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Does the electron have an anomalous electric dipole moment?

Engel Roza
Philips Research Labs, Eindhoven, The Netherlands (retired)
Email: engel.roza@onsbrabantnet.nl

Summary
An analysis is presented of the possible existence of a second anomalous dipole moment of Dirac's particle next to the angular one. It includes a discussion why, in spite of his own derivation, Dirac has denied its relevancy. It is shown why since then it has been overlooked and why it has vanished from leading textbooks. A critical survey is given on the reasons of its reject, including the failure of attempts to measure and the perceived violations of time reversal symmetry and charge-parity symmetry. Moreover, by reference from literature, the possible impact is discussed in the nuclear domain and in the gravitational domain.

Keywords: anomalous electric dipole moment; Dirac particle; Pauli's spin vector; isospin

Introduction

In his classic paper on electrons, Paul Dirac has derived a basic 4-dimensional wave equation for an electron in motion subject to a vector potential $\boldsymbol{A}(A_0, A_x, A_y, A_z)$. In this equation [1, eq. 15/16], an anomalous electric dipole moment shows up, next to the well-known anomalous magnetic dipole moment. Dirac doubted whether it could have a physical interpretation, the more because it appeared in a quantity with an imaginary sign as compared with a similar expression for the magnetic dipole moment. Where a magnetic dipole moment makes sense as a manifestation of angular spin, a similar physical manifestation for an electric dipole moment is not obvious.

This is a first reason why, since then, Dirac's electric dipole moment of an electron has been ignored. The second reason is, that experimental attempts to reveal an electric dipole moment of an electron (eEDM), if it would exist, all failed. Presently, the Particle Data Group (PDG) has set an upper limit for its value as [2],

eEDM $< 0.87\ 10^{-30}\ q$ m,

where q is the elementary charge.

The third reason why an electron dipole moment for an electron has been put into doubt is due to the perceived violations of time reversal symmetry (T) and charge-parity symmetry (CP), [3].

There is somewhat more. It is quite curious that in the highly reputed textbook of Bjorken and Drell, the electric dipole moment is no longer mentioned. Bjorken and Drell have

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decomposed Dirac's four-component wave function $\psi(\psi_1,\psi_2,\psi_3,\psi_4)$ into two two-components wave equations for the non-relativistic domain, a dominant one $\psi(\psi_1,\psi_2)$ and a minor one $\chi(\chi_1,\chi_2)$. See [4, eq. 1.32 and 1.33]. Dirac's electric dipole moment no longer shows up, while the magnetic dipole moment is clearly present. One might guess that its disappearance is due to the non-relativistic restriction. In Griffiths textbook [5], the electrons's electric dipole moment is not mentioned.

Let us inspect all those arguments step by step. First of all, Dirac's doubt is not a proof. The imaginary signed term pointing to a possible existence of an electric dipole moment is just a term in a wave equation with complex quantities. It might well be that in a proper energetic interpretation of the wave equation, the term will contribute a real quantity. Furthermore, the difficulty of physical interpretation might be due to an unexpected property of an electron. Let us suppose that the electron, similarly like all physical particles, is subject to the Heisenberg uncertainty. Let us suppose, just by hypothesis, that its position d in its center of mass frame can be explained as the result of a motion with ultra-relativistic speed near vacuum light velocity c, such that

$$d = c\Delta t\,. \tag{1}$$

Applying Heisenberg's relationship $\Delta E \Delta t = \hbar/2$, [6], on (1), we get

$$d = c\Delta t = c\frac{\hbar}{2}\frac{1}{\Delta E} \rightarrow d = c\frac{\hbar}{2}\frac{1}{mc^2} = c\frac{\hbar}{2}\frac{1}{mc^2} \rightarrow \mu_{mass} = md = \frac{\hbar}{2c}, \tag{2}$$

where μ_{mass} has the dimensions of a (mass) dipole moment, expressed in terms of Planck's reduced constant $\hbar$ and a virtual mass m (not to be confused with the particle's rest mass m_0).

Hence, it might well be that Dirac's doubt might be fed by not considering the possibility of a Heisenberg type vibration as an elementary motion of a Dirac particle next to an elementary angular motion.

Let us proceed by discussing the failure of measurement. Now we have suggested, by argumentation, the possible existence of a mechanical vibration moment **$\hbar/c$**, next to the mechanical angular momentum **$\hbar$**, the question has to be addressed how to relate these mechanical motions with the hypothetical existence of an electric dipole moment $\boldsymbol{\mu}_{el}$ and the existence of a magnetic dipole moment $\boldsymbol{\mu}_{\mathbf{m}}$ of an electron with its elementary charge q and its mass m_0. The magnitude of the magnetic one is well known from textbooks as [5],

$$|\boldsymbol{\mu}_{\mathbf{m}}| = \frac{q}{2m_0}\,|\boldsymbol{\hbar}|,\ (\approx 9.27\ 10^{-24}\ \mathrm{C\ m^2\ s^{-1}}). \tag{3}$$

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The magnitude of the anomalous electric dipole moment **AEDM** as derived by Dirac [1, eq. 15/16] amounts to,

$$|\boldsymbol{\mu}_{el}| = \frac{q}{2m_0}\,|\hbar/c|,\ (\approx 3.09\ 10^{-32}\ \text{C m}). \qquad (4)$$

This is quite different from the PDG value quoted before. Obviously, the discrepancy must be due to a basic difference between the electric dipole moment **eEDM** as defined in the context of PDG and the anomalous electric dipole moment **AEDM** as meant by Dirac. The latter one, be it imaginary or not (to be discussed later) is a pure quantum mechanical phenomenon, while eEDM is not quite. Instead, a classical EDM is a consequence of a presupposed *spatial* structure of an electron with some charge distribution [7,8]. If the electron is pointlike indeed, there is no classical EDM. Dirac's anomalous one, on the other hand, shows up as a quantum mechanical vector with eigenvalues, even if the particle is pointlike. This difference remains, in spite of the present less classical definition in terms of a form factor that models the charge cloud around a pointlike source [8].

Let us now discuss the perceived parity violations. It will make the difference between the eEDM and the AEDM more clear. Let use the arguments quoted in [3]. Here, the interaction Hamiltonians, H_E and H_M for the electric dipole moment and the magnetic dipole moment are, respectively, expressed as,

$$H_E = -d_E\mathbf{S}.\cdot\mathbf{E} \quad \text{and} \quad H_M = -d_M\mathbf{S}.\cdot\mathbf{B}, \qquad (5)$$

where $\mathbf{S},\mathbf{E},\mathbf{B},d_E$ and d_M, respectively, are the spin angular momentum, the electric field strength and the magnetic field strength, and where d_E and d_M are the strengths of the dipoles. Let as usual $\mathbf{S} = \hbar\sqrt{s(s+1)}$, d_E and d_M proportional with the elementary electric charge q and let us consider, in Table I, the T,C and P symmetries of electromagnetism [9].

Table I

spin dependent dipole moments						
		Time reversal	Charge inversion	Parity reversal	CP	CPT
Magnetic mom	$-d_M\mathbf{S}$	sign change	sign change	no sign change	change	no change
$\mathbf{B}$		sign change	sign change	no sign change		
H_M	$-d_M\mathbf{S}.\cdot\mathbf{B}$	no sign change	no sign change	no sign change	no change	no change
EDM	$-d_E\mathbf{S}$	sign change	sign change	no sign change	change	no change
$\mathbf{E}$		no sign change	sign change	sign change		
H_E	$-d_E\mathbf{S}.\cdot\mathbf{E}$	sign change	no sign change	sign change	sign change	no change

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From this table, it is concluded that an eEDM c.f. (5) violates the time reversal symmetry and the CP symmetry of its interaction Hamiltonians, albeit that CPT symmetry remains conserved. On the one hand, it gives a reason to deny its possible existence, while on the other hand, it raises a particular interest, because CP symmetry violation is believed being a condition for the origin of the matter/antimatter asymmetry in the universe [3,10,11]. This explains why there is a considerable amount of experimental research that attempts to prove the existence of a non-zero eEDM.

However, a similar table, composed on the basis of Dirac's anomalous dipole moments, does not show such a different behaviour of the electric interaction Hamiltonian from the magnetic one. Dirac's anomalous AEDM doesn't violate time reversal symmetry nor CP symmetry.

Table II

Dirac's anomalous dipole moments						
		Time reversal	Charge inversion	Parity reversal	CP	CPT
Magnetic mom	$(q/2m_0)\|\boldsymbol{\hbar}\|$	sign change	sign change	no sign change	change	no change
$\mathbf{B}$		sign change	sign change	no sign change		
$\mathbf{B}\cdot\boldsymbol{\mu}_{magn}$		no sign change	no sign change	no sign change	no change	no change
Electric mom	$(q/2m_0)\|\boldsymbol{\hbar}/\boldsymbol{c}\|$	no sign change	sign change	sign change	no change	no change
$\mathbf{E}$		no sign change	sign change	sign change		
$\mathbf{E}\cdot\boldsymbol{\mu}_{el}$		no sign change	no sign change	no sign change	no change	no change

Understanding it properly, requires recognition of the differences between the various dipoles and dipole moments. The anomalous magnetic dipole moment is a pseudo vector orthogonal to the angular momentum. The magnetic dipole itself is non-rotating and aligned along the axis of its pseudo vector. The electric dipole moment eEDM is a pseudo vector collinear with the anomalous magnetic dipole moment. The electric dipole iself is rotating with the angular momentum. The anomalous electric dipole moment AEDM is a pseudo vector as well. However, unlike the eEDM dipole, the AEDM dipole is non-rotating. The orientation of its pseudo vector is not determined by orthogonality to the angular momentum **ħ**, but is determined by a non-angular *isospin* vector **ħ**/**c**. That marks a basic difference between eEDM and AEDM. Where the eEDM and the anomalous magnetic dipole moment compose the *same* vector, the AEDM and the anomalous magnetic dipole moment compose *different* vectors. Where the spin vector is subject to a change of sign under time reversal, the isospin vector is not. Where the vector properties of an eEDM depend on the *angular momentum vector* **ħ**, the vector properties of an AEDM depend on the *position vector* **ħ/c.** The former one represents an angular motion, while the latter one represents a (Heisenberg) vibration. It is a (position) vector that can be directed under influence of an electric field, independent from the angular momentum vector.

It is fair to suppose that (in 1928) Dirac was not aware that his wave equation of electrons implicitly embodied Heisenberg's uncertainty (1927), because if so, he wouldn't have so easily waived away his anomalous electric dipole moment. It is David Hestenes who, in his studies on the "zitterbewegung" of electrons, recognized it [12,13].

Where present experiments so far failed to probe the existence of an eEDM, an experimental proof that reveals an AEDM doesn't exist either, in spite of the fact that its magnitude is many orders of magnitude larger than the present established upper limit of eEDM. It could be that the AEDM is imaginary indeed. It could also be that the AEDM could not be experimentally proved, because no experiments have been devised so far on the basis of a proper understanding of its origin. It might well be that present eEDM experiments are unable to detect an AEDM. Later in this article, these issues will be re-addressed.

After having discussed the three arguments physical interpretation, magnitude and parity violation, we are left with the problem why the electric dipole moment does not show up in the Bjorken-Drell (2 x 2)- wave function, while it does in Dirac's 4-component one. Eventually, it will be an Hamiltonian expression that determines its viability. Hence, we shall straightforwardly analyze the problem by an inspection of the Hamiltonian. This will reveal the actual crux of the problem. The approach that we shall adopt is meant to get some understanding of three puzzles. All three have to do with my difficulty to understand Dirac's result properly. The puzzles that I would like to address are,

1. Why does Dirac's result not show a full symmetry over the four space-time dimensions? A three component Pauli vector $\overline{\sigma} = \overline{\sigma}(\sigma_1, \sigma_2, \sigma_3)$ is mixed up in in-products with, respectively, an electric field and a magnetic field. Hence, over three dimensions. A full symmetry would show a mix up of a four component Pauli-type vector with the four components of a vector potential. Hence, the question is whether, under the separation of the p_0 component from the p_i components in the 4D momentum vector $\boldsymbol{p} = \boldsymbol{p}(p_0, p_x, p_y, p_z)$, the full general relativistic character has been maintained.

2. It is not immediately obvious why the interaction terms of Pauli's vector with the vector potential would result in the rather simple energetic interpretation as quoted in (5). In (5), the additional energy due to a magnetic dipole moment in a magnetic field, is proportional to an in-product of a three dimensional spin vector **S** with a three-dimensional magnetic field vector. This allows a simple interpretation of the effect of the electron's angular spin,

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while the interaction term with the in-product of the Pauli vector with the magnetic field vector in Dirac's wave equation does not immediately allow an energetic interpretation.

3. The third issue that I would like to address is on the composition of the Pauli vector. It is well known that Majorana made a different choice. In this text, I wish to put Dirac's particle in a more general text but electrons. In particular, I wish to model the particle in a general approach by considering different choices for the composition of the Pauli vector.

As just noted, it is rather curious that Dirac's article shows the possible existence of two different dipole moments, while this doesn't show up in most, if not all, textbooks. It might well be that this is related with the general relativity issue. As already stated, from that point of view, one should expect an analysis that includes a four-component Pauli-type vector instead of a three-component one. In fact, Dirac did so, by defining a four-component Pauli-type vector $\overline{\alpha}=\overline{\alpha}(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, functionally related with the Pauli-matrices, but with a higher dimensionality than the Pauli-vector $\overline{\mathrm{o}}=\overline{\mathrm{o}}(\mathrm{o}_1,\mathrm{o}_2,\mathrm{o}_3)$. However, the full symmetry in his analysis disappeared due to the asymmetry in the metric (-,+,+,+) for space-time (ct,x,y,z). In spite of this, Dirac maintained all features. Dirac could have avoided the seeming asymmetry by using the "Hawking metric" (+,+,+,+) for space-time ($\mathrm{i}ct,x,y,z$), which would have considerably simplified his analysis. Nevertheless, the rigor of his analysis showed the arise of two dipole moments, while other texts don't, possibly by not recognizing the subtlety of the metric.

A general approach for covering these issues is rather complicated because of the 4D dimensionality. However, a simplified analysis in 2D might give a lead. Where a 2D analysis cannot provide full knowledge, the results of a proper 4D analysis should encompass the results of a proper 2D analysis. Hence, it might well be that useful insight can be provided from a 2D view. The basic elements in such an analysis are a two-dimensional momentum vector $\boldsymbol{p}=\boldsymbol{p}(p_0,p_x)$, and a two-dimensional Pauli-vector $\overline{\mathrm{o}}=\overline{\mathrm{o}}(\mathrm{o}_1,\mathrm{o}_2)$.

Dirac/Majorana particles in free space

Let us first consider a generic free moving particle.

Its Einsteinean energy is given as,

$$E_W=\sqrt{(m_0c^2)^2+(c|\mathbf{p}|)^2}\,, \tag{6}$$

where m_0 is the particle's rest mass and where $\mathbf{p}$ is the three-vector momentum (ds/dt, not be confused with the fourvector momentum $\boldsymbol{p}$). Without loss of generality, the particle's free motion can be aligned along the $x-$axis in a system of Cartesian coordinates, for which we shall adopt the Hawking metric ($\mathrm{i}ct,x,y,z$), $\mathrm{i}=\sqrt{-1}$. Squaring (6) gives,

$$E_W^2 = -p_0^2 c^2 = (m_0 c^2)^2 + c^2 p_1^2 , \tag{7}$$

which can be normalized as,

$$p_0'^2 + p_1'^2 + 1 = 0 ; \quad p'_\mu = \frac{p_\mu}{m_0 c} . \tag{8}$$

Note: As long as the temporal dimension is included, the bold italic notation for the vector $\boldsymbol{p}$ will be maintained.

In retrospect, in Dirac's single mode representation, this equation can be written as,

$$p_0'^2 + p_1'^2 + 1 = (\beta + \overline{\alpha} \cdot \boldsymbol{p}')(\beta + \overline{\alpha} \cdot \boldsymbol{p}') = 0 , \text{ with } \overline{\alpha} = \overline{\alpha}(\alpha_0, \alpha_1) \text{ and } \boldsymbol{p}'(p'_0, p'_1) , \tag{9a}$$

while in Majorana's dual mode representation, this equation can be written as,

$$p_0'^2 + p_1'^2 + 1 = (\beta + \mathrm{i}\overline{\alpha} \cdot \boldsymbol{p}')(\beta - \mathrm{i}\overline{\alpha} \cdot \boldsymbol{p}') = 0 , \tag{9b}$$

where the components of the two-dimensional vector $\overline{\alpha} = \overline{\alpha}(\alpha_0, \alpha_1)$ are given by the Pauli matrices,

$$\alpha_0 = \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \quad \alpha_1 = \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} ,$$

and where β is a matrix that is slightly different for Dirac's case and Majorana's case, respectively as

$$\beta = \sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \text{ and } \beta = [I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \tag{10}$$

where $[I]$ is the identity matrix.

After transforming the momenta into operators on wave functions,

$$p'_\mu \to \hat{p}_\mu \psi \quad \text{with} \quad \hat{p}'_\mu = \frac{1}{m_0 c} \frac{\hbar}{\mathrm{i}} \frac{\partial}{\partial x_\mu} , \tag{11}$$

(axiomatic quantum mechanical hypothesis), the momentum relationship (9) is transformed under consideration (11) into the following two two-dimensional wave equations

$$[\alpha_0] \begin{bmatrix} \hat{p}'_0 \Psi_0 \\ \hat{p}''_0 \Psi_1 \end{bmatrix} + [\alpha_1] \begin{bmatrix} \hat{p}'_1 \Psi_0 \\ \hat{p}'_1 \Psi_1 \end{bmatrix} \pm [\beta] \begin{bmatrix} \Psi_0 \\ \Psi_1 \end{bmatrix} = 0 . \tag{12}$$

The single mode condition is met for a single equation with a $+$ sign in front of β, while the dual mode falls apart into two simultaneous equations.. This set can be heuristically solved by $\psi(\psi_0, \psi_1)$ such that,

$$\psi_i(x,t) = u_i \exp i(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t), \tag{13}$$

resulting into,

$$\psi_0(x,t) = u_0 \exp \pm i(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\psi_1(x,t) = u_1 \exp \pm i(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\frac{u_1}{u_0} = \mp i \frac{p_1}{(W/c + m_0 c)}.$$

and

$$\psi_2(x,t) = u_2 \exp \pm i(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\psi_3(x,t) = u_3 \exp \pm i(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\frac{u_2}{u_3} = \pm i \frac{p_1}{(W/c + m_0 c)}.$$

The dual mode allows equating $u_1 = u_3$ and $u_0 = u_2$ and subsequent combining to the wave function set $\psi(\psi_a, \psi_b)$, where

$$\psi_a = u_a \cos(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\psi_b = u_b \sin(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$
$$\frac{u_a}{u_b} = \frac{p_1}{(W/c + m_0 c)}. \tag{14}$$

Hence, where the Dirac's single mode solution is a complex wave function, Majorana's dual mode allows a real wave function solution.

The impact of the vector potential on Dirac/Majorana particles in motion

Let us proceed by considering the impact of a field vector potential $\boldsymbol{A}$ on the free moving particle. To do it properly, an extension of the time-space dimension is required. To avoid the complexity of a full 4D analysis, we shall adopt a second spatial dimension next to the $x-$axis. Hence, we wish to consider now a slightly less simplified system with as basic elements: a "1 + 2"- dimensional momentum vector $\boldsymbol{p} = \boldsymbol{p}(p_0, p_x, p_y)$, a three-dimensional Pauli-vector $\overline{\sigma} = \overline{\sigma}(\sigma_1, \sigma_2, \sigma_3)$ and a three-dimensional vector potential $\boldsymbol{A}(A_0, A_x, A_y)$. In these terms, the free-space Einsteinean energy (9) is expressed as

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$$[I]+(\overline{\sigma}\cdot \boldsymbol{p}')(\overline{\sigma}\cdot \boldsymbol{p}')=0\,, \tag{15}$$

A basic property of the Pauli-vector says,

$$(\overline{\sigma}\cdot \mathbf{v})(\cdot\overline{\overline{\alpha}}\cdot \mathbf{w})=[I](\mathbf{v}\cdot \mathbf{w})+\mathrm{i}(\mathbf{v}\times \mathbf{w})\cdot\overline{\sigma}\,, \tag{16}$$

where $\mathbf{v}$ and $\mathbf{w}$ are two generic three-component vectors.

Applying (16) in (15) yields two possible results,

$$(\overline{\sigma}\cdot \mathbf{v})(\cdot\overline{\sigma}\cdot \mathbf{w})=\mathbf{v}\cdot\mathbf{w}\pm|\mathbf{v}\times\mathbf{w}|\,. \tag{17}$$

Hence, from (15) and (17)

$$[I]+(\overline{\sigma}\cdot \boldsymbol{p}')(\overline{\sigma}\cdot \boldsymbol{p}')=0\rightarrow 1+\boldsymbol{p}'\cdot\boldsymbol{p}'\pm|\boldsymbol{p}'\times\boldsymbol{p}'=0|\,. \tag{18}$$

This might seem a trivial result, because the vector product of a vector with itself is zero. Hence, this is just a retrieval of the Einsteinean energy expression (8). However, under influence of the presence of a conservative field of forces, characterized by a (normalized) vector potential A', the expression changes under the change of momenta components,

$$\boldsymbol{p}'\rightarrow \boldsymbol{p}'+A'\,. \tag{19}$$

such that (18) transforms to,

$$1+\boldsymbol{p}'\cdot\boldsymbol{p}'\pm|\boldsymbol{p}'\times\boldsymbol{p}'|=0\rightarrow 1+(\boldsymbol{p}'+A')\cdot(\boldsymbol{p}'+A')\pm|(\boldsymbol{p}'+A')\times(\boldsymbol{p}'+A')|=0 \tag{20}$$

The vector product in this expression still seems being irrelevant, because of its zero value. This, however, changes after the quantum mechanical transform from momenta to operations on a wave function, defined by

$$p'_\mu\rightarrow \hat{p}_\mu\psi \quad \text{with} \quad \hat{p}'_\mu=\frac{1}{m_0c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x_\mu}\,. \tag{21}$$

Applying these transforms on the generic identity

$$(\mathbf{v}+\mathbf{w})\times(\mathbf{v}+\mathbf{w})=(\mathbf{v}\times\mathbf{x})+(\mathbf{w}\times\mathbf{v})$$

we have

$$(\boldsymbol{p}'+\boldsymbol{A}')\times(\boldsymbol{p}'+\boldsymbol{A}')\rightarrow(\hat{\boldsymbol{p}}'\times\boldsymbol{A}')\psi+(\boldsymbol{A}'\times\hat{\boldsymbol{p}}')\psi\,. \tag{22}$$

Where the operator in the first term operates on ψ as well as on $\boldsymbol{A}'$, the operator in the second term only operates on ψ. As a consequence (22) is evaluated as,

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$$(\boldsymbol{p}'\times\boldsymbol{A}')+(\boldsymbol{A}'\times\boldsymbol{p}')=\frac{\hbar}{\mathrm{i}m_0c}\psi(\nabla\times A')\,. \tag{23}$$

Where the vector product of the momentum representation is zero, the equivalent wave function representation is not. Apparently, the Einsteinean energy of a particle moving with a momentum $\boldsymbol{p}=\boldsymbol{p}(p_0,p_x,p_y)$ under influence of a three-dimensional vector potential $\boldsymbol{A}(A_0,A_x,A_y)$ is given by,

$$(\boldsymbol{p}'+\boldsymbol{A}')(\boldsymbol{p}'+\boldsymbol{A}')+1\pm\left|\frac{\hbar}{im_0c}(\nabla\times\boldsymbol{A}')\right|=0\,. \tag{24}$$

The last term in the right hand part of this expression represents the energy added by the interaction of the particle's spin with the vector potential.

If we would have considered a 4D dimensionality in our analysis instead of the simplified "1 + 2" one, we would have met the difficulty to generalize the curl operation, which shows up in (24), to a 4D equivalent. In many texts on the subject, the problem is avoided by giving up symmetry and separating the temporal momentum from the spatial momenta, thereby excluding the temporal momentum from the curl operation. This, for instance, can be seen in Bjorken and Drell's textbook by moving from [4, eq. 1.32) to [4, eq.1.34]. Here, the temporal momentum is not included in the curl operation. As a consequence only a single dipole moment shows up, while in Dirac's comparable expression [1, eq. 15/16] two dipole moments are shown, albeit that Dirac could not maintain full symmetry in his representation because of the asymmetry in the metric (-,+,+,+). At this point, it is interesting to note that Lanczos [14] has been able to maintain full symmetry in the curl operation owing to his special quaternion algebra, which enabled him to give an interpretation to "isospin" and that Hestenes [12,13] developed his special STA algebra for the purpose, which enabled to explain the "zitterbewegung" of electrons. Dirac's second dipole moment, Lanczos' isospin and Hestenes' zitterbewegung are related and can be traced back to the inclusion of the temporal moment in the curl operation that shows up as a consequence of the interaction of the Pauli vector with the vector potential. The simplification from 4D space-time to "1 + 2" space-time may have its limitations, but, on the other hand, it enables maintaining full symmetry under the use of the "Hawking" metric (+,+,+,+) for space-time ($\mathrm{i}ct,x,y,z$).

Generically, the vector potential $\boldsymbol{A}$ consists of a scalar component Φ next to a vector component $\mathbf{A}$. If the particle would be spin less, a free motion along the $x-$axis would be possible under the equilibrium of a force from a transversal component of the scalar field Φ and the Lorentz force from a orthogonal component B_z created from $\nabla\times\mathbf{A}$, i.e. from components A_x and A_y. The interaction of the vector potential with the Pauli spin vector, however disturbs such an equilibrium. Let us consider the energetic influence of the interaction. Let the momentum vector and the vector potential, respectively be given by,

$$\boldsymbol{p}'=\boldsymbol{p}'(p_0',p_x) \text{ and } \boldsymbol{A}'=\boldsymbol{A}'(\mathrm{i}\frac{\Phi/c}{m_0c},A_x',A_y')\,. \tag{25}$$

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Note: The i factor in the scalar component is due to the (Hawking) metric choice (+,+,+,+) / (ict,x,y,z). It can be easily seen from the Lorenz gauge

$$\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}=0\rightarrow\nabla\cdot\mathbf{A}+\mathrm{i}\frac{\partial\Phi/c}{\partial ict}=0\,. \tag{26}$$

Note also that Φ/m_0c^2 is a dimensionless quantity. Under the conditions of stationarity of $\boldsymbol{A}$ and exclusive dependence of Φ on y, we have, under consideration of (25)

$$\nabla\times\boldsymbol{A}'=\begin{bmatrix}\mathbf{e}_\mathbf{t} & \mathbf{e}_\mathbf{x} & \mathbf{e}_\mathbf{y}\\ \partial/ict & \partial/\partial x & \partial/\partial y\\ \mathrm{i}\Phi/m_0c^2 & A'_x & A'_y\end{bmatrix}=-\frac{\partial}{\partial y}\mathrm{i}\frac{\Phi}{m_0c^2}\mathbf{e}_\mathbf{x}+(\frac{\partial}{\partial x}A'_y-\frac{\partial}{\partial y}A'_x)\mathbf{e}_\mathbf{t} \tag{27}$$

where $\mathbf{e}_\mathbf{x}$,$\mathbf{e}_\mathbf{y}$ and $\mathbf{e}_\mathbf{t}$, respectively, are unit vectors along the two spatial axes and the temporal axis.

Because of $\mathbf{B}=\nabla\times\mathbf{A}$, we have

$$\frac{\partial}{\partial x}A'_y-\frac{\partial}{\partial y}A'_x=B'_z\,. \tag{28}$$

Because $\mathbf{e}_\mathbf{t}$ is a unit vector along the imaginary axis, both contributions in (27) are imaginary, thereby making a real contribution in (24). Hence, the additional energy ΔE due to the interaction of the Pauli vector with the vector potential amounts to

$$\Delta E=\left|\frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}+\frac{\hbar}{2c}\frac{B_z}{m_0c}\right|. \tag{29}$$

Adding this amount to the energy of the moving particle, we have

$$E_W'^2=p_1'^2+1\pm\left|\frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}+\frac{\hbar}{2c}\frac{B_z}{m_0c}\right|. \tag{30}$$

After denormalization,

$$(\frac{E_W}{m_0c^2})^2=(\frac{m_0v_x}{m_0c})^2\pm\left|\frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}+\frac{\hbar}{2c}\frac{B_z}{m_0c}\right|+1=0\,. \tag{31}$$

Supposing that the term between brackets is much smaller than 1,

$$E_W \approx (m_0c^2)(1+\frac{1}{2}\frac{v^2}{c^2} \pm \left|\frac{\hbar}{m_0 c}\frac{\partial}{\partial y}\frac{\Phi}{m_0 c^2} + \frac{\hbar}{2c}\frac{B_z}{m_0 c}\right|) = \\ = m_0c^2)(1+\frac{1}{2}\frac{v^2}{c^2}) \pm (\frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\Phi + \frac{\hbar}{2}\frac{B_z}{m_0}). \tag{32}$$

So far, the Dirac particle has been considered in general terms, i.e. without identifying it as an electron. To do so, the potential Φ has to be interpreted by the force equity F as,

$$F = q\frac{\partial}{\partial y}\Phi_e = \frac{\partial}{\partial y}\Phi \rightarrow \Phi_e = \frac{\Phi}{q};\ B_z^{em} = \frac{B_Z}{q}. \tag{33}$$

where q is the electric charge of an electrical particle under consideration and where Φ_e and B_z^{em}, respectively, are the electric potential and the magnetic field. Hence, from (32) and (33),

$$E_W = m_0c^2)(1+\frac{1}{2}\frac{v^2}{c^2}) \pm (\frac{\hbar}{2c}\frac{q}{m_0}\frac{\partial}{\partial y}\Phi_e + \frac{\hbar q}{2m_0}B_z^{em}). \tag{34}$$

From (34) the influence of the Pauli spin on the motion of the particle under influence of the balance between the force of a transversal electric field and the Lorentz force from a magnetic field perpendicular on the motion as well as on the electric field, becomes clear from the last two terms in the right-hand part (34). Here, two dipole moments are identified. Apart from the well-known dipole moment $\hbar q/2m_0$ operating on the magnetic field, as mentioned before in (3), there appears a second dipole moment $\hbar q/2cm_0$ operating on the electric field, as mentioned before in (4).

Comparison with Dirac's analysis

Similar as in Dirac's analysis two dipole moments show up. However, where Dirac concluded that the electric dipole moment is not feasible, because of the imaginary term that shows up after evaluation of [1, eq. (15)], the electric dipole moment in (34) shows up as real. Let us try to understand what the origin could be of the difference. To do so, let us closely inspect Dirac's [1,eq.(15)]. His Hamiltonian contains the term

$$\text{Term} = \rho_1[\{\overline{\sigma}\cdot(\hat{\mathbf{p}}'+q\mathbf{A}')\}(\hat{p}_0'+qA_0') - (\hat{p}_0'+qA_0')\{\overline{\sigma}\cdot(\hat{\mathbf{p}}'+q\mathbf{A}')\}\ , \tag{35}$$

where ρ_1 is a 4 x 4 matrix derived from the 2 x 2 identity matrix to account for the four modalities of Dirac's equation.

Let us simplify this term by omitting all time dependencies. Hence Term evolves as,

$$\text{Term} = \rho_1[\{\overline{\sigma}\cdot(\hat{\mathbf{p}}'+q\mathbf{A}')\}qA_0' - qA_0'\{\overline{\sigma}\cdot(\hat{\mathbf{p}}'+q\mathbf{A})\}]. \tag{36}$$

(Note that, in Dirac's analysis A_0 is real as $A_0 = \Phi_e$).

Let us now skip the terms that obviously cancel. This gives,

Term = $\rho_1[(\overline{\sigma}\cdot\hat{\mathbf{p}}')qA_0' - qA_0'(\overline{\sigma}\cdot\hat{\mathbf{p}}')]$. (37)

This term evolves as,

Term = $\mathrm{i}q\rho_1\hbar\overline{\sigma}\cdot(\frac{\partial A_0'}{\partial x}\mathbf{i}+\frac{\partial A_0'}{\partial y}\mathbf{j}+\frac{\partial A_0'}{\partial z}\mathbf{k})$. (38)

Expanding this term gives

Term = $\mathrm{i}q\hbar\rho_1(\cdot\sigma_1\mathbf{i}+\sigma_2\mathbf{j}+\sigma_3\mathbf{k})\cdot(\frac{\partial A_0'}{\partial x}\mathbf{i}+\frac{\partial A_0'}{\partial y}\mathbf{j}+\frac{\partial A_0'}{\partial z}\mathbf{k})$. (39)

Let us simplify even more by considering the same condition as in our "1 + 2" approach. This gives

Term = $\mathrm{i}q\hbar\rho_1\begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}\frac{\partial A_0'}{\partial y}=\frac{\mathrm{i}q\hbar\rho_1}{m_0c}\begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}\frac{\partial \Phi_e}{\partial y}$. (40)

Hence, in spite of Dirac's opinion, Term shows different modes that reveal a real electric dipole moment. Hence, after all, the conclusion drawn from the simplified analysis in the "1 + 2" domain is not in conflict with Dirac's generic result as shown in [1,eq.(15)]. However, Dirac's interpretation that the electric dipole moment is imaginary seems doubtful.

Discussion

At first glance, it might seem that the impact of the conclusion that Dirac's second dipole moment is real, is of limited value. The view on its possible impact might change if we put a Dirac/Majorana particle in a more general context. Let us suppose that a quark can be conceived as a Dirac/Majorana particle as well. In such a scope, we may assign a nuclear field Φ_{qu} to a quark and, in addition, we may suppose that an antiquark may couple to this field by a coupling factor g (from the electromagnetic fine constant expression $q^2 = 4\pi\varepsilon_0\hbar c g^2$) such that,

$$F = g\frac{\partial}{\partial y}\Phi_{qu} = \frac{\partial}{\partial y}\Phi \rightarrow \Phi_{qu} = \frac{\Phi}{g}. \quad (41)$$

In a recent preprint [15], I have demonstrated the consequences of such a model, thereby revealing that it gives an adequate explanation for the mass spectrum of leptons and hadrons. Because the second dipole moment of the quark is shown as being the origin of isospin as well as being the gluing force between the quarks, such an approach is considered being in conflict with the QCD color force binding by gluons. Hence, this theory meets fierce opposition, in spite of the unprecedented successful calculation of mass values with an undeniable match with experimental evidence. Some years ago, before being aware of the second dipole moment I have used the very same model for successfully expressing Newton's gravitational constant into quantum mechanical quantities [16]. Another generalization of Dirac/Majorana particles is conceivable in the gravitational domain with gravitational potential Φ_G.

$$F = m_0 \frac{\partial}{\partial y}\Phi_G = \frac{\partial}{\partial y}\Phi \rightarrow \Phi_G = \frac{\Phi}{m_0}. \qquad (42)$$

In attempts to give an explanation for cosmological phenomena as dark matter and dark energy, the universe is modeled as a tenuous gaseous medium with background energy [17]. By modeling the gaseous molecules as Dirac/Majorana particles, these cosmological phenomena can be explained as a consequence of vacuum polarization and verified by observational evidence [18].

Although these studies have led the author to the rediscovery of Dirac's second dipole moment, a decisive experimental proof would be most welcome. How to construct such a proof is not obvious. One thing is clear: it can't be done in the way how present high-precision eEDM measurements are set up [19,20]. These measurements aim to measure the precession effect from the pre-supposed electric dipole moment contribution to the effects from the nuclear spin of an electron. Because the AEDM is not caused by an angular motion, it does not contribute to such effects. Hence, it cannot be detected by the instrumentation that aims to measure the eEDM. It remains a challenge for further research. The phenomenon to be shown and measured is the second spin-flip of an electron under influence of a vector potential. To do so, one might consider to measure the hyperfine split effect due to the spin-spin interaction of the electron with the atomic nucleus, which gives rise to the well-known 21 cm line in the cosmological electromagnetic spectrum of atomic hydrogen [21]. Unfortunately, the interaction energy between the spins due to the anomalous electric dipole moments is just equal to the interaction energy due to the anomalous magnetic dipole moments. This can be seen as follows. According to Griffiths [5,22], the interaction energy ΔE between the magnetic dipoles μ_m^e and μ_m^p of, respectively, the electron (mass m_e) and the proton (mass m_p; "g"-factor g_p = 5.59) amounts to

$$\Delta E = \mu_0 \frac{g_p \mu_m^e \mu_m^p}{3\pi a_0^3} = \frac{4 g_p \hbar^4}{3 a_0^4 m_p m_e^2 c^2} \; ; \; a_0 = \frac{\hbar}{g^2 m_e c}, \qquad (43)$$

where a_0 is the Bohr radius [23].

Hence, from

$$\Delta E = \hbar\omega = hf \rightarrow f \approx 1.42 \text{ GHz} \rightarrow \lambda \approx 21 \text{ cm}. \qquad (44)$$

The interaction energy ΔE_{el} between the electric dipoles μ_{el}^{e} and μ_{el}^{p} of, respectively, the electron and the proton amounts to

$$\Delta E_{el} = \varepsilon_0 \frac{g_p \mu_{el}^{e} \mu_{el}^{p}}{3\pi a_0^3}. \tag{45}$$

Because of the relationship between the magnetic dipoles and electric dipoles as expressed by (3) and (4) and because $c^2 = (\varepsilon_0 \mu_0)^{-1}$, the two interaction energies ΔE_{el} and ΔE are just the same.

In view of this, a proof for the existence of an anomalous electric dipole moment of electrons is far from easy. It might even be a reason to deny its relevancy, like Dirac did. This might be different for quarks, if one is willing to consider the hypothesis that a quark is a Dirac particle as well, albeit of a different kind. Let the quark be conceived as a pointlike source with an elementary angular momentum ***ħ***, and an elementary linear momentum ***ħ/c***, and let this quark spread a field $\Phi(r) = \Phi_F(r) + \Phi_N(r)$, with a far field component $\Phi_F(r)$ and a near field component $\Phi_N(r)$. As can be expected, the most simple expression for the (near) field from the linear momentum will show up along the axis x set up between the poles. Under consideration of a generic dipole moment $m_p d$, this potential field can be readily derived as,

$$\frac{\Phi_N(x)}{m_0} = \frac{Gm_p d}{x^2} \rightarrow \frac{\Phi_N(x)}{m_0} = \frac{\hbar}{2c}\frac{\lambda^2 G}{(\lambda x)^2} \rightarrow \frac{\Phi_N(x)}{m_0} = \frac{\Phi_0}{m_0}\frac{1}{(\lambda x)^2}; \quad \frac{\Phi_0}{m_0} = \frac{\hbar}{2c} G\lambda^2. \tag{46}$$

Whether the proportionality constant G is the Newtonian one or not, is not relevant for the present discussion. What matters is the generic expression in terms of a strength parameter Φ_0 and a spatial parameter λ. Note that the mass m_p of the fictitious pole in the elementary dipole is different from the effective mass m_0 in the massive energy of the Dirac particle. Because it is just an auxiliary quantity, the relevancy of mass m_p is restricted.

The far field is the result of an effective mass from the elementary angular moment. Interpreting the angular momentum as a rotation with light speed at a fictitious radius $r_0 = 1/g_m\lambda$, we have

$$\frac{\hbar}{2} = \frac{m_p c}{g_{m\lambda}\lambda} \rightarrow m_p = g_m \frac{\hbar}{2}\frac{\lambda}{c}. \tag{47}$$

The quantity g_m is an unknown *gyrometric* constant.
Hence, from classical field theory,

$$\frac{\Phi_F(r)}{m_0} = \frac{m_p G}{r} = g_m \frac{\hbar}{2}\frac{\lambda}{c}\frac{G}{r} = g_m \frac{\hbar}{2}\frac{G\lambda^2}{c}\frac{1}{\lambda r}, \tag{48}$$

and, under consideration of Φ_0 as defined in (46),

$$\frac{\Phi_F(r)}{m_0} = g_m \frac{\hbar}{2} \frac{G\lambda^2}{c} \frac{1}{\lambda r} = \frac{\Phi_0}{m_0} \frac{g_m}{(\lambda r)}. \tag{49}$$

These two potential fields, $\Phi_N(x)$ in (46) and $\Phi_F(r)$ in (49), are the results of the energetic flow from the Dirac particle-type source. It can be influenced by a background (Higgs) field. Assuming such influence and assuming that the near field is attracting and that the far field is repulsive, the quark's potential field along the axis set up between the poles of the linear momentum has the format,

$$\Phi(\lambda x) = \Phi_0 \exp(-\lambda x)\{\frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x}\} \ . \tag{50}$$

The potential field has the well known liquid drop format, known from the potential field between nucleons. Each of two quarks in a meson are coupled to the field of the other with the generic quantum mechanical coupling factor g. Because of the electroweak hypothesis this coupling factor is taken as the square root from the electromagnetic fine structure constant g^2, as shown in (41). Hence, the combined field from two quarks in a meson aligned along the $x-$axis, spaced at a $2d$ distance, can be expanded as,

$$V(x) = \Phi(d+x) + \Phi(d-x) = g\Phi_0(k_0 + k_2\lambda^2 x^2 + ...), \tag{51}$$

where $k_0$and k_2 are dimensionless coefficients with magnitudes that depend on the spacing d. The interesting feature of the meson configuration now is, that its center of mass is subject to a potential that (almost) depends on the square of x. The two quarks in the meson align themselves in the condition of minimum energy at $d\lambda = d'_{\min}$. Hence, the meson shows the characteristics of a quantum mechanical oscillator. It is therefore subject to excitation, which is the underlying mechanism for the systematic characteristic of the mass spectrum of mesons [24].

The intriguing conclusion therefore is that Dirac's second dipole moment may give a physical interpretation to the gluing force between quarks and antiquark, which in the Standard Model of particle physics is axiomatically conceived from a mathematical framework as the QCD color force. It is also obvious that an another axiomatic attribute in the Standard Model, namely the isospin, gets a physical interpretation as well. This does not necessarily imply that the two approaches are necessarily incompatible. They might be two sides of the same medal.

Conclusion

A Dirac/Majorana particle has two anomalous dipole moments. One of these is the consequence of an elementary angular moment assigned to the pointlike particle. For electrons It becomes manifest as a magnetic dipole moment. The second one comes forward as the result of Dirac's modeling, but it remained forgotten because of a number of reasons.

The main one is Dirac's perception that it has no physical relevance as an electric dipole moment, because of its seeming imaginary value. A second one is its disappearance in Dirac-type analyses in standard textbooks. A third reason is the failure of proof by measurements. A fourth reason is the perceived violation of time reversal symmetry and CP symmetry. In this article, first of all a proof is given for the inconsistency between Dirac's result on the dipole moments and the Bjorken and Drell textbook result. The reason why has been shown. By simplifying the analysis to a domain with, next to the temporal axis, only two spatial axes and comparing the result with the result from Dirac's analysis, it could be shown that Dirac's conclusion on the perceived imaginary value of the second dipole moment is doubtful, if not incorrect. Subsequently, the remaining issues of the failure of experimental evidence and the violations of T-symmetry and PC-symmetry have been analyzed thereby revealing the fundamental difference between Dirac's anomalous non-rotating electric dipole, giving rise to AEDM, and the rotating electric dipole, giving rise to eEDM as defined by the references quoted by the Particle Data Group [25]. Where the eEDM can be viewed as the strength of a pseudo vector orthogonal to the angular momentum vector caused by an off-set between the center of mass and the center of charge, the AEDM can be viewed as the strength of a position vector due to the Heisenberg vibration of the center of charge. Quantitatively the AEDM is much larger than the eEDM. However, probably due to its unrecognized origin, no attempts have been made as yet to measure the AEDM by dedicated instrumentation.

Finally, I would like to emphasize that Dirac's anomalous linear dipole moment is relevant for other Dirac-type elementary particles but electrons. It is a stepping stone to understand the properties of quarks in the nuclear domain and to understand the constituents ("darks") of the cosmological background energy [18].

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References

[1] P.A.M. Dirac, Proc.Royal Soc. London, A 117, 610 (1928)
[2] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)
[3] Y.K. Semertzidis, Nuclear Phys. B (Proc. Suppl.), 131, 244 (2004)
[4] J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill Book Cie (1964)
[5] D. Griffiths, *Introduction to Elementary Particles*, ISBN 3527406018, Wiley (2008)
[6] L.I. Mandelshtam, I.E. Tamm, J. Phys (USSR) 9, 249 (1945)
[7] https://en.wikipedia.org/wiki/Electron_electric_dipole_moment
[8] Nowakowski, M.; Paschos, E.A.; Rodriguez, Europ. Journal of Physics. **26** (4): 545–560. arXiv:physics/0402058. doi:10.1088/0143-0807/26/4/001 (2005)
[9] J.S. Bell, *Fundamental Symmetries*, eds. P. Bloch, P. Pavlopoulos and B. Klapisch, Plenum Press, N.Y and London (1987)
[10] S. Eckel, A. O. Sukhov, S.K. Lamoreaux, Phys. Rev. Lett., vol. 109, issue 19 (2012); arXiv: 1208.4420v2 (2012)
[11] A.D. Sakharov, Prisma Zh. Eksp. Teor. Fiz. 5,32 (1967)
[12] D. Hestenes, Found Phys. 40, doi.org/10.1007/s10701-009-9360-3 (2010)
[13] D. Hestenes, ArXiv: 0.802.2728v1 (2008)
[14] C. Lanczos, F. Phys, 57, 447 (1929), reprint by W.R. Davis et al., North Car. St. Univ., Raleigh, Vol III, p2-1248 (1988)

[15] E. Roza, doi: 10.20944/preprints201905.0118.v1(2019)
[16] E. Roza, Results in Physics, 6 DOI: 10.1016/j.rnp 2016.03.001 (2016)
[17] T. Jacobson, Phys.Rev. Lett., 116, no. 20, 201101 (2016)
[18] E. Roza, Astrophys. And Space Sci., 364:73, doi.org/10.1007/s10509-019-3561-9 (2019)
[19] A.A. Bondarevskaya *et al*., arXiv 1303.65584v1 (2014)
[20] R. Talman, *Measuring Electron and Proton Electric Dipole Moments*, https://www.classe.cornell.edu
[21] C.A. Muller, J.H. Oort, Nature 168 (4270), 357 (1951)
[22] https://physicspages.com/pdf/Griffiths QM/ Griffiths Problems 06.27.pdf
[23] "CODATA Value Bohr radius" Fund. Phys. Constants, NIST, retrieved Feb. 2016
[24] E. Roza, preprints.org, doi: 10.20944/preprints201701.0076.v1 (2017)
[25] pdglive.lbl.gov*, Electric Dipole Moment* (2019)

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