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Applying Compressive Sensing to the Massive MIMO Channel Estimation Problem

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***Abstract*— This paper proposes the use of compressive sensing to tackle the Massive MIMO channel estimation problem. As our results show compressive sensing-based estimators perform as well as the optimum MMSE estimator.**

Keywords: massive MIMO; compressive sensing; channel estimation

I. Introduction

Accurate channel estimation is very important for massive MIMO systems, once they are necessary to provide significant improvements in spectral and energy efficiency. In massive MIMO systems, the base station (BS) estimates the channels of all its connected users. These estimates, whcih are obtained during the uplink transmission phase are used to generate pre-coding and decoding matrices. These matrices are used to receive and transmit data. Therefore, accurate estimation of the channels is a very important task for massive MIMO systems. In this work, we present a comparative study on the application of compressive sensing to the massive MIMO channel estimation problem.

II. System Model

Let $x_k(n)$ denote the transmitted time-domain samples of the kth MTC device, $k = 1, \ldots, K$, *i.e.* the Orthogonal Frequency-Division Multiplexing (OFDM) symbol transmitted by the kth MTC device. OFDM symbols are normalized to unitary variance, so $E[|x_k(n)|^2] = 1$. In the uplink, the signals from a cluster of K MTC devices is collected into the vector

$$\mathbf{x}(n) = [x_1(n), \ldots, x_K(n)]^T, \quad (1)$$

where $(\cdot)^T$ denotes transposition and $\mathbf{x} \in \mathbb{C}^{K\times 1}$ [16].

Consider now a Massive MU-MIMO setup, where $\mathbf{x}(n)$ is detected by a BS equipped with M receive antennas, $M \gg K$. Between every transmit antenna k at the MTC device and every receive antenna m at the BS there is a complex single-input single-output (SISO) channel impulse response $h_{m,k}(n)$ of length $L+1$, described by the vector:

$$\mathbf{h}_{m,k} = [h_{m,k}(0), \ldots, h_{m,k}(L)]^T. \quad (2)$$

Under the assumption that all SISO channels have channel order L, the frequency selective MIMO channel can be described by a number of $L+1$ $M \times K$ complex channel matrices:

$$\mathbf{H}(n) = \begin{bmatrix} h_{1,1}(n) & \cdots & h_{1,K}(n) \\ \vdots & \ddots & \vdots \\ h_{M,1}(n) & \cdots & h_{M,K}(n) \end{bmatrix}, n = 0, \ldots, L \quad (3)$$

If the signal received by the mth antenna at the nth time instant is denoted by $y_m(k)$, the signals received by all M antennas can be represented in vector form as

$$\mathbf{y}(n) = [y_1(n), \ldots, y_M(n)]^T, \quad (4)$$

which we then rewrite in terms of (1) and (3) as

$$\mathbf{y}(n) = \sum_{i=0}^{L} \mathbf{H}(i)\mathbf{x}(n-i) + \mathbf{w}(n), \quad (5)$$

where the noise vector $\mathbf{w}(n)$ has length M is assumed additive white Gaussian noise (AWGN) with zero mean and variance σ_w^2 per receive antenna. For each of the K MTC devices there are M PNSCH signal versions in (5). Hence, the task of the BS consists of detecting K simultaneous MTC transmissions on the basis of estimates of the channel coefficients in (3).

III. Channel Estimation

The Massive MIMO channel is modeled as a superposition of $M \times K$ single-input single-output (SISO) channels. Each SISO channel has $L+1$ unknowns. For pilot-assisted channel estimation in OFDM systems we will employ the comb-type pilot pattern on the time-frequency 2-D grids [17].

Consider an OFDM system with N subcarriers in each OFDM symbol, among which N_p pilot subcarriers indicated by $p_1, p_2, \ldots, p_{N_p}$ are used for frequency-domain pilot-assisted channel estimation. Without loss of generality, we assume that $1 \leq p_1 < p_2 < \ldots < p_{N_p} < N$. The corresponding transmit pilot symbols are denoted as $s(p_1), s(p_2), \ldots, s(p_{N_p})$. Let $h(0), h(1), \ldots, h(L+1)$ be the equivalent discrete channel impulse response (CIR) with the maximum multipath delay spread being $L+1$ samples. The received signals on the pilot subcarriers can be written as

$$\begin{bmatrix} y(p_1) \\ y(p_2) \\ \vdots \\ y(pN_p) \end{bmatrix} = \begin{bmatrix} s(p_1) & 0 & 0 & 0 \\ 0 & s(p_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & s(p_{N_p}) \end{bmatrix} .\mathbf{F}_{N_p\times L+1} \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L+1) \end{bmatrix} + \begin{bmatrix} w(1) \\ w(2) \\ \vdots \\ w(N_p) \end{bmatrix} \quad (6)$$

where $w(i) \sim \mathcal{CN}(0, \sigma_w^2), i = 1, 2, \ldots, N_p$ is the i.i.d additive white Gaussian noise, and $\mathbf{F}_{N_p\times L+1}$ is a discrete Fourier transform (DFT) sub-matrix given by

$$\mathbf{F}_{N_p\times L+1} = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & w^{p_1} & \cdots & w^{p_1(L-1)} \\ 1 & w^{p_2} & \cdots & w^{p_2(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{p_{N_p}} & \cdots & w^{p_{N_p}(L-1)} \end{bmatrix} \quad (7)$$

where $w = e^{-j2\pi/N}$. We denote

$$\begin{aligned} \mathbf{S} &\triangleq \mathrm{diag}\{s(p_1), s(p_2), \ldots, s(p_{N_p})\} \\ \mathbf{y} &\triangleq [y(p_1), y(p_2), \ldots, y(p_{N_p})]^T \\ \mathbf{h} &\triangleq [h(0), h(1), \ldots, h(L+1)]^T \\ \mathbf{w} &\triangleq [w(1), w(2), \ldots, w(N_p)]^T. \end{aligned}$$

Furthermore, we let

$$\mathbf{D} \triangleq \mathbf{S}\mathbf{F}_{N_p\times L+1}. \quad (8)$$

Then, (6) can be rewritten as

$$\mathbf{y} = \mathbf{D}\mathbf{h} + \mathbf{w}. \quad (9)$$

Next we briefly describe the channel estimation techniques adopted here for comparison. They are employed to estimate one of the SISO channels.

A. Least Squares

The linear Least Square (LS) channel estimator is given by

$$\hat{\mathbf{h}}^{\mathbf{LS}}_{\mathbf{m,k}} = [\mathbf{D}^\dagger\mathbf{D}]^{-1}\mathbf{D}^\dagger\mathbf{y}_m. \quad (10)$$

where $(.)^\dagger$ denotes transpose-conjugate (Hermitian) operation.

LS employs no knowledge of the statistics of the channels. It presents very low complexity, but has a high mean-square error [17].

B. Minimum Mean Squared Error

$$\hat{\mathbf{h}}^{\mathbf{MMSE}}_{\mathbf{m,k}} = \left[\mathbf{D}^\dagger\mathbf{D} + \frac{\sigma_n^2}{\sigma_h^2}\mathbf{I}\right]^{-1}\mathbf{D}^\dagger\mathbf{y}_m. \quad (11)$$

where σ_n^2 is the noise variance and σ_h^2 is the variance of the SISO channel, $\mathbf{h}_{m,k}$. MMSE estimators employ second-order statistics of the channels in order to minimize the mean-square error. These estimators present better performance than the LS ones, especially at low SNR values [17].

C. Compressed Sensing

The technique for sparse signal recover known as compressive sensing has been under heavy investigation since its inception a few years ago [18, 19]. Sparse channel estimation can be more efficient than the conventional channel estimation approaches, $i.e.$, LS, MMSE, etc., due to the sparse nature of multipath wireless channels [20, 21].

IV. Signal Detection

Detection techniques are needed to separate the data streams transmitted by each MTC device in our Massive MU-MIMO setup. Maximum likelihood detection is theoretically optimum but its complexity grows exponentially with the modulation order and the number of transmit antennas K (hard to implement in case of thousands of MTC devices). One way to circumvent this limitation is to use sub-optimal alternatives with reduced computational complexity [22]. Maximum Ratio Combining (MRC) chooses the linear detection matrix using $\mathbf{A}_{\text{MRC}} = \mathbf{H}$, which requires $\mathcal{O}(MK)$ multiplications. Constrained to $\mathbf{AH} = \mathbf{I}$, Zero Forcing (ZF) chooses $\mathbf{A}_{\text{ZF}} = \mathbf{H}(\mathbf{H}^\dagger\mathbf{H})^{-1}$ and poses an associated complexity of $\mathcal{O}(MK + MK^2 + K^3)$ [23].

In contrast to ZF, which minimizes interference but fails to treat noise, and to MRC, which minimizes noise but fails to treat interference, MMSE achieves an optimal balance between interference suppression and noise enhancement at the same cost of ZF [23, 24]. As the name suggests, the MMSE detector chooses the $\mathbf{A}$ that minimizes $e = E[\|\mathbf{A}^\dagger\mathbf{y} - \mathbf{x}\|^2]$ without any additional constraints

$$\mathbf{A}_{\text{MMSE}} = \mathbf{H}\left(\mathbf{H}^\dagger\mathbf{H} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right)^{-1}, \quad (12)$$

where σ_x^2 and σ_n^2 denote the variances of transmitted signal vector and noise vector, respectively.

V. Simulation Work

In this section, we assess the performances of LS, MMSE and OMP channel estimators in terms of their Mean Square Error (MSE) and Bit Error Rate (BER) over a range of Signal-to-Noise Ratios (SNRs).

As can be seen in Figure 1 (a) OMP has a better MSE performance than MMSE and LS channel estimator for the whole range of SNR values. On Figure 1 (b), we see the results of the BER comparison. As we notice, for low SNR values the MMSE estimator performs better than the OMP, however, as the SNR increases, the OMP estimator surpasses the MMSE estimator and for SNR values greater than 30 dB they both achieve a floor value and from that point on both of them present the same performance. The floor achieve by all the three estimators is caused by the type of combining adopted in this work and it can be mitigated by increasing the number of antennas deployed at the BS.

VI. Conclusions

This paper has proposed the use of compressive sensing to tackle the Massive MIMO channel estimation problem. As can be seen by analyzing the results the OMP based estimator performs as well as the MMSE estimator.

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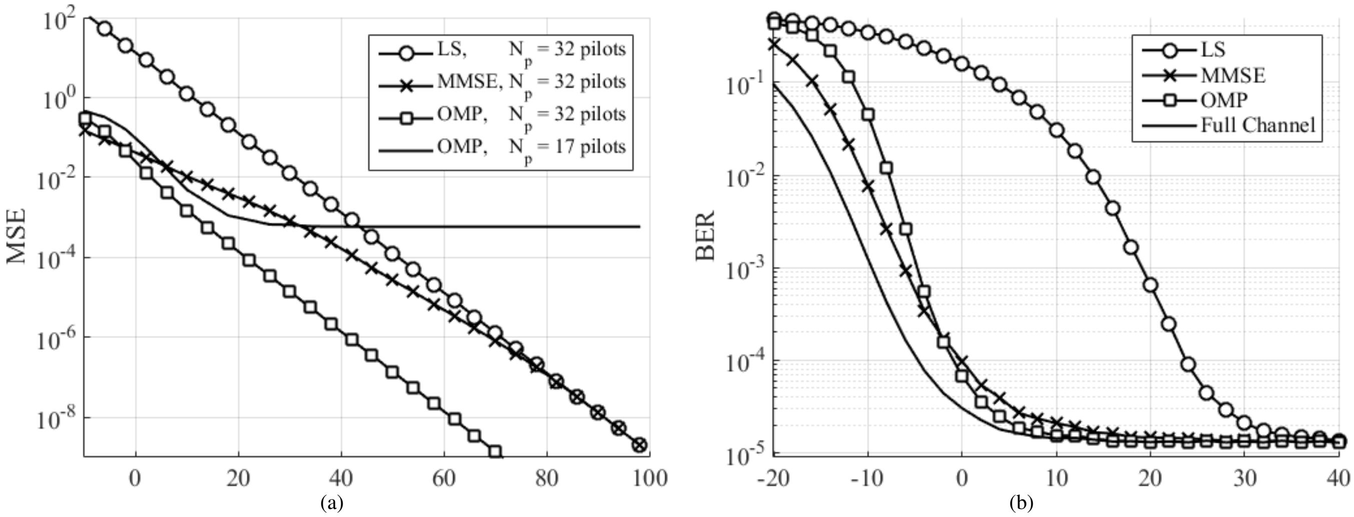

Fig. 1. Comparison of MSE and BER performance for different channel estimation techniques.

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