On Stability of Iterative Sequences with Error

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Abstract

In this paper, it is proved that s-iteration with error and Picard-Mann iteration with error iterative processes converge strongly to the unique fixed point of Lipschitzian strongly pseudo-contractive map. This convergence is almost F-stable and F-stable. Applications of these results to the operator equations$\mathrm{F}x = \mathrm{f}$, $x + \mathrm{F}x = \mathrm{f}$, where F is a strongly accretive and accretive mappings of X into itself.

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1. Introduction and Preliminaries

Consider a normed space $\mathrm{X}, \mathrm{F}: \mathrm{X} \longrightarrow \mathrm{X}$ is a mapping, M is a function and $\lambda_n, \eta_n \in (0,1)$, we present the following iterative sequences

$\mathrm{w}_0 \in \mathrm{X}$,

$$\mathrm{w}_{n+1} = \mathrm{M}(\mathrm{F}, \mathrm{w}_n),$$

is called **s-iteration** [1] if

$$\mathrm{w}_{n+1} = \lambda_n \mathrm{F} z_n + (1-\lambda_n)\mathrm{F}\mathrm{w}_n,$$
$$z_n = \eta_n \mathrm{F}\mathrm{w}_n + (1-\eta_n)\mathrm{w}_n, \forall\, n \geq 0\,. \tag{1.1}$$

$x_0 \in \mathrm{X}$,

$$x_{n+1} = \mathrm{M}(\mathrm{F}, x_n)$$

is called **Picard-Mann** iteration [2] if

$$x_{n+1} = \mathrm{F}y_n,$$
$$y_n = \lambda_n \mathrm{F}x_n + (1-\lambda_n)x_n, \forall\, n \geq 0\,. \tag{1.2}$$

$\mathrm{w}_0 \in \mathrm{X}$,

$$\mathrm{w}_{n+1} = \mathrm{M}(\mathrm{F}, \mathrm{w}_n),$$

is called **s-iteration with errors** if

$$\mathrm{w}_{n+1} = \lambda_n \mathrm{F} z_n + (1-\lambda_n)\mathrm{F}\mathrm{w}_n + a_n,$$
$$z_n = \eta_n \mathrm{F}\mathrm{w}_n + (1-\eta_n)\mathrm{w}_n + c_n, \forall\, n \geq 0\,. \tag{1.3}$$

where $\sum_{n=0}^{\infty}\|a_n\| < \infty, \sum_{n=0}^{\infty}\|c_n\| < \infty$.

$$x_0 \in \mathrm{X},$$
$$x_{n+1} = \mathrm{M}(\mathrm{F}, x_n)$$

is called **Picard-Mann** iteration with errors if

$$x_{n+1} = \mathrm{F}y_n + a_n,$$
$$y_n = \lambda_n \mathrm{F}x_n + (1-\lambda_n)x_n, \forall\, n \geq 0\,. \tag{1.4}$$

where $\sum_{n=0}^{\infty}\|a_n\| < \infty$.

Throughout this paper, we study three cases, convergence, almost stability and stability of schemes of sequences define in (1.3) and (1.4). In the following, we recall needed definitions and lemmas.

Definition 1.1: [3]

An arbitrary iteration scheme $x_{n+1} = \mathrm{M}(\mathrm{F}, x_n)$ where M is function and $\langle x_n \rangle$ converges to a fixed point p of F. Suppose that $\langle \mathrm{q}_n \rangle$ be a sequence in X, then $\langle x_n \rangle$ is called **Stable with respect to F (or F - stable)** if $\lim_{n\to\infty} \delta_n = 0$, implies to $\lim_{n\to\infty} \mathrm{q}_n = \mathrm{p}$,

where

$$\delta_n = \|\mathrm{q}_{n+1} - \mathrm{M}(\mathrm{F}, x_n)\|, n \geq 0 .$$

Definition 1.2: [4]

Let X, F, $\langle x_{n+1} \rangle, \delta_n, \mathrm{q}_n$, and p be as of definition (1.1), then the fixed point iteration procedure $\langle x_n \rangle$ is **Almost stable with respect to F (or almost F-stable)** if $\sum_{n=0}^{\infty} \delta_n < \infty$ implies that $\lim_{n\to\infty} \mathrm{q}_n = \mathrm{p}$.

Definition 1.3:[5]

Let X be a normed space, and F : X ⟶ X be a mapping then for fixed m, $0 \leq m < \infty$, F is said Lipschitzian if

$$\|\mathrm{F}x - \mathrm{F}y\| \leq m\, \|x - y\|, \forall x, y \in \mathrm{X} . \tag{1.5}$$

Definition 1.4: [6], [7]

Let X be a normed space, F : X ⟶ X be a mapping. Then F is called **strongly pseudo-contractive** if there exist $\mathfrak{r} = \frac{1}{l}$, where, $l > 1$. such that

$$\langle \mathrm{F}x - \mathrm{F}y, \mathrm{j}(x - y)\rangle \leq \mathfrak{r}\|x - y\|^2, \forall \mathrm{x}, y \in \mathrm{X} . \tag{1.6}$$

Definition 1.5: [8], [9], [10]

A mapping F: X ⟶ X, where X normed linear space is said to be

i-**Strongly accretive,** if there $\mathfrak{r} > 0$ such that for each $x, y \in X$ there exists $\mathrm{j}(x - y) \in \mathrm{J}(x - y)$

$$\langle \mathrm{F}x - \mathrm{F}y, \mathrm{j}(x - y)\rangle \geq \mathfrak{r}\|x - y\|^2. \tag{1.7}$$

ii- **Accretive** if $\|x - y\| \leq \|x - y + \mathfrak{r}(\mathrm{F}x - \mathrm{F}y)\|$, (1.8)

also if $\mathfrak{r} = 0$ in (1.7) .

Proposition 1.6: [7],[9], [10]

The relation between (strongly) pseudo-contraction mapping and (strongly) accretive mapping is that:

i- F is (strongly) pseudo-contraction if and only if (I − F) is (strongly) accretive.

ii- Also, if (I − F) is (strongly) pseudo-contraction then F is (strongly) accretive, and the converse is true.

Lemma 1.7: [11]

Let $\{\rho_n\}$be a nonnegative sequence which satisfies the following inequality, $\rho_{n+1} \leq (1 - \gamma_n)\rho_n + \mu_n$ where $\gamma_n \in (0,1)$, for each $n \in \mathbb{N}$, $\sum \gamma_n = \infty$ and $\mu_n = o(\gamma_n)$. Then $\lim_{n\to\infty} \rho_n = 0$.

Lemma 1.8: [12]

Let $\{\xi_n\}$ be a nonnegative sequence that satisfies the inequality $\xi_{n+1} \leq (1 - \gamma_n)\xi_n + b_n + \mu_n, n \geq 0$.

Where $\gamma_n \in [0,1]$, $\forall n \in \mathbb{N}$, $\sum \gamma_n = \infty$, and $b_n = o(\gamma_n)$, $\sum_{n=0}^{\infty} \mu_n < \infty$. Then $\lim_{n\to\infty} \xi_n = 0$.

Lemma 1.9: [13], [14]

Let X real Banach space, $F: X \longrightarrow X$ be a mapping such that :

i- If F continuous and strongly pseudo-contractive, then F has a unique fixed point .

ii- If F continuous and strongly accretive, then the equation $Fx = f$ has a unique solution for any $f \in X$.

iii- If F continuous and accretive, then F is m-accretive. Also the equation $x + Fx = f$ has a unique solution for any $f \in X$.

2. Main Results

Firstly, define condition (Δ_1) as

If $\lambda_n, \eta_n \in (0,1)$, $\mathrm{r} \in (0,1)$ and $m > 0$, then

$$m\big((m+1)(1+\eta_n) + \lambda_n m^2(2+(m-1)\eta_n)\big) - (2-\mathrm{r})\lambda_n(2m + m(m-1)\eta_n) \le \mathrm{r}m - e,$$

where $e \in (0, m)$.

Theorem 2.1: Let X real Banach space, let $F: X \longrightarrow X$ is Lipschitzian strongly pseudo-contractive mapping . Suppose that $\langle w_n \rangle$ be in (1.3), $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = 0$ and condition (Δ_1) satisfied .

Then:

1- $\langle w_n \rangle$ converges strongly to the unique fixed pointp .

2- $\|q_{n+1} - p\| \le \delta_n + \|a_n\| + [1 - \frac{\lambda_n e}{1+\lambda_n}]\|q_n - p\| + [3m + m^2]\|c_n\|, \forall n \ge 0$.

Proof:

From Lemma (1.9), we obtain F has unique fixed point, and from (1.3), (1.6), and proposition (1.6), we have

$$\begin{aligned} Fw_n &= w_{n+1} + \lambda_n Fw_n - \lambda_n Fz_n - a_n \\ &= w_{n+1} + \lambda_n Fw_n - \lambda_n Fz_n - a_n + 2\lambda_n w_{n+1} - 2\lambda_n w_{n+1} - \mathrm{r}\lambda_n w_{n+1} + \mathrm{r}\lambda_n w_{n+1} \\ &\qquad - \lambda_n Fw_{n+1} + \lambda_n Fw_{n+1} \\ &= (1+\lambda_n)w_{n+1} + \lambda_n(I - F - \mathrm{r}I)w_{n+1} - (1-\mathrm{r})\lambda_n Fw_n + (2-\mathrm{r})\lambda_n^2(Fw_n - Fz_n) + \\ &\lambda_n(Fw_{n+1} - Fz_n) - (1+(2-\mathrm{r})\lambda_n)a_n \end{aligned} \tag{2.1}$$

$$p = (1+\lambda_n)p + \lambda_n(I - F - \mathrm{r}I)p - (1-\mathrm{r})\lambda_n p \tag{2.2}$$

So that

$$Fw_n - p = (1+\lambda_n)(w_{n+1} - p) + \lambda_n[(I - F - \mathrm{r}I)w_{n+1} - (I - F - \mathrm{r}I)p] - (1-\mathrm{r})\lambda_n(Fw_n - p) + (2-\mathrm{r})\lambda_n^2(Fw_n - Fz_n) + \lambda_n(Fw_{n+1} - Fz_n) - (1+(2-\mathrm{r})\lambda_n)a_n \tag{2.3}$$

$$\|Fw_n - p\| \ge (1+\lambda_n)\left\|(w_{n+1} - p) + \frac{\lambda_n}{1+\lambda_n}[(I - F - \mathrm{r}I)w_{n+1} - (I - F - \mathrm{r}I)p]\right\|$$

$$- (1-\mathrm{r})\lambda_n\|Fw_n - p\| - (2-\mathrm{r})\lambda_n^2\|Fw_n - Fz_n\| - \lambda_n\|Fw_{n+1} - Fz_n\| - 3\|a_n\|$$

Thus

$$(1+\lambda_n)\|w_{n+1} - p\|$$

$$\le (1+(1-\mathrm{r})\lambda_n)\|Fw_n - p\| + (2-\mathrm{r})\lambda_n^2\|Fw_n - Fz_n\| + \lambda_n\|Fw_{n+1} - Fz_n\| + 3\|a_n\|$$

$$\|w_{n+1} - p\| \leq \frac{1}{1+\lambda_n}[(1 + (1-\mathfrak{r})\lambda_n)\|Fw_n - p\| + (2-\mathfrak{r})\lambda_n^2\|Fw_n - Fz_n\|$$
$$+ \lambda_n\|Fw_{n+1} - Fz_n\| + 3\|a_n\|]$$
$$\|w_{n+1} - p\| \leq \frac{1}{1+\lambda_n}[(1 + (1-\mathfrak{r})\lambda_n)m\|w_n - p\| + (2-\mathfrak{r})\lambda_n^2\|Fw_n - Fz_n\| +$$
$$\lambda_n\|Fw_{n+1} - Fz_n\| + 3\|a_n\|] \qquad (2.4)$$

Observe that

$$\|Fw_n - Fz_n\| \leq \|Fw_n - p\| + \|p - Fz_n\| \leq m\|w_n - p\| + m\|z_n - p\|$$
$$\leq 2m + m(m-1)\eta_n\|w_n - p\| + m\|c_n\| \qquad (2.5)$$
$$\|Fw_{n+1} - Fz_n\| \leq m\|w_{n+1} - z_n\|$$
$$\leq [m(m+1) + \lambda_n m(2m + m(m-1)\eta_n\|w_n - p\|) + \eta_n m(m+1)]\|w_n - p\| + m\|a_n\| +$$
$$m\|c_n\| + \lambda_n m^2\|c_n\| \qquad (2.6)$$

Now, putting (2.6), (2.5) in (2.4)

$$\|w_{n+1} - p\| \leq \frac{1}{1+\lambda_n}[(1 + (1-\mathfrak{r})\lambda_n)m\|w_n - p\|$$
$$+ \lambda_n([m(m+1) + \lambda_n m(2m + m(m-1)\eta_n) + \eta_n m(m+1)]\|w_n - p\|$$
$$+ m\|a_n\| + m\|c_n\| + \lambda_n m^2\|c_n\|) +$$
$$(2-\mathfrak{r})\lambda_n^2((2m + m(m-1)\eta_n\|w_n - p\| + m\|c_n\|) + 3\|a_n\|]$$
$$= \frac{1}{1+\lambda_n}[(1 + (1-\mathfrak{r})\lambda_n)m + \lambda_n m(m+1)(1+\eta_n) + 2\lambda_n^2 m^2 +$$
$$\lambda_n^2 m^2(m-1)\eta_n + (2-\mathfrak{r})\lambda_n^2((2m + m(m-1)\eta_n]\|w_n - p\| +$$
$$\left[\frac{\lambda_n}{1+\lambda_n}m + \frac{3}{1+\lambda_n}\right]\|a_n\| + \left[\frac{(2-\mathfrak{r})\lambda_n^2}{1+\lambda_n}m + \frac{\lambda_n^2}{1+\lambda_n}(m^2+m)\right]\|c_n\|$$
$$\leq [1 - \frac{\lambda_n}{1+\lambda_n}[m\mathfrak{r} - m((m+1)(1+\eta_n) + \lambda_n m^2(2 + (m-1)\eta_n)) +$$
$$-(2-\mathfrak{r})\lambda_n((2m + m(m-1)\eta_n]\|w_n - p\| + [m+3]\|a_n\| + [3m + m^2]\|c_n\|$$
$$= [1 - \frac{\lambda_n e}{1+\lambda_n}\|w_n - p\| + [m+3]\|a_n\| + [3m + m^2]\|c_n\|$$

By applying Lemma (1.7), we yield $\lim_{n\to\infty} w_n = p$.

For (2) :

Let$\langle q_n \rangle$be a sequence inX,defined $\{\delta_n\}$by $\delta_n = \|q_{n+1} - g_n - a_n\|$,where

$$g_n = \lambda_n Fz_n + (1-\lambda_n)Fq_n, z_n = \eta_n Fq_n + (1-\eta_n)q_n + c_n, n \geq 0 .$$
$$\|q_{n+1} - p\| \leq \|q_{n+1} - g_n - a_n\| + \|a_n\| + \|g_n - p\| \leq \delta_n + \|a_n\| + \|g_n - p\| \qquad (2.7)$$

Since

$$Fq_n = g_n + \lambda_n Fq_n - \lambda_n Fz_n$$
$$= (1+\lambda_n)g_n + \lambda_n(I - F - rI)g_n - (2-r)\lambda_n g_n + \lambda_n Fq_n + \lambda_n(Fg_n - Fz_n)$$
$$= (1+\lambda_n)g_n + \lambda_n(I - F - rI)g_n - (1-r)\lambda_n Fq_n + (2-r)\lambda_n^2(Fq_n - Fz_n) + \lambda_n(Fg_n - Fz_n) \quad (2.8)$$

Thus

$$p = (1+\lambda_n)p + \lambda_n(I - F - rI)p - (1-r)\lambda_n p \quad (2.9)$$
$$Fq_n - p = (1+\lambda_n)(g_n - p) + \lambda_n[(I - F - rI)g_n - (I - F - rI)p] - (1-r)\lambda_n(Fq_n - p) + (2-r)\lambda_n^2(Fq_n - Fz_n) + \lambda_n(Fg_n - Fz_n).$$

So that

$$\|Fq_n - p\| \geq (1+\lambda_n)\left\|(g_n - p) + \frac{\lambda_n}{1+\lambda_n}[(I - F - rI)g_n - (I - F - rI)p]\right\| - (1-r)\lambda_n \times \|Fq_n - p\| - (2-r)\lambda_n^2\|Fq_n - Fz_n\| - \lambda_n\|Fg_n - Fz_n\|$$
$$\geq (1+\lambda_n)\|g_n - p\| - (1-r)\lambda_n\|Fq_n - p\| - (2-r)\lambda_n^2\|Fq_n - Fz_n\| - \lambda_n\|Fg_n - Fz_n\|$$

Thus

$$\|g_n - p\| \leq \frac{1}{1+\lambda_n}[(1+(1-r)\lambda_n)\|Fq_n - p\| + (2-r)\lambda_n^2\|Fq_n - Fz_n\| + \lambda_n\|Fg_n - Fz_n\|]$$
$$\|g_n - p\| \leq \frac{1}{1+\lambda_n}[(1+(1-r)\lambda_n)m\|q_n - p\| + (2-r)\lambda_n^2\|Fq_n - Fz_n\| + \lambda_n\|Fg_n - Fz_n\|] \quad (2.10)$$

Observe that

$$\|Fq_n - Fz_n\| \leq \|Fq_n - p\| + \|p - Fz_n\| \leq m\|q_n - p\| + m\|z_n - p\|$$
$$\leq 2m + m(m-1)\eta_n\|q_n - p\| + m\|c_n\| \quad (2.11)$$
$$\|Fg_n - Fz_n\| \leq m\|g_n - z_n\|$$
$$\leq m[\|Fq_n - q_n\| + \lambda_n\|Fq_n - Fz_n\| + \eta_n\|q_n - Fq_n\| + \|c_n\|]$$
$$\leq [m(m+1) + \lambda_n m(2m + m(m-1)\eta_n) + \eta_n m(m+1)]\|q_n - p\| + m\|c_n\| + \lambda_n m^2\|c_n\| \quad (2.12)$$

Now, putting (2.12), (2.11) in (2.10), we get

$$\|g_n - p\| \leq \frac{1}{1+\lambda_n}[(1+(1-r)\lambda_n)m\|q_n - p\| + \lambda_n([m(m+1) + \lambda_n m(2m + m(m-1)\eta_n) + \eta_n m(m+1)]\|q_n - p\| + m\|c_n\| + \lambda_n m^2\|c_n\|) + (2-r)\lambda_n^2((2m + m(m-1)\eta_n\|q_n - p\| + m\|c_n\|)]$$

$= \frac{1}{1+\lambda_n}[(1+(1-\mathfrak{r})\lambda_n)m + \lambda_n m(m+1)(1+\eta_n) + 2\lambda_n^2 m^2 + \lambda_n^2 m^2(m-1)\eta_n +$

$(2-\mathfrak{r})\lambda_n^2((2m + m(m-1)\eta_n]\|q_n - p\| + \left[\frac{(2-\mathfrak{r})\lambda_n^2}{1+\lambda_n}m + \frac{\lambda_n^2}{1+\lambda_n}(m^2+m)\right]\|c_n\|$

$$\leq [1 - \frac{\lambda_n}{1+\lambda_n}[m\mathfrak{r} - m\big((m+1)(1+\eta_n) + \lambda_n m^2(2+(m-1)\eta_n)\big) +$$

$(2-\mathfrak{r})\lambda_n((2m + m(m-1)\eta_n]\|q_n - p\| + [(2-\mathfrak{r})\lambda_n^2 m + (m^2+m)\lambda_n^2]\|c_n\|$

$= [1 - \frac{\lambda_n e}{1+\lambda_n}\|q_n - p\| + [3m + m^2]\|c_n\|\|c_n\|.$ (2.13)

Substituting(2.13) in (2.7) to obtain

$$\|q_{n+1} - p\| \leq \|q_{n+1} - g_n - a_n\| + \|a_n\| + \|g_n - p\| \leq \delta_n + \|a_n\| + \|g_n - p\|$$

$\leq \delta_n + \|a_n\| + \left[1 - \frac{\lambda_n e}{1+\lambda_n}\right]\|q_n - p\| + [3m + m^2]\|c_n\|.$ (2.14)

Theorem 2.2: Assume that X, F, p, $\{w_n\}$, $\{z_n\}$, $\{q_n\}$, $\{\lambda_n\}$, $\{\eta_n\}$, and $\{\delta_n\}$ be as in Theorem (2.1). Then $\langle w_n \rangle$ is almost F-stable.

Proof: Assume that $\sum_{n=0}^{\infty}\delta_n < \infty$. Then, we prove that $\lim_{n\to\infty} q_n = p$.

Now, using (2.14) such that denote $\xi_n = \|q_n - p\|, \gamma_n = \frac{\lambda_n e}{1+\lambda_n},$

$b_n = [3m + m^2]\|c_n\| + \|a_n\|$, and $\mu_n = \delta_n, \forall\, n \geq 0$.

Note that $\lim_{n\to\infty} b_n = 0$, thus Lemma (1.8) holds such that

$\lim_{n\to\infty}\xi_n = 0$ yields $\lim_{n\to\infty} q_n = p$.

Theorem 2.3: Let X, F, p, $\{q_n\}$, $\{\lambda_n\}$, $\{\eta_n\}$, $\{a_n\}$, $\{c_n\}$, and $\{\delta_n\}$ be as in Theorem (2.1) . Then $\{w_n\}$ is F-stable.

Proof: Assume that $\lim_{n\to\infty}\delta_n = 0$. Then, we prove that $\lim_{n\to\infty} q_n = p$.

By application lemma (1.7) on (2.14) of Theorem (2.1), we obtain $\lim_{n\to\infty} q_n = p$.

Example 2.4:

Let $X = [0,1]$, $F: X \longrightarrow X$ by $Fx = \frac{x}{2}$, hence the conditions (1.5), (1.6) satisfy as shown

$$\|Fx - Fy\| = \left\|\frac{x}{2} - \frac{y}{2}\right\| \leq \frac{1}{2}\|x - y\|$$

$$\langle Fx - Fy, j(x-y)\rangle \leq \mathfrak{r}\|x-y\|^2 \leq (Fx - Fy)(x-y)$$

$$\leq \left|\frac{x}{2} - \frac{y}{2}\right| |x - y| = \frac{1}{2}\|x-y\|^2$$

Now, put $\lambda_n = \frac{1}{2}$, $q_n = \frac{1}{n}$, $\forall\, n \geq 0$, since $\lim_{n\to\infty} q_n = 0$, to show that $\lim_{n\to\infty}\delta_n = p = 0$.

$$\delta_n = \|q_{n+1} - x_{n+1}\| = \|q_{n+1} - Fq_n + a_n\| = \left\|\frac{1}{n+1} - \frac{q_n}{2}\right\| = \left\|\frac{1}{n+1} - \frac{(1-\lambda_n)}{2}q_n - \frac{\lambda_n}{2}\frac{q_n}{2}\right\|$$

$= \left\|\frac{1}{n+1} - \frac{1}{4n} - \frac{1}{8n}\right\| \Longrightarrow \lim_{n\to\infty}\delta_n = 0$.

Corollary 2.5 : Let X, F, p, m, $\{q_n\}$, $\{\lambda_n\}$, $\{\eta_n\}$, $\{a_n\}, \{c_n\}, \{\delta_n\}$ be as in Theorem (2.1), and $\{w_n\}$ defined by (1.1), Then $\{w_n\}$:
1- converges strongly to the unique fixed point p .
2- Almost F-stable .
3- F-stable.

To prove the next results, we replace the inequality in the condition (Δ_1) by
$m\left(1+m^2+\lambda_n(1+m)\right) \le \mathfrak{r}m^2 - e,$
and call it (Δ_2) .

Theorem 2.6: Suppose X real Banach space,let $F: X \longrightarrow X$ is Lipschitzian strongly pseudo-contractive mapping. For $w_0 \in X$, let $\langle x_n \rangle$ be in (1.4), $\lim_{n\to\infty} a_n = 0$ and condition (Δ_2) satisfied .Then:
1- $\langle x_n \rangle$ converges strongly to the unique fixed pointp .
2- $\|q_{n+1} - p\| \le \delta_n + [1 - \frac{\lambda_n e}{1+\lambda_n}]\|q_n - p\| + \|a_n\|, \forall n \ge 0.$

Proof: From Lemma (1.9), we obtain F has a unique fixed point..

$$F y_n = x_{n+1} - a_n$$
$$= x_{n+1} + 2\lambda_n x_{n+1} - 2\lambda_n x_{n+1} - \mathfrak{r}\lambda_n x_{n+1} + \mathfrak{r}\lambda_n F x_{n+1} - \lambda_n F x_{n+1} + \lambda_n F x_{n+1} - a_n$$
$$= (1+\lambda_n)x_{n+1} + \lambda_n(I - F - \mathfrak{r}I)x_{n+1} + \lambda_n(F x_{n+1} - F y_n) - (1-\mathfrak{r})\lambda_n F y_n$$
$$-(1+(2-\mathfrak{r})\lambda_n)a_n \quad (2.15)$$
$$= (1+\lambda_n)p + \lambda_n(I - F - \mathfrak{r}I)p - (1-\mathfrak{r})\lambda_n p \quad (2.16)$$

So that

$$F y_n - p = (1+\lambda_n)(x_{n+1} - p) + \lambda_n[(I - F - \mathfrak{r}I)x_{n+1} - (I - F - \mathfrak{r}I)p] - (1-\mathfrak{r})\lambda_n(F y_n - p) + \lambda_n(F x_{n+1} - F y_n) - (1+(2-\mathfrak{r})\lambda_n)a_n$$

$$\|F y_n - p\| \ge (1+\lambda_n)\left\|(x_{n+1} - p) + \frac{\lambda_n}{1+\lambda_n}[(I - F - \mathfrak{r}I)x_{n+1} - (I - F - \mathfrak{r}I)p]\right\| -(1-\mathfrak{r})\lambda_n\|F y_n - p\| - \lambda_n\|F x_{n+1} - F y_n\| - 3\|a_n\|$$

Thus

$$(1+\lambda_n)\|x_{n+1} - p\|$$
$$\le (1+(1-\mathfrak{r})\lambda_n)\|F y_n - p\| + \lambda_n\|F x_{n+1} - F y_n\| + 3\|a_n\|$$
$$\|x_{n+1} - p\| \le \frac{1}{1+\lambda_n}[(1+(1-\mathfrak{r})\lambda_n)\|F y_n - p\| + \lambda_n\|F x_{n+1} - F y_n\| + 3\|a_n\|] \quad (2.17)$$

Observe that

$$\|F y_n - p\| \le m[(1-\lambda_n)\|x_n - p\| + \lambda_n\|F x_n - p\|]$$
$$= m(1-\lambda_n + m\lambda_n)\|x_n - p\| \le m^2\|x_n - p\| \quad (2.18)$$

(Since $1 \le m$ yields $(1-\lambda_n + m\lambda_n) \le m$

$$\|F x_{n+1} - F y_n\| \le m\|x_{n+1} - y_n\|$$
$$\le m[\|F y_n - x_n\| + \lambda_n\|x_n - F x_n\| + \|a_n\|]$$
$$= m[(1+m^2+\lambda_n(1+m))\|x_n - p\| + \|a_n\|] \quad (2.19)$$

By substituting (2.19) and (2.18) in (2.17), we yield

$$\|x_{n+1} - p\| \le \frac{1}{1+\lambda_n}[[(1+(1-\mathfrak{r})\lambda_n)m^2 + \lambda_n m[(1+m^2+\lambda_n(1+m))]\|x_n - p\| + \|a_n\|] + 3\|a_n\|]$$

$$\|x_{n+1}-\mathrm{p}\| = \frac{1}{1+\lambda_n}[(1+(1-\mathfrak{r})\lambda_n)m^2+\lambda_n m((1+m^2+\lambda_n(1+m))]\|x_n-\mathrm{p}\| + \left[\frac{\lambda_n}{1+\lambda_n}m+\frac{3}{1+\lambda_n}\right]\|a_n\|$$

$$\leq [1-\frac{\lambda_n}{1+\lambda_n}[m^2\mathfrak{r}-m(1+m^2+\lambda_n(1+m))]]\|x_n-\mathrm{p}\|+[m+3]\|a_n\|$$

$$= [1-\frac{\lambda_n e}{1+\lambda_n}\|x_n-\mathrm{p}\|+[m+3]\|a_n\|$$

By applying Lemma (1.7), we get $\lim_{n\to\infty} x_n = \mathrm{p}$.

For (2) :

Let $\langle \mathrm{q}_n \rangle \subset X$, defined $\{\delta_n\}$ by $\delta_n = \|\mathrm{q}_{n+1}-\mathfrak{g}_n-a_n\|$, where

$\mathfrak{g}_n = \mathrm{F}y_n, y_n = \lambda_n \mathrm{F}\mathrm{q}_n+(1-\lambda_n)\mathrm{q}_n+c_n, n\geq 0$.

$$\|\mathrm{q}_{n+1}-\mathrm{p}\| \leq \|\mathrm{q}_{n+1}-\mathfrak{g}_n-a_n\|+\|a_n\|+\|\mathfrak{g}_n-\mathrm{p}\| \leq \delta_n+\|a_n\|+\|\mathfrak{g}_n-\mathrm{p}\| \quad (2.20)$$

Since

$$\mathrm{F}y_n = \mathfrak{g}_n$$

$$= \mathfrak{g}_n+2\lambda_n\mathfrak{g}_n-2\lambda_n\mathfrak{g}_n-\mathfrak{r}\lambda_n\mathfrak{g}_n+\mathfrak{r}\lambda_n\mathrm{F}\mathfrak{g}_n-\lambda_n\mathrm{F}\mathfrak{g}_n+\lambda_n\mathrm{F}\mathfrak{g}_n$$

$$= (1+\lambda_n)\mathfrak{g}_n+\lambda_n(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathfrak{g}_n-(2-\mathfrak{r})\lambda_n\mathrm{F}y_n+\lambda_n\mathrm{F}\mathfrak{g}_n$$

$$= (1+\lambda_n)\mathfrak{g}_n+\lambda_n(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathfrak{g}_n+\lambda_n(\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n)-(1-\mathfrak{r})\lambda_n\mathrm{F}y_n \quad (2.21)$$

$$= (1+\lambda_n)\mathrm{p}+\lambda_n(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathrm{p}-(1-\mathfrak{r})\lambda_n\mathrm{p} \quad (2.22)$$

So that

$$\mathrm{F}y_n-\mathrm{p} = (1+\lambda_n)(\mathfrak{g}_n-\mathrm{p})+\lambda_n[(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathfrak{g}_n-(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathrm{p}]-(1-\mathfrak{r})\lambda_n(\mathrm{F}y_n-\mathrm{p}) + \lambda_n(\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n)$$

$$\|\mathrm{F}y_n-\mathrm{p}\| \geq (1+\lambda_n)\left\|(\mathfrak{g}_n-\mathrm{p})+\frac{\lambda_n}{1+\lambda_n}[(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathfrak{g}_n-(\mathrm{I}-\mathrm{F}-\mathfrak{r}\mathrm{I})\mathrm{p}]\right\| -(1-\mathfrak{r})\lambda_n\|\mathrm{F}y_n-\mathrm{p}\|-\lambda_n\|\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n\|$$

$$\geq (1+\lambda_n)\|\mathfrak{g}_n-\mathrm{p}\|-(1-\mathfrak{r})\lambda_n\|\mathrm{F}y_n-\mathrm{p}\|-\lambda_n\|\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n\|$$

This implies to

$$\|\mathfrak{g}_n-\mathrm{p}\| \leq \frac{1}{1+\lambda_n}[(1+(1-\mathfrak{r})\lambda_n)\|\mathrm{F}y_n-\mathrm{p}\|+\lambda_n\|\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n\|] \quad (2.23)$$

Hence

$$\|\mathrm{F}y_n-\mathrm{p}\| \leq m[(1-\lambda_n)\|\mathrm{q}_n-\mathrm{p}\|+\lambda_n\|\mathrm{F}\mathrm{q}_n-\mathrm{p}\|]$$

$$= m(1-\lambda_n+m\lambda_n)\|\mathrm{q}_n-\mathrm{p}\| \leq m^2\|\mathrm{q}_n-\mathrm{p}\| \quad (2.24)$$

(Since $1\leq m$ yields $(1-\lambda_n+m\lambda_n)\leq m$

$$\|\mathrm{F}\mathfrak{g}_n-\mathrm{F}y_n\| \leq m\|\mathfrak{g}_n-y_n\|$$

$$\leq m[\|\mathrm{F}y_n-\mathrm{q}_n\|+\lambda_n\|\mathrm{q}_n-\mathrm{F}\mathrm{q}_n\|]$$

$$= m[(1+m^2+\lambda_n(1+m))]\|\mathrm{q}_n-\mathrm{p}\| \quad (2.25)$$

By substituting (2.25) and (2.24) in (2.23), we yield

$$\|\mathfrak{g}_n-\mathrm{p}\| \leq \frac{1}{1+\lambda_n}[(1+(1-\mathfrak{r})\lambda_n)m^2+\lambda_n m(1+m^2+\lambda_n(1+m))]\|\mathrm{q}_n-\mathrm{p}\|$$

$$\leq [1-\frac{\lambda_n}{1+\lambda_n}[m^2\mathfrak{r}-m(1+m^2+\lambda_n(1+m))]]\|\mathrm{q}_n-\mathrm{p}\| = [1-\frac{\lambda_n e}{1+\lambda_n}]\|x_n-\mathrm{p}\| \quad (2.26)$$

substitute (2.26) in (2.20) to obtain

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$$\|q_{n+1} - p\| \leq \delta_n + [1 - \frac{\lambda_n e}{1+\lambda_n}]\|q_n - p\| + \|a_n\| . \quad (2.27)$$

Theorem 2.7: Assume that X, F, p, $\{x_n\}$, $\{q_n\}$, $\{\lambda_n\}$, and $\{\delta_n\}$ be as in Theorem (2.6). Then $\langle x_n \rangle$ is almost F-stable .

Proof: Let $\sum_{n=0}^{\infty} \delta_n < \infty$, to prove that $\lim_{n\to\infty} q_n = p$.

By using the conclusion (2.27) of Theorem (2.6) and an application of Lemma (1.8), we get

$$\lim_{n\to\infty} q_n = p .$$

Theorem 2.8: Let X, F, p, $\{q_n\}$, $\{\lambda_n\}$, $\{a_n\}$ and $\{\delta_n\}$ be as in Theorem (2.6) . Then $\{x_n\}$ is F-stable.

Proof: Suppose that $\lim_{n\to\infty} \delta_n = 0$, to show that $\lim_{n\to\infty} q_n = p$.

By expresses (2.27) in the form $\rho_{n+1} \leq (1-\gamma_n)\rho_n + \mu_n$, of Lemma(1.7),where $\gamma_n = \frac{\lambda_n e}{1+\lambda_n}$, $\rho_n = \|q_n - p\|$ and $\mu_n = \delta_n + \|a_n\|$, this implies to $\lim_{n\to\infty} q_n = p$.

Corollary 2.9: Let X, F, p, m, $\{q_n\}$, $\{\lambda_n\}$, $\{a_n\}$, $\{\delta_n\}$ be as in Theorem (2.6) and $\langle x_n \rangle$ be in (1.2) . Then $\langle x_n \rangle$:

1- converges strongly to the unique fixed point p .

2- Almost F-stable .

3- F-stable .

3. Applications

Theorem 3.1: An arbitrary real Banach space X, let $F: X \longrightarrow X$ is Lipschitzian strongly accretive mapping . Define $\mathcal{S}: X \longrightarrow X$ by $\mathcal{S}x = f + x - Fx$. Let $\langle \lambda_n \rangle, \langle \eta_n \rangle, \langle a_n \rangle, \langle c_n \rangle$as are in Theorem (2.1) . For $w_0, f \in X$,

$$w_{n+1} = \lambda_n \mathcal{S} z_n + (1-\lambda_n)\mathcal{S} w_n + a_n,$$
$$z_n = \eta_n \mathcal{S} w_n + (1-\eta_n) w_n + c_n, \forall\, n \geq 0 .$$

Then $\langle w_n \rangle$ is :

1- converge strongly to the fixed point p^* the unique solution of the equation $Fx = f$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Proof: The mapping $\mathcal{S}$ is Lipschitzian with a constant $m_* = 1 + m$, and from Lemma (1.9) the equation $Fx = f$has unique solution p^*, that implies $\mathcal{S}$ has a unique fixed point p^* .
From (1.7) and proposition (1.6), hence
$\langle (I-\mathcal{S})x - (I-\mathcal{S})y, j(x-y) \rangle = \langle Fx - Fy, j(x-y) \rangle \geq \mathfrak{r}\|x-y\|^2$, this implies $\mathcal{S}$ is strongly pseudo-contractive, therefore the proof follows from Theorems (2.1), (2.2), (2.3) .

Corollary 3.2: Let X, F, $\mathcal{S}$, $p^*, m, \{q_n\}$, $\{\lambda_n\}$, $\{\eta_n\}, \{\delta_n\}$ be as in Theorem (3.1) and $\langle w_n \rangle$ defined by

$$w_{n+1} = \lambda_n \mathcal{S} z_n + (1-\lambda_n)\mathcal{S} w_n,$$
$$z_n = \eta_n \mathcal{S} w_n + (1-\eta_n) w_n, \forall\, n \geq 0 .$$

Then $\langle w_n \rangle$ is :

1- Converge strongly to the fixed point p^* the unique solution of the equation $Fx = f$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Theorem 3.3: Let X real Banach space, $\mathrm{F}: \mathrm{X} \longrightarrow \mathrm{X}$ is Lipschitzian accretive mapping . Define $\mathcal{S}: \mathrm{X} \longrightarrow \mathrm{X}$ by $\mathcal{S}x = \mathrm{f} - \mathrm{F}x$. Let $\langle\lambda_n\rangle, \langle\eta_n\rangle, \langle a_n\rangle, \langle c_n\rangle$ as are in Theorem (2.1) . For $\mathrm{w}_0, \mathrm{f} \in \mathrm{X}$,

$$\mathrm{w}_{n+1} = \lambda_n \mathcal{S} z_n + (1-\lambda_n)\mathcal{S}\mathrm{w}_n + a_n,$$
$$z_n = \eta_n \mathcal{S}\mathrm{w}_n + (1-\eta_n)\mathrm{w}_n + c_n, \forall\, n \geq 0\,.$$

Then $\langle \mathrm{w}_n \rangle$ is :

1- converge strongly to the unique solution $\mathrm{p}^*$of the equation $x + \mathrm{F}x = \mathrm{f}$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Proof: From Lemma (1.9), hence the equation $x + \mathrm{F}x = \mathrm{f}$ has a unique fixed point p^*, (i.e. $\mathcal{S}$ has aunique fixed point p^*) . By using (1.8), we obtain

$$\|x - y\| \leq \|x - y + \mathrm{r}(\mathrm{F}x - \mathrm{F}y)\| = \|x - y + \mathrm{r}(\mathcal{S}x - \mathcal{S}y)\| \tag{3.1}$$

Since

$$\mathcal{S}\mathrm{w}_n = \mathrm{w}_{n+1} + \lambda_n \mathcal{S}\mathrm{w}_n - \lambda_n \mathcal{S} z_n - a_n$$
$$= (1+\lambda_n)\mathrm{w}_{n+1} - \lambda_n \mathcal{S}\mathrm{w}_{n+1} + \lambda_n(\mathcal{S}\mathrm{w}_{n+1} - \mathcal{S}z_n)\mathcal{S}\mathrm{w}_n + {\lambda_n}^2(\mathcal{S}\mathrm{w}_n - \mathcal{S}z_n) - (1+\lambda_n)a_n$$
$$\mathrm{p}^* = (1+\lambda_n)\mathrm{p}^* - \lambda_n \mathcal{S}\mathrm{p}^*$$

By using (3.1), we get

$$\|\mathcal{S}\mathrm{w}_n - \mathrm{p}^*\| \geq (1+\lambda_n)\left\|(\mathrm{w}_{n+1} - \mathrm{p}^*) + \frac{\lambda_n}{1+\lambda_n}(\mathcal{S}\mathrm{w}_{n+1} - \mathcal{S}\mathrm{p}^*)\right\| - \lambda_n\|\mathcal{S}\mathrm{w}_{n+1} - \mathcal{S}z_n\|$$
$$- \lambda_n^2\|\mathcal{S}\mathrm{w}_n - \mathcal{S}z_n\| - (1+\lambda_n)\|a_n\|$$
$$\geq (1+\lambda_n)\|\mathrm{w}_{n+1} - \mathrm{p}^*\| - \lambda_n^2\|\mathcal{S}\mathrm{w}_n - \mathcal{S}z_n\| - \lambda_n\|\mathcal{S}\mathrm{w}_{n+1} - \mathcal{S}z_n\| - (1+\lambda_n)\|a_n\|$$

This implies to

$$\|\mathrm{w}_{n+1} - \mathrm{p}^*\| \leq \frac{1}{1+\lambda_n}\|\mathcal{S}\mathrm{w}_n - \mathrm{p}^*\| + \frac{\lambda_n}{1+\lambda_n}\|\mathcal{S}\mathrm{w}_{n+1} - \mathcal{S}z_n\| + \frac{\lambda_n^2}{1+\lambda_n}\|\mathcal{S}\mathrm{w}_n - \mathcal{S}z_n\| + \|a_n\|$$

The proof completes by a same way of Theorems (2.1), (2.2), (2.3) .

Corollary 3.4: Let X, $\mathrm{F}, \mathcal{S}$, $\mathrm{p}^*, m, \{\mathrm{q}_n\}$, $\{\lambda_n\}$, $\{\eta_n\}, \{\delta_n\}$ be as in Theorem (3.3) and $\langle \mathrm{w}_n \rangle$ defined by

$$\mathrm{w}_{n+1} = \lambda_n \mathcal{S} z_n + (1-\lambda_n)\mathcal{S}\mathrm{w}_n,$$
$$z_n = \eta_n \mathcal{S}\mathrm{w}_n + (1-\eta_n)\mathrm{w}_n, \forall\, n \geq 0\,.$$

Then $\langle \mathrm{w}_n \rangle$ is :

1- converge strongly to the unique solution $\mathrm{p}^*$of the equation $x + \mathrm{F}x = \mathrm{f}$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Theorem 3.5: Suppose Xreal Banach space, $\mathrm{F}: \mathrm{X} \longrightarrow \mathrm{X}$Lipschitzian strongly accretive mapping .Define $\mathcal{S}: \mathrm{X} \longrightarrow \mathrm{X}$ by $\mathcal{S}x = \mathrm{f} + x - \mathrm{F}x$. Let $\langle\lambda_n\rangle, \langle a_n\rangle$, as are in Theorem (2.6) . For $x_0, \mathrm{f} \in \mathrm{X}$,

$$x_{n+1} = \mathcal{S} y_n + a_n,$$

$$y_n = \lambda_n \mathcal{S} x_n + (1-\lambda_n)x_n, \forall\, n \geq 0.$$

Then $\langle x_n \rangle$ is :

1- converge strongly to the fixed point p^* the unique solution of the equation $\mathrm{F}x = \mathrm{f}$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Proof: The proof follows by a same way of Theorem (3.1) .

Corollary 3.6: Let X, F, $\mathcal{S}$, p^*, m, $\{q_n\}$, $\{\lambda_n\}$, $\{\delta_n\}$ be as in Theorem (3.5) and $\langle x_n\rangle$ defined by

$$x_{n+1} = \mathcal{S}y_n,$$

$$y_n = \lambda_n \mathcal{S}x_n + (1-\lambda_n)x_n, \forall\, n \geq 0.$$

Then $\langle x_n\rangle$ is :

1- converge strongly to the fixed point p^* the unique solution of the equation $Fx = f$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Theorem 3.7: Let Xreal Banach space, $F: X \longrightarrow X$ is Lipschitzian accretive mapping . Define $\mathcal{S}: X \longrightarrow X$ by $\mathcal{S}x = f - Fx$. Let $\langle\lambda_n\rangle, \langle a_n\rangle$, be as in Theorem (2.6) . For $x_0, f \in X$,

$$x_{n+1} = \mathcal{S}y_n + a_n,$$

$$y_n = \lambda_n \mathcal{S}x_n + (1-\lambda_n)x_n, \forall\, n \geq 0.$$

Then $\langle x_n\rangle$ is :

1- converge strongly to the unique solution $p^*$of the equation $x + Fx = f$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Proof: The proof follows by a same way of Theorem (3.3) .

Corollary 3.8: Let X, F, $\mathcal{S}$, p^*, m, $\{q_n\}$, $\{\lambda_n\}$, $\{\delta_n\}$ be as in Theorem (3.7) and $\langle x_n\rangle$ defined by

$$x_{n+1} = \mathcal{S}y_n,$$

$$y_n = \lambda_n \mathcal{S}x_n + (1-\lambda_n)x_n, \forall\, n \geq 0.$$

Then $\langle x_n\rangle$ is :

1- converge strongly to the unique solution $p^*$of the equation $x + Fx = f$.

2- Almost $\mathcal{S}$-stable .

3- $\mathcal{S}$-stable.

Open Problem

Let B be a nonempty closed convex subset of a Banach space X and $\{T_i, S_i, \forall i = 1, 2, \dots, k\}$ be two families of total asymptotically quasi-nonexpansive self-maps. Abed and Hasan [16] studied the convergence of following iteration sequence (a_n) :

$$w_1 \in B$$
$$w_{n+1} = (1-\alpha_{in})S_i^n w_n + w_{in}T_i^n b_{in}$$
$$b_{in} = (1-w_{in})S_i^n a_n + w_{in}T_i^n b_{(i-1)n}$$
$$b_{(i-1)n} = \left(1-\alpha_{(i-1)n}\right)S_{i-1}^n w_n + \alpha_{(i-1)n}T_{i-1}^n b_{(i-2)n}$$

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$$b_{2n} = (1-w_{2n})S_2^n a_n + \alpha_{2n}T_2^n b_{1n}$$
$$b_{1n} = (1-\alpha_{1n})S_1^n w_n + \alpha_{1n}T_1^n b_{on}$$

Where $b_{0n} = w_n \ and \ (\alpha_n)_{n=1}^{\infty}$ are sequence in [0, 1].
We suggest studying the stability of this sequence.

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