

Article

Smart Meter Data-based Three-Stage Algorithm to Calculate Power and Energy Losses in Low Voltage Distribution Networks

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Abstract: In the paper, an improved smart meter data-based three-stage algorithm to calculate the power/energy losses in the three-phase low voltage (LV) distribution networks was proposed. In the first stage, an loading procedure of input data was built, being able to work simultaneously with files containing the active and reactive power profiles provided by smart meters and typical profiles associated to consumers without smart meters, based on the energy consumption categories, day type (weekend and working), and season type, knowing the daily energy indexes, in the second stage, a structure vectors-based algorithm was implemented to recognize the network topology, and in the third stage, an improved version of forward/backward sweep-based algorithm was proposed to calculate fast the power/energy losses to three-phase LV distribution networks in balanced and unbalanced regime. A real LV rural distribution network from a pilot zone belonging to a Distribution Network Operator (DNO) from Romania was used to verify the accuracy of the proposed algorithm. The results were compared with those obtained using the DigSilent PowerFactory Professional Software, the MAPE being by 0.94%.

Keywords: smart meters; low voltage distribution networks; energy losses; three-stage algorithm; typical load profiles

1. Introduction

Until a few years ago, the electric distribution networks were generally characterized by the lack of technical possibilities represented by smart devices that can help the Distribution Networks Operators (DNOs) in the supervisory, control and decision making processes. Although the low voltage (LV) distribution networks feed a high number of consumers, few information could have been gathered from inside (from the consumers and producers), with a delayed response time. In order to obtain as much data from the network, it is necessary to be installed smart meters which would allow the storage of supervised data (energy consumptions, active and reactive powers, voltages, power factors, harmonics etc.) and their transmission to the DNOs level.

The Smart Metering technology is essential for achieving the targets regarding the energy efficiency and renewable energy set for 2020, as well as the delineation of future smart grids. The implementation of smart metering system (SMS) at the European Union level is finished in some countries and is in different stages in others [1-5]. Thus, a special attention should be paid to the management of databases built with the help of information provided by smart meters from consumers and producers for improving the energy efficiency in the low voltage networks. The benefits of smart meters consist in the fact that, in addition to the metering function, they also provide a whole range of applications, such as the following [6, 7]:

- secure transmission of data to the consumer or a third party (for example Metering Operator), respectively to the DNO;
- bidirectional communication between the smart meters installed at consumer/prosumer sites and the concentrators (information management points) belonging to the DNO;
- remotely controlled connection/disconnection from the grid or demand limitation at consumer sites;
- implementing of differentiated time-of-use tariffs.

In these circumstances, the DNOs can get accurate online information regarding energy consumptions and productions from renewable sources, which allows them to calculate the energy losses and then to take some measures which will enable the low voltage networks to operate more energy efficient and better plan their investments. In this context, the energy losses are related to the losses in network elements (overhead power lines, power cables and distribution transformers MV/LV). The accurate evaluation of these losses could lead to appropriate measures for increasing the energy efficiency and capacity of lines, optimal loading of transformers, sustainable planning and development of electric networks. In order to achieve these goals, a database is needed that contains as much information as possible on the network configuration, consumers' energy consumption, the accurate allocation of consumers to the supply points and phases, etc. Also, the used calculation method is very important because the choice depends on the available data for the network and the energy consumptions.

More methods are presented in the literature, majority being based on known theoretical relationships. The input information about the mean and variation components of the load curve to calculate the loss coefficient were used in [8] to estimate the energy losses. Another method which used the formulas derived from the Elgerd's power loss relations was proposed in [9] to estimate active and reactive power losses in distribution networks. The average of load demand and load limitations obtained from the load profiles of consumers were considered in the inside of this method. The load profile of feeders and characteristics (length, peak demand to installed capacity ratio, and load distribution profile) have constituted the used data in [10] for the simulation studies in order to determine the peak power loss functions of medium voltage distribution feeders based on different feeder characteristics. The difference between the known amounts of the incoming and outgoing energy led to evaluate the energy losses at the whole level of distribution network in [11]. The loss factors were calculated in [12] and [13] to evaluate the energy losses in distribution networks. Other approaches [14-16] were used to estimate the power losses a scaling factor for typical load profile with attached pseudo measurements for a real balanced LV grid with distributed generations. In [17,18], the power losses computation used the loss factor and the load profile. The drawback of this method are the estimation of the two coefficients A and B as constants for given homogeneous loads or load profile. Moreover, in [19,20] the active power losses for three balanced (neglecting the neutral) LV feeders from a microgrid were calculated. All the aforementioned methods uses for Joule power losses estimation a simplified methodology based on Kirchhoff laws.

The Artificial Intelligence techniques represented an alternative to estimate the energy losses in the cases of uncertain factors regarding the material characteristics of networks and the load. Thus, the loss estimation in the low-voltage networks is made in [21] based on the load characteristics similarity of distribution feeders using clustering. A similar approach is proposed in [22] to determine the levels of the power/energy losses using the clustering techniques in the distribution networks. The difference from the previous approach is represented by the following considered variables: the rated voltage, the installed transformer capacity, the number of transformation points, and the loading level. A hybrid approach formed by clustering techniques and modified extreme Gradient Boosting algorithm was proposed in [23]. This use the characteristic data of feeders that are collected in the smart power distribution and utilization system. A genetic algorithm based approach regarding the power losses computation for a LV distribution grid was proposes in [24]. The total active power losses are considered as a sum of power losses computed for each phase (a, b, c, and 0). The particularity of the method refers at the loads from the consumers, defined through the installed active and reactive power, the utilization factor, phase location and bus connection.

In relation to the approaches presented in the literature, the proposed algorithm has three stages: the first stage is based on an access algorithm of input data, being able to work simultaneously with active and reactive power profiles provided by smart meters and typical profiles associated to consumers without smart meters, depending by the energy consumption categories and day type (weekend and working), knowing the daily energy indexes, the second stage is based on a structure vectors-based algorithm to recognize the network topology, and the third stage is based on the improved version of forward/backward sweep-based algorithm to calculate fast the power/energy losses to three-phase LV distribution networks in balanced and unbalanced regime. Regarding the forward/backward sweep-based algorithm, it was mainly used in the medium voltage distribution networks to calculate the balanced, symmetric steady-state regime [25-27], [30]. But, the proposed version was adapted to the LV distribution networks that most often operate in unbalanced regime due to the chaotic allocation of the 1-phase consumers on the phases.

Our approach use real data about the active and reactive power for both consumers and small-scale sources integrated in the considered LV power networks. Another particularity refer at the power/energy losses computation using a modified branch and bound (forward backward sweep) method. A real low voltage network belonging to a DNO from Romania was used to test the proposed algorithm, and the results were compared with those obtained using a professional software, namely DigSilent PowerFactory (DSPF). Even if the DSPF is one of the most powerful software in analysing generation, transmission, and distribution, it presents some disadvantages represented by the introduction of network elements (lines, loads, buses) requiring a long time period depending by the size of network and the loading of load profiles which should have a certain format. Therefore, this software is especially recommended in the off-line analyses.

The rest of paper is organized as following: Section 2 reveals the stages of proposed algorithm, detailing the load profiling process used for the consumers without smart meters, Section 3 presents the case study in which a real low voltage network from a pilot rural zone of a DNO from Romania is used to test the new approach and a comparison with the obtained results from the simulations with DSPF software is made, and Section 4 highlights the conclusions.

2. Three-Stage Algorithm to Energy Losses Calculation in Three-Phase LV Distribution Networks

The steady-state regime calculation using the very well-known methods Newton-Raphson or Seidel Gauss could be difficult in the LV electric distribution networks to obtain the energy losses due to the particular features represented by the ill-conditioning given by the structure of networks: radial or weakly meshed topologies, high ratios between resistance and reactance, operation with multi-phase lines (1, 2 or 3-phases) with unbalanced distributed loads.

In these conditions, another approach should be used to eliminate these drawbacks. The proposed algorithm contains the following stages:

Stage 1. The input data for the consumers referring to the energy characteristics are read from the database which contains for each consumer, in function by integration in the SMS, the active and reactive power profiles recorded with smart meters or the typical active and reactive power profiles associated taking into account the consumption category, the day type, and the season type, and assigning the daily energy indexes). Also, the active and reactive power profiles of the generators from the network are loaded by the input algorithm.

Stage 2. The architecture of network is established using an efficient algorithm based on two structure vectors.

Stage 3. The power/energy losses are determined using an improved variant forward/backward sweep algorithm which can work both in the balanced and unbalanced regimes, with or without distributed generation.

2.1. The first stage

For an electric consumer, the behaviour is represented by a load profile representing the electric power consumption for some period of time. Availability of such data depends on the type of

consumer and if this is implemented in the SMS. The vast majority of the DNOs from the UE countries developed pilot projects to evaluate the efficiency of smart meters implementation programme. These pilot projects were implemented in the regions covered by DNOs and the results should refer at the following [1]:

- The energy consumption is monitored on-line with benefits for both parts: the DNOs to implement the energy efficiency measures, and the consumers to establish their pricing mode in function by the energy consumption behaviour.
- The analysis of recorded data must lead to optimal strategies regarding the increase of energy efficiency in the LV distribution networks of the DNOs.
- Extended the processed data (as load type profiles) at the networks from same area, without SMS implementation, to analyse their operation regime.

The proposed algorithm accepts the input data using a similar format to the one available in the database of the DNOs. The records with the associated fields from the input file are indicated in Table 1.

Table 1. The format of input data

Number	Pole	Branching	Phase	Type	Category	Integration	Meter ID
1	1	1-phase	b	1	3	1	3002864374
2	2	1-phase	b	1	1	1	3002864354
3	2	1-phase	c	2	1	1	3002864393
4	3	1-phase	a	1	1	0	3002864386
5	3	3-phase	abc	1	1	1	3002864504
-	-	-	-	-	-	-	-
N	3	1-phase	b	1	1	1	3002108670

Each field of a record will be detailed in the following:

- *Number* represents the allocated record for a certain consumer in the database of DNO.
- *Pole* represents the identification number of each pole made by the DNO for a rural LV distribution network. The poles are numbered in all rural LV distribution networks to know where are connected the consumers. For example in Table 1, the consumer 1 is connected at the pole 1, and the consumer 4 is connected at the pole 3.
- *Branching* identifies the type of electric branching for each consumer: 1-phase or 3-phases. These can be identified in the database with 1 (1-phase) or 3 (3-phases).
- *Phase* allow to identify the phase(s) at which a consumer is allocated (if it is a 1-phase consumer then in the field the notations *a*, *b*, or *c* can be seen, and if it is a 3-phase consumer then the notation *abc* can be observed).
- *Type* emphasizes if the consumer belongs the following consumption patterns: residential (ID is 1), non-residential, namely community buildings, hospitals, town halls, schools, etc. (ID is 2), commercial (ID is 3), and industrial (ID is 4).
- *Category* belongs to a certain *Type* of the consumer identified by an annually electric energy consumption. The DNO can classify the consumers in a given number of consumer categories in function by different criteria. As an example, a DNO from Romania has a division in five consumption categories for the residential/non-residential consumers [28]: Category 1 (0-400 kWh), Category 2 (400 – 1250 kWh), Category 3 (1250 – 2500 kWh), category 4 (2500 – 3500 kWh), and Category 5 (> 3500 kWh).
- *Integration* allows the user to know if the meter is (value is equal with 1) or not (value is equal with 0) in the database of the SMS. If the meter is integrated, based on the *ID of meter*, the active and reactive power profiles of consumer can be loaded from the database. If a meter is not integrate then the daily energy indexes will be loaded from the database. In this last case, a typical load profile will be allocate the consumer using the algorithm presented in the next section in function by the records *Type* and *Category*. The associated active and reactive power profile will be finally obtained based on the loaded energy indexes.

A matrix with the number of rows equal to the sampling size of active and reactive power profiles (24, 48, or 96 values) and the number of columns equal to $6 \times N$, where N represents the number of consumers from the network, will be loaded. The signification of the six columns is given by the fact that for each consumer will be allocated three columns for active power and three columns for reactive power. Only columns associated the connection phase of the consumers will have values different by 0 in the input matrix. Also, the algorithm can be used in the on-line calculations, the data being read as soon as they reached the data center.

2.1.1. Load Profiling Process based on Smart Meters Data

If a consumer is not integrated in the SMS, the DNO can assigned a typical load profile depending the different consumer' types (residential, commercial and public), energy consumption category, and seasons (spring, summer, autumn and winter) which are obtained based on the processed data from the consumers with smart meter available [28, 29].

Thus, based on the typical load profiles and daily energy consumption of each consumer, without a smart meter implemented, the load profiles could be computed using the following algorithm. The deformed load profile at consumer l is calculated with the relation:

$$P_l^{(h)} = W_l \cdot TLP_{tc}^{(h)} \quad (1)$$

where:

$P_l^{(h)}$ – the denormalized load profile at consumer l for each hour $h = 1, \dots, 24, l = 1, \dots, N_c$;

tc – the type of the l -th consumer, $l = 1, \dots, N_c$;

$TLP_{tc}^{(h)}$ – the typical load profile for the tc type of consumer (tc can be residential, commercial or industrial), for each hour $h = 1, \dots, 24$;

W_l – the daily energy consumption for the consumer l ;

N_c – total number of consumers without smart meter installed or with missing data in the SMS.

Next, the denormalized profiles calculated above are adjusted by using the hourly measured values at the main feeder of the MV/LV electric substation, as follows:

$$P_{tc}^{(h)} = \sum_{l=1}^{N_c} P_l^{(h)} \quad h = 1, \dots, 24 \quad (2)$$

$$P_{SM}^{(h)} = \sum_{n=1}^{N_{SM}} P_{sm,n}^{(h)} \quad h = 1, \dots, 24 \quad (3)$$

$$\Delta P^{(h)} = P_{tc}^{(h)} - P_m^{(h)} - P_{SM}^{(h)} \quad h = 1, \dots, 24 \quad (4)$$

$$P_{cor,l}^{(h)} = P_l^{(h)} \cdot \left(1 + \frac{\Delta P^{(h)}}{\sum_{l=1}^{N_c} P_l^{(h)}} \right) \quad h = 1, \dots, 24 \quad (5)$$

where:

$P_m^{(h)}$ – three-phase feeder measured load profile for the analysed period;

$P_{sm,n}^{(h)}$ – active power measured with the smart meter at the consumer $n, n = 1, \dots, N_{SM}$;

N_{SM} – total number of consumers integrated in the SMS.

$\Delta P^{(h)}$ – deviation between the measured and computed load profiles for the analysed period;

$P_{cor,l}^{(h)}$ – denormalized load profiles adjusted by measured load profiles for the analysed period at the consumer l , for each hour $h = 1, \dots, 24, l = 1, \dots, N_c$.

2.2. The second stage

The topology of the analysed network will be very ease identified based on an algorithm which build two structure vectors (V1 and V2). Their structure is described hereinafter.

The process allows the clustering of each section at a hierarchical level, starting with the first section. To exemplify the procedure, a radial LV distribution network with 9 nodes (poles) and 8 sections was considered. The three steps are highlighted in Figure. 1.

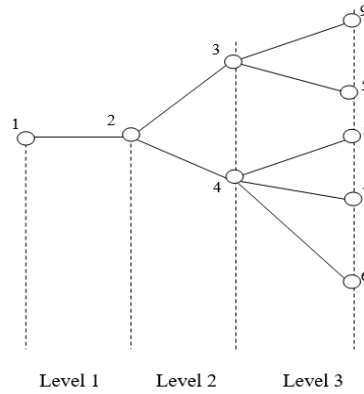


Figure 1. The topology of a radial LV distribution network

If it is adopted the following order to numbering the sections: 1-2 (I), 2-3 (II), 3-9 (III), 3-5 (IV), 2-4 (V), 4-6 (VI), 4-7 (VII), and 4-8 (VIII), then the size of the vector $V1$ is identically with the number of levels, each element signifying the number of sections from each level. The relation between each level from $V1$ and the sections of each level from $V2$ can be observed in Table 2, where the structure vectors is shown.

Table 2. The structure vectors for the LV distribution network from Figure 1

V1	1	2	5					
V2	I	II	V	III	IV	VI	VII	VIII
	(1-2)	(2-3)	(2-4)	(3-9)	(3-5)	(4-6)	(4-7)	(4-8)

2.3. The third stage

The calculations for the steady state regime from the each hour h , $h = 1, \dots, H$ (in our case $H = 24$) will be made using an improved variant forward/backward sweep algorithm which can work both in the balanced and unbalanced regimes, with or without distributed generation. The following steps are taken to calculate the energy losses:

1. *The active and reactive powers to each node (pole) are aggregated* using the total active and reactive power of all consumers n_c allocated at the pole i , on the each phase:

$$P_i^{a,b,c} = \sum_{j=1}^{n_c^i} P_j^{i(a,b,c)} \quad i = 1, \dots, N_p \quad (6)$$

$$Q_i^{a,b,c} = \sum_{j=1}^{n_c^i} Q_j^{i(a,b,c)}, \quad i = 1, \dots, N_p \quad (7)$$

where N_p represents the total number of poles from the analysed networks.

If a generator is located in a node j (it can be at the same time and the consumer), connected at the pole i then:

$$P_j^{a,b,c} = P_{c,j}^{i(a,b,c)} - P_{g,j}^{i(a,b,c)} \quad (8)$$

$$Q_j^{a,b,c} = Q_{c,j}^{i(a,b,c)} - Q_{g,j}^{i(a,b,c)} \quad (9)$$

where $P_{g,j}^{i(a,b,c)}$, $Q_{g,j}^{i(a,b,c)}$ are the active and reactive power of the generator from the node j , connected at the pole i , on the phases a , b , and c , and $P_{c,j}^{i(a,b,c)}$, $Q_{c,j}^{i(a,b,c)}$ are the active and reactive absorbed by node j .

2. *The phase voltages* are initialized at each node (pole) of distribution network using the values of the phase voltages from the slack bus (\underline{U}_s) represented by the LV bus of electric substation (the values could be different by the nominal voltage):

$$\underline{U}_i^{a,b,c(0)} = \underline{U}_s^{a,b,c}, \quad i = 1, \dots, N_p, \quad i \neq s \quad (10)$$

3. *Backward sweep:*

3.1. The currents on each phase (a , b , and c) at each node (pole) i are calculated:

$$\underline{I}_i^{a,b,c(k)} = \frac{\underline{S}_i^{a,b,c^*}}{\left(\underline{U}_i^{a,b,c^*}\right)^{(k-1)}}, \quad k = 1, \dots, K_{max}, \quad i = 1, \dots, N_p \quad (11)$$

$$\underline{S}_i^{a,b,c} = P_i^{a,b,c} + j \cdot Q_i^{a,b,c} \quad (12)$$

where k is the index of current iteration and K_{max} represents the maximum number of iterations introduced initially by user.

3.2. The total currents on each phase (a , b , and c) and section of network are calculated:

$$\underline{I}_{v,i}^{a,b,c(k)} = \underline{I}_i^{a,b,c(k)} + \sum_{n \in Next(i)} \underline{I}_{i,n}^{a,b,c(k)} \quad (13)$$

where: v is the pole in up stream of pole i ; $Next(i)$ is the set of poles next to the pole i ;

4. Forward sweep:

4.1. The voltage drop on the phases (a , b , and c) of all sections is calculated:

$$\underline{\Delta U}_{v,i}^{a,b,c(k)} = \underline{Z}_{v,i} \cdot \underline{I}_{v,i}^{a,b,c(k)} + \underline{Z}_{v,i}^0 \cdot \underline{I}_{v,i}^{0(k)} \quad i = 1, \dots, N_p, \quad v \neq i \quad (14)$$

$$\underline{Z}_{v,i} = R_{v,i} + j \cdot X_{v,i} \quad (15)$$

where: $\underline{Z}_{v,i}$ is impedance of the phase conductor, $\underline{Z}_{v,i}^0$ is the impedance of neutral conductor; $\underline{I}_{v,i}^0$ represents the current flow from the neutral conductor.

$$\underline{I}_{v,i}^0 = \underline{I}_{v,i}^a + \underline{I}_{v,i}^b + \underline{I}_{v,i}^c \quad (16)$$

4.2. The voltage on the phases (a , b , and c) for each pole i is calculated:

$$\underline{U}_i^{a,b,c(k)} = \underline{U}_v^{a,b,c(k)} - \underline{\Delta U}_{v,i}^{a,b,c(k)}, \quad i = 1, \dots, N_p \quad (17)$$

4.3. The total apparent power injected from the slack bus (LV side of electric substation) is calculated:

$$\underline{S}_s^{a,b,c(k)} = U_s \cdot \sum_{m \in Next(s)} \underline{I}_{s,m}^{a,b,c^*(k)} \quad (18)$$

4.4. Testing the convergence condition:

$$\left| \underline{S}_s^{a,b,c(k)} - \underline{S}_s^{a,b,c(k-1)} \right| \leq \epsilon_s \quad (19)$$

5. If the convergence condition is verified, the power loss on each section is calculated:

$$\underline{\Delta P}_{v,i}^{a,b,c(k)} = R_{v,i} \cdot \left(\underline{I}_{v,i}^{a,b,c(k)} \right)^2 + R_{v,i}^0 \cdot \left(\underline{I}_{v,i}^{0(k)} \right)^2 \quad (20)$$

where $\underline{R}_{v,i}$ is resistance of the phase conductor, $R_{v,i}^0$ is the resistance of neutral conductor.

The flow-chart of proposed algorithm with the three steps is presented in Figures 2a (the first and second stages) and Figures 2b (the third stage).

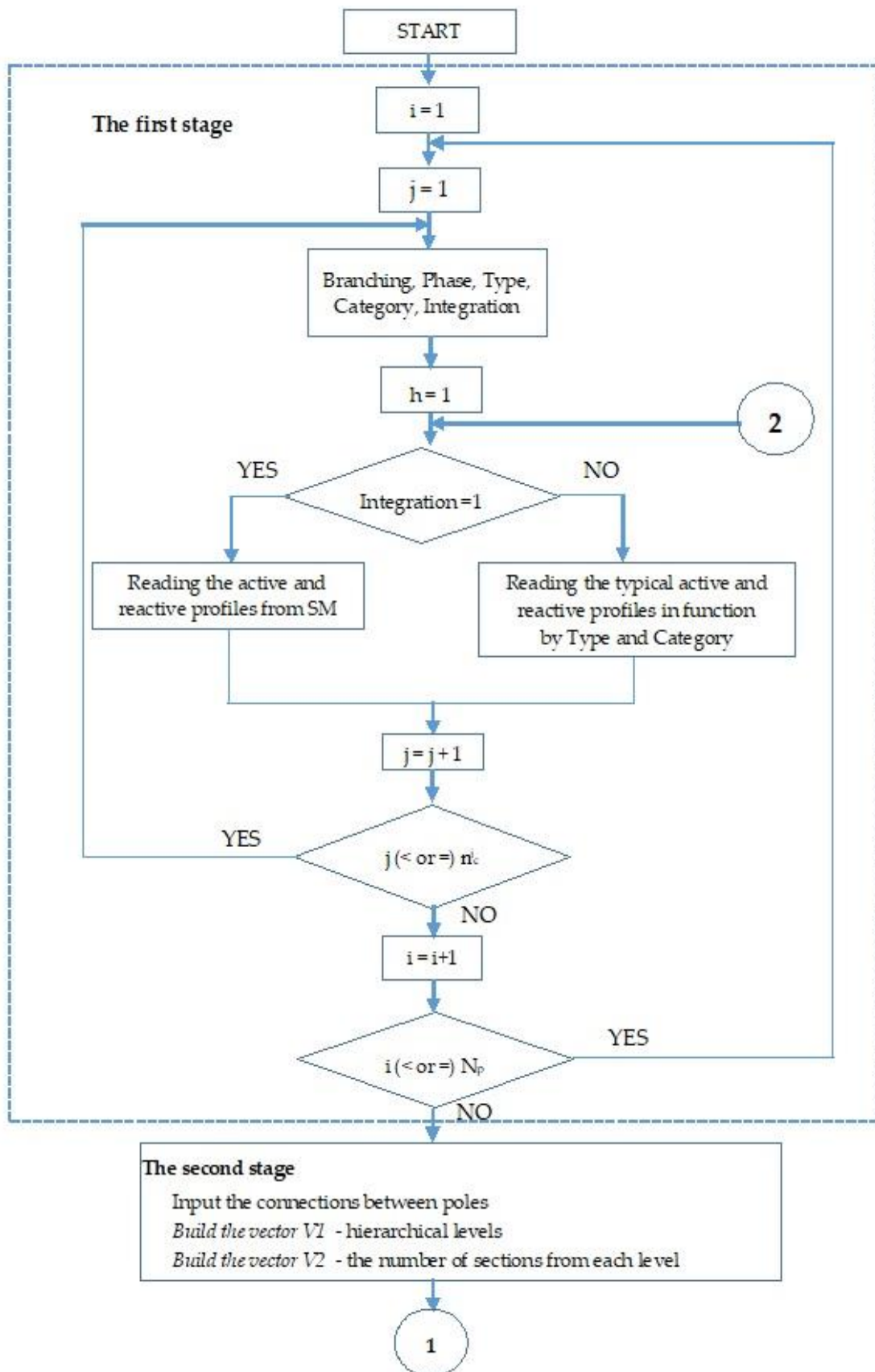


Figure 2.a. The flow-chart of proposed algorithm – the first and second stages

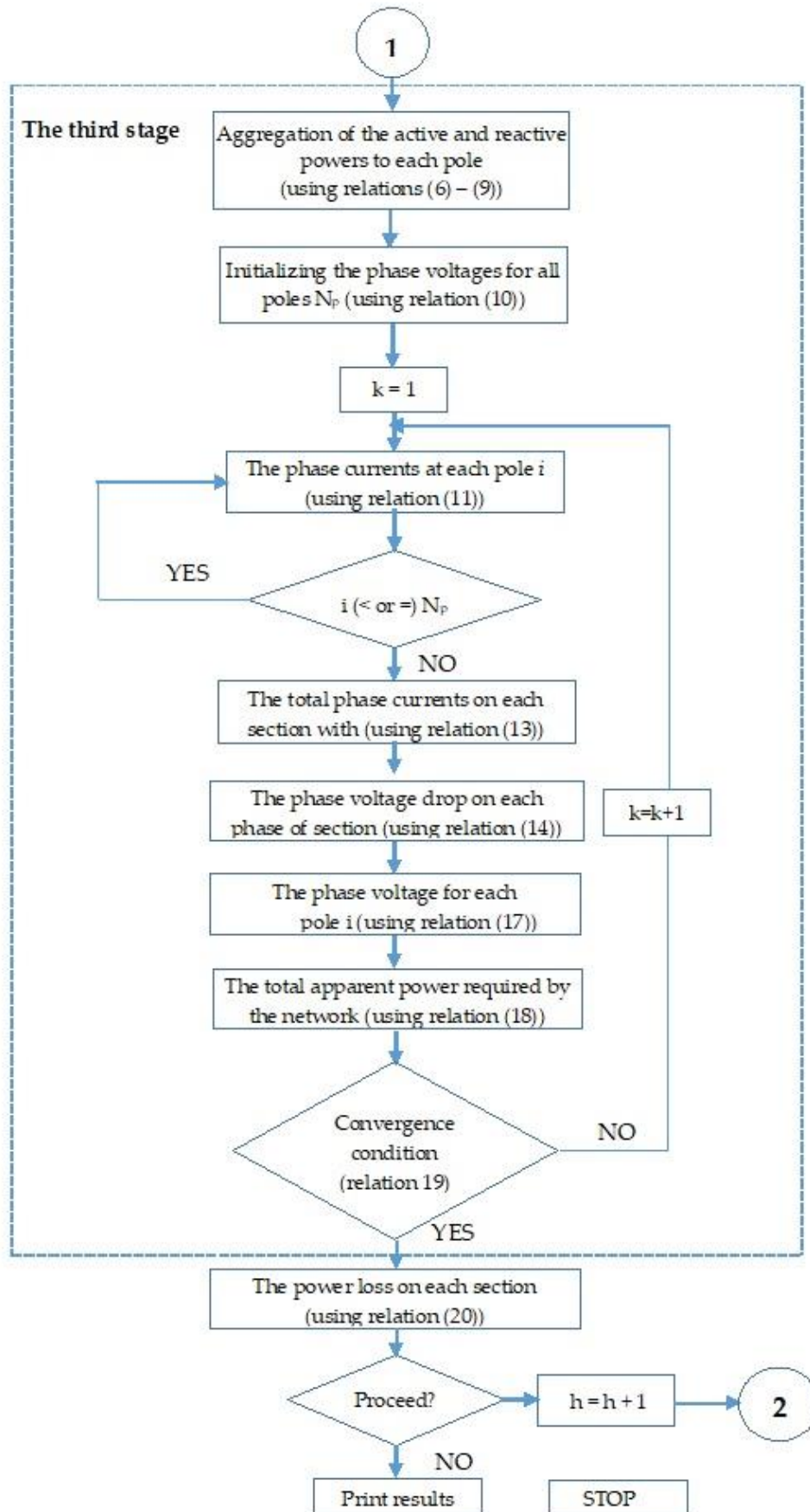


Figure 2.b. The flow-chart of proposed algorithm – the third stage

3. Case Study

The proposed algorithm was tested on a real pilot LV electric distribution network belonging to a DNO from Romania. The topology of analysed network can be seen in Figure 3. The electric substation MV/LV supplies 3 distribution feeders.

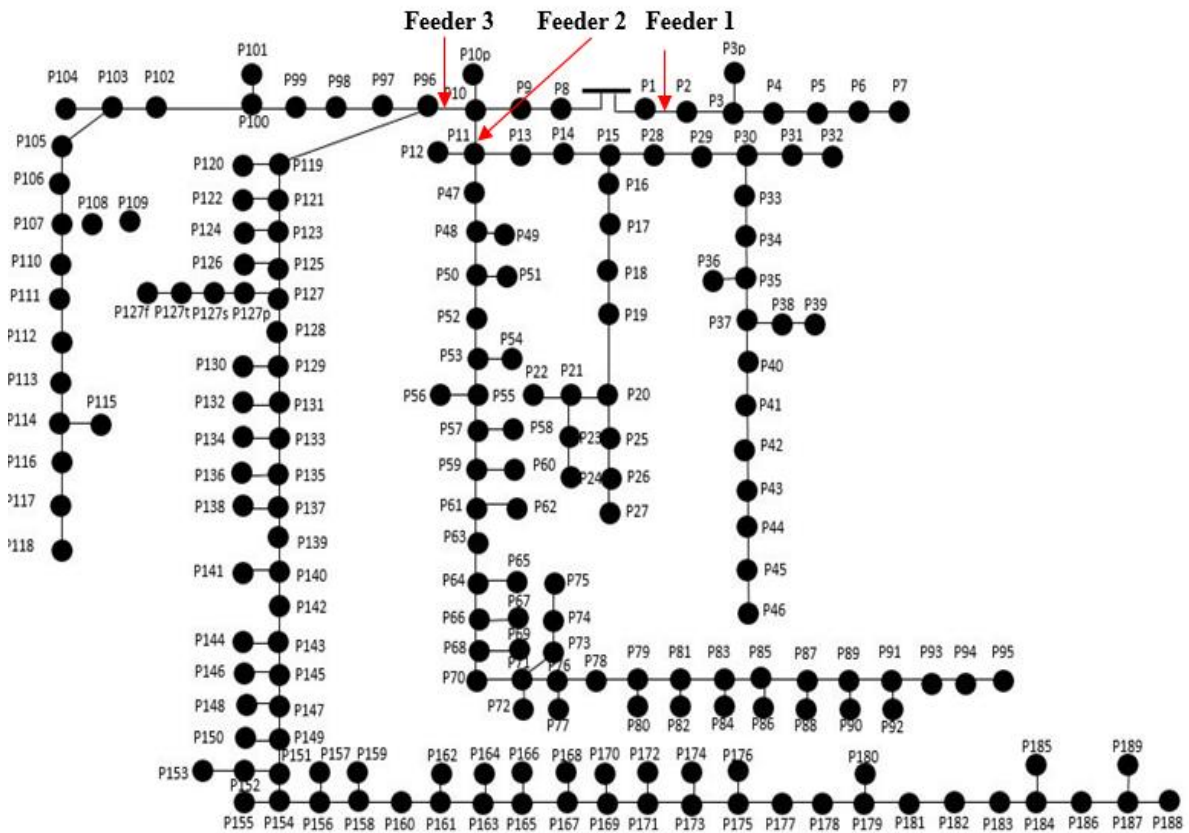


Figure 3. The analysed LV distribution network

The three feeders have 189 poles together. The poles represent points where the consumers are connected using the 1-phase or 3-phase branching at the network, and these are identified through black circles. Each section has 40 meters, representing the distance between two poles. The primary characteristics (number of poles, total length, cable type, cable size, length of sections using given cable types, and number of consumers) are shown in Table 3. Also, consumers' characteristics can be identified in Table 4.

Table 3. The characteristics of feeders

Feeder	Length [m]	Conductor Type	Cross-section (phase+neutral) [mm ²]	Length [m]	r ₀ [Ω/km]	x ₀ [Ω/km]
Feeder 1	280	Classical	1x50+50	280	0.61	0.298
Feeder 2	3,880	Stranded	3x35+35	120	0.871	0.055
		Classical	3x50+50	3,760	0.61	0.298
Feeder 3	3,520	Stranded	3x50+50	120	0.605	0.05
		Classical	3x50+50	2,080	0.61	0.298
		Classical	3x35+35	960	0.871	0.055
		Classical	1x25+25	280	1.235	0.319
Total	7,680	Classical		7,440		
		Stranded		240		

Table 4. The characteristics of consumers

Consumers' type	1-phase	3- phase	
Phase a	83	-	
Phase b	15	-	
Phase c	100	-	
Total	335	8	
	0 – 400	150	5
Consumption	400-1000	108	2
Category	1000-2500	65	0
[kWh/year]	2500-3500	5	0
	> 3500	7	1

The details regarding the allocations at poles and phases, and the type of consumers are indicated in Appendix A, Table A1. The connection phase of each consumer reflects the real situation, and this aspect helped to establish the true-to-reality unbalanced model. The hourly load records (active and reactive power profiles) for each consumer integrated in the SM system were imported from the database of the DNO for the day when the analysis was made. Based on these profiles, the phase loading at the LV level of electric substation was calculated for each feeder, see Figures 4, 5 and 6.

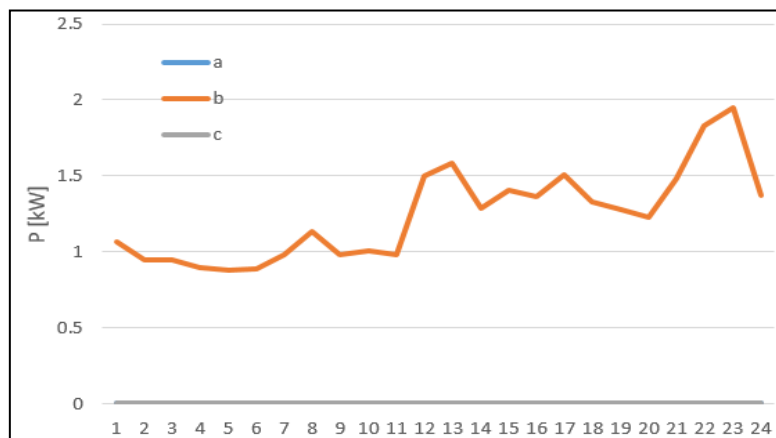


Figure 4. The phase loading on the first section of Feeder 1

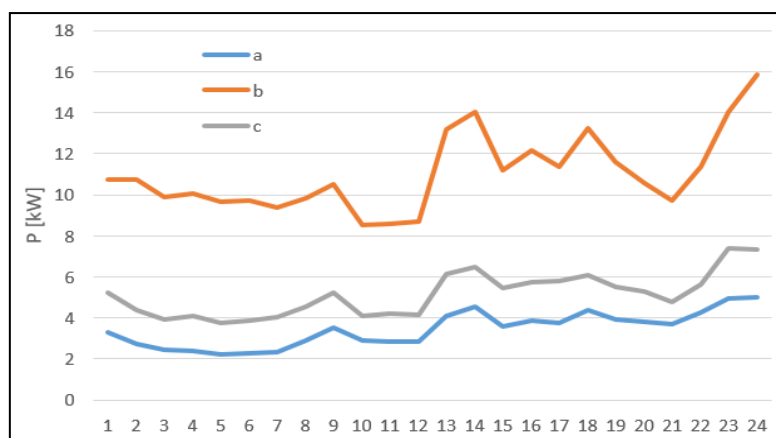


Figure 5. The phase loading on the first section of Feeder 2

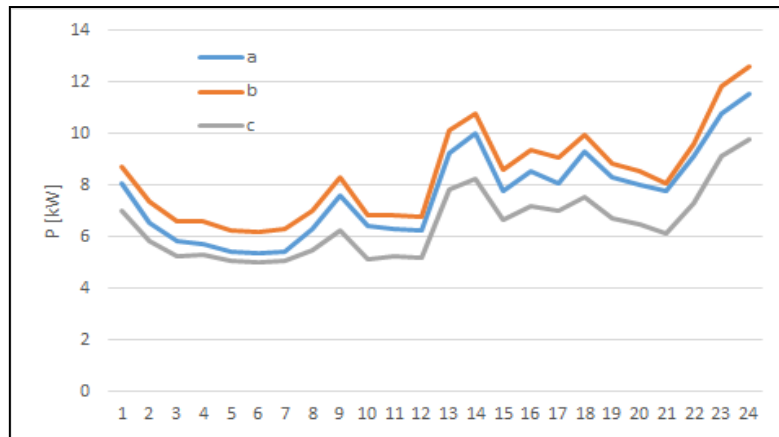


Figure 6. The phase loading on the first section of Feeder 3

From Figure 4, it can be observed that all consumers from Feeder 1 are only allocated on the phase *b*. Feeder 2 has a high unbalance, the phase *b* is more loaded than the other two phases (*a* and *c*), see Figure 5. In this case, the current flow on the neutral conductor will lead at the high additional losses. For Feeder 3, the allocation of consumers on the phases of the feeder is more balanced, see Figure 6.

The steady state regime calculations were made for each hour from the analysed day. The total energy losses calculated with proposed algorithm for each feeder, on each phase and neutral conductor, on branching and main conductors are presented in Tables 5. The obtained results with the DSPF software are presented in Table 6 to emphasize the accuracy of proposed algorithm.

The detailed results obtained with proposed algorithm for each feeder are presented in Tables B1, B2, and B3 from Appendix B.

The absolute errors (ϵ) and percentage errors (δ) between both approaches, DSPF software and proposed algorithm (PA) are presented in Table 7, Figure 7, and Figure 8. The calculation relations are the following:

$$\epsilon = |\Delta W_{\text{DSPF}} - \Delta W_{\text{PA}}|, [\text{kWh}] \quad (21)$$

$$\delta = \left| \frac{\Delta W_{\text{DSPF}} - \Delta W_{\text{PA}}}{\Delta W_{\text{DSPF}}} \right| \cdot 100, [\%] \quad (22)$$

Table 5. The energy losses calculated with PA algorithm, [kWh]

	Feeder	Phase			Neutral	TOTAL
		a	b	c		
Main conductors	Feeder 1	0.000	0.047	0.000	0.058	0.105
	Feeder 2	0.529	9.973	2.455	8.055	21.012
	Feeder 3	6.370	5.411	5.726	1.586	19.092
	TOTAL	6.900	15.430	8.180	9.699	40.209
Branching conductors	Feeder 1	0.000	0.006	0.000	0.004	0.010
	Feeder 2	0.055	0.173	0.019	0.162	0.410
	Feeder 3	0.072	0.052	0.052	0.086	0.263
	TOTAL	0.127	0.232	0.072	0.253	0.682
TOTAL		7.026	15.662	8.252	9.951	40.892

Table 6. The energy losses calculated with DSPF software, [kWh]

	Feeder	Phase			Neutral	TOTAL
		a	b	c		
Main conductors	Feeder 1	0.000	0.043	0.000	0.054	0.097
	Feeder 2	0.509	9.647	2.433	7.765	20.354
	Feeder 3	6.099	5.438	6.184	1.572	19.293
	TOTAL	6.608	15.129	8.616	9.391	39.744
Branching conductors	Feeder 1	0.000	0.006	0.000	0.004	0.010
	Feeder 2	0.053	0.218	0.020	0.187	0.479
	Feeder 3	0.069	0.055	0.059	0.090	0.273
	TOTAL	0.122	0.279	0.079	0.281	0.762
TOTAL		6.731	15.408	8.696	9.671	40.506

Table 7. The values of energy losses and errors

Approach	Phase	Neutral	Total		
PA [kWh]	7.03	15.66	8.25	9.95	40.89
DSPF [kWh]	6.73	15.41	8.70	9.67	40.51
ε [kWh]	0.3	0.25	0.45	0.28	0.38
δ [%]	4.45	1.62	5.17	2.90	0.94

From the analyse of results presented in Table 7, it can be observed that the percentage errors are in the range [1.62 – 5.17] for the total energy losses on the phase and neutral conductor, and below 1 percent (0.94) for the total energy losses. Also, it can highlighted a high value of energy losses on the neutral conductor. These represent about 25 percent from the total energy losses which means that the DNO should take the balancing measures (especially in the case of Feeder 2).

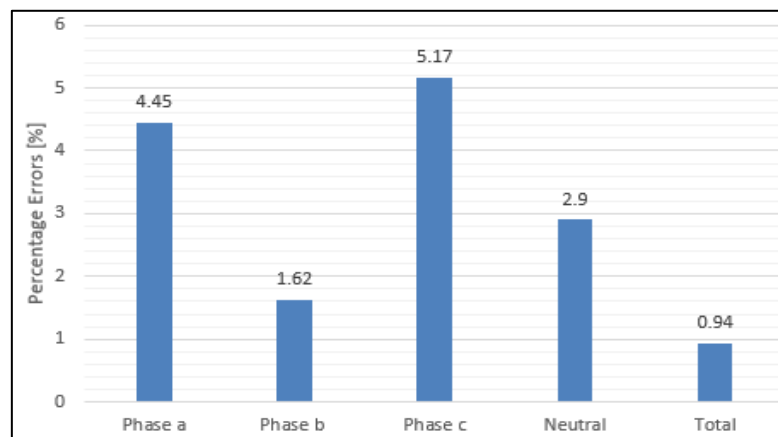


Figure 7. The errors between both approaches, [%]

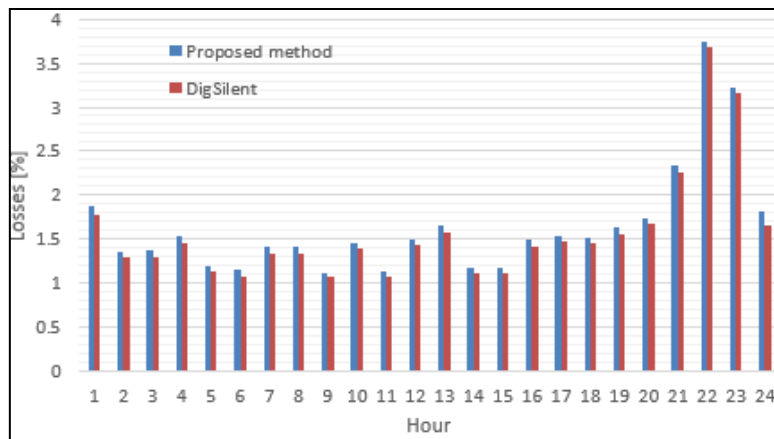


Figure 8. The hourly energy losses computed with both approaches, [%]

In terms of phase voltages, these were calculated for each pole. The obtained values for the farthest poles are represented in the Figures 9-11 (pole P95 – Feeder 2) and Figures 12-14 (pole P189 – Feeder 3).

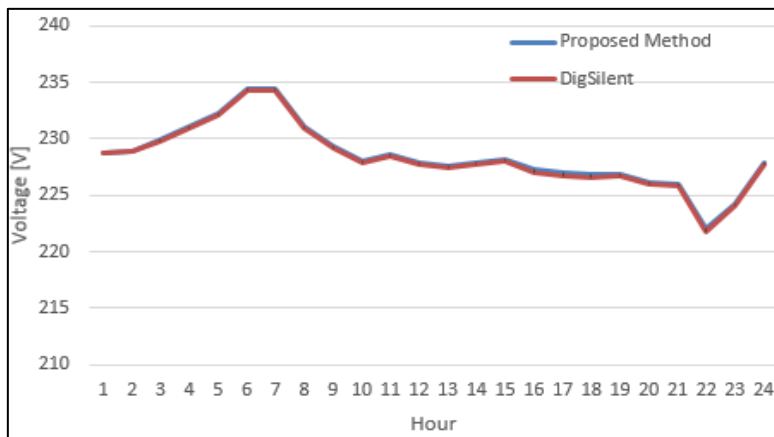


Figure 9. The voltage on the phase *a*, Pole P95 – Feeder 2

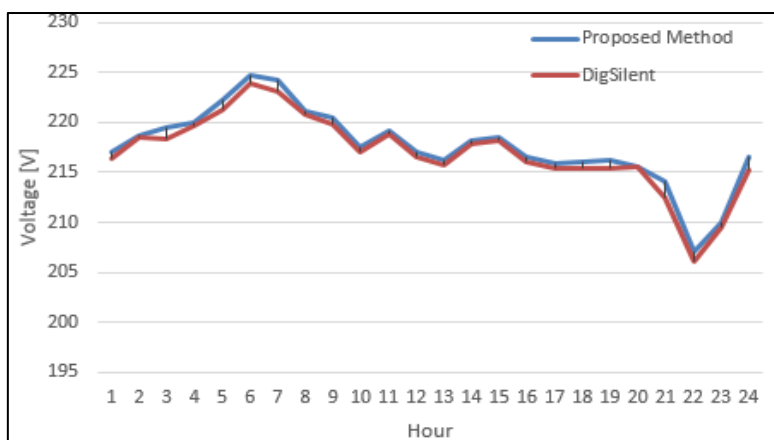


Figure 10. The voltage on the phase *b*, Pole P95 – Feeder 2

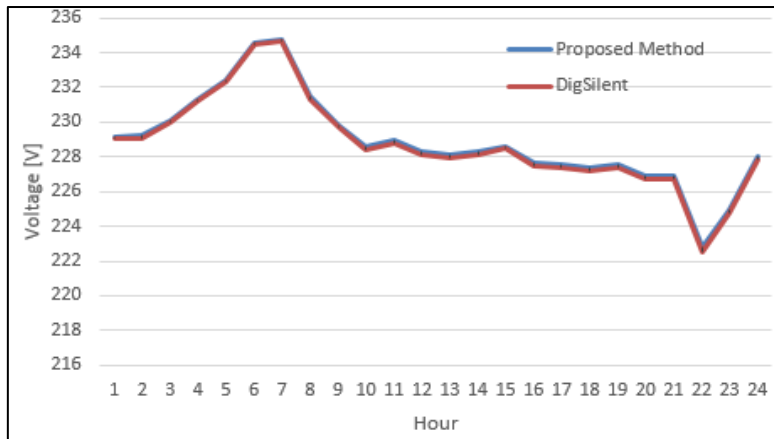


Figure 11. The voltage on the phase *c*, Pole P95 – Feeder 2

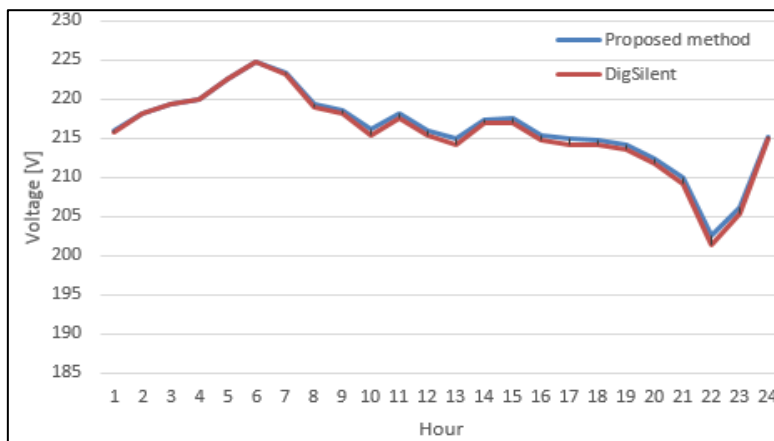


Figure 12. The voltage on the phase *a*, Pole P189 – Feeder 3

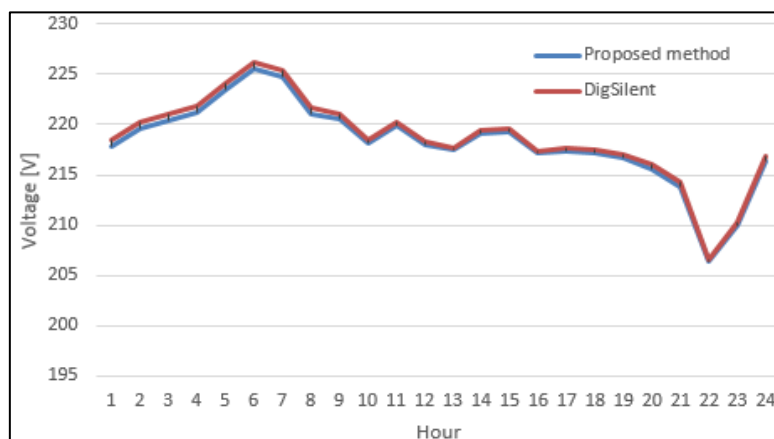


Figure 13. The voltage on the phase *b*, Pole P189 – Feeder 3

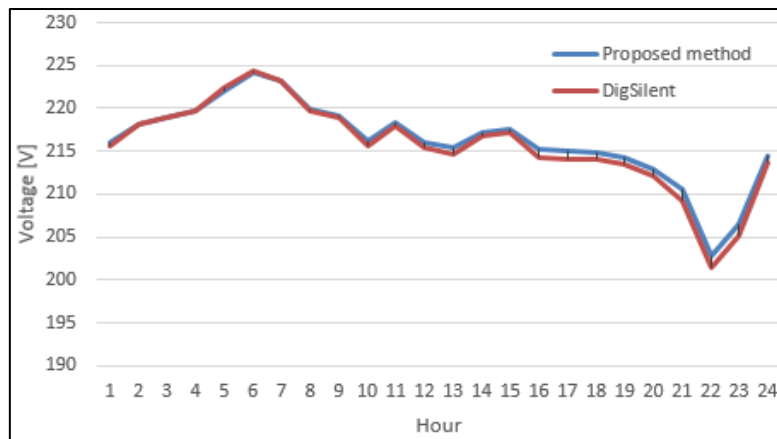


Figure 14. The voltage on the phase *c*, Pole P189 – Feeder 3

The detailed results obtained with proposed algorithm for each pole are presented in Table 4B from Appendix B.

An analyse of Figures 9 - 14 highlighted that at the pole P95 the phase voltages are inside of admissible limits (nominal voltage $\pm 10\%$), and at the pole P189 only the voltage on the phase *b* is corresponding, but equal with minimum value (nominal voltage -10%). The voltages on the phases *a* and *c* are slightly below the minimum limit with 0.02%. The nominal phase voltage in Romania is 230 V. Thus, the DNO should take the measures to improve the voltage level in this final node (tap changing of transformer from the electric substation).

The mean percentage errors (MPE) of the phase voltages are presented in Figures 15 and 16. These were calculate with the following relation:

$$MPE = \frac{100}{24} \sum_{t=1}^{24} \frac{\Delta W_{DSPF} - \Delta_{PA}}{\Delta W_{DSPF}}, [\%] \quad (23)$$

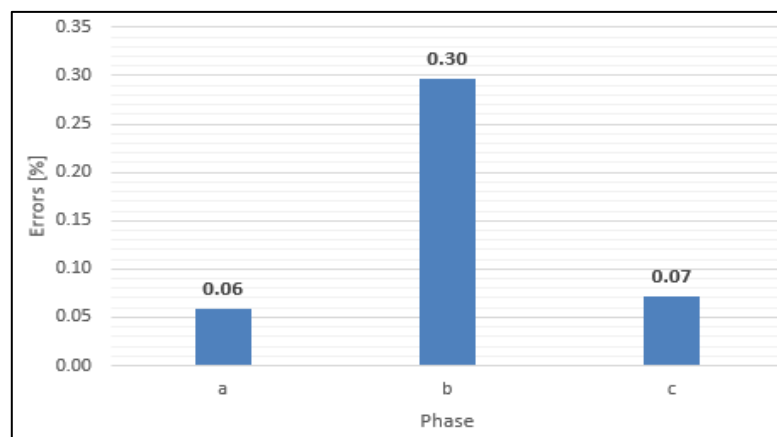


Figure 15. The MPE on the phase, Pole P95

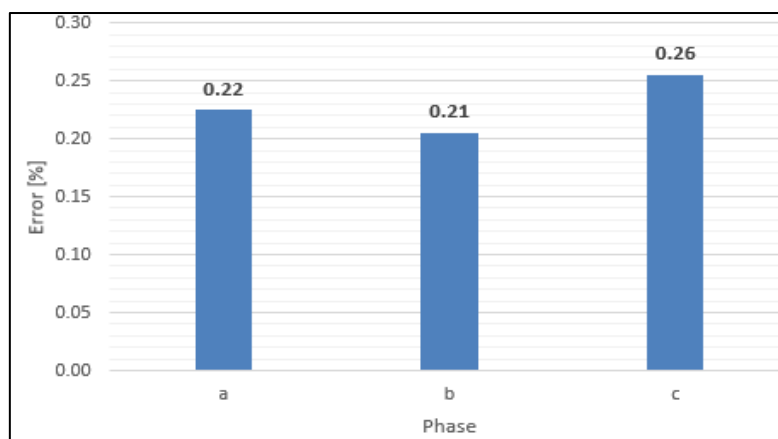


Figure 16. The MPE on the phase, Pole P189

It can be observed that the MPEs is below 0.3 percent on each phase at both poles (P95 and P189).

4. Conclusions

The paper presents an improved smart meter data-based three-stage algorithm to power/energy losses calculation in three-phase LV distribution networks. The three stages refer to: *The first stage.* The data files are loaded from the database on the DNO and read by the algorithm in function by the recorded information referring to the energy consumption (active and reactive power profiles if the consumers are integrated in the SMS, or typical power profiles for the other consumers assigned in function by the consumption category, day type, and season type assign the daily energy indexes); *The second stage.* The network topology is recognized using the structure vectors-based algorithm; and *The third stage.* The power/energy losses are calculated using the forward/backward sweep algorithm.

In relation to the approaches presented in the literature, the method has additionally an access procedure of input data, being able to work simultaneously with active and reactive power profiles provided by smart meters and typical profiles associated to consumers without smart meters. Also, the proposed algorithm eliminates the difficulties from the steady state calculations, with the classical methods (Seidel-Gauss and Newton-Raphson, represented by the ill-conditioning of LV distribution networks with the radial or weakly meshed topologies, high R/X ratios, operation with multi-phase lines (one, two or three phases), unbalanced operation, unbalanced distributed load. In these conditions, DNOs have possibility to take some technical measures which enable the LV distribution networks to operate more energy efficient and better plan their investments.

The obtained results using a real LV distribution network from a pilot rural zone belonging to a DNO from Romania were compared with those obtained using the PFDS software and the mean absolute percentage error was by 0.94 %. The advantages of proposed algorithm are represented by the comfortable introduction of network elements (lines, transformers or nodes) whatever by the size of network and the loading of active and reactive power profiles, working simultaneously with files containing data provided by smart meters and typical load profiles, and by the

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Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature:

0 The neutral

a, b, c	1-phase consumers (or three phases)
abc	3-phase consumer
DNO	Distribution Network Operator
DSPF	DigSilent Power Factory
i	The index for bus
I_v	The branch impedance
j	The index for pole
k	The index for current iteration
K_{max}	The maximum number of iterations
h	The current hour ($h = 1, \dots, H$)
LV	Low Voltage
l	The consumer ($l = 1, \dots, N_c$)
MAPE	Mean Average Percentage Error, [%]
MPE	Mean Percentage Errors, [%]
MV	Medium Voltage
n	Consumer with smart meter ($n = 1, \dots, N_{SM}$)
N_c	Total number of consumers
N_p	Total number of poles
N_{SM}	Total number of consumers integrated in the SMS
v	The pole in up stream of pole i
PA	Proposed algorithm
P_g, Q_g	The active and reactive power of the generator, [kW], [kVAr]
P_c, Q_c	The active and reactive absorbed power, [kW], [kVAr]
P_l	The denormalized load profile at consumer l , [kW/kWh]
P_m	Three-phase feeder measured load profile, [kW]
P_{sm}	Active power measured with the smart meter, [kW]
P_{cor}	Denormalized load profiles adjusted by measured load profiles, [kW/kWh]
s	The slack bus
S_s	Total apparent power, [kVA]
SMS	Smart Metering System
R	Resistance, [Ω]
tc	Type of consumer (residential, non-residential, commercial, and industrial)
TLP	The Typical Load Profile
U_i	The phase voltages from each pole $i = 1, \dots, N_p$, [V]
U_s	The phase voltages from the slack bus, [V]
V1, V2	Structure vectors
Z_v	The branch impedance, [Ω]
X	Reactance, [Ω]
W	The daily energy consumption, [kWh]
ΔP	The deviation between the measured and computed load profiles, [kW]
ΔU	The voltage drop, [V]
ΔW	The energy losses, [kWh]
ϵ_s	The error for the convergence test (Absolute error), [kVA]
ϵ	The absolute error, [kWh]
δ	The percentage error, [%]

Appendix A

Table A1. The allocation on pole, phase, and the type of the consumers

Pole	Branching		Phase			Consumers Type			Pole	Branching		Phase			Consumers Type		
	1-Phase	3-Phase	a	b	c	1	2	3		1-Phase	3-Phase	a	b	c	1	2	3
1	1	-	-	1	-	1	-	-	96	1	1	1	2	1	1	-	-
2	2	-	-	2	-	1	-	-	97	1	-	-	1	-	1	-	-
3	4	-	-	4	-	1	-	-	98	1	-	-	1	-	1	-	-
4	3	-	-	2	-	1	-	-	99	6	-	-	4	2	1	-	-
7	3	-	-	3	-	1	-	-	100	4	-	-	3	1	1	-	-
8	2	-	-	2	-	1	-	-	101	1	-	-	1	-	1	-	-
9	2	-	-	2	-	1	-	-	102	3	-	-	3	-	1	-	-
10	3	-	2	1	-	1	-	-	103	1	-	-	1	-	1	-	-
11	1	-	-	1	-	1	-	-	104	1	-	-	1	-	1	-	-
12	2	-	-	2	-	1	-	-	106	2	-	1	-	1	1	-	-
13	1	-	-	1	-	1	-	-	107	3	-	1	-	2	1	-	-
14	2	-	-	-	2	1	-	-	109	1	-	-	-	1	1	-	-
15	2	-	-	1	1	1	-	-	110	1	-	-	-	1	1	-	-
17	-	1	1	1	1	1	-	-	111	3	-	-	-	3	1	-	-
18	2	-	-	-	2	1	-	-	112	4	-	-	-	4	1	-	-
19	2	-	2	-	-	1	-	-	113	1	-	-	-	1	1	-	-
20	2	-	2	-	-	1	-	-	114	3	-	-	-	3	1	-	-
21	1	-	1	-	-	1	-	-	115	1	-	-	-	1	2	-	-
22	2	-	1	1	-	1	-	-	116	1	-	-	-	1	2	-	-
23	2	-	2	-	-	1	-	-	117	-	1	1	1	1	1	-	-
24	1	-	-	-	1	1	-	-	118	-	1	1	1	1	2	-	-
26	2	-	-	-	2	1	-	-	119	1	-	1	-	-	1	-	-
27	3	-	1	-	2	1	-	-	120	1	-	1	-	-	1	-	-
28	2	-	-	1	1	1	-	-	121	2	-	2	-	-	1	-	-
29	4	-	-	1	3	1	-	-	122	2	-	1	1	-	1	-	-
30	2	-	-	-	2	1	-	-	123	4	1	2	3	1	1	-	-
31	2	-	-	-	2	1	-	-	124	3	-	2	1	-	1	-	-
32	1	-	-	-	1	1	-	-	125	2	-	-	2	-	1	-	-
33	4	-	-	-	4	1	-	-	126	2	-	-	2	-	1	-	-
34	5	-	-	-	5	1	-	-	127	2	-	1	-	1	1	-	-
35	4	-	1	1	2	1	-	-	128	2	-	2	-	-	1	-	-
36	1	-	-	1	-	1	-	-	129	4	-	4	-	-	1	-	-
37	3	-	-	-	3	1	-	-	130	1	-	-	1	-	1	-	-
38	1	-	-	-	1	1	-	-	131	3	-	-	3	-	1	-	-
39	4	-	-	1	3	1	-	-	133	2	-	1	1	-	1	-	-
40	3	-	-	-	3	1	-	-	134	1	-	-	1	-	1	-	-
41	1	-	-	-	1	1	-	-	135	3	-	3	-	-	1	-	-
42	1	-	-	-	1	1	-	-	136	3	-	3	-	-	1	-	-
43	2	-	-	-	2	1	-	-	137	3	-	-	3	-	1	-	-
44	2	-	-	1	1	1	-	-	138	2	-	-	2	-	1	-	-
45	4	-	-	-	4	1	-	-	139	1	-	-	1	-	1	-	-
46	2	-	-	-	2	1	-	-	140	3	1	2	3	1	1	-	-

47	3	-	1	2	-	1	-	-	141	4	-	1	3	-	1	-	-
48	3	-	1	2	-	1	2	-	142	1	-	1	-	-	1	-	-
49	2	-	-	2	-	1	-	-	143	2	-	1	1	-	1	-	-
50	1	-	-	-	1	1	-	-	144	2	-	1	1	-	1	-	-
51	1	-	-	1	-	-	2	-	145	2	-	1	-	1	1	-	-
52	3	-	-	3	-	1	2	-	146	2	-	1	1	-	1	-	-
53	1	-	-	1	-	-	2	-	147	1	-	-	1	-	1	-	-
54	6	-	-	-	6	1	2	-	148	2	-	-	1	1	1	-	-
55	2	-	1	1	-	1	-	-	149	1	-	1	-	-	1	-	-
56	2	-	-	2	-	1	-	-	150	3	-	-	2	1	1	-	-
57	1	-	-	1	-	1	-	-	151	2	-	1	1	-	1	-	-
58	1	-	1	-	-	1	-	-	152	3	-	1	2	-	1	-	-
59	2	-	-	2	-	1	-	-	153	1	-	1	-	-	1	-	-
60	2	-	1	1	-	1	-	-	154	1	-	1	-	-	1	-	-
61	1	-	-	1	-	1	-	-	155	2	-	2	-	-	1	-	-
62	1	-	-	-	1	1	-	-	156	2	-	-	1	1	1	-	-
63	2	-	2	-	-	1	-	-	157	2	-	1	1	-	1	-	-
65	1	-	-	1	-	1	-	-	158	2	-	1	1	-	1	-	-
66	4	-	1	3	-	1	-	-	159	1	-	-	1	-	1	-	-
67	2	-	-	2	-	1	-	-	161	1	-	1	-	-	1	-	-
68	2	-	-	2	-	1	-	-	162	2	-	-	2	-	1	-	-
69	2	-	1	1	-	1	-	-	163	1	-	-	-	1	1	-	-
70	1	-	-	1	-	1	-	-	164	3	-	2	-	1	1	-	-
71	1	-	-	1	-	1	-	-	165	1	-	1	-	-	1	-	-
72	1	-	-	1	-	1	-	-	166	2	-	1	-	1	1	-	-
75	2	-	-	2	-	1	-	-	168	2	-	2	-	-	1	-	-
76	2	-	-	2	-	1	-	-	169	3	-	2	-	1	1	-	-
77	2	-	1	1	-	1	-	-	170	1	-	1	-	-	1	-	-
78	4	-	1	3	-	1	-	-	171	2	-	-	-	2	1	-	-
79	1	1	1	2	1	1	-	-	172	2	-	-	1	1	1	-	-
80	2	-	-	2	-	1	-	-	173	2	1	2	1	2	1	-	-
82	2	-	-	2	-	1	-	-	174	2	-	1	-	1	1	-	-
83	1	-	1	-	-	1	-	-	175	2	-	-	-	2	1	-	-
84	2	-	-	2	-	1	-	-	176	2	-	1	-	1	1	-	-
86	1	-	-	1	-	1	-	-	177	2	-	1	-	1	1	-	-
87	2	-	-	2	-	1	-	-	179	1	-	-	-	1	1	-	-
88	1	-	-	1	-	1	-	-	180	1	-	1	-	-	1	-	-
89	2	-	-	2	-	1	-	-	181	1	-	-	-	1	1	-	-
90	1	-	-	1	-	1	-	-	183	1	-	-	-	1	1	-	-
91	2	-	-	2	-	1	-	-	184	1	-	-	-	1	1	-	-
92	1	-	-	1	-	1	-	-	185	1	-	-	1	-	1	-	-
93	2	-	-	2	-	1	-	-	187	1	-	1	-	-	1	-	-
94	1	-	1	-	-	1	-	-	188	1	-	1	-	-	1	-	-
95	1	-	-	1	-	1	-	-	189	1	-	-	-	1	1	-	-

Appendix B

Table B1. The energy losses calculated with proposed algorithm - Feeder 1, [kWh]

Hour	Branching conductors				Main conductors				Total
	a	b	c	Neutral	a	b	c	Neutral	
1	0	0.002018	0	0.002518	0	0.000253	0	0.000166	0.004955
2	0	0.00142	0	0.001774	0	0.000175	0	0.000115	0.003484
3	0	0.001438	0	0.001797	0	0.000176	0	0.000115	0.003527
4	0	0.001555	0	0.001947	0	0.000189	0	0.000124	0.003815
5	0	0.0012	0	0.001499	0	0.000145	0	9.53e-05	0.00294
6	0	0.001208	0	0.001508	0	0.000146	0	9.58e-05	0.002958
7	0	0.0015	0	0.001867	0	0.000183	0	0.00012	0.00367
8	0	0.00154	0	0.001916	0	0.000189	0	0.000124	0.003768
9	0	0.001351	0	0.001679	0	0.000174	0	0.000114	0.003317
10	0	0.001747	0	0.002171	0	0.000225	0	0.000148	0.004291
11	0	0.001319	0	0.00164	0	0.000167	0	1.10e-04	0.003237
12	0	0.001761	0	0.002189	0	0.000227	0	0.000149	0.004326
13	0	0.001889	0	0.002347	0	0.000245	0	0.000161	0.004641
14	0	0.001428	0	0.001771	0	0.000185	0	0.000121	0.003505
15	0	0.001427	0	0.001773	0	0.000185	0	0.000121	0.003506
16	0	0.001832	0	0.002273	0	0.000236	0	0.000155	0.004497
17	0	0.00184	0	0.002271	0	0.000255	0	0.000167	0.004533
18	0	0.001808	0	0.002231	0	0.000247	0	0.000162	0.004448
19	0	0.002043	0	0.002524	0	0.000277	0	0.000181	0.005026
20	0	0.00227	0	0.002808	0	0.000312	0	0.000205	0.005595
21	0	0.003143	0	0.003882	0	0.000443	0	0.00029	0.007758
22	0	0.004848	0	0.005993	0	0.000661	0	0.000433	0.011936
23	0	0.004099	0	0.005075	0	0.000568	0	0.000372	0.010114
24	0	0.002171	0	0.002703	0	0.000268	0	0.000176	0.005318
Total	0	0.046854	0	0.058157	0	0.006134	0	0.00402	0.115165

Table B2. The energy losses calculated with proposed algorithm - Feeder 2, [kWh]

Hour	Branching conductors of consumers				Main conductors of distribution feeder				Total
	a	b	c	Neutral	a	b	c	Neutral	
1	0.0201	0.4897	0.1064	0.3975	0.0019	0.0103	0.0008	0.0085	1.0350
2	0.0145	0.3723	0.0782	0.3027	0.0014	0.0084	0.0006	0.0068	0.7848
3	0.0139	0.3780	0.0837	0.3103	0.0011	0.0086	0.0006	0.0068	0.8030
4	0.0149	0.4240	0.0879	0.3472	0.0013	0.0102	0.0006	0.0079	0.8940
5	0.0122	0.3414	0.0728	0.2799	0.0010	0.0083	0.0005	0.0064	0.7226
6	0.0129	0.3174	0.0776	0.2614	0.0011	0.0067	0.0006	0.0055	0.6833
7	0.0190	0.3678	0.0983	0.3005	0.0019	0.0065	0.0007	0.0060	0.8008
8	0.0213	0.3475	0.0993	0.2827	0.0025	0.0053	0.0008	0.0056	0.7648
9	0.0172	0.2792	0.0716	0.2236	0.0021	0.0042	0.0006	0.0045	0.6030
10	0.0206	0.3828	0.0930	0.3076	0.0022	0.0065	0.0007	0.0062	0.8197

11	0.0164	0.3121	0.0734	0.2506	0.0019	0.0056	0.0006	0.0053	0.6659
12	0.0194	0.4158	0.0902	0.3339	0.0020	0.0081	0.0007	0.0071	0.8774
13	0.0226	0.4602	0.0966	0.3671	0.0026	0.0091	0.0007	0.0081	0.9670
14	0.0163	0.3305	0.0772	0.2664	0.0016	0.0061	0.0006	0.0055	0.7042
15	0.0159	0.3303	0.0738	0.2654	0.0017	0.0062	0.0006	0.0056	0.6995
16	0.0205	0.4027	0.1010	0.3263	0.0020	0.0070	0.0008	0.0064	0.8666
17	0.0222	0.4306	0.0886	0.3414	0.0025	0.0083	0.0007	0.0075	0.9019
18	0.0228	0.4079	0.0920	0.3241	0.0026	0.0072	0.0008	0.0069	0.8642
19	0.0257	0.3984	0.1046	0.3184	0.0029	0.0057	0.0008	0.0062	0.8628
20	0.0292	0.3904	0.1056	0.3099	0.0037	0.0048	0.0009	0.0061	0.8506
21	0.0373	0.4999	0.1377	0.3974	0.0044	0.0058	0.0011	0.0074	1.0911
22	0.0519	0.7618	0.2340	0.6188	0.0050	0.0085	0.0019	0.0100	1.6919
23	0.0407	0.7067	0.1783	0.5657	0.0040	0.0099	0.0014	0.0100	1.5168
24	0.0217	0.4255	0.1328	0.3558	0.0016	0.0062	0.0010	0.0058	0.9505
Total	0.5294	9.9731	2.4546	8.0549	0.0549	0.1735	0.0191	0.1621	21.4215

Table B3. The energy losses calculated with proposed algorithm - Feeder 3, [kWh]

Hour	Branching conductors of consumers				Main conductors of distribution feeder				Total
	a	b	c	Neutral	a	b	c	Neutral	
1	0.2724	0.2458	0.2612	0.0673	0.0031	0.0025	0.0026	0.0035	0.8582
2	0.1935	0.1774	0.1897	0.0475	0.0022	0.0018	0.0019	0.0025	0.6165
3	0.1849	0.1788	0.1940	0.0464	0.0021	0.0018	0.0020	0.0024	0.6123
4	0.2051	0.1948	0.2117	0.0512	0.0024	0.0020	0.0022	0.0026	0.6719
5	0.1583	0.1535	0.1666	0.0394	0.0018	0.0015	0.0017	0.0020	0.5249
6	0.1580	0.1549	0.1668	0.0392	0.0018	0.0015	0.0017	0.0021	0.5258
7	0.2129	0.1937	0.2020	0.0506	0.0025	0.0018	0.0020	0.0029	0.6684
8	0.2373	0.2054	0.2011	0.0549	0.0029	0.0020	0.0019	0.0033	0.7088
9	0.2023	0.1654	0.1649	0.0482	0.0024	0.0017	0.0015	0.0028	0.5892
10	0.2445	0.2059	0.2120	0.0592	0.0028	0.0020	0.0019	0.0033	0.7317
11	0.1882	0.1584	0.1632	0.0450	0.0022	0.0015	0.0015	0.0026	0.5626
12	0.2395	0.2037	0.2130	0.0586	0.0027	0.0020	0.0020	0.0032	0.7246
13	0.2684	0.2191	0.2275	0.0653	0.0031	0.0021	0.0021	0.0036	0.7913
14	0.1870	0.1620	0.1727	0.0461	0.0020	0.0015	0.0016	0.0025	0.5754
15	0.1907	0.1627	0.1706	0.0466	0.0021	0.0016	0.0016	0.0026	0.5784
16	0.2372	0.2082	0.2198	0.0584	0.0026	0.0019	0.0020	0.0032	0.7333
17	0.2506	0.2007	0.2141	0.0632	0.0028	0.0019	0.0018	0.0036	0.7387
18	0.2505	0.2025	0.2133	0.0625	0.0028	0.0019	0.0018	0.0036	0.7389
19	0.2839	0.2279	0.2396	0.0706	0.0033	0.0021	0.0020	0.0040	0.8334
20	0.3297	0.2479	0.2627	0.0829	0.0040	0.0024	0.0023	0.0047	0.9366
21	0.4392	0.3315	0.3553	0.1130	0.0051	0.0032	0.0030	0.0062	1.2564
22	0.6306	0.5135	0.5523	0.1628	0.0067	0.0047	0.0047	0.0085	1.8838
23	0.5370	0.4351	0.4719	0.1395	0.0057	0.0042	0.0040	0.0072	1.6047

24	0.2686	0.2618	0.2799	0.0676	0.0028	0.0024	0.0027	0.0035	0.8892
Total	6.3702	5.4105	5.7257	1.5859	0.0718	0.0520	0.0524	0.0865	19.3550

Table B4. The phase voltages at the farthest poles (P95 and P189), calculated cu both algorithms [V]

Hour	Pole P95						Pole P189					
	Proposed Algorithm			DigSilent			Proposed Algorithm			DigSilent		
	a	b	c	a	b	c	a	b	c	a	b	c
1	228.81	216.95	229.17	228.72	216.34	229.04	216.05	217.82	216.03	215.85	218.48	215.70
2	228.92	218.59	229.20	228.83	218.45	229.10	218.20	219.56	218.12	218.11	220.23	218.22
3	229.89	219.45	230.09	229.81	218.26	229.99	219.39	220.40	218.92	219.34	221.08	219.01
4	231.04	220.02	231.27	230.97	219.64	231.16	219.97	221.15	219.68	219.92	221.86	219.74
5	232.20	222.31	232.39	232.13	221.26	232.29	222.54	223.42	222.11	222.53	224.11	222.39
6	234.37	224.80	234.56	234.30	223.86	234.46	224.75	225.55	224.22	224.73	226.22	224.46
7	234.44	224.16	234.78	234.35	223.10	234.64	223.37	224.73	223.24	223.22	225.37	223.21
8	231.03	221.04	231.46	230.90	220.80	231.31	219.37	221.13	219.82	219.04	221.66	219.64
9	229.33	220.39	229.82	229.20	219.83	229.69	218.50	220.52	219.03	218.15	221.03	218.99
10	228.06	217.59	228.54	227.89	217.08	228.36	216.09	218.15	216.30	215.44	218.47	215.68
11	228.54	219.11	228.97	228.39	218.79	228.82	218.09	219.85	218.29	217.57	220.21	218.04
12	227.88	216.99	228.34	227.71	216.49	228.16	215.98	217.98	216.09	215.29	218.26	215.41
13	227.60	216.19	228.15	227.41	215.65	227.95	215.05	217.43	215.45	214.22	217.65	214.61
14	227.93	218.20	228.33	227.77	217.91	228.16	217.46	219.08	217.23	216.90	219.39	216.81
15	228.21	218.49	228.62	228.05	218.20	228.46	217.60	219.35	217.58	217.03	219.66	217.20
16	227.25	216.49	227.68	227.08	216.06	227.47	215.43	217.18	215.14	214.72	217.42	214.30
17	226.95	215.94	227.57	226.76	215.43	227.38	214.90	217.34	214.97	214.12	217.60	214.15
18	226.79	216.05	227.40	226.62	215.46	227.22	214.78	217.14	214.87	214.10	217.46	214.15
19	226.92	216.25	227.56	226.76	215.46	227.38	214.10	216.65	214.28	213.47	217.06	213.50
20	226.10	215.56	226.89	225.96	215.51	226.72	212.28	215.53	212.96	211.68	216.05	212.21
21	226.01	214.06	226.90	225.86	212.51	226.71	209.95	213.77	210.61	209.17	214.25	209.18
22	221.99	207.14	222.81	221.84	206.00	222.51	202.55	206.42	202.70	201.45	206.58	201.50
23	224.20	209.92	224.98	224.06	209.50	224.75	206.21	209.91	206.49	205.34	210.25	205.22
24	227.82	216.61	228.06	227.71	215.31	227.88	215.16	216.30	214.37	214.90	216.84	213.70

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