open is semi-open.

2 *is**-open set and some properties of *is**-open set

Definition 2.1 A subset *A* of the topological space *X* is said to be *is**-open set if there exists a non-empty subset *O* of *X*, $O \in SO(X)$, such that $A \subseteq Cl(A \cap O)$. The complement of the is-open set is called is-closed. We denote the family of all *is**-open sets of a topological space *X* by $iSO(X)$.

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Example 2.2 Let X={1,3,5}, τ={∅,{5},{1,5},X},$SO(X)$=$\alpha O(X)$={∅,{1},{1,5}, {3,5}, X} and $iSO(X)$ = i$\alpha O(X)$={∅,{1},{3},{5},{1,3},{1,5},{3,5},X}. Note that $SO(X)$ =$\alpha O(X) \subset iSO(X)$ = i$\alpha O(X)$.

Example 2.3 Let X={1,2,3,4}, τ={∅,{1,3},{2,4}, X}=$SO(X)$
$iSO(X)$={∅,{1},{2},{3},{4},{1,4},{2,3},X}.

Example 2.4 Let X={7,8,9}, τ={∅,{7},{8},{7,8},X}
$isO(X)$={∅,{7},{8},{7,8},{7,9},{8,9},X}.

Propositions 2.5 Every iα-open set in any topological space is an *is**-open set.

Proof. Let X be any topological space and $A \subseteq X$ be any iα-open set. Therefore, $A \subseteq Cl(A \cap O)$, where $\exists O \in \alpha O(X)$ and $O \neq X, \phi$. Since, every α-open is semi-open , then $\exists O \in SO(X)$. We obtain $A \subseteq Cl(A \cap O)$, where $\exists O \in SO(X)$ and $O \neq X, \phi$. Thus, A is an *is**-open set■

Corollary 2.6 Every i-open set in any topological space is an *is* ∗-open set.

Proof. Clear, Since every i-open set is iα- open sets by **Lemma 1.4,** and Since Every iα-open set in any topological space is an *is**-open set by **Propositions 2.5** ■

Corollary 2.7 Every semi-open set in any topological space is an *is* ∗-open set.

Proof. Clear, since semi-open is an iα-open sets by **Lemma 1.4**, and Since Every iα-open set in any topological space is an *is**-open set by **Propositions 2.5** ■

Corollary 2.8 Every α-open set in any topological space is an *is* ∗-open set.

Proof. Clear, since α-open is an iα-open sets **Lemma 1.4**, and Since Every iα-open set in any topological space is an *is**-open set by **Propositions 2.5** ■

Corollary 2.9 Every open set in any topological space is an *is* ∗-open set.

Proof. Clear, since open is an iα-open sets **Lemma 1.4**, and Since Every iα-open set in any topological space is an *is**-open set by **Propositions 2.5** ■The following **example 2.2** shows that is-open set need not be (res. semi, α, i) open sets

Remark 2.10. The intersection and the union of *is* ∗-open sets is not necessary to be *is* ∗-open set as shown in the examples 2.4 and 2.3 respectively.

Proposition 2.11 If the topological space X is extermally Disconnected, then every pre -open set $is\,*$-open set.

Proof. Let X be any topological space and $A\subseteq X$ be any pre-open set. Since X is externally Disconnected **Lemma 1.5**, then A is semi- open sets. Hence, Since every semi-open is an $is\,*$ – open by Corollary **2.7**. Therefore, A is an $is*$–open set■

Proposition 2.12 If the topology X is externally Disconnected, then every (b, sp,γ)-open set $is*$ -open set.

Proof. Let X be any topological space and $A\subseteq X$ be any (b, sp,γ) -open set. Then A either pre –open, or semi –open, or it is the union of pre –open and semi –open. Since X is externally Disconnected by **Lemma 1.5**, then any way we have A is semi- open sets . Hence, Since every semi-open is an $is*$– open by **Corrllary 2.7**. Therefore, A is an $is*$–open set ■

Corollary 2.13 If the topology X is externally Disconnected, then every β-open set $is*$-open set.

Proof. Clear, since every β-open is pre-open set■ The following example shows that if X is not externally Disconnected, (pre, β, (b, sp, γ) - open sets are not is -open set.

Example 2.14 Let X={7,8,9},τ={∅,{7},{8,9},X} iSO(X)={∅,{7},{8},{9},{8,9},X}⊂ PO(X)= BO(X)=βO(X)={ ∅,{7},{8},{9},{7,8},{7,9}{8,9},X}.

3 *is**-homeomorphism and *is**- irresolute

Definition 3.1 Let X, Y be a topological spaces, a mapping $f: X \to Y$ is said to be$is*$-continuous, if the inverse image of every open subset of Y is an $is*$-open set in X.

Example 3.2 Let X=Y={0,2,4}, τ={∅,{2},{4},{2,4},X}, iSO(X)={∅,{2},{4},{0,2},{0,4},{2,4},X} and σ={∅,{0,2},X}. Clearly, the identity mapping $f: X \to Y$ is an $is*$-continuous.

Proposition 3.3 Every iα-continuous mapping is an $is*$-continuous.

Proof. Let $f: X \to Y$ be an iα-continuous mapping and V be any open subset in Y. Since, f is an iα-continuous, then $f^{-1}(V)$ is an iα-open set in X. Since, every iα-open set is an is*-open set by **proposition 2.5**, then $f^{-1}(V)$ is an is*-open set in X. Therefore, f is an is*-continuous■

Corollary 3.4 Every continuous mapping is an is*-continuous.

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Proof. Clear, since every open is an is*-open set **corollary 2.9**■

Corollary 3.4 Every smei-continuous mapping is an is*-continuous.

Proof. Clear, since every semi-open is an is*-open set by **corollary 2.7**■

Corollary 3.4 Every α-continuous mapping is an is*-continuous.

Proof. Clear, since every α-open is an is-open set by **corollary 2.8**■

Corollary 3.4 Every i-continuous mapping is an is*-continuous.

Proof. Clear, since every i-open is an is*-open set by **corollary 2.8**■

Remark 3.4 The following example shows that *is**-continuous mapping need not be continuous, semi-continuous, α-continuous and i-continuous mappings.

Example 3.5 Let X={n,m,r} and Y={n,m,r}, τ={∅,{m},X}, $SO(X)=\alpha O(X)=iO(X)=$ {∅,{m},{n,m},{m,r},X},$iSO(X)$={∅,{n},{m},{r},{n,m},{m,r},{n,r},X}, σ={∅,{r},Y}. A mapping $f: X \to Y$ is defined by f{n}={r},f{m}={n},f{r}={m}. Clearly, f is an *is**-continuous, but f is not continuous, f is not semi-continuous, f is not α-continuous and f is not i-continuous because for open subset {m}, f^{-1}{r}={n}∉τ and f^{-1}{m}={r}∉SO(X)=$\alpha O(X)=iO(X)$.

Definition 3.6 Let X and Y be a topological space, a mapping $f: X \to Y$ is said to be *is**-open, if the image of every open set in X is an *is*-open set in Y.

Example 3.7 Let X=Y{h,r,k}, τ ={∅,{r,k},X}, σ ={∅,{h},Y}, and $iSO(Y)$={ ∅,{h},{r},{k},{h,r},{h,k},{r,k},Y}. Clearly, the identity mapping $f: X \to Y$ is an *is**-open.

Proposition 3.8 Every iα-open mapping is an *is**-open.

Proof. Let $f: X \to Y$ be an iα-open mapping and V be any open set in X. Since, f is an iα-open, then $f(V)$ is an iα-open set in Y. Since, every iα-open set is an *is**-open set by **proposition 2.5**, then $f(V)$is an *is*-open set in Y. Therefore, f is an *is**-open ■

Corollary 3.8 Every open mapping is an *is**-open.

Proof. Same the proof of **Proposition 3.8**, and Since every open set is an *is**-open set by **corollary 2.9**■

Corollary 3.8 Every semi-open mapping is an *is**-open.

Proof. Clear, Since, every semi-open set is an *is**-open set by **corollary 2.7**■

Corollary 3.8 Every α-open mapping is an *is**-open.

Proof. Clear, Since, every α-open set is an *is**-open set by **corollary 2.8**■

Remark 3.9 The following example shows that semi-open mapping need not be semi (resp. α, i) -open mappings.

Example3.10 Let Let X=Y={5,6,7}, τ={∅,{7},X} σ={∅,{5},Y},$SO(Y)$=$\alpha O(Y)$=$iO(Y)$, ={∅,{5},{5,6},{5,7},Y}, $iSO(Y)$={∅,{5},{6},{7},{5,6},{5,7},{6,7},Y}. A mapping $f : X \to Y$ is defined by f (5)=6, f (6)=5, f (7) = 7. Clearly, f is an *is**-open, but f is not open, semi-open, α-open and f is not i-open because for open subset{7}, f^{-1}{7}={7}∉σ and f^{-1}{7}={7}∉$SO(Y)=\alpha O(Y)=iO(Y)$.

Definition 3.11 Let X and Y be a topological space, a mapping $f : X \to Y$ is said to be *is**-irresolute, if the inverse image of every *is**-open subset of Y is an *is**-open subset in X.

Example 3.12 Let X=Y={d,e,f}, τ={∅,{e},X}, $iSO(X)$={∅,{d},{e},{f},{d,e},{d,f} ,{e,f},X}, σ={∅,{f},Y} and $iSO(Y)$={∅,{d},{e},{f},{d,e},{d,f},{e,f},Y}. Clearly, the identity mapping $f : X \to Y$ is an *is*-irresolute.

Proposition 3.13 Every iα-irresolute mapping is an *is*-irresolute.

Proof. Let $f : X \to Y$ be an iα-irresolute mapping and V be any iα-open set in Y and Since every iα-open set is an *is** – open set **proposition 2.5**, then is an *is** - open set. Since, f is an iα-irresolute, then $f^{-1}(V)$ is an iα-open set in X. Since every iα-open set is an is*–open set. Hence, is*-open set in X . Therefore, f is an is*-irresolute■

Corollary 3.13 Every irresolute mapping is an is*-irresolute.

Proof. Same the proof of **proposition3.13** and by **corollary 2.7** ■

Corollary 3.13 Every i-irresolute mapping is an is*-irresolute.

Proof. Clear by **corollary 2.6** ■

Corollary 3.14 semi α-irresolute mapping is an is*-irresolute.

Proof. Clear by **corollary 2.8** and **corollary 2.7**■

Remark 3.14 this example show that *is**-irresolute mapping need not be irresolute, semi α-irresolute, α-irresolute and i-irresolute mappings.

Example 3.15 Let *X*=*Y*={o,y,k}, τ={∅,{o},*X*},*SO*(*X*)=*αO*(*X*)=*iO*(*X*)={∅,{o},{o,y},{o,k},*X*}, *iSO*(*X*)={∅,{o},{y},{k},{o,y},{o,k},{y,k},*X*}, σ ={∅,{k},*Y*}, *SO*(*Y*)=*αO*(*Y*)=*iO*(*Y*)={∅,{k}, {o,k}, {y,k},*Y*} and *iSO*(*Y*) ={ ∅, {o} ,{y} ,{k} ,{o,k} ,{o,y} ,{y,k} ,*Y*}. Clearly , the identity mapping $f: X \to Y$ is an *is**-irresolute, but *f* is not irresolute, *f* is not α-irresolute, *f* is not semi α-irresolute, *f* is not i-irresolute because for semi-open, α-open and i-open subset {c}, f^{-1}{c}={c} ∉*SO*(*X*)=*αO*(*X*)=*iO*(*X*).

Proposition 3.16 Every *is**-irresolute mapping is an *is**-continuous.

Proof. Let $f: X \to Y$ be an *is**-irresolute mapping and *V* be any open set in *Y*. Since, every open set is an is- open set by **proposition 2.5**, then *V* is an *is**-open set. Since, *f* is an *is**-irresolute, then $f^{-1}(V)$ is an is*-open set in *X*. Therefore f is an is*-continuous■ The converse of the above proposition need not be true as shown in the following example

Example 4.17 Let *X*=*Y*={h,i,j},τ={∅,{h,i},*X*},*iSO*(*X*)={∅,{h},{i},{h,i},{h,j},{i,j} ,*X*}, σ={∅,{h,j},*Y*} and *iSO*(*Y*)={∅,{h},{j},{h.i},{h,j},{i,j},*Y*}. Clearly, the identity mapping $f: X \to Y$ is an *is**- continuous, but *f* is not *is**-irresolute because for *is**-open set {j}, f^{-1}{j}={j} ∉*iSO*(*X*).

Definition 3.18 Let *X* and *Y* be a topological space, a bijective mapping $f: X \to Y$ is said to be *is**-homeomorphism if *f* is an *is**-continuous and *is**-open.

Theorem 3.19 If $f: X \to Y$ is an *iα*-homomorphism, then $f: X \to Y$ is an *is*-homomorphism.

Proof. Since, every iα -continuous mapping is an *is**-continuous by proposition 3.3. Also, since every *is**-open mapping is an *is*-open 3.8. Further, since *f is* bijective. Therefore, *f* is an *is**-homomorphism ■

Corollary 3.20 If $f: X \to Y$ is an homomorphism, then $f: X \to Y$ is an *is*-homomorphism.

Proof. Clear, every continuous mapping is an *is*-continuous by corollary 3.3. Also, since every open mapping is an *is*-open by corollary 3.8. Further, since *f is* bijective. Therefore, *f* is an *is*-homomorphism ■

Corollary 3.21 If $f: X \to Y$ is semi-homomorphism, then $f: X \to Y$ is an *is*-homomorphism.

Proof. Clear, every semi-continuous mapping is an *is*-continuous by corollary 3.3. Also, since every semi-open mapping is an *is*-open corollary 3.8. Further, since f *is* bijective. Therefore, f is an *is*- homomorphism ■

Corollary 3.20 If $f: X \to Y$ is α-homomorphism, then $f: X \to Y$ is an *is*-homomorphism.

Proof. Clear, every α-continuous mapping is an *is*-continuous by corollary 3.3. Also, since every α-open mapping is an *is*-open corollary 3.8. Further, since f *is* bijective. Therefore, f is an *is*- homomorphism ■ The converse of the above theorems need not be true as shown in the following example

Example3.20 X=Y ={e,f,g},τ={∅,{e},X},$SO(X)$=α(X)=$iO(X)$={∅,{e},{e,f},{e,g},X}, $iSO(X)$={∅,{e},{f},{g},{e,f},{e,g},{f,g},X}, σ ={∅,{f},Y}, $SO(X)$ = α(Y)=$iO(Y)$={∅, {f} , {e,f} ,{ f,g} ,Y } and $iSO(Y)$={∅,{e},{f},{g},{e,f},{e,g},{f,g},Y}. Clearly, the identity mapping $f: X \to Y$ is an *is*-homomorphism, but it is not i-homomorphism and it is not α-homomorphism because f is not i-continuous and α-continuous , since for open subset {f}, f^{-1}{f}={f} ∉$iO(X)$ = $SO(X)$=α(X).

4- Advanced *is*-continuous mappings

In this section, we introduce new classes of mappings called iα-totally continuous, i-contra-continuous and iα-contra-continuous.

Definition 4.1 Let X and Y be a topological space, a mapping $f: X \to Y$ is said to be *is*-totally continuous, if the inverse image of every *is*-open subset of Y is clopen set in X.

Example 4.2 Let X=Y={l,m,n},τ={∅,{l},{m,n},X}, σ={∅,{l},Y} and $iSO(Y)$= {∅,{l},{m},{n},{l,n},{m,l},{m,n},Y}. The mapping $f: X \to Y$ is defined by f{l}={l} ,f{n}=f{n}=m. Clearly, f is an iα-totally continuous mapping.

Theorem 4.3 Every $i\alpha$-totally continuous mapping is *is*-totally continuous.

Proof. Let $f: X \to Y$ be iα-totally continuous and V be any open set in Y. Since, every iα open set is an *is*-open set, then V is an iα-open set in Y. Since, f is an iα-totally continuous mapping, then $f^{-1}(V)$ is clopen set in X. Therefore, f is *is*-totally continuous■ The converse of the above theorem need not be true as shown in the following example

Corollary 4.5 Every totally continuous mapping is *is*-totally continuous.

Proof. Same the proof therorem 4.3, and since every open set is an *is*-open set by corollary 2.9■

Example 4.6 Let $X=Y=\{l,m,n\}, \tau=\{\emptyset,\{a\},\{b,c\},X\}$ $\sigma=\{\emptyset,\{l\},Y\}$ and $isO(Y)=\{\emptyset,\{l\},\{m\},\{n\},\{l,m\},\{l,n\},\{m,n\},Y\}$. Clearly, the identity mapping is $f:X\rightarrow Y$ totally continuous, but f is not iα-totally continuous because for iα-open set $\{l,n\}$, $f^{-1}\{l,n\}=\{l,n\}\notin CO(X)$.

Theorem 4.7 Every *is*-totally continuous mapping is an *is*-irresolute.

Proof. Let $f:X\rightarrow Y$ be *is*-totally continuous and V be an *is*-open set in Y. Since, f is an *is*-totally continuous mapping, then $f^{-1}(V)$ is clopen set in X, which implies $f^{-1}(V)$ open, it follow $f^{-1}(V)$ *is*-open set in X by corollary 2.9. Therefore, f is an *is*-irresolute■
The converse of the above theorem need not be true as shown in the following example

Example 4.8 Let $X=Y=\{0,2,4\}, \tau=\{\emptyset,\{2\},X\}$, $iSO(X)=\{\emptyset,\{0\},\{2\},\{4\},\{0,2\},\{0,4\},\{2,4\},X\}$, $\sigma=\{\emptyset,\{0,2\},Y\}$, and $iSO(Y)=\{\emptyset,\{0\},\{2\},\{0,2\},\{0,4\},\{2,4\},Y\}$. Clearly, the identity mapping $f:X\rightarrow Y$ is an is-irresolute, but f is not is-totally continuous because for is-open subset$\{0,4\}$, $f^{-1}\{0,4\}=\{0,4\}\notin CO(X)$.

Theorem 4.9 The composition of two *is*-totally continuous mapping is also *is*-totally continuous.

Proof. Let be any two $g:Y\rightarrow Z$ and $f:X\rightarrow Y$ *is*-totally continuous. Let V be any is-open in Z. Since, g is an *is*-totally continuous, then $g^{-1}(V)$ is clopen set in Y, which implies $f^{-1}(V)$ open set, it follow $f^{-1}(V)$ *is*-open set. Since, f is an *is*-totally continuous, then $f^{-1}(g^{-1}(V))=(g \text{ o } f)^{-1}(V)$ is clopen in X. Therefore, $gof:X\rightarrow Z$ is an *is*-totally continuous■

Corollary 4.10 If $f:X\rightarrow Y$ be an *is*-totally continuous andbe an $g:Y\rightarrow Z$ *is*-irresolute, then $gof:X\rightarrow Z$ is an *is*-totally continuous.

Proof. Let $f:X\rightarrow Y$ be *is*-totally continuous and $g:Y\rightarrow Z$ be *is*-irresolute . Let V be is-open set in Z. Since, g *is* an is-irresolute, then $g^{-1}(V)$ is an *is*-open set in Y. Since, f is an *is*-totally continuous, then $f^{-1}((g^{-1}(V))=(g \text{ o } f)^{-1}(V)$ is clopen set in X. Therefore, $gof:X\rightarrow Z$ is an is-totally continuous■

Theorem 4.11 If $f:X\rightarrow Y$ is an *is*-totally continuous and-is an is $g:Y\rightarrow Z$ continuous, then $gof:X\rightarrow Z$ is totally continuous.

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Proof. Let *V*continuous . Let -is an is $g:Y\rightarrow Z$ totally continuous and -be is $f:X\rightarrow Y$ be an open set in *Z*. Since, *g* is an is-continuous, then $g^{-1}(V)$ is an is-open set in *Y*. Since, *f* is an is-totally continuous, then $f^{-1}(g^{-1}(V))=(g \text{ o } f)^{-1}(V)$ is clopen set in *X*. Therefore, $gof:X\rightarrow Z$ is totally continuous ■

Definition 4.12 Let *X*, *Y* be a topological spaces, a mapping $f:X\rightarrow Y$ is said to be is-contra-continuous, if the inverse image of every open subset of *Y* is an is-closed set in *X*.

Example 4.13 Let *X*=*Y*={o,p,q},τ={∅,{o},*X*}, σ={∅,{q},*Y*} and *iSO*(*X*)= {∅,{o},{p},{q},{o,p},{o,q},{p,q},*X*}. Clearly, the identity mapping $f:X\rightarrow Y$ is an *is*-contra-continuous.

Proposition 4.14 Every contra-continuous mapping is an is-contra-continuous.

Proof. Let $f:X\rightarrow Y$ be contra continuous mapping and *V* any open set in *Y*. Since, *f* is contra continuous, then $f^{-1}(V)$ is an is-closed sets in *X*. Since, every closed set is an is-closed set, then $f^{-1}(V)$ is an is-closed set in *X*. Therefore, *f* is an is-contra-continuous■ Similarly we have the following results.

Corollary 4.15 Every contra-continuous mapping is an is-contra-continuous.

Proof. Clear, since every closed set is an is-open set■

Corollary 4.16 Every i-contra-continuous mapping is an is-contra-continuous.

Proof. Clear, since every i-closed set is an is-open set■

Proposition 4.17 Every contra semi-continuous mapping is an i-contra-continuous.

Proof. Clear since every semi-open set is an i-open set■

Proposition 4.18 Every contra α-continuous mapping is an is-contra-continuous.

Proof. Clear since every α-open set is an is-open set■ The converse of the propositions 4.12, 4.13 and 4.14 need not be true in general as shown in the following example

Example 4.19 Let*X*=*Y*={o,p,q},τ={∅,{o,q},*X*}, *isO*(*X*)={∅,{o},{p},{o,p},{o,q},{p,q}, *X*} and σ={∅,{q},*Y*}. Clearly, the identity mapping $f:X\rightarrow Y$ is is an *is*-contra continuous, but *f* is not contra-continuous, *f* is not contra semi-continuous, *f* is not contra α-continuous because for open subset f^{-1}{q}={q} is not closed in *X*, f^{-1}{q} ={q}is not semi-closed in *X* and f^{-1}{q}={q}is not α-closed in *X*.

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Proposition 4.20 Every iα -contra-continuous mapping is an *is*-contra-continuous.

Proof. Let $f: X \to Y$ be an *iα*-contra continuous mapping and *V* any open set in *Y*. Since, *f* is an iα-contra continuous, then $f^{-1}(V)$ is an iα-closed sets in *X*. Since, every iα -closed set is an *is*-closed, then $f^{-1}(V)$ is an is-closed set in *X*. Therefore, *f* is an is-contra-continuous■

Remark 4.21 The following example shows that *is*-contra-continuous mapping need not be contra-continuous, contra semi-continuous, contra-α-continuous and i-contra-continuous mappings.

Example 4.22 Let *X*=*Y*={o,p,q},τ={∅,{o},X}, *SO*(X)=*αO*(*X*)=*iO*(*X*)={∅,{o},{o,p},{o,q},*X*}, iS*O*(*X*)= {∅,{o},{p},{q},{o,p},{o,q},{p,q},*X*}, σ={∅,{q}, *Y*}. A mapping continuous, but -contra-is an is *f* (q)=o. Clearly,*f*(p)=p,*f*(o)=q, *f*is defined by $f: X \to Y$ *f* is not contra-continuous, *f* is not contra semi continuous, *f* is not contra α-continuous and *f* is not i-contra-continuous because for open subset {q}, f^{-1}{q}={o} is not closed, f^{-1}{q}={o} is not semi-closed , f^{-1}{q}={o} is not α-closed and f^{-1}{q}={o} is not i-closed in *X*.

Definition 4.23 Let *X*, *Y* be a topological spaces, a mapping $f: X \to Y$ is said to be *is*-totally continuous, if the inverse image of every *is*- open subset of *Y* is clopen set in *X*.

Theorem 4.24 Every *is*-totally continuous mapping is an iα- totally continuous.

Proof. Let $f: X \to Y$ be *is* -totally continuous and *V* be any iα-open set in *Y*. Since, every iα-open set is an *is* -open set by proposition 2.4, then *V* is an is-open sets in *Y*. Since, *f* is an *is*-totally continuous mapping, then $f^{-1}(V)$ is clopen set in *X*. Therefore, *f* is an *iα*-totally continuous■

Corollary 4.25 Every *is*-totally continuous mapping is an i- totally continuous.

Proof. Same proof of **Theorem 4.24** , and since every i-open set is an *is* -open set by corollary 2.5. ■

Corollary 4.24 Every *is*-totally continuous mapping is totally continuous.

Proof. Same proof of **Theorem 4.24** , and since every open set is an *is* -open set by corollary 2.5. ■

Theorem 4.26 Every totally continuous mapping is an iα-continuous.

Proof. Let $f: X \to Y$ be totally continuous and *V* be any open set in *Y*. Since, *f* is an *is*-totally continuous mapping, then $f^{-1}(V)$ is clopen set in *X*, and hence $f^{-1}(V)$ is open set, Since, every open set is an *is* -open set by corollary 2.4, then V is an *is*-open sets in Y . Therefore, f is an $i\alpha$-continuous■

Corollary 4.26 Every is-totally continuous mapping is an is-continuous.

Proof. Let $f : X \to Y$ be *is*-totally continuous, Since Every *is*-totally continuous mapping is totally continuous by **Corollary 4.24,** then f is totally continuous. Since, Every totally continuous mapping is an iα-continuous by **Theorem 4.26,** Therefore f is *is* -continuous■ The converse of the above theorem 4.24 and corollary 4.25 need not be true as shown in the following example

Example 4.24 Let X=Y={o,p,q},τ ={∅,{o,q},X}, σ={∅,{q},Y} and $iSO(X)$={∅,{o} ,{q},{o},{o,p},{o,q},{p,q},X}. A mapping (p)=o,f(o)=p, f yis defined b $f : X \to Y$ f(q)=q, f is an iα-continuous, but f is not totally continuous and is - totally continuous because for open subset {q}, f^{-1}{q}={q}∉CO(X) and *is*-open subset{p}, f^{-1}{p}={o}∉CO(X).

Corollary 4.27 Every is-totally continuous mapping is an is-irresolute.

proof. Let $f : X \to Y$ be is -totally continuous and V be any is -open set in Y. Since, f is an *is*-totally continuous mapping, then $f^{-1}(V)$ is clopen set in X, and hence $f^{-1}(V)$ is open set, Since, every open set is an *is* -open set by corollary 2.4, then V is an *is*-open sets in Y . Therefore, f is an *is*- irresolute■ The converse of the above theorem 4.24 need not be true as shown in the following example

Example 4.24 Let X=Y={o,p,q},τ ={∅,{o},X}, σ={∅,{b},Y} and $iSO(X)$={∅,{o} ,{q},{o},{o,p},{o,q},{p,q},X}. A mapping (p)=o,f p, = (o) f yis defined b $f : X \to Y$ f(q)=q, f is an *is*- irresolute, but f *is*- totally continuous because for *is*-open subset {p}, f^{-1}{p}={o}∉CO(X) .

Theorem 4.28 Every totally continuous mapping is an *is*-contra continuous.

Proof. Let $f : X \to Y$ be totally continuous and V be any open set in Y. Since, f is an totally continuous mapping, then $f^{-1}(V)$ is clopen set in X, and hence closed, it follows is-closed set. Therefore, f is an iα-contra-continuous■

Corollary4.29 Every is-totally continuous mapping is an *is*-contra continuous.

Proof. Let $f : X \to Y$ be is totally continuous. Since Every *is*-totally continuous mapping is totally continuous by **Corollary 4.24**. then f is totally continuous . Since, Every totally continuous mapping is an *is*-contra continuous by **Theorem**

4.28.Therefore, f is an iα-contra-continuous■ The converse of the above theorem need and corollary not be true as shown in the following example

Example 4.24 Let X=Y={4,6,8},τ={∅,{6},X}, σ={∅,{4},Y} and is$O$(X)={∅,{4},{6},{8},{4,6},{4,8},{6,8},X}. Clearly, the identity mapping -contra-is an iα $f:X\rightarrow Y$ continuous, but f is not totally continuous because for open subset f^{-1}{2}={2}∉CO(X).

5 Some Separation axioms with is-open Set

Definition 5.1 A topological space X is said to be

(i) $i\alpha$-T_o [] if for each pair distinct points of X, there exists $i\alpha$ -open set containing one point but not the other.
(ii) $i\alpha$ -T_1 [ ](resp. clopen -T_1 [3]) if for each pair of distinct points of X, there exists two is-open (resp. clopen) sets containing one point but not the other .
(iii) $i\alpha$ -T_2 [](resp. ultra hausdorff (UT_2)[10]) if for each pair of distinct points of X can be separated by disjoint is-open (resp. clopen) sets.
(iv) $i\alpha$–regular [](resp. ultra regular [9]) if for each closed set F not containing a point in X can be separated by disjoint is-open (resp. clopen) sets.
(v) clopen regular [10] if for each clopen set F not containing a point in X can be separated by disjoint open sets.
(vi) $i\alpha$–normal [](resp. ultra normal[10], s-normal[13], α-normal[2]ᵞ-normal [15]) if for each of non-empty disjoint closed sets in X can be separated by disjoint iα-open (resp. clopen, semi-open, α-open, ᵞ-open) sets.
(vii) clopen normal [10] if for each of non-empty disjoint clopen sets in X can be separated by disjoint open sets.
(viii) $i\alpha$-$T_{1/2}$ [ ]if every is-closed is $i\alpha$ -closed in X.

Definition 5.2 A topological space X is said to be

(ix)is-T_o if for each pair distinct points of X, there exists is-open set containing one point but not the other.
(x) is-T_1, if for each pair of distinct points of X, there exists two is-open sets containing one point but not the other .
(xi) is-T_2, if for each pair of distinct points of X can be separated by disjoint is-open sets.
(xii) is–regular, if for each closed set F not containing a point in X can be separated by disjoint is-open sets.
(xiii) is–normal, if for each of non-empty disjoint closed sets in X can be separated by disjoint is-open sets.

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(xiv) is-$T_{1/2}$ if every is-closed is is-closed in X.

Proposition 5.3 Every $i\alpha$–normal is an is – normal

Proof. Let $F_1$and F_2 be disjoint closed subsets of topological space (X,τ). Since, (X,τ) is an $i\alpha$- normal, there are disjoint c subsets of $i\alpha$-open sets U and V such that $F1 \subset$ U and $F2 \subset V$, Since every $i\alpha$-open is an is- open by proposition 2.5, Then (X,τ) is is-Normal space■

Corollary 5.3 Every ultra normal is an is – normal

Proof. Let $F_1$and F_2 be disjoint closed subsets of topological space (X,τ). Since, (X,τ) is an ultra normal, there are disjoint c subsets of clopen sets U and V, and hence U and V open sets such that $F1 \subset$ U and $F2 \subset V$, Since every open is an is- open by Corollary 2.6, Then (X,τ) is is-Normal space■

Corollary 5.3 Every normal is an is – normal

Proof. Same the proof of proposition 5.3, and since every open is an is- open by corollary 2.7 ■

Corollary 5.3 Every s-normal is an is – normal

Proof. Same the proof of proposition 5.3, and since every semi-open is an is- open by corollary 2.8 ■

Corollary 5.3 Every α-normal is an is – normal

Proof. Same the proof of proposition 5.3, and since every α -open is an is- open by corollary 2.7 ■

Propostion 5.3 if A topological (X,τ) is extermally disconnected, then every γ-normal is an is – normal

Proof. Let (X,τ) is a topological space, and extermally disconnected with $F_1$and F_2 disjoint closed subsets of topological space. Since, (X,τ) is γ- normal, there are disjoint subsets of γ-open sets U and V, Since (X,τ) is extermally disconnected , then U and V are semi-open sets such that $F1 \subset$ U and $F2 \subset V$, Since every semi-open is an is-open by Corollary 2.8, Then (X,τ) is is-Normal space■

Remark 5.2 The following example shows that *is*-normal need not be normal, s-normal, α-normal spaces

Example 5.3 Let *X*={a,b,c,d,e}, τ={∅,{a,b,c},{a,b,c,d},{a,b,c,e},*X*} and is*O*(*X*)= {∅,{a},{b},{c},{d},{e},{a,b},{a,c},{a,d},{a,e},{b,c},{b,d},{b,e},{c,d},{c,e},{d,e}, {a,b,c},{a,b,d},{a,b,e},{a,c,d},{a,c,e},{a,d,e},{b,c,d},{b,c,e},{b,d,e},{c,d,e},{a,b,c,d} ,{a,b,c,e},{a,c,d,e},{a,b,d,e},{b,c,d,e},*X*}. Clearly, the space *X* is is-T_o, is-T_1, is-T_2, is–regular, is–normal and is-$T_{1/2}$, but *X* is not normal, s-normal, ultra normal, γ-normal and α-normal.

Lemma 5.4 if a mappingcontinuous mapping and the space –contra-is an iα $f:X\rightarrow Y$ *X* is an iα-$T_{1/2}$, then *f* is an iα-contra-continuous [ ].

Theroem 5.4 if a mappingis an $f:X\rightarrow Y$ *is*-contra–continuous mapping and the space *X* is an *is*-$T_{1/2}$, then *f* is an *is*-contra-continuous .

Proof. Clear, since every iα-open set is an is –open set.

Theorem 5.5 If $f:X\rightarrow Y$ is an is-totally continuous mapping from any topological *X* to finite space *Y* T_1, then *f* is strongly continuous.

Proof. Let $f:X\rightarrow Y$ an is-totally continuous mapping, and *V* is any subset in is finite space T_1 *Y*. Since *Y*, is finite space T_1, then *Y* is discret topology space, so that *V* is open set and wich implies *V* is open set , then *V* is an is-open set by corollary 2.7. Since *f* is an is-totally continuous, then $f^{-1}(V)$ is clopen in *X* . Therefore f is strongly continuous■

Theorem 5.5 If $f:X\rightarrow Y$ is an is-totally continuous injection mapping and *Y* is an is-T_1, then *X* is clopen-T_1.

Proof. Let *x* and *y* be any two distinct points in *X*. Since, *f* is an injective, we have $f(x)$ and $f(y)\in Y$ such that $f(x)\neq f(y)$. Since, *Y* is an is-T_1, there exists is-open sets *U* and *V* in *Y* such that $f(x)\in U$, $f(y)\notin U$ and $f(y)\in V$, $f(x)\notin V$. Therefore, we have $x\in f^{-1}(U)$,$y\notin f^{-1}(U)$ and $y\in f^{-1}(V)$ and $x\notin f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are clopen subsets of *X* because *f* is an is-totally continuous. This shows that *X* is clopen-T_1■

Corollary 5.5 If $f:X\rightarrow Y$ is an strongly continuous injection mapping and *Y* is an is-T_1, then *X* is clopen-T_1.

Proof. Same the proof of the Theorem 5.7■

Theorem 5.6 If $f: X \to Y$ is an is-totally continuous injection mapping and Y is an is-T_o, then X is ultra-Hausdorff (UT_2).

Proof. Let a and b be any pair of distinct points of X and f be an injective, then $f(a) \neq f(b)$ in Y. Since Y is an is-T_o, there exists is-open set U containing $f(a)$ but not $f(b)$, then we have $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$. Since, f is an is-totally continuous, then $f^{-1}(U)$ is clopen in X. Also $a \in f^{-1}(U)$ and $b \in X\text{-}f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint clopen sets in X. Therefore, X is ultra-Hausdorff■

Corollary 5.6 If $f: X \to Y$ is an is-totally continuous injection mapping and Y is an is-T_2, then X is ultra-Hausdorff (UT_2).

Proof. Same the proof of Theorem 5.8■

Theorem 5.7 Let $f: X \to Y$ be a closed is-continuous injection mapping. If Y is an is–normal, then X is an is–normal.

Proof. Let $F_1$and F_2 be disjoint closed subsets of X. Since, f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since, Y is an is–normal , $f(F_1)$ and $f(F_2)$ are separated by disjoint is-open sets V_1 and V_2 respectively. Therefore, we obtain, $F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since, f is an is-continuous, then $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are is-open sets in X. Also, $f^{-1}(V_1) \cap f^{-1}(V_1) = f^{-1}(V_1 \cap V_2) = \emptyset$. Thus, for each pair of non-empty disjoint closed sets in X can be separated by disjoint is-open sets. Therefore, X is an is–normal ■

Corollary 5.7 Let $f: X \to Y$ be a closed is-irresolute injection mapping. If Y is an is–normal, then X is an is–normal.

Proof. Same the proof of theorem and since every is-irresolute is an is-continuous mapping by corollary ■

Proof. Same the proof of theorem and since every is-totally continuous is an is-continuous mapping by corollary ■

Corollary 5.7 Let $f: X \to Y$ be a closed is-totally continuous injection mapping. If Y is an is–normal, then X is an is–normal

Theorem 5.8 If $f: X \to Y$ is an is-totally continuous closed injection mapping and Y is an is–normal, then X is ultra-normal.

Proof. Let $F_1$and F_2 be disjoint closed subsets of X. Since, f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since, Y is an is–normal , $f(F_1)$ and $f(F_2)$ are separated by disjoint is-open sets $V_1$and V_2 respectively. Therefore, we obtain, $F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since, f is an is-totally continuous, then $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are clopen sets in X. Also, $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \emptyset$. Thus, for each pair of non-empty disjoint closed sets in X can be separated by disjoint clopen sets in X. Therefore, X is ultra-normal■

Corollary 5.8 If $f: X \to Y$ is an totally continuous closed injection mapping and Y is an is–normal, then X is ultra-normal.

Proof. Same the proof of the theorem 5. 12, And since every totally continuous is an is-totally continuous mapping ■

Theorem 5.9 Let $f: X \to Y$ be an is-totally continuous closed injection mapping, if Y is an is–regular, then X is ultra-regular.

Proof. Let F be a closed set not containing x. Since, f is closed, we have $f(F)$ is a closed set in Y not containing $f(x)$. Since, Y is an is–regular, there exists disjoint is-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$, which imply $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are clopen sets in X because f is an is-totally continuous. Moreover, since f is an injective, we have $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. Thus, for a pair of a point and a closed set not containing a point in X can be separated by disjoint clopen sets. Therefore, X is ultra-regular■

Corollary 5.9 Let $f: X \to Y$ be a totally continuous closed injection mapping, if Y is an is–regular, then X is ultra-regular.

Proof. Same the proof of the theorem 5. 12, And since every totally continuous is an is-totally continuous mapping ■

Theorem 5.10 Ifopen mapping from a -is totally continuous injective iα $f: X \to Y$ clopen regular space X into a space Y, then Y is an is–regular.

Proof. Let F be a closed set in Y and $y \notin F$. Take $y = f(x)$. Since, f is totally continuous, $f^{-1}(F)$ is clopen in X. Let $G = f^{-1}(F)$, then we have $x \notin G$. Since, X is clopen regular, there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$. This implies $F = f(G) \subset f(U)$ and $y = f(x) \in V$. Further, since f is an injective and is-open, we have $f(U) \cap f(V) = f(U \cap V) = f(\emptyset) = \emptyset$, $f(U)$ and $f(V)$ are an is-open sets in Y. Thus, for each closed set F in Y and each $y \notin F$, there exists disjoint is-open sets $f(U)$ and $f(V)$ in Y such that $F \subset f(U)$ and $y \in f(V)$. Therefore, Y is an *is*–regular■

Theorem 5.11 If $f : X \to Y$ is a totally continuous injective and iα-open mapping from clopen normal space *X* into a space *Y*, then *Y* is an iα–normal.

Proof. Let $F_1$and F_2 be any two disjoint closed sets in *Y*. Since, *f* is totally continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are clopen subsets of *X*. Take $U= f^{-1}(F_1)$ and $V= f^{-1}(F_2)$. Since, *f* is an injective, we have $U \cap V= f^{-1}(F_1) \cap f^{-1}(F_2)= f^{-1}(F_1 \cap F_2)= f^{-1}(\emptyset)=\emptyset$. Since, *X* is clopen normal, there exists disjoint open sets *A* and *B* such that $U \subset A$ and $V \subset B$. This implies $F_1= f(U) \subset f(A)$ and $F_2= f(V) \subset f(B)$. Further, since *f* is an injective is-open, then *f* (*A*) and *f* (*B*) are disjoint is-open sets. Thus, each pair of disjoint closed sets in *Y* can be separated by disjoint iα-open sets. Therefore, *Y* is an is–normal ■

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