Does the electron have an anomalous electric dipole moment?

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Summary
An analysis is presented of the possible existence of a second anomalous dipole moment of Dirac's particle next to the angular one. It includes a discussion why, in spite of his own derivation, Dirac has doubted about its relevancy. It is shown why since then it has been overlooked and why it has vanished from leading textbooks. A critical survey is given on the reasons of its reject, including the failure of attempts to measure and the perceived violations of time reversal symmetry and charge-parity symmetry. It is emphasized that the anomalous electric dipole moment of the pointlike electron (AEDM) is fundamentally different from the quantum field type electric dipole moment of an electron (eEDM) as defined in the standard model of particle physics and that its measurement requires different instrumentation. A proposal has been described how to prove or disprove its existence by experiment. Moreover, by reference from literature, the possible impact is discussed in the nuclear domain and in the gravitational domain.

Keywords: anomalous electric dipole moment; Dirac particle; Pauli's spin vector; isospin

Introduction

In his classic paper on electrons, Paul Dirac has derived a basic 4-dimensional wave equation for an electron in motion subject to a vector potential $\boldsymbol{A}(A_0, A_x, A_y, A_z)$. In this equation [1, eq. 15/16], an anomalous electric dipole moment shows up, next to the well-known anomalous magnetic dipole moment. Dirac doubted whether it could have a physical interpretation, the more because it appeared in a quantity with an imaginary sign as compared with a similar expression for the magnetic dipole moment. Where a magnetic dipole moment makes sense as a manifestation of angular spin, a similar physical manifestation for an electric dipole moment is not obvious.

This is a first reason why, since then, Dirac's electric dipole moment of an electron has been ignored. The second reason is, that experimental attempts to reveal an electric dipole moment of an electron (eEDM), if it would exist, all failed. Presently, the Particle Data Group (PDG) has set an upper limit for its value as [2],

eEDM $<$ 0.87 10^{-30} e m,

where e is the elementary charge.

The third reason why an electron dipole moment for an electron has been put into doubt is due to the perceived violations of time reversal symmetry (T) and charge-parity symmetry (CP), [3].

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There is somewhat more. It is quite curious that in the highly reputed textbook of Bjorken and Drell, the electric dipole moment is no longer mentioned. Bjorken and Drell have decomposed Dirac's four-component wave function $\psi(\psi_1,\psi_2,\psi_3,\psi_4)$ into two two-components wave equations for the non-relativistic domain, a dominant one $\psi(\psi_1,\psi_2)$ and a minor one $\chi(\chi_1,\chi_2)$. See [4, eq. 1.32 and 1.33]. Dirac's electric dipole moment no longer shows up, while the magnetic dipole moment is clearly present. One might guess that its disappearance is due to the non-relativistic restriction. In Griffiths textbook [5], the electrons's electric dipole moment is not mentioned. As I wish to show later, the basic reason is different.

Dirac's anomalous electric dipole moment (AEDM)

Let us inspect all those arguments step by step. On page 619 of his famous article, Dirac concludes that the Hamiltonian of an electron in a magnetic field shows two excessive energy contributions as a consequence the particular characteristics of his equation of motion that identifies a four component wave equation. The excessive add-on ΔH_a appears being

$$\Delta H_a = \frac{e\hbar}{c}(\overline{\sigma}\cdot\mathbf{H}) + \mathrm{i}\frac{e\hbar}{c}\rho_1(\overline{\sigma}\cdot\mathbf{E}). \tag{1}$$

As usual, $\mathrm{i}=\sqrt{-1}$. This expression, expressed in Gaussian units, contains, apart from e as the elementary electric charge, c the vacuum light velocity, $\mathbf{E}$ and $\mathbf{H}$, respectively the electric field vector and the magnetic field vector, a Pauli vector $\overline{\sigma}(\sigma_1,\sigma_2,\sigma_3)$ and a matrix ρ_1. The latter two compose a system of four (4 x 4) matrices, defined by Dirac as,

$$\sigma_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix};\ \sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix};\ \sigma_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix};\ \rho_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

In the first term of this expression, Dirac recognized the anomalous magnetic dipole moment $e\hbar/2m_0c$ of an electron (m_0 being its rest mass). He had a difficulty, however, to interpret the second term, which seemed to him an imaginary electric dipole moment $\mathrm{i}e\hbar(\rho_1\overline{\sigma})/2m_0c$ without a physical meaning. The matrices shown in (2) are somewhat different from the $\gamma-$ matrices that show up in a canonical representation of Dirac's wave equation, which reads as,

$$[\gamma_0\hat{p}_0' + (\overline{\gamma}\cdot\hat{\mathbf{p}}') - \mathrm{i}I]\psi = 0;\ \overline{\gamma} = \overline{\gamma}(\gamma_1,\gamma_2,\gamma_3), \tag{3}$$

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where I is the 4 x 4 unity matrix and where $\hat{p}_0'$ and $\hat{\mathbf{p}}'(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ are the quantum field wave operators obtained by transforms on the momenta such that,

$$\hat{p}_i' = \frac{1}{m_0 c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x_i} \quad \text{and} \quad \hat{p}_0' = \frac{1}{m_0 c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial ct},$$

and where γ_μ are the gamma matrices, which are closely related with the α_i matrices and the ρ_i matrices used by Dirac in his 1928 paper. These gamma matrices are defined as,

$$\gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix};\ \gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix};\ \gamma_2 = \begin{bmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \end{bmatrix};\ \gamma_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(4)

As elaborated in the Appendix, rewriting Dirac's result (1) in SI units and in terms of the $\gamma-$ matrices gives an energy representation ΔE,

$$\Delta E = \frac{e\hbar}{2m_0}(\overline{\sigma}_B \cdot \mathbf{B}) + \mathrm{i}\frac{e\hbar}{2m_0 c}(\overline{\sigma}_E \cdot \mathbf{E}), \tag{5}$$

where $\overline{\sigma}_B = \overline{\sigma}_B\{(\gamma_1\gamma_2),(\gamma_1\gamma_3),(\gamma_2\gamma_3)\}$; $\overline{\sigma}_E = \overline{\sigma}_E(\gamma_0\gamma_1,\gamma_0\gamma_2,\gamma_0\gamma_3)$.

The interpretation of $\overline{\sigma}_B \cdot \mathbf{B}$ and $\mathrm{i}\overline{\sigma}_E \cdot \mathbf{E}$ is crucial. The format is the in-product of two vectors, such that

$$\overline{\sigma}_i = \sigma_{i1}\mathbf{i} + \sigma_{i2}\mathbf{j} + \sigma_{i3}\mathbf{k}\ , \tag{6}$$

where $(\mathbf{i},\mathbf{j},\mathbf{k})$ are the spatial unit vectors. The format of the coefficients poses a difficulty, because their number typing is a 4 x 4 matrix. One thing, however, will be clear: it is too easy to say that the first (= magnetic) term represents a real contribution to the energy and that the second (= electric) term represents an imaginary contribution. Because γ_2 is imaginary and $\gamma_0,\gamma_1,\gamma_3$ are real , the real contribution to the excessive magnetic energy comes from B_x and B_z and the real contribution to the excessive electric energy comes from E_y. The impact of these particular vector components is due to the particular arbitrary assignment of the three matrix coefficients $(\sigma_{i1},\sigma_{i2},\sigma_{i3})$ over the three spatial axes. Another distribution would have made other vector components effective, the inefficiency of which now is shown by the imaginary sign. The complexity of the coefficients reflect the wave state of the particle.

Obviously, Dirac's doubt cannot be a proof for the physical non-existence of his anomalous electric dipole moment, which, as will be stipulated later, is not the same as the eEDM mentioned before. The difficulty of the physical interpretation might be due to an unexpected property of an electron. Let us suppose that the electron, similarly like all physical particles, is subject to the Heisenberg uncertainty. Let us suppose, just by hypothesis, that its position d in its center of mass frame can be explained as the result of a motion with ultra-relativistic speed near vacuum light velocity c, such that

$$d = c\Delta t \,. \tag{7}$$

Applying Heisenberg's relationship $\Delta E \Delta t = \hbar / 2$, [6], on (7), we get

$$d = c\Delta t = c\frac{\hbar}{2}\frac{1}{\Delta E} \rightarrow d = c\frac{\hbar}{2}\frac{1}{mc^2} \rightarrow \mu_{mass} = md = \frac{\hbar}{2c}, \tag{8}$$

where μ_{mass} has the dimensions of a (mass) dipole moment expressed in terms of Planck's reduced constant $\hbar$ and the vacuum light velocity c. The virtual mass m should not be confused with the particle's rest mass m_0. It is fair to suppose that (in 1928) Dirac was not aware that his wave equation of electrons implicitly embodied Heisenberg's uncertainty (1927), because if so, he wouldn't have so easily waived away his anomalous electric dipole moment. It is David Hestenes who, in his studies on the "zitterbewegung" of electrons, recognized it [7,8].

Let us proceed by discussing the failure of measurement. Now we have suggested, by argumentation, the possible existence of a mechanical vibration moment $\boldsymbol{\hbar/c}$, next to the mechanical angular momentum $\boldsymbol{\hbar}$, the question has to be addressed how to relate these mechanical motions with the hypothetical existence of an electric dipole moment $\boldsymbol{\mu}_{el}$ and the existence of a magnetic dipole moment $\boldsymbol{\mu}_{\mathbf{m}}$ of an electron with its elementary charge e and its mass m_0. The magnitude of the magnetic one is well known from textbooks as [5],

$$\left|\boldsymbol{\mu}_{\mathbf{m}}\right| = \frac{e}{2m_0} |\boldsymbol{\hbar}|, (\approx 9.27\ 10^{-24}\ \mathrm{C\ m^2\ s^{-1}}). \tag{9}$$

The magnitude of the anomalous electric dipole moment **AEDM** as derived by Dirac [1, eq. 15/16], amounts to,

$$\left|\boldsymbol{\mu}_{el}\right| = \frac{e}{2m_0} |\boldsymbol{\hbar/c}|, (\approx 3.09\ 10^{-32}\ \mathrm{C\ m}). \tag{10}$$

This is quite different from the PDG value quoted before. Obviously, the discrepancy must be due to a basic difference between the electric dipole moment **eEDM** as defined in the context of PDG and the anomalous electric dipole moment **AEDM** as meant by Dirac. The latter one, be it imaginary or not (to be discussed later) is a pure quantum mechanical phenomenon, while eEDM is not quite. Instead, a classical EDM is a consequence of a presupposed *spatial* structure of an electron with some charge distribution [9,10]. If the electron is pointlike indeed, there is no classical EDM. Dirac's anomalous one, on the other hand, shows up as a quantum mechanical vector with eigenvalues, even if the particle is pointlike. This difference remains, in spite of the present less classical definition in terms of a QFT-based form factor that models the charge cloud around a pointlike source [10]. Where present experiments so far failed to probe the existence of an eEDM, an experimental proof that reveals an AEDM doesn't exist either, in spite of the fact that its magnitude is many orders of magnitude larger than the present established upper limit of eEDM. It could be that the AEDM could not be experimentally proved, because no experiments have been devised so far on the basis of a proper understanding of its origin. It might well be that present eEDM experiments are unable to detect an AEDM. Later in this article, these issues will be addressed further, as well as the parity violation of eEDM as compared with AEDM and the reason why Dirac's AEDM vanished from textbooks.

Parity violation difference between eEDM and AEDM

Let us now discuss the perceived parity violations. It will make the difference between the eEDM and the AEDM more clear. Let us use the arguments quoted in [3]. Here, the interaction Hamiltonians, H_E and H_M for the electric dipole moment and the magnetic dipole moment are, respectively, expressed as,

$$H_E = -d_E \mathbf{S}.\cdot\mathbf{E} \quad \text{and} \quad H_M = -d_M \mathbf{S}.\cdot\mathbf{B}, \tag{11}$$

where $\mathbf{S}, \mathbf{E}, \mathbf{B}, d_E$ and d_M, respectively, are the spin angular momentum, the electric field strength and the magnetic field strength, and where d_E and d_M are the strengths of the dipoles. Let, in terms of a spin number s, as usual $\mathbf{S} = \hbar\sqrt{s(s+1)}$, d_E and d_M proportional with the elementary electric charge e and let us consider, in Table I, the T, C and P symmetries of electromagnetism [11].

Table I

spin dependent dipole moments						
		Time reversal	Charge inversion	Parity reversal	CP	CPT
Magnetic mom	$-d_M\mathbf{S}$	sign change	sign change	no sign change	change	no change
$\mathbf{B}$		sign change	sign change	no sign change		
H_M	$-d_M\mathbf{S}.\cdot\mathbf{B}$	no sign change	no sign change	no sign change	no change	no change
EDM	$-d_E\mathbf{S}$	sign change	sign change	no sign change	change	no change
$\mathbf{E}$		no sign change	sign change	sign change		
H_E	$-d_E\mathbf{S}.\cdot\mathbf{E}$	sign change	no sign change	sign change	sign change	no change

From this table, it is concluded that an eEDM c.f. (5) violates the time reversal symmetry and the CP symmetry of its interaction Hamiltonians, albeit that CPT symmetry remains conserved. On the one hand, it gives a reason to deny its possible existence, while on the other hand, it raises a particular interest, because CP symmetry violation is believed being a condition for the origin of the matter/antimatter asymmetry in the universe [3,12,13]. This explains why there is a considerable amount of experimental research that attempts to prove the existence of a non-zero eEDM.

However, a similar table, composed on the basis of Dirac's anomalous dipole moments, does not show such a different behaviour of the electric interaction Hamiltonian from the magnetic one. Dirac's anomalous AEDM doesn't violate time reversal symmetry nor CP symmetry.

Table II

Dirac's anomalous dipole moments						
		Time reversal	Charge inversion	Parity reversal	CP	CPT
Magnetic mom	$(e/2m_0)\lvert\boldsymbol{\hbar}\rvert$	sign change	sign change	no sign change	change	no change
$\mathbf{B}$		sign change	sign change	no sign change		
$\mathbf{B}\cdot\boldsymbol{\mu}_{magn}$		no sign change	no sign change	no sign change	no change	no change
Electric mom	$(e/2m_0)\lvert\boldsymbol{\hbar}/\boldsymbol{c}\rvert$	no sign change	sign change	sign change	no change	no change
$\mathbf{E}$		no sign change	sign change	sign change		
$\mathbf{E}\cdot\boldsymbol{\mu}_{el}$		no sign change	no sign change	no sign change	no change	no change

Understanding it properly, requires recognition of the differences between the various dipoles and dipole moments. The anomalous magnetic dipole moment is a pseudo vector orthogonal to the angular momentum. The magnetic dipole itself is non-rotating and aligned along the axis of its pseudo vector. The electric dipole moment eEDM is a pseudo vector collinear with the anomalous magnetic dipole moment. The electric dipole iself is rotating with the angular momentum. The anomalous electric dipole moment AEDM is a pseudo vector as well. However, unlike the eEDM dipole, the AEDM dipole is non-rotating. The orientation of its pseudo vector is not determined by orthogonality to the angular momentum $\boldsymbol{\hbar}$, but is determined by a non-angular *isospin* vector $\boldsymbol{\hbar}/\boldsymbol{c}$. That marks a basic difference between eEDM and AEDM. Where the eEDM and the anomalous magnetic dipole moment compose the *same* vector, the AEDM and the anomalous magnetic dipole moment compose *different* vectors. Where the spin vector is subject to a change of sign under time reversal, the isospin vector is not. Where the vector properties of an eEDM depend on the *angular momentum vector* $\boldsymbol{\hbar}$, the vector properties of an AEDM depend on the *position* *vector* $\boldsymbol{\hbar/c}$. The former one represents an angular motion, while the latter one represents a (Heisenberg) vibration. It is a (position) vector that can be directed under influence of an electric field, independent from the angular momentum vector.

The disappearance of Dirac's AEDM in textbooks

After having discussed the three arguments physical interpretation, magnitude and parity violation, we are left with the problem why the electric dipole moment does not show up in the Bjorken-Drell (2 x 2)- wave function, while it does in Dirac's 4-component one. In the Bjorken and Drell's textbook, similarly like in many other ones, Dirac's trick is applied on a curl operation, such as, for instance, can be seen in their textbook by moving from [4, eq. 1.32) to [4, eq.1.34]. The same, for instance, holds for Shankar's textbook (see [14, eq. 20.2.16 in relation with eq. 20.2.2]. Because the temporal momentum is not included and cannot be included in the curl operation, only a single dipole moment shows up, while in Dirac's comparable expression [1, eq. 15/16] two dipole moments are shown. At this point, it is interesting to note that Lanczos [15] has been able to maintain full symmetry in the curl operation owing to his special quaternion algebra, which enabled him to give an interpretation to "isospin" and that Hestenes [7,8] developed his special STA algebra for the purpose, which enabled to explain the "zitterbewegung" of electrons. In the Appendix I have shown that the curl operation problem can be avoided as well by adopting the "Hawking metric" (+,+,+,+) for space-time (ict, x, y, z) as a useful instrument for maintaining full parity over the four dimensions.

How to measure Dirac's AEDM

Where present experiments so far failed to probe the existence of an eEDM, an experimental proof that reveals an AEDM doesn't exist either, in spite of the fact that its magnitude is many orders of magnitude larger than the present established upper limit of eEDM. How to prove the existence or non-existence of the AEDM is not obvious. One thing is clear: it can't be done in the way how present high-precision eEDM measurements are set up [16,17]. These measurements aim to measure the precession effect from the pre-supposed electric dipole moment contribution to the effects from the nuclear spin of an electron. Because the AEDM is not caused by an angular motion, it does not contribute to such effects. Hence, it cannot be detected by the instrumentation that aims to measure the eEDM. It remains a challenge for further research. The phenomenon to be shown and measured is the second spin-flip of an electron under influence of a vector potential. To do so, one might consider to measure the hyperfine split effect due to the spin-spin interaction of the electron with the atomic nucleus, which gives rise to the well-known 21 cm line in the cosmological electromagnetic spectrum of atomic hydrogen [18]. Unfortunately, the interaction energy between the spins due to the anomalous electric dipole moments is just equal to the interaction energy due to the anomalous magnetic dipole moments. This can be seen as follows. According to Griffiths [5,19], the interaction energy ΔE between the magnetic dipoles μ_m^e and μ_m^p of, respectively, the electron (mass m_e) and the proton (mass m_p; "g"-factor g_p = 5.59) amounts to

$$\Delta E = \mu_0 \frac{g_p \mu_m^e \mu_m^p}{3\pi a_0^3} = \frac{4 g_p \hbar^4}{3 a_0^4 m_p m_e^2 c^2} \quad ; \quad a_0 = \frac{\hbar}{g^2 m_e c}, \tag{12}$$

where a_0 and g^2 ,respectively, are the Bohr radius [20] and the electromagnetic fine structure constant.

Hence, from (12),

$$\Delta E = \hbar\omega = hf \rightarrow f \approx 1.42 \text{ GHz} \rightarrow \lambda \approx 21 \text{ cm}. \tag{13}$$

The interaction energy ΔE_{el} between the electric dipoles μ_{el}^e and μ_{el}^p of, respectively, the electron and the proton amounts to

$$\Delta E_{el} = \varepsilon_0 \frac{g_p \mu_{el}^e \mu_{el}^p}{3\pi a_0^3}. \tag{14}$$

Because of the relationship between the magnetic dipoles and electric dipoles as expressed by (9) and (10) and because $c^2 = (\varepsilon_0 \mu_0)^{-1}$, the two interaction energies ΔE_{el} and ΔE are just the same. In view of this, a proof for the existence of an anomalous electric dipole moment of electrons is far from easy. It might even be a reason to deny its relevancy, like Dirac did. Let me propose how a possible solution for a decisive proof for the existence or no-existence of the AEDM could look like. Unfortunately, the experiment itself is beyond my capabilities as an individual researcher. But it may challenge interested experimenters. It might be done with a hydrogen maser modified for the purpose, or something similar.

An experimental set-up proposal for proving Dirac's AEDM

Figure 1 shows the well known configuration of the hydrogen maser [21]. The hydrogen pump is an assembly that dissociates hydrogen molecules into atoms and pumps those into an assembly that focuses the beam of atoms after selection of atoms in a particular quantum state. These atoms enter a bulb in which their quantum state falls back to another quantum state under emission of electromagnetic quanta (photons) with a wavelength corresponding with a frequency of 1.4 GHz. These quanta interact with the standing electromagnetic waves in a microwave cavity taken up in a phase-locked loop configuration that synchronizes a quartz oscillator. The key element to be discussed within the scope of this article in this configuration is the selection section.

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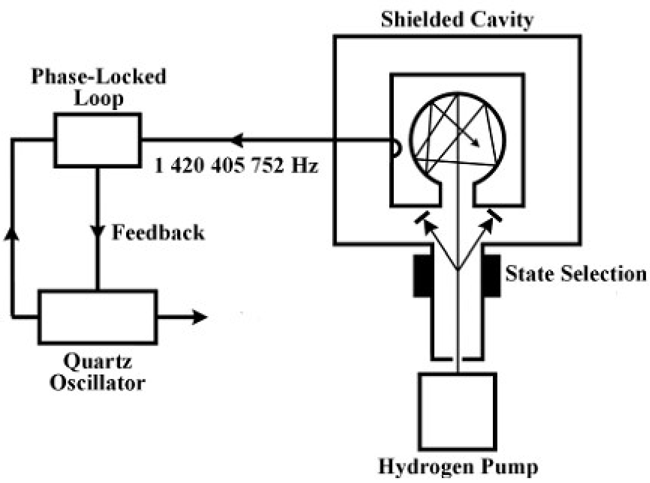

Fig. 1: Hydrogen maser

The purpose of the selection is to provide a spatial discrimination between the hydrogen atoms in the triplet state of their nuclear spins and the singlet state. In the triplet state, the spin of the electron is in parallel with the spin of the proton nucleus, in the singlet state those spins are in antiparallel. Figure 2 shows how the three energy levels of the triplet state (|1,1>,|1,-1> or|1,0>) and the singlet state energy (|0,0>) depend in a magnetic field. Two of the four levels are "low field seeking" and two of the four levels are "high field seeking".

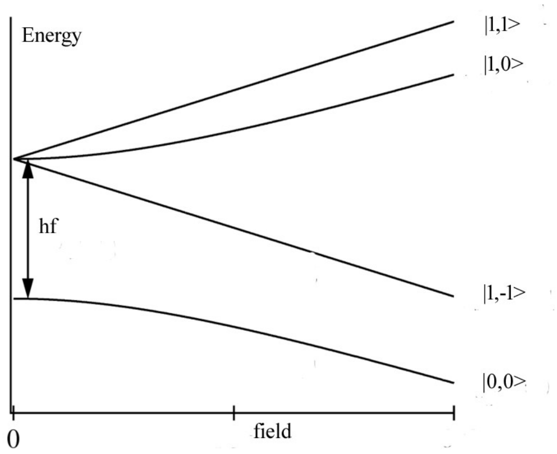

Fig.2 : Hyperfine split due to spin of the energy levels of an electron in atomic hydrogen

A cylindrical hexapole magnet configuration produces a radial symmetric field that behaves as, [22]

$$B(r) = B_0 (\frac{r}{R})^2, \tag{15}$$

where B_0 is a bias and where R is the radius of the cylinder. As a result, the atoms in two of the three triplet states are focused along the cylinder axis and enter into the bulb. Atoms in the other states are spatially dispersed. Typical values for B_0 are around 1 Tesla [22]. If the electron has an electric dipole moment indeed, the maser would operate under an electrostatic equivalent of the hexapole magnet as well. Electrostatic hexapole electrodes can be constructed indeed. They produce a radial symmetric electric field with the format [23],

$$E(r) = 3V_0(\frac{r^2}{R^3}). \tag{16}$$

An estimate for the required voltage on the hexapole electrodes can be obtained from requiring an energy relationship

$$\mu_m B_0 = \mu_{el}\frac{3V_0}{R} \rightarrow V_0 = \frac{RcB_0}{3}. \tag{17}$$

Assuming $B_0 \approx 1$ Tesla and $R \approx 1$ mm as practical values for actual instrumentation [22], the applied voltage V_0 on the electrostatic hexapole should amount to $V_0 \approx 30$ kV. This is larger than the operational voltages of about 15 kV that are used in electrostatic hexapoles for measuring the Stark effect in molecular physics [24]. It might well be that voltage breakdown prevents operation at the level required for substituting the magnetic hexapole by an electrostatic one in attempts to proof the existence of Dirac's second dipole moment of electrons by means of a modified hydrogen maser. Anyhow, a proof for the existence of an anomalous electric dipole moment of electrons is far from easy. It might even be a reason to deny its relevancy.

Discussion

At first glance, it might seem that the impact of the conclusion that Dirac's second dipole moment cannot be immediately rejected, is of limited value. The view on its possible impact might change if we put a Dirac particle in a more general context. As shown in the Appendix, Dirac's result (5) is general for any pointlike particle with mass m_0 moving in a conservative field $\boldsymbol{A}(A_0,\mathbf{A})$, such that

$$\Delta E = g\frac{\hbar}{2m_0}\{\overline{\sigma}_B \cdot (\nabla\times\mathbf{A})\} + \mathrm{i}g\frac{\hbar}{2m_0 c}\{\overline{\sigma}_E \cdot (\nabla A_0 - \frac{\partial\mathbf{A}}{\partial(ct)})\}. \tag{18}$$

The coupling factor g to the field and the energetic characteristics of the field might be specific for the particle under consideration, being an electron or else. Let us suppose that a quark can be conceived as a Dirac particle as well. It would mean that a quark possesses next to its nuclear spin associated with its quantum mechanical angular moment ***ħ*,** an additional spin associated with its dipole moment ***ħ/c*.** This suggests that this additional spin is an explanation for the axiomatic attribute isospin in quantum particle physics. But if so, there is no reason for regarding the d-quark as an elementary particle different from the u-quark. It is illogical to accept nuclear spin as a normal attribute without a need for further differentiation, while not doing so for isospin.

There is more. If a quark is an energetic Dirac particle, it spreads an energetic field. In classical physics, the origin of this field would be assigned to the mass m_0 of the quark, in quantum physics the origin of the mass m_0 would be assigned to the field. This might be just a dual description of the same thing. In quantum field theory (QFT), the energetic origin of all physical particles is assigned to an omni-present field of energy, known as the Higgs field. It might also be that both views can be unified in one way or another. Let us suppose that the quark mass m_0 spreads a classical field of energy. Such a field shows a r^{-1} dependency of its energetic potential. The dipole moment ***ħ/c***, with its dimension of mass times spacing, is due to a tiny virtual mass Δm, different from m_0. Such a dipole spreads an en energetic potential with x^{-2} dependency along the orientation axis of the dipole. As a consequence an equilibrium of forces can arise between a repelling force from the r^{-1} field dependency and the attractive force with x^{-2} field dependency from suitable aligned dipoles from two quarks. The Higgs field shields these fields with an exponential decay, such as can be shown from a numerical solution of the Lagrangian density of the omni-present Higgs field. Because the two quarks each have non-integer spin, the described structure has integer spin. It means that the described structure is composed by a quark and a antiquark, known as meson. It will be clear that the viability of this view heavily depends upon the awareness of the quark's dipole moment ***ħ/c***, hence on the viability of Dirac's second dipole moment. In [25], this view has been studied in detail. In this view, the quarks are mutually coupled with a coupling factor g, related with the electromagnetic fine structure relationship $g^2 = 1/\sqrt{137}$. It has led to the surprising result that the gravitational constant can be expressed and verified in terms of quantum mechanical parameters. In spite of this connection between gravity and quantum mechanics, this result, if noticed, is just considered as a curiosity. More about this view of mesons has been documented in a preprint [26], the publication of which is prevented so far because a theory that interprets the axioms of isospin and the color binding force in quantum chrome dynamics (QCD) between quarks by an alternative mechanism, is considered as a violation of the canon instead as an explanation of an underlying physical layer next to QED. In this study it is shown a.o. that the recognition of Dirac's second dipole moment explains that all quarks have a common origin. Their attributes can be traced back to those of a single archetype, which is the only true elementary quark.

Apart from the possible role of Dirac's second dipole moment in particle physics, there is a possible role in cosmology as well, next to its impact on the gravitational constant as just mentioned. Where in particle physics, the Higgs field is omni-present, canonical cosmology theory accepts an omni-present field of energy as well. It is expressed by Einstein's gauge parameter Λ, which at the level of the "visible" universe is known as the Cosmological Constant. It would be illogical if those omni-present fields of energy would not have a common root. In a recent article [27], this cosmological field of background energy has been described in terms of a low density gas of vacuum particles that show a Heisenberg vibration of uncertainty. If this article, particles are conceived as Dirac particles possessing a ***ħ/c*** dipole moment. In galaxies, these dipole moments are directed under influence of the gravitational field spread by the center of the galaxy. The result is an anti-screening effect on the gravitational force, just opposite to the Debije-screening effect of the electrical field of a charged particle in an ionized plasma. Rather than assigning the excess of the gravitational force to undetectable dark matter, the increase of the gravitational force can now be explained as the result of vacuum polarization due to the aligned $\hbar/c$ dipole moments of the vacuum particles. It has been shown that the result of the calculation of this effect matches with observational evidence expressed by Milgrom's empirical acceleration constant. Identification of the cosmological Λ field with the Higgs field remains a challenge for further research.

Although these studies have led the author to the rediscovery of Dirac's second dipole moment, it might be not enough for a finite proof for its existence, nor for its role in particle physics and cosmology. Hence a a decisive experimental proof would be most welcome, either of a type described in the previous paragraph, either by a clever alternative. Note that for the actual scope of this article, this discussion paragraph is not really relevant. Nevertheless, in the author's view it is a stepping stone to understand the properties of quarks in the nuclear domain and to understand the constituents ("darks") of the cosmological background energy [27]. The real issue of this article, however, is showing that ignorance of Dirac's second dipole moment is not scientifically justified.

Conclusion

A Dirac particle has two anomalous dipole moments. One of these is the consequence of an elementary angular moment assigned to the pointlike particle. For electrons It becomes manifest as a magnetic dipole moment. The second one comes forward as the result of Dirac's modeling, but it remained forgotten because of a number of reasons. The main one is Dirac's perception that it has no physical relevance as an electric dipole moment, because of its seeming imaginary value. A second one is its disappearance in Dirac-type analyses in standard textbooks. A third reason is the failure of proof by measurements. A fourth reason is the perceived violation of time reversal symmetry and CP symmetry. In this article, first of all a proof is given for the inconsistency between Dirac's result on the dipole moments and the Bjorken and Drell textbook result. The reason why has been shown. It is due to the difficulty to include the temporal momentum into a 3D curl operation, which in most textbooks is invoked to implement Dirac's trick. In this article it has been shown that adopting the Hawking metric is an effective instrument that allows to maintain the full symmetry over the four dimensions as obtained by Dirac, while it is lost in many textbooks. Subsequently, the remaining issues of the failure of experimental evidence and the violations of T-symmetry and PC-symmetry have been analyzed thereby revealing the fundamental difference between Dirac's anomalous non-rotating electric dipole that gives rise to AEDM, and the rotating electric dipole that gives rise to eEDM as defined by the references quoted by the Particle Data Group [28]. Where the eEDM can be viewed as the strength of a pseudo vector orthogonal to the angular momentum vector caused by an off-set between the center of mass and the center of charge, the AEDM can be viewed as the strength of a position vector due to the Heisenberg vibration of the center of charge. Quantitatively the AEDM is much larger than the eEDM. It cannot be stressed enough that Dirac's anomalous electric dipole moment is a pure quantum mechanical effect of a pointlike particle that not should be confused with the electrical dipole moment defined as eEDM. It has been argued in this article that the experimental proof of Dirac's AEDM is not trivial and that it requires different instrumentation from the ones that aim to measure the eEDM. It has also been argued that an AEDM does not exclude an eEDM. However, probably due to its unrecognized origin, no attempts have been made as yet to measure the AEDM by dedicated instrumentation. In this article a proposal has been described for devising one. The final conclusion has to be that there is no decisive reason why Dirac's AEDM has to be rejected from a theoretical point of view for Dirac particles in general sense and that still a challenge exists to proof or disproof its existence by experiment.

Appendix: Derivation of Dirac's anomalous electric dipole moment (AEDM)

The aim in this appendix is to give a refreshment of Dirac's analysis that resulted into the conclusion that a Dirac particle, in this appendix not necessarily an electron, possesses two anomalous dipole moments, both purely quantum mechanical in nature. In this analysis, the original matrices will be invoked that Dirac defined in his classic paper. Afterwards, these matrices will be transformed into the now canonical gamma matrices. To symmetrise the analysis, the Hawking metric $(+,+,+,+)$ for ($\mathrm{i}ct, x, y, z$), $\mathrm{i} = \sqrt{-1}$ will be adopted and justified later by showing that the final result is the same as in the conventional metric $(-,+,+,+)$ for (ct, x, y, z).

It all starts from the Einsteinean energy expression of a generic free moving particle with rest mass m_0. This reads as,

$$E_W = \sqrt{(m_0 c^2)^2 + (c|\mathbf{p}|)^2}\,, \tag{A1}$$

where $\mathbf{p}$ is the three-vector momentum ($\mathrm{d}s/\mathrm{d}t$, not be confused with the fourvector momentum $\boldsymbol{p}$). Under adoption of the Hawking metric

$$E_W^2 = -p_0^2 c^2 = (m_0 c^2)^2 + c^2 p_1^2 + c^2 p_2^2 + c^2 p_3^2\,, \tag{A2}$$

which can be normalized as,

$$p_0'^2 + p_1'^2 + p_2'^2 + p_3'^2 + 1 = 0\,; \quad p'_\mu = \frac{p_\mu}{m_0 c}\,. \tag{A3}$$

As long as the temporal dimension is included, the bold italic notation for the vector $\boldsymbol{p}$ will be maintained.

Note: In the Hawking metric, time shows up as an imaginary quantity [29]. The merit of it is the full symmetry over the four dimensions as shown by (A3). In most textbooks a preference is given to real time, hence a metric $(-,+,+,+)$ for (ct, x, y, z). Perkins [30] prefers the Hawking metric. As will be shown, it simplifies Dirac's analysis substantially.

Under particular number typing of a coefficient vector $\overline{\alpha}(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$, eq. (A3) can be rewritten as a full square,

$$(\overline{\alpha} \cdot \boldsymbol{p}' + \beta) \cdot (\overline{\alpha} \cdot \boldsymbol{p}' + \beta) = 0\,. \tag{A4}$$

Another possibility is factorizing as,

$$(\overline{\alpha}\cdot\boldsymbol{p}'+\beta)\cdot(\overline{\alpha}\cdot\boldsymbol{p}'-\beta)=(\overline{\alpha}\cdot\boldsymbol{p}')\cdot(\overline{\alpha}\cdot\boldsymbol{p}')-\beta^2=0\,.$$

This reflects the energy relationship (A3) under the condition

$$\alpha_\mu\alpha_\nu+\alpha_\nu\alpha_\mu=0 \text{ if } \mu\neq\nu;\text{ and } \alpha_\mu^2=1;\ \beta^2=-1\,. \tag{A5}$$

This condition can be met if a coefficient vector $\overline{\alpha}(\alpha_0,\alpha_1,\alpha_2,\alpha_3)$ is constructed from

$$\alpha_1=-\mathrm{i}\gamma_1;\alpha_2=-\mathrm{i}\gamma_2;\alpha_3=-\mathrm{i}\gamma_3;\ \alpha_0=\gamma_0;\ \beta=\mathrm{i}I\,,$$

with the gamma matrices γ_μ shown before in eq. (4) of the main text, supplemented by the identity matrix I,

$$I=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}. \tag{A6}$$

Hence,

$$\begin{aligned}&(\overline{\alpha}\cdot\boldsymbol{p}')\cdot(\overline{\alpha}\cdot\boldsymbol{p}')=(\alpha_0p_0'+\alpha_1p_1'+\alpha_2p_2'+\alpha_3p')\cdot(\alpha_0p_0'+\alpha_1p_1'+\alpha_2p_2'+\alpha_3p')=\\&(\alpha_0p_0')^2+(\alpha_0p_0')(\alpha_1p_1'+\alpha_2p_2'+\alpha_3p_3')+\\&(\alpha_1p_1')^2+(\alpha_1p_1')(\alpha_0p_0'+\alpha_2p_2'+\alpha_3p_3')+\\&(\alpha_2p_2')^2+(\alpha_2p_2')(\alpha_0p_0'+\alpha_1p_1'+\alpha_3p_3')+\\&(\alpha_3p_3')^2+(\alpha_3p_3')(\alpha_0p_0'+\alpha_1p_1'+\alpha_2p_2')=U^2+\varepsilon,\end{aligned} \tag{A7}$$

where

$U^2=(\alpha_0p_0')^2+(\alpha_1p_1')^2+(\alpha_2p_2')^2+(\alpha_3p_3')^2$, and

$$\begin{aligned}&\varepsilon=\\&\{(a_0p_0')(a_1p_1')+(a_1p_1')(a_0p_0')\}+\{(a_0p_0')(a_2p_2')+(a_2p_2')(a_0p_0')\}+\{(a_0p_0')(a_3p_3')+(a_3p_3')(a_0p_0')\}+\\&\{(a_1p_1')(a_2p_2')+(a_2p_2')(a_1p_1')\}+\{(a_1p_1')(a_3p_3')+(a_3p_3')(a_1p_1')\}+\\&\{(a_2p_2')(a_3p_3')+(a_3p_3')(a_2p_2')\}.\end{aligned}$$

Obviously, $\varepsilon=0$, because of (A5). This remains so for a particle moving under influence of a conservative field of forces with a (generic) field potential $\boldsymbol{A}'(A_0',A_1',A_2',A_3')$. As before, $\boldsymbol{A}'$ is signed for indicating the normalization by m_0c. The field influence can be accounted for by,

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$$p'_\mu \rightarrow p'_\mu + A'_\mu . \qquad \text{(A8)}$$

The triviality $\varepsilon = 0$ disappears if the momenta are transformed into wave operators, like Dirac did by adopting the basic transform of quantum electrodynamics (QED),

$$p'_\mu \rightarrow \hat{p}_\mu \psi \quad \text{with} \quad \hat{p}'_\mu = \frac{1}{m_0 c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x_\mu} . \qquad \text{(A9)}$$

As a consequence of the QED transform (A9) and the minimum substitution rule (A8), together known as the gauge covariant transform, the first term in ε of (A7) transforms as,

$$a_0 a_1 (\hat{p}'_0 + A'_0)(\hat{p}'_1 + A'_1) + a_1 a_0 (\hat{p}'_1 + A_1)(\hat{p}'_0 + A'_0) = a_0 a_1 \hat{p}'_0 A'_1 + a_1 a_0 \hat{p}'_1 A'_0 . \qquad \text{(A10)}$$

Note that quite some terms have disappeared because of $a_0 a_1 = -a_1 a_0$, see (A5) and, more importantly now, because of the sequence sensitivity of the operator action.

Applying this on all terms of (A7), the result is,,

$$\begin{aligned}
&\varepsilon = \\
&\hat{p}'_0 (a_0 a_1 A'_1 + a_0 a_2 A'_2 + a_0 a_3 A'_3) + \{a_1 a_0 \hat{p}'_1 A'_0 + a_2 a_0 \hat{p}'_2 A'_0 + a_3 a_0 \hat{p}'_3 A'_0\} + \\
&\{\hat{p}'_1 (a_1 a_2 A'_2) + \hat{p}'_2 (a_2 a_1 A'_1\} + \{\hat{p}'_1 (a_1 a_3 A'_3) + \hat{p}'_3 (a_3 a_1 A'_1)\} + \{\hat{p}'_2 (a_2 a_3 A'_3) + \hat{p}'_3 (a_3 a_2 A'_2)\}.
\end{aligned}$$

which can be rewritten as,

$$\begin{aligned}
&\varepsilon = \\
&(a_0 a_1 \hat{p}'_0 A'_1 + a_1 a_0 \hat{p}'_1 A'_0) + (a_0 a_2 \hat{p}'_0 A'_2 + a_2 a_0 \hat{p}'_2 A'_0) + (a_0 a_3 \hat{p}'_0 A'_3 + a_3 a_0 \hat{p}'_3 A'_0) + \\
&(a_1 a_2 \hat{p}'_1 A'_2 + a_2 a_1 \hat{p}'_2 A'_1) + (a_1 a_3 \hat{p}'_1 A'_3 + a_3 a_1 \hat{p}'_3 A'_1) + \\
&(a_2 a_3 \hat{p}'_2 A'_3 + a_3 a_2 \hat{p}'_3 A'_2).
\end{aligned} \qquad \text{(A11)}$$

Regrouping under consideration of (A5) gives,

$$\begin{aligned}
&\varepsilon = \\
&a_0 a_1 (\hat{p}'_0 A'_1 - \hat{p}'_1 A'_0) + a_0 a_2 (\hat{p}'_0 A'_2 - \hat{p}'_2 A'_0) + a_0 a_3 (\hat{p}'_0 A'_3 - \hat{p}'_3 A'_0) + \\
&a_1 a_2 (\hat{p}'_1 A'_2 - \hat{p}'_2 A'_1) + a_1 a_3 (\hat{p}'_1 A'_3 - \hat{p}'_3 A'_1) + \\
&a_2 a_3 (\hat{p}'_2 A'_3 - \hat{p}'_3 A'_2).
\end{aligned} \qquad \text{(A12)}$$

Hence, from (A12) and (A9),

$$\varepsilon =$$
$$\frac{\hbar}{\mathrm{i}m_0c}\{\sigma_{E1}(\frac{\partial A_0'}{\partial x}-\frac{\partial A_1'}{\partial(\mathrm{i}ct)})+\sigma_{E2}(\frac{\partial A_0'}{\partial y}-\frac{\partial A_2'}{\partial(\mathrm{i}ct)})+\sigma_{E3}(\frac{\partial A_0'}{\partial z}-\frac{\partial A_3'}{\partial(\mathrm{i}ct)})\}+$$
$$\frac{\hbar}{\mathrm{i}m_0c}\{\mathrm{i}\sigma_{B1}(\frac{\partial}{\partial x}A_y'-\frac{\partial}{\partial y}A_x')-\mathrm{i}\sigma_{B2}(\frac{\partial}{\partial x}A_z'-\frac{\partial}{\partial z}A_x')+\mathrm{i}\sigma_{B3}(\frac{\partial}{\partial y}A_z'-\frac{\partial}{\partial z}A_y')\};$$
$$\sigma_{E1}=a_0a_1;\sigma_{E2}=a_0a_2;\sigma_{E3}=a_0a_3;$$
$$\sigma_{B1}=a_1a_2;\sigma_{B2}=a_1a_3;\sigma_{B3}=a_2a_3. \qquad \text{(A13)}$$

Elementary matrix multiplications reveal the following identities,

$$\sigma_{E1}=a_0a_1=-\mathrm{i}\gamma_0\gamma_1=-\mathrm{i}(\rho_1\sigma_1);\sigma_{E2}=a_0a_2=-\mathrm{i}\gamma_0\gamma_2=-\mathrm{i}(\rho_1\sigma_2);\sigma_{E3}=a_0a_3=-\mathrm{i}\gamma_0\gamma_3=-\mathrm{i}(\rho_1\sigma_3);$$
$$\sigma_{B1}=a_1a_2=-\gamma_1\gamma_2=\mathrm{i}\sigma_3;\sigma_{B2}=a_1a_3=-\gamma_1\gamma_3=-\mathrm{i}\sigma_2;\sigma_{B3}=a_2a_3=-\gamma_2\gamma_2=\mathrm{i}\sigma_1.$$
(A14)

It can be written in terms of the grad operator, the curl operator and Dirac's Pauli vector as

$$\varepsilon=\frac{\hbar}{\mathrm{i}m_0c}(\overline{\sigma}_E\cdot\nabla A_0'-\overline{\sigma}_E\cdot\frac{\partial\mathbf{A}'}{\partial(\mathrm{i}ct)})+\frac{\hbar}{\mathrm{i}m_0c}\overline{\sigma}_B\cdot\mathrm{i}(\nabla\times\mathbf{A}'). \qquad \text{(A15)}$$

Note that ε still is a dimensionless quantity. It is an excess term to be included in the energy expression (6) as a consequence of the particular characteristics of Dirac's equation of motion. Hence,

$$\frac{E_W^2}{(m_0c^2)^2}=1+\frac{v^2}{c^2}+\varepsilon, \qquad \text{(A16)}$$

where v is the velocity of the particle in motion. As long as $v/c<<1$ and $\varepsilon<<1$, E_W can be approximated as,

$$E_W=m_0c^2(1+\frac{v^2}{2c^2}+\frac{\varepsilon}{2})=m_0c^2(1+\frac{v^2}{2c^2})+\Delta E;\;\;\Delta E=\varepsilon\frac{m_0c^2}{2}, \qquad \text{(A17)}$$

where ε is given by (A15).

We are almost done, but not quite. So far, the Dirac particle has been considered in general terms, i.e., without identifying it as an electron. To do so, a first step to do so is defining the four-vector potential as,

$$\boldsymbol{A}'=\boldsymbol{A}'(\mathrm{i}\frac{\Phi/c}{m_0c},A_x',A_y',A_z'). \qquad \text{(A18)}$$

Note: The i factor in the scalar component is due to the (Hawking) metric choice (+,+,+,+) / (ict,x,y,z). It can be easily seen from the Lorenz gauge

$$\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}=0\rightarrow\nabla\cdot\mathbf{A}+\mathrm{i}\frac{\partial\Phi/c}{\partial\mathrm{i}ct}=0. \quad \text{(A19)}$$

Note also that in (A18) the dimension of Φ is energy. It is not the same as the electric potential Φ_{e} .The relationship between the two can be found from the force equity,

$$F=e\frac{\partial}{\partial y}\Phi_{\mathrm{e}}=\frac{\partial}{\partial y}\Phi\rightarrow\Phi_{\mathrm{e}}=\frac{\Phi}{e}. \quad \text{(A20)}$$

From the Lorenz gauge and (A20) obviously,

$$\mathbf{A}_{\mathbf{e}}=\frac{\mathbf{A}}{e}. \quad \text{(A21)}$$

Hence, from (A17), (A21) and (A15),

$$\begin{aligned}\Delta E&=\varepsilon\frac{m_0c^2}{2}=\frac{\hbar c}{2\mathrm{i}}(\overline{\sigma}_E\cdot\nabla A_0'-\overline{\sigma}_E\cdot\frac{\partial\mathbf{A}'}{\partial(\mathrm{i}ct)})+\frac{\hbar c}{2\mathrm{i}}\overline{\sigma}_B\cdot\mathrm{i}(\nabla\times\mathbf{A}')\\&=\frac{\hbar c}{2\mathrm{i}}\overline{\sigma}_E\cdot(\nabla(e\frac{\mathrm{i}\Phi_{\mathrm{e}}/c}{m_0c}-\frac{\partial\mathbf{A}'}{\partial(\mathrm{i}ct)})+\frac{\hbar c}{2\mathrm{i}}\overline{\sigma}_B\cdot\mathrm{i}(\nabla\times\mathbf{A}')=\frac{e\hbar}{2m_0c}(-\overline{\sigma}_E\cdot\mathbf{E})+\frac{e\hbar}{2m_0}(\overline{\sigma}_B\cdot\mathbf{B}).\end{aligned} \quad \text{(A22)}$$

Note: in the eq. (5) of the main text $\overline{\sigma}_E(-\mathrm{i}\gamma_0\gamma_1,-\gamma_0\gamma_2,-\gamma_0\gamma_3)$ has been replaced by $\mathrm{i}\overline{\sigma}_E'(\gamma_0\gamma_1,\gamma_0\gamma_2,\gamma_0\gamma_3)$.

Because of the relationships (A14), (A22) can be written as well as,

$$\Delta E=\varepsilon\frac{m_0c^2}{2}=\frac{e\hbar}{2m_0c}\mathrm{i}(\rho_1\overline{\sigma}\cdot\mathbf{E})+\frac{e\hbar}{2m_0}(\overline{\sigma}\cdot\mathbf{B}). \quad \text{(A23)}$$

This is identical with Dirac's result. There is one issue to be resolved still. That is the difference between the format of the wave function used in this appendix with the format used by Dirac. The format shown in (A4) is,

$$(\alpha_0\hat{p}_0+\alpha_1\hat{p}_1+\alpha_2\hat{p}_2+\alpha_3\hat{p}_3+\beta)\psi=0, \quad \text{(A24)}$$

and, under consideration of the matrix relationships,

$$[\gamma_0\hat{p}_0'-\mathrm{i}(\overline{\gamma}\cdot\hat{\mathbf{p}}')+\mathrm{i}I]\psi=0;\overline{\gamma}=\overline{\gamma}(\gamma_1,\gamma_2,\gamma_3) \quad \text{(A25a)}$$

where $\hat{p}_0'=\frac{1}{m_0c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial(\mathrm{i}ct)};\hat{p}_i'=\frac{1}{m_0c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x_i}$. (A25b)

Hence, from (A25a,b),

$$\frac{1}{m_0 c}(\mathrm{i}\gamma_0\hbar\frac{\partial}{\partial(ct)}+\hbar\mathrm{i}\overline{\gamma}\frac{\partial}{\partial x_i}+I)\psi=0\,. \tag{A26}$$

Dirac has used the format (see p. 615 of his article),

$$[\hat{p}'_0+\rho_1(\overline{\sigma}\cdot\hat{\mathbf{p}}')+\rho_3]\psi=0\,, \tag{A27}$$

where $\hat{p}'_0=\frac{\hbar}{m_0 c}\mathrm{i}\frac{\partial}{\partial(ct)}$.

Because $\rho_3\equiv\gamma_0$, it is found after multiplication of (A27) with ρ_0 that under recognition of the relationships shown in (A14) and the different definition of $\hat{p}'_0$, that (A27) is equivalent with (A26) as well.

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