

# Considering Product Life Cycle Cost Purchasing Strategy for Solving Vendor Selection Problems

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## Abstract

The framework of product life cycle (PLC) cost analysis is one of the most important evaluation tools for a contemporary high-tech company in an increasingly competitive market environment. The PLC-purchasing strategy provides the framework for a procurement plan and examines the sourcing strategy of a firm. The marketing literature emphasizes that ongoing technological change and shortened life cycles are important elements in commercial organizations. From a strategic viewpoint, the vendor has an important position between supplier, buyer and manufacturer. The buyer seeks to procure the products from a set of vendors to take advantage of economies of scale and to exploit opportunities for strategic relationships. However, previous studies have seldom considered vendor selection (VS) based on PLC cost (VSPLCC) analysis. The purpose of this paper is to solve the VSPLCC problems considering the situation of a single-buyer-multiple-supplier. For this issue, a new VSPLCC procurement model and solution procedure are derived by this paper to minimize net cost, rejection rate, late delivery and PLC cost subject to vendor capacities and budget constraints. Moreover, a real case in Taiwan is provided to show how to solve the VSPLCC procurement problem.

*Key words:* Vendor selection; Product life cycle; Multi-objective linear programming; Multi-choice goal programming.

## 1. Introduction

Modern businesses face an increasingly competitive market environment, in which companies need to shorten product life cycle (**PLC**) to bring their good products to market quickly, and thereby increase their competitive advantages. In particular, the PLC of electronic products has become shorter to support the timing of marketing [1]. A significant challenge faced by the vendor–buyer supply chain (**SC**) is how to deal with the arrangement of the vendor’s uncertain lead time and the buyer’s random demand over the selling season [2]. Accurately determining timing for purchasing is an important issue for procurement plans. The PLC-purchasing strategy (**PS**) offers a framework for procurement plans and examines the sourcing strategy of a firm [3,4]. PLC is a descriptive framework that classifies the development of product-markets into four stages: introduction, growth, maturity, and decline. In the introduction stage, there are few competitors in the market. This provides innovators with a chance to use a price-skimming strategy to recoup their product development costs and encourage knowledge of the new product. In the growth stage, overall market sales increase radically, attracting many new market entrants. The decline stage is entered when overall market sales begin to fall. During this stage products are withdrawn from the market and firms reduce their marketing expenditures to cut costs [5]. It can be seen that using the framework of PLC can act as a guideline to aid purchasing managers in fitting the performance of their ever-expanding duties and tasks for the optimal profit of the company. Purchasing planners have known that what they want to achieve this desired elasticity by fitting procurement actions to each PLC phase. The emphasis in this procurement planning is on the timing of the changes in purchasing activities to create the best utilization of company resources [6].

Another important issue faced by firms is the vendor selection (**VS**) problem. The purchasing firm's preferences or weights associated with various vendor attributes may vary during different stages of the PLC. Supply professionals must balance their firm's quality and delivery policies with the cost saving and flexibility profit offered by vendors, so a vendor's product manufacturing skills are attractive early on the relationship but efficiency dictates in later stages [7]. The concept of PLC cost (**PLCC**) originates from the US Department of Defense which are focused on a product's entire value chain from a cost perspective since the development phase of a product's life, through design, manufacturing, marketing/distribution and finally customer services [8]. In brief, the PLCC methodology aims to assist the producer to forecast and manage costs of a product during its life cycle. PLCC is a good technique used to assess the performance of a PLC. It can evaluate the total cost incurred in a PLC and assists managers in making decisions in all stages [9]. Elmark and Anatoly (2006) indicated that the PLCC is the total cost of acquiring and utilizing a system over its complete life span [10]. Vasconcellos and Yoshimura (1999) proposed a breakdown structure to identify the main activities for the active life cycle of automated systems [11]. Spickova and Myskova (2015) proposed activity based costing, target costing and PLC techniques for optimal costs management [12]. Sheikhalishahi and Torabi (2014) proposed a VS model considering PLCC analysis for manufacturer to deal with different vendors offering replaceable/spare parts [13]. We integrate VS and PLCC (**VSPLCC**) procurement planning into a mode for enterprise to reduce their purchasing cost.

Based on the literature reviews and discussions with experts in this field, we obtain important criteria in the VSPLCC problem including price, transportation cost, quality, quality certification, lead time, necessary buffer stock, goodwill, PLC cost, vendor

reliability, and vendor-area-specific experience. In addition, we would like to maximize the benefit of the procurement process, and must continue to reduce purchasing costs, as well as aiming to achieve minimal cost to obtain the maximum benefit. To help purchasing managers effectively perform and coordinate these responsibilities with their jobs, we need to reconceptualize their role for procurement [14,15]. Schematically, the PLC can be approximated by a bell-shaped curve that is divided into several stages. The PLC is typically depicted as a unit sales curve of a product category over time [7,16].

Narasimhan and Mahapatra (2006) developed a multi-objective decision model that incorporates a buyer's PLC-oriented relative preferences regarding multiple procurement criteria for a portfolio of products [3]. Life cycle costing is concerned with optimizing the total costs in the long run, which consider the trade-offs between different cost elements during the life stages of a product [17]. Their research aims to obtain a comprehensive estimation of the total costs of alternative products or activities in the long run. It is usually possible to affect the future costs beforehand by either planning the use of an asset or by improving the product or asset itself [18]. Previous studies, however, have seldom examined the VSPLCC procurement problem in the situation of single-buyer-multiple-supplier.

The purpose of the study is to consider a VSPLCC problem with a single-buyer multiple-supplier procurement problem. A new VSPLCC procurement model is then proposed to solve the problem in considering the following goals: for more realistic applications, net cost minimization, rejection rate minimization, and late delivery minimization, minimization of PLCC, and vendor capacities and budget constraints. Moreover, multi-objective linear programming (MOLP) and multi-choice goal programming (MCGP) approaches are integrated to solve this problem.

The paper is organized as follows. We review the literature regarding the quantitative methods for the VS decision in Section 1. Section 2 presents the formulations and solutions to the VSPLCC procurement problem using both MOLP and MCGP approaches. In Section 3, the solution procedures of the two approaches for VSPLCC procurement problem are presented based on the modified dataset of the auto parts manufacturers' example and a numerical example is adopted from a light-emitting diode company in Taiwan [19]. In Sections 4 discussions of MOLP and MCGP are provided, conclusions regarding the advantages of solving the VSPLCC procurement problem in the four stages of the PLC with MOLP and MCGP approaches are addressed in Sections 5.

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Please insert Figure 1 here  
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## **2. The VSPLCC Procurement Approaches**

### ***2.1 Linear Programming Technique***

Linear programming (LP) is a powerful mathematical technique which can be used to solve PLC problem. Azapagic and Clift (1998) applied LP to assess the environmental performance of a product system [20]. Dowlatshahi (2001) developed a conceptual framework to tactically consider PLC costs [21]. Zimmermann (1978) showed that a problem with fuzzy goals and constraints can be reformulated as conventional LP problem [22]. Ghodsypour and O'Brien (1998) utilized AHP and LP to develop a decision support system for solving VS problems [23]. Kumar et al. (2004) used fuzzy GP to address the effects of information uncertainty on the VS problems [24]. Amid et al. (2006) developed a fuzzy multi-objective LP model to overcome the VS problem with vague information [25]. In addition, Kagnicioglu (2006) first compared two fuzzy multi-objective methods for VS problems [26]. Chang (2007, 2008) proposed the

MCGP method which allows one goal mapping multiple aspiration levels to find the best achievement levels for multiple objective decision making (**MODM**) problems [27, 28]. Accordingly, in order to improve the quality of decision making for solving the VSPLCC problem, we integrate AHP and MCGP methods, wherein both qualitative and quantitative issues are considered for more realistic VSPLCC applications. The AHP-MCGP method is also used to aid decision makers (DMs) in obtaining appropriate weights and solutions for the VSPLCC problem. The proposed VSPLCC model can be easily used to select an appropriate vendor from a number of potential alternatives. The framework adopted for this study is shown in Figure 1.

The formulation of the VSPLCC model requires the following assumptions, indices, decision variables and parameters.

## ***2.2 Fuzzy Multi-Objective Models for the VSPLCC Procurement***

### ***Problem***

- (i) Only one item is purchased from each vendor.
- (ii) Quantity discounts are not considered.
- (iii) No shortage of the item is allowed for any of the vendors.
- (iv) The lead time and demand for the item are constant and known with certainty.

The sets of indices, parameters, and decision variables for the VSPLCC model are listed in Table 1.

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Please insert Table 1 here  
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## ***2.3 VSPLCC Procurement Model***

The multi-objective VSPLCC procurement problem with four fuzzy objectives and some constraints are as follows:

$$\text{Min } Z_{1t} \cong \sum_{i=1}^n \sum_{t=1}^4 P_{it} X_{it} \quad \text{the total net cost} \quad (1)$$

$$\text{Min } Z_{2t} \cong \sum_{i=1}^n \sum_{t=1}^4 Q_{it} X_{it} \quad \text{the rejects items for vendor } i \quad (2)$$

$$\text{Min } Z_{3t} \cong \sum_{i=1}^n \sum_{t=1}^4 L_{it} X_{it} \quad \text{the late delivered items for vendor } i \quad (3)$$

$$\text{Min } Z_{4t} \cong \sum_{i=1}^n \sum_{t=1}^4 C_{it} X_{it} \quad \text{the product life cycle cost for vendor } i \quad (4)$$

The following constraints are given for the VSPLCC procurement problem:

$$\sum_{i=1}^n \sum_{t=1}^4 X_{it} \geq \tilde{D}_t \quad (\text{aggregate demand constraint}) \quad (5)$$

$$X_{it} \leq \tilde{U}_{it} \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4, \quad (\text{capacity constraint}) \quad (6)$$

$$\sum_{i=1}^n \sum_{t=1}^4 r_{it}(X_{it}) \geq p_{it}; \quad t = 1, 2, 3, 4, \quad (\text{total items purchasing constraint}) \quad (7)$$

$$\sum_{i=1}^n \sum_{t=1}^4 f_{it}(X_{it}) \leq F_{it}; \quad t = 1, 2, 3, 4, \quad (\text{quota constraint}) \quad (8)$$

$$P_{it} X_{it} \leq B_{it}; \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4, \quad (\text{budget constraint}) \quad (9)$$

$$X_{it} \geq 0, \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4. \quad (\text{non-negativity constraint}) \quad (10)$$

Eq. (5) presents the aggregate demand constraint large than quantity of items is supplied over a fixed planning period. Eq. (6) presents the vendor product capacity constraint based on the uncertain aggregate demand. Eq. (7) presents the incorporate total item purchasing value constraint. Eq. (8) presents the flexibility of the vendors' quota. Eq. (9) presents the budgetary constraint where no vendor can exceed the budgeted allocated to vendors. Finally, Eq. (10) presents the non-negativity constraint prohibits negative orders. Generally, the tilde sign ( $\sim$ ) indicates that the environment objectives function and constraints are fuzzy [29,30]. The fuzzy decision can be either symmetric or asymmetric depending on whether the objectives and constraints have equal or unequal weights [26,30,31]. These weights can be derived using techniques



such as the AHP with a geometric mean (see details of the process in Chakraborty, Majumder; Sarkar, 2005) [32].

### ***2.3 The Solution of the VSPLCC Procurement Problem Using the Weight Additive Approach***

In this section, we present the general multi-objective model for solving the VS problem. To specify the weights of the goals and constraints in a fuzzy environment, we can use a fuzzy approach, instead of having the DM subjectively assign values to these weights. To obtain the supertransitive approximation of the previous comparison matrix, we construct supplementary matrices  $A^1, A^2, \dots, A^n$ . The  $j$ th row of matrix  $A^j$  is the same as the  $j$ th row of the initial matrix  $A$ , where the supplementary matrix  $(A^j)^T = [a_1^j, a_2^j, \dots, a_n^j]$  and each row of the matrix  $A^j$  is computed as follows ( $T^*$ : Transpose):  $a_j^j = a_j$ ,  $a_1^j = (a_{j1})^{-1} a_j^j$ ,  $a_2^j = (a_{j2})^{-1} a_j^j, \dots, a_n^j = (a_{jn})^{-1} a_j^j$ . Next, we construct the supertransitive approximation,  $A^s = \|a_{ij}^s\|$ ,  $i, j = 1, 2, \dots, n$ , by taking the geometric mean of the corresponding elements from the supplementary matrices  $A^1, A^2, \dots, A^n$ . More formally,  $a_{ij}^s = (a_{ij}^1 \times a_{ij}^2 \times \dots \times a_{ij}^n)^{1/n}$ . Then we obtain the largest value of  $A^s$  with an eigenvector method. The corresponding eigenvector is the optimal weight for the criteria [26,33]. In the solution to the VSPLCC problem model, the AHP with weighted geometric mean (WGM) is calculated using a supertransitive approximation. Thus these weights are assigned separately. In these equations,  $\alpha_{jt}$  is the weighting coefficient that shows the relative importance at the four stages of the PLC.

The following crisp simplex objective programming function used to solve VSPLCC procurement problem.

**Model 1:** The weighted additive (WA) approach [34], which is formulated as follows:

$$\text{Max } \sum_{j=1}^s \sum_{t=1}^4 \alpha_{jt} \lambda_{jt}^* \quad (11)$$

s.t

$$\lambda_{jt} \leq \mu_{zjt}(x), \quad j = 1, 2, \dots, q, \quad t = 1, 2, 3, 4, \quad (12)$$

$$\gamma_{rt} \leq \mu_{hrt}(x), \quad r = 1, 2, \dots, h, \quad t = 1, 2, 3, 4, \quad (13)$$

$$g_{mt}(x) \leq b_{mt}, \quad m = 1, \dots, p, \quad t = 1, 2, 3, 4, \quad (14)$$

$$\lambda_t \in \{0, 1\}, \quad t = 1, 2, 3, 4, \quad (15)$$

$$\sum_{j=1}^s \sum_{t=1}^4 \alpha_{jt} = 1, \quad \alpha_{jt} \geq 0, \quad t = 1, 2, 3, 4, \quad (16)$$

$$x_{nt} \geq 0, \quad n = 1, 2, \dots, i, \quad t = 1, 2, 3, 4. \quad (17)$$

See Amid et al. (2011) [35] for a more detail.

## 2.4 The Solution of the VSPLCC Procurement Problem Based on Lin's

### Weighted Max-Min Approach

Lin (2004) proved that a weighted max-min (WMM) approach could find an optimal solution such that the ratio of the achievement level approximates the ratio of the weight as closely as possible [41]. He noted that the WA model gives heavier weights to objectives of higher achievement levels than do others models. However, the ratio of the achievement levels is not necessarily the same as that of the objectives' weights [35, 36]. Thus, to obtain the solution of the VSPLCC problem model, WMM model is used as follows:

**Model 2:** Lin's WMM approach (Lin, 2004) [36]:

$$\text{Max } \lambda_t, \quad (18)$$

$$\text{s.t. } w_{jt} \lambda_{jt} \leq \mu_{zjt}(x), \quad j = 1, 2, \dots, q, \quad t = 1, 2, 3, 4, \quad (19)$$

$$\gamma_{rt} \leq \mu_{hrt}(x), \quad r = 1, 2, \dots, h, \quad t = 1, 2, 3, 4, \quad (20)$$

$$g_{mt}(x) \leq b_{mt}, \quad m = 1, \dots, p, \quad t = 1, 2, 3, 4, \quad (21)$$

$$\lambda_t \in \{0,1\}, \quad t = 1, 2, 3, 4, \quad (22)$$

$$\sum_{j=1}^s \sum_{t=1}^4 \alpha_{jt} = 1, \quad \alpha_{jt} \geq 0, \quad t = 1, 2, 3, 4, \quad (23)$$

$$x_{nt} \geq 0, \quad n = 1, 2, \dots, i, \quad t = 1, 2, 3, 4. \quad (24)$$

## 2.5 The Solution of the VSPLCC Procurement Problem Based on MCGP Approaches

In real decision-making problems, goals are often interrelated in which DMs can set more aspiration levels using the idea of multi-choice aspiration level (MCAL) to find more appropriate resources so as to reach the higher aspiration level in the initial stage of the solution process Chang (2007) [27]. To address this issue, the MCGP AFM models are developed below.

**Model 3:** The MCGP AFM (achievement function model) (case I) is used in the case of “the more, the better” as follows.

$$\text{Minimize } \sum_{i=1}^n \sum_{t=1}^4 [w_{it}(d_{it}^+ + d_{it}^-) + \alpha_{it}(e_{it}^+ + e_{it}^-)]$$

$$\text{s.t. } f_{it}(X)b_{it} - d_{it}^+ + d_{it}^- = b_{it}y_{it}, \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4, \quad (25)$$

$$y_{it} - e_{it}^+ + e_{it}^- = g_{it,\max}, \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4, \quad (26)$$

$$g_{it,\min} \leq y_{it} \leq g_{it,\max}, \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4, \quad (27)$$

$$d_{it}^+, d_{it}^-, e_{it}^+, e_{it}^- \geq 0, \quad i = 1, 2, \dots, n, \quad t = 1, 2, 3, 4. \quad (28)$$

$X \in F$  where  $F$  is a feasible set and  $X$  is unrestricted in sign.

where  $b_{it} \in \{0, 1\}$  is a binary variable attached to  $|f_{it}(X) - y_{it}|$ , which can be either achieved or released in Eq.(25). In terms of real conditions,  $b_{it}$  is subject to some

appropriate constraints according to real needs.

**Model 4:** The MCGP AFM (case II) is used in the case of “the less, the better” as follows.

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{t=1}^4 [w_{it}(d_{it}^+ + d_{it}^-) + \alpha_{it}(e_{it}^+ + e_{it}^-)]$$

$$\text{s.t.} \quad f_{it}(X)b_{it} - d_{it}^+ + d_{it}^- = b_{it}y_{it}, \quad i = 1, 2, \dots, n, t = 1, 2, 3, 4, \quad (29)$$

$$y_{it} - e_{it}^+ + e_{it}^- = g_{it,\max}, \quad i = 1, 2, \dots, n, t = 1, 2, 3, 4, \quad (30)$$

$$g_{it,\min} \leq y_{it} \leq g_{it,\max}, \quad i = 1, 2, \dots, n, t = 1, 2, 3, 4, \quad (31)$$

$$d_{it}^+, d_{it}^-, e_{it}^+, e_{it}^- \geq 0, \quad i = 1, 2, \dots, n, t = 1, 2, 3, 4. \quad (32)$$

where all variables are defined as in model 3. The mixed-integer terms Eqs.(29) and (32) can easily be linearized using the linearization method (Chang, 2008) [28]. As seen in Eqs.(25), (29), (30) and (31), there are no selection restrictions for a single goal, but some dependent relationships exist among the goals. For instance, we can add the auxiliary constraint  $b_{it} \leq b_{i+1,t} + b_{i+2,t}$  to the MCGP AFM, where  $b_{it}$ ,  $b_{i+1,t}$  and  $b_{i+2,t}$  are binary variables. As a result,  $b_{i+1,t}$  or  $b_{i+2,t}$  must equal 1 if  $b_{it} = 1$ . This means that if goal 1 has been achieved, then either goal 2 or goal 3 has also been achieved.

## 2.6 The Solution Procedure of VSPLCC Procurement Problem

In order to solve the VSPLCC problem, the following procedure is then proposed.

Step 1: Construct the model for VSPLCC procurement.

Step 2: A WGM technique is used to determine the criteria for MOLP model [37]. A

WGM technique with a supertransitive approximation is used to obtain the

binary comparison matrixes (Narasimhan, 1982) [33].

Step 3: Calculate the criteria of weighted geometric mean for solving VSPLCC problem.

Step 4: Repeat the process individually for each of the remaining objectives. It determines the lower and upper bounds of the optimal values for each objectives corresponding to the set of constraints.

Step 5: Use these limited values (see Table 3) as the lower and upper bounds for the crisp formulation of the VSPLCC problem.

Step 6: Based on steps 4-5 we can find the lower and upper bounds corresponding to the set of solutions for each objective. Let  $Z_{jt}^-$  and  $Z_{jt}^+$  denote the lower and upper bound, respectively, for the  $jt$  th objective ( $Z_{jt}$ ) (Amid, Ghodsypour; O'Brien, 2011) [35].

Step 7: Using the weighted geometric mean with a supertransitive approximation to solve Model 1 by following Eqs. (11) to (17).

Step 8: Formulate and solve the equivalent crisp model of the weighted geometric mean max-min for the VSPLCC problem to solve Model 2 by following Eqs. (18) to (24).

Step 9: Use the weighted geometric mean and the no-PW (penalty weights) formulation of the fuzzy optimization problem to solve Model 3 by following Eqs. (25) to (28).

Step 10: Formulate Model 4 using the weighted geometric mean and the PW formulation of the fuzzy optimization problem by following Eqs. (29) to (32). Assume that the purchasing company manager sets a PW of five for a vendor missing the net cost goal, four for missing the rejection goal, of three for missing the late deliveries goal, and two for exceeding the PLC cost goal (Chang, 2008) [28].

Step 11: The four stages of the PLC cost matrix are given as follows (Demirtas; Ustun, 2009) [37]:

$$\begin{pmatrix} 1.92 & 1.52 & 1.23 & 1.82 \\ 1.04 & 0.92 & 0.86 & 1.00 \\ 3.94 & 3.52 & 3.05 & 3.56 \end{pmatrix}$$

Step 12: Assume that the four stages of the PLC budget matrix are given as follows:

$$\begin{pmatrix} 25,000 & 26,500 & 27,400 & 26,000 \\ 100,000 & 120,000 & 125,000 & 110,000 \\ 35,000 & 36,000 & 37,500 & 35,200 \end{pmatrix}$$

Step 13: Solve the MOLP and MCGP models for the fuzzy optimization problem.

Step 14: Analyze the PLCCs and capacity limitations for the four stages. The procedure of the VSPLCC problem-solving model is illustrated through a numerical example. Figure 2 shows the use of the AHP with a WGM and supertransitive approximation with a WGM technique to the MOLP and MCGP approach models are used to solve VSPLCC problems.

**Please insert Figure 2 here**

### 3. Numerical Example

As global warming intensifies and carbon dioxide emissions are important issues in the warming caused by greenhouse gases. Reducing the greenhouse effect and protecting the Earth's environment are import goals associated with the use of white light-emitting diodes (LED) since they consume substantially less electrical power than other light sources. White-light LED power can reduce the amount of crude oil used in power plants and substantially reduce the generation of CO<sub>2</sub> emissions, which helps to significantly reduce contributions to the greenhouse effect. Thus, according to the estimated from the optoelectronics industry development association's (OIDA), using white LED lighting technology could reduce emissions worldwide by 2.5 billion

tons of CO<sub>2</sub> annually.

We use the VSPLCC procurement model solve a real case in the distribution department for the Everlight Company (the leading LED manufacturer in Taiwan), which is part of a multi-national group for LED research and development (R&D) sector. External purchases account for more than 75% of the total annual costs, and the firm works on a make-to-order basis. The company's management aimed to improve the efficiency of the purchasing process and reconsidering the company's sourcing strategies. A manager felt that the company must evaluate and certify the company's vendors to ensure reductions in product inventory and time to market. The company sought to develop longer-term, trust-based relationships with a smaller group of vendors, and the company manager appointed a team to recommend three or four suitable vendors. This team consisted of several managers from various departments, including purchasing, marketing, quality control, production, engineering and R& D. The members of the team organized several meetings to create profiles for the competing vendors and constructed an initial set of three vendors for evaluation purposes. A VSPLCC procurement model is then developed to select the appropriate vendors and to determine their quota allocations in uncertain environments.

The team considered some objective functions, and constraints as follows: minimizing the net cost, minimizing the net rejections, minimizing the net late deliveries, minimizing the PLC cost, vendor capacity limitations, vendor budget allocations. The other considerations were: price quoted ( $P_i$  in \$), the percentage of rejections ( $R_i$ ), the percentage of late deliveries ( $L_i$ ), the PLCC ( $C_i$ ), the PLC of the vendors' capacities ( $U_i$ ), the vendors' quota flexibility ( $F_i$ , on a scale from 0 to 1), the vendors' ratings ( $R_i$ , on a scale from 0 to 1), and the budget allocations for the vendors

( $B_i$ ) were also considered.

The least amount of flexibility in the vendors' quotas is calculated as  $Q = F \times D$  and the smallest total purchase value is calculated as  $P = R \times D$ . If the overall flexibility ( $F$ ) is 0.03 on a scale of 0–1, if the overall vendor rating ( $R$ ) is 0.92 on a scale of 0–1 and if the aggregate demand ( $D$ ) is 20,000, then the least amount of flexibility in the vendors' quotas ( $F$ ) and the smallest total purchase value of the supplied items ( $P$ ) are 600 and 18,400, respectively. The three vendor profiles are shown in Table 2.

**Please insert Table 2 here**

In this case, the linear membership function is used to fuzzify the right-hand side of the constraints in the VSPLCC problem. The values of the uncertainty levels for all of the fuzzy parameters were taken as 10% of the corresponding values of the deterministic model. The datasets for the values at the lowest and highest aspiration levels of the membership functions are given in Table 3.

**Please insert Table 3 here**

### ***3.1 Application of the WA Approach to the Numerical Example***

We obtained the solution using the WA approach of Tiwari et al. (1987) and in the next section we show the procedure by using the WGM AHP to construct a WGM supertransitive approximation to obtain the binary comparison matrixes.

#### ***3.1.1 Using the WGM AHP Process to Solve the VSPLCC Procurement***

##### ***Problem***

Before determining the solution, we determined the weights of the AHP with the geometric mean process (see Chakraborty et al. 2005 [32]). Evaluating and selecting vendors is a typical MCDM problem involving multiple criteria that can be formulated by both qualitative and quantitative [38]. The VS problem involve tangible and intangible criteria, which may vary depending on the type of product being



considered and may include many judgmental factors [39,24,40].

**Please insert Figure 3 here**

These criteria are shown in Figure 3. The VSPLCC problem addresses how optimally performing vendors can be selected given the desired criteria. The AHP is one of the most widely used MCDM methods it can be used to handle multiple criteria. The criteria for the VS problem are shown in Table 4. Based on the ratings obtained using the questionnaire, the average matrix is shown in Table 5. The maximum value of the eigenvector for the above matrix  $\lambda_{\max}$  is 10.77 [32]. The consistency index C.I. is given by  $(\lambda_{\max} - n) / (n - 1) = 0.09$ . The random index for the matrix of order 10 [41]. R.I. is 1.49. The consistency ratio C.R. is given by  $C.I. / R.I. = 0.06$ , which is not greater than 0.1 (<0.1 acceptable).

**Please insert Tables 4-6 here**

$$A = \Rightarrow \begin{pmatrix} 1 & 6 & 4 & 9 & 3 & 4 & 9 & 9 & 8 \\ 1/6 & 1 & 1/2 & 3 & 1/3 & 1/3 & 2 & 4 \\ 1/4 & 2 & 1 & 4 & 1/2 & 1/2 & 3 & 5 \\ 1/9 & 1/3 & 1/4 & 1 & 1/5 & 1/2 & 2 & 3 \\ 1/3 & 3 & 2 & 5 & 1 & 1 & 4 & 6 & 7 \\ 1/4 & 3 & 2 & 5 & 1 & 1 & 4 & 6 & 7 \\ 1/9 & 1/2 & 1/3 & 2 & 1/4 & 1/4 & 1 & 3 \\ 1/9 & 1/4 & 1/5 & 1/2 & 1/6 & 1/6 & 1/3 & 1 \\ 1/8 & 1/5 & 1/6 & 1/3 & 1/7 & 1/7 & 1/4 & 1 \\ 1/2 & 4 & 3 & 6 & 2 & 2 & 5 & 8 & 9 \end{pmatrix}$$

The AHP process with a geometric mean was applied to this comparison matrix, and the following weights were obtained [53]:  $w_1 = 0.2958$ ,  $w_2 = 0.0579$ ,  $w_3 = 0.0863$ ,  $w_4 = 0.0365$ ,  $w_5 = 0.1291$ ,  $w_6 = 0.1254$ ,  $w_7 = 0.0392$ ,  $w_8 = 0.0199$ ,  $w_9 = 0.0151$ , and  $w_{10} = 0.1949$  (see Section 4.1.1: Using the AHP process with a geometric mean).

The supertransitive approximation method is only used with the WA approach with a geometric mean to matrix A. Supertransitive approximation matrix A is constructed

using the following algorithm described in Section 3.2: Solution to the VSPLCC problem via the WA approach. The ten supplementary matrices corresponding to  $A$  are:

$$A = \begin{bmatrix} 1 & 5.1080 & 3.4278 & 9.2397 & 2.2919 & 2.5851 & 8.6768 & 15.4672 & 19.6386 & 1.3830 \\ 0.1958 & 1 & 0.6711 & 3 & 0.3333 & 0.3333 & 2 & 4 & 4.5622 & 0.25 \\ 0.2917 & 1.4902 & 1 & 4 & 0.5 & 0.5 & 3 & 3.6239 & 4.2701 & 0.3333 \\ 0.1082 & 0.3333 & 0.25 & 1 & 0.2 & 0.3789 & 0.2192 & 3 & 1.6161 & 0.1667 \\ 0.4363 & 3 & 0.7579 & 5 & 1 & 1 & 4.0903 & 6 & 7 & 0.5 \\ 0.3868 & 3 & 2 & 5 & 1 & 1 & 3 & 6 & 7 & 0.5 \\ 0.1153 & 0.5 & 0.2805 & 1 & 0.25 & 0.3333 & 1 & 2.8808 & 4 & 0.2 \\ 0.0647 & 0.25 & 0.2 & 0.5 & 0.1667 & 0.1667 & 0.3471 & 1 & 2 & 0.125 \\ 0.0509 & 0.2 & 0.1667 & 0.3333 & 0.1429 & 0.1429 & 0.25 & 0.5 & 1 & 0.1111 \\ 0.7231 & 4 & 3 & 6 & 2 & 2 & 5 & 8 & 9 & 1 \end{bmatrix}$$

The supertransitive approximation method [54] was applied to this comparison matrix, and the following weights were obtained:  $w_1 = 0.3020$ ,  $w_2 = 0.0611$ ,  $w_3 = 0.0810$ ,  $w_4 = 0.0272$ ,  $w_5 = 0.1226$ ,  $w_6 = 0.1294$ ,  $w_7 = 0.0376$ ,  $w_8 = 0.01936$ ,  $w_9 = 0.0142$ , and  $w_{10} = 0.2057$  and its corresponding eigenvalue  $\lambda_{\max}$  is 9.94. [33]. Table 6 shows the AHP method weight with geometric mean and the supertransitive approximation with the geometric mean. For this VSPLCC problem, we obtained the optimal quota allocations (i.e., the purchasing order), vendor product capacity limitations, and the budget constraints of the different vendors by using the WA approach model (i.e., model 1) in accordance with Eqs. (11) to (17).

### 3.2 Using Lin's WMM Approach to Solve the Numerical Example

For this VSPLCC illustrative example, we obtained the optimal quota allocations (i.e., the purchasing order) subject to vendor product capacity limitations, and budget constraints among the different vendors with Lin's WMM [36].

### 3.2.1 Using a MCGP AFM (model 3: case I) to Solve the Numerical

#### Example

For this VSPLCC procurement problem, we obtained the optimal quota allocations (i.e., the purchasing order), supplier product capacity limitations and budget constraints among the different vendors by using the MCGP method and a no-PW approach (according to Eqs. (25) to (28)). This VSPLCC problem was then formulated as follows (using the first stage of the PLC for Model 1):

$$\text{Max} = 0.2958 \lambda_1 + 0.0579 \lambda_2 + 0.0863 \lambda_3 + 0.0365 \lambda_4 + 0.1291 \lambda_5 + 0.1254 \lambda_6 + \\ 0.0392 \lambda_7 + 0.0199 \lambda_8 + 0.0151 \lambda_9 + 0.1949 \lambda_{10}$$

Main Goals:

$$(G_{11}) \quad 3x_{11} + 2x_{21} + 6x_{31} = 57,000 (G_{11, \text{MIN.}}) \text{ or } 71,833 (G_{11, \text{MAX.}}),$$

$$(G_{21}) \quad 0.05x_{11} + 0.03x_{21} + 0.01x_{31} = 413 (G_{21, \text{MIN.}}) \text{ or } 521 (G_{21, \text{MAX.}}),$$

$$(G_{31}) \quad 0.04x_{11} + 0.02x_{21} + 0.08x_{31} = 604 (G_{31, \text{MIN.}}) \text{ or } 816 (G_{31, \text{MAX.}}),$$

$$(G_{41}) \quad 1.92x_{11} + 1.04x_{21} + 3.94x_{31} = 10,000 (G_{41, \text{MIN.}}) \text{ or } 90,000 (G_{41, \text{MAX.}}).$$

Capacity Constraints Goals:

$$(G_{51}) \quad x_{11} = 5,000 (G_{51, \text{MIN.}}) \text{ or } 5,500 (G_{51, \text{MAX.}}) (X_{11}, \text{ Vendor 1's product capacity}),$$

$$(G_{61}) \quad x_{21} = 15,000 (G_{61, \text{MIN.}}) \text{ or } 165,000 (G_{61, \text{MAX.}}) (X_{21}, \text{ Vendor 2's product capacity}),$$

$$(G_{71}) \quad x_{31} = 6,000 (G_{71, \text{MIN.}}) \text{ or } 165,000 (G_{71, \text{MAX.}}) (X_{31}, \text{ Vendor 3's product capacity}),$$

$$x_{11} + x_{21} + x_{31} = 20,000 \quad (\text{Total demand constraint}).$$

Budget Constraints Goals:

$$(G_{81}) \quad 3x_{11} = 25,000 (G_{81, \text{MIN.}}) \text{ or } 27,500 (G_{81, \text{MAX.}}) (X_{11}, \text{ Vendor 1's budget constraint}),$$

$$(G_{91}) \quad 2x_{21} = 100,000 (G_{91, \text{MIN.}}) \text{ or } 110,000 (G_{91, \text{MAX.}}) (X_{21}, \text{ Vendor 2's budget constraint}),$$

$$(G_{101}) \quad 6x_{31} = 35,000 (G_{101, \text{MIN.}}) \text{ or } 110,000 (G_{101, \text{MAX.}}) (X_{31}, \text{ Vendor 3's budget constraint}).$$

constraint).

### **3.2.2 Using a MCGP AFM (model 4: case II) to Solve the Numerical**

#### ***Example***

The subjectivity inherent to the determination of both the desired level of attainment for each goal and the penalty weights assigned to deviations from the goal may present a problem [19,36]. Suppose that the purchasing company's manager sets a penalty weight of five for the vendor missing the net cost goal, four for missing the rejection goal, three for missing the late deliveries goal, and two for exceeding the PLC cost goal [28]. For this VSPLCC problem, we obtained the optimal quota allocations (i.e., the purchasing order), supplier product capacity limitations and budget constraints among the different vendors using the MCGP method and a PW approach in accordance with Eqs.(29) to (32). After using MOLP and MCGP approaches to solve the VSPLCC problem at the four stages of the PLC, we summarized the results of the problem in Tables 7 to 15. From  $Z_{4t}$  (i.e., the PLC cost goal) of Figure 4, we can see that the maturity stage has the lowest PLC cost, in contrast the growth and decline stages have similar costs and the introduction stage has a high PLC cost.

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**Please insert Tables 7-12 here**  
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**Please insert Figure 4 here**  
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## **4. Discussion of the Results of the Two Types of MOLP and MCGP**

### **Model Approaches**

After solving the VSPLCC procurement problem, we found that Lin's (2004) [36] WMM approach and the MCGP method with the geometric mean and the PW

approach have the same results in the first stage of the PLCs. With regards to the MCGP approaches with the geometric mean (no-PW restrictions),  $x_{11} = 5,000$  (due to the absence of PW constraints),  $b_{11}=1$  and  $b_{51}=1$ . The forced bound order quantity of vendor 1 was 5,000 (i.e., for model 3 at the first stage (Introduction),  $x_{11}=5,000$ ) (see Tables 7 and 11). With regards to the other approaches (i.e., the MCGP approach with the geometric mean and the PW approach),  $b_{12}=1$  and  $b_{62}=1$ . The forced bound order quantity of vendor 2 was greater than 15,000 (i.e., for model 4 at the second period (Growth),  $x_{22}=15,750$ ). To guarantee the net cost goal, the rejection goal or the late delivery goal, zero value should be achieved (e.g., if  $b_{12}=1$  and  $b_{62}=1$ , then forces  $b_{it}$  equal to zero used to adjust the purchasing quantity) (see Tables 8 and 12). We found the MCGP model to be stable with regard to the PLCC in all of the stages (see Tables 13-15).

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Based on the solutions to the two type's goal-programming models, we found that the MCGP model demonstrated more stable control of the PLC cost over all of the stages. We also found that the weighted geometric mean with AHP and PW methods have good control conditions for constructing an MCGP model (model 4) within four stages.

## 5. Conclusions and Managerial Implications

The results obtained using the MOLP and MCGP approaches for determining vendor quotas in SCM if the capacity and budget constraints of each vendor are not

known with certainty. The effectiveness of the VSPLCC procurement model was demonstrated with a real-world problem adopted from a leading LED company in Taiwan. Managers in high-tech companies can easily apply our proposed model to select their vendors in a fuzzy environment using the MOLP and MCGP approaches.

Some managerial implications are found as follows: (i) Doing so is practical because the no-PW and PW MCGP model approaches (MCGP AFM models 3 and 4) do not require precise knowledge of all of the parameters and they make the application of a fuzzy methodology more understandable [35,27,28], (ii) the No-PW and PW MCGP models are demonstrated more stable over all of the PLC stages, and (iii) company managers can easily use MOLP and MCGP approaches to solve VSPLCC procurement problems. In addition, integrating other mathematical models, such as the Pareto concept with AHP [43], ANP [37] with DEAHP [44], or AHP-QFD [45] with the MOGP [46] and MCGP [27,28,47] models to solve the VSPLCC problems in a multi-item/multi-vendor environment that can be performed in conjunction with the various models [48].

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**Conflicts of Interest:** The authors declare no conflict of interest.

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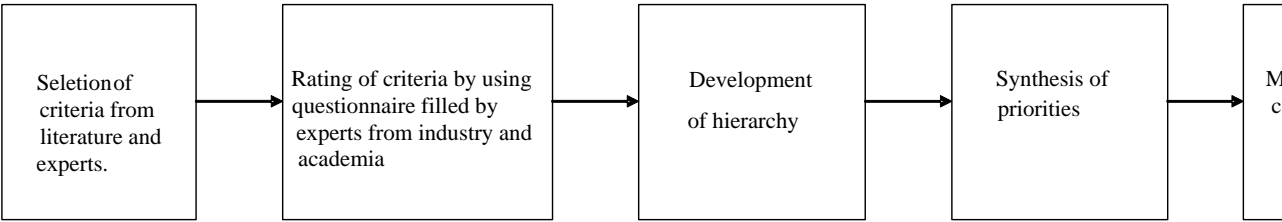
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Selection of vendor using AHP

Figure 1: Framework of the study

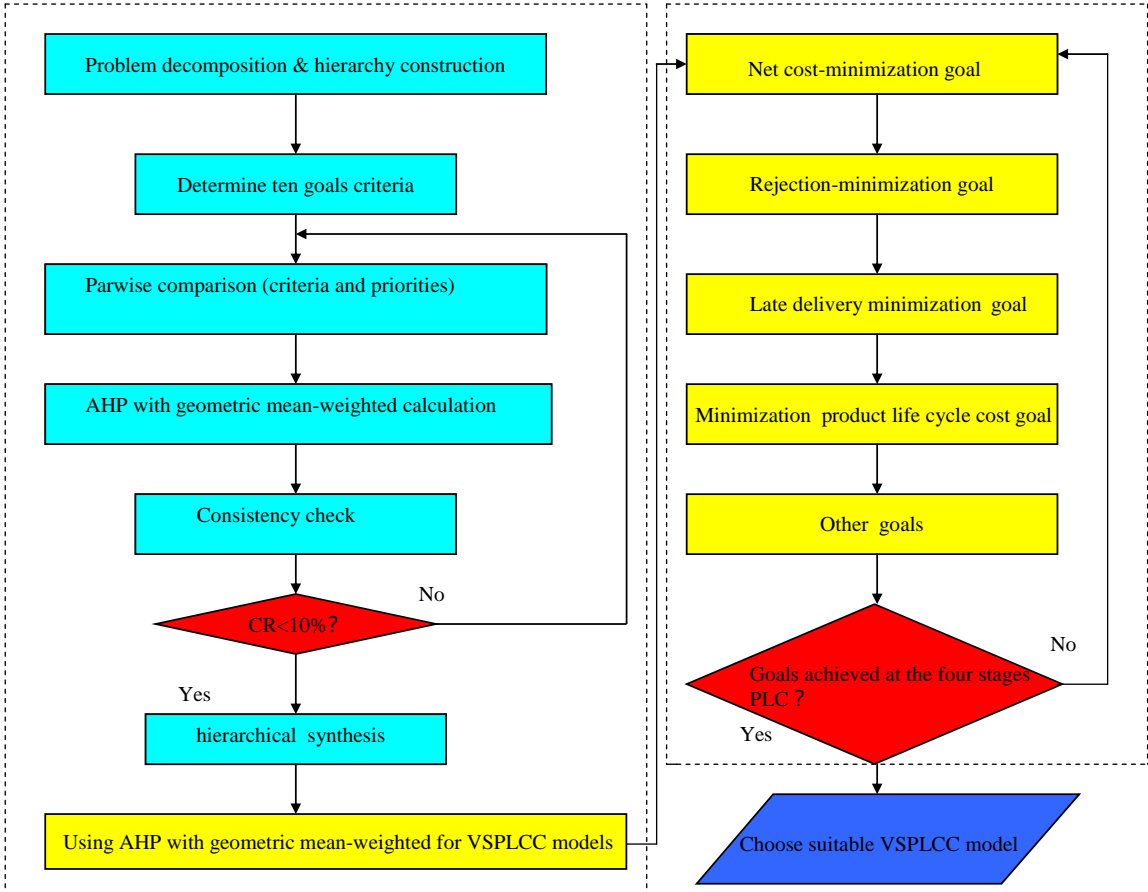


Figure 2. Using AHP and supertransitive approximation with a WGM algorithm with the MOLP and MCGP approach models to solve VSPLCC problems

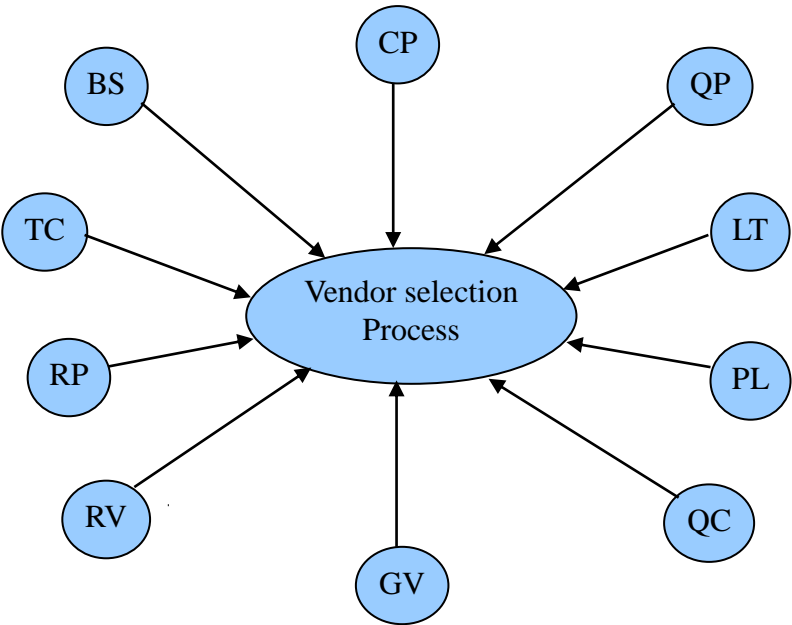


Figure 3. Criteria for the VSPLCC problem

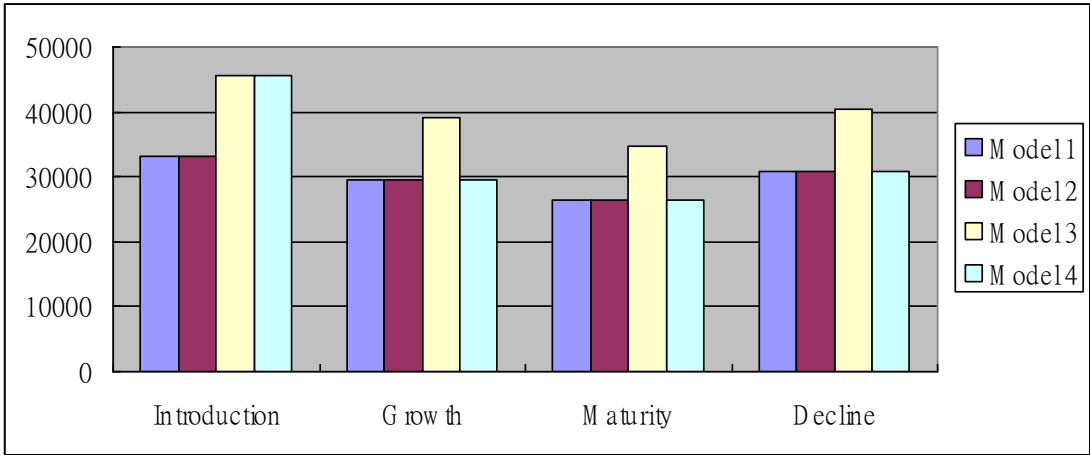


Figure 4.  $Z_{4t}$ : The results of the four VSPLCC models' solutions to the PLCC goal

**Table 1** Nomenclature [fuzzy parameters are shown with a tilde (~)]

<b>Index</b>	
$i$	Index for vendor, for all $i = 1, 2, \dots, n$
$j$	Index for objectives, for all $j = 1, 2, \dots, J$
$k$	Index for constraints, for all $k = 1, 2, \dots, K$
$t$	index objectives and constraints for all at four PLC stages $t = 1, 2, 3, 4$
<b>Decision variable</b>	
$X_{it}$	Ordered quantity given to the vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
<b>Parameters</b>	
$\tilde{D}_t$	Aggregate demand for the item over a fixed planning period, $t = 1, 2, 3, 4$ index for all at four PLC stages
$n$	Number of vendors competing for selection
$p_{it}$	Price of a unit item of ordered quantity $x_i$ for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$Q_{it}$	Percentage of the rejected units delivered for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$L_{it}$	Percentage of the units delivered late for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$C_{it}$	Product life cycle cost of ordered for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$\tilde{U}_{it}$	Upper limit of the quantity available for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$r_{it}$	Vendor rating value for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four

	PLC stages
$P_{it}$	The total purchasing value that a vendor can have, $t = 1, 2, 3, 4$ index for all at four PLC stages
$f_{it}$	Vendor quota flexibility for vendor $i$ , $t = 1, 2, 3, 4$ index for all at four PLC stages
$F_{it}$	The value of flexibility in supply quota that a vendor should have, $t = 1, 2, 3, 4$ index for all at four PLC stages
$B_{it}$	Budget constraints allocated to each vendor, $t = 1, 2, 3, 4$ index for all at four PLC stages

Table 2 Vendor source data for the problem

Vendor No.	$P_i$ (\$)	$R_i$ (%)	$L_i$ (%)	$C_i$ (\$)	$U_i$ (Units)	$r_i$	$F_i$	$B_i$ (\$)
1	3	0.05	0.04	1.92	5,000	0.88	0.02	25,000
2	2	0.03	0.02	1.04	15,000	0.91	0.01	100,000
3	6	0.01	0.08	3.94	6,000	0.97	0.06	35,000

Table 3 Limiting values in the membership function for net cost, rejections, late deliveries, PLC cost, vendor capacities and budget information. (Data for all four stages: introduction, growth, maturity, decline)

	(min.) $\mu = 1$	(max.) $\mu = 0$
Main Goals		
( $G_1$ ) Net cost objective	57,000	71,833
( $G_2$ ) Rejection objective	413	521
( $G_3$ ) Late deliveries objective	604	816
( $G_4$ ) PLC cost objective	10,000	90,000
( $G_5$ ) Vendor 1	5,000	5,500
( $G_6$ ) Vendor 2	15,000	16,500
( $G_7$ ) Vendor 3	6,000	6,600



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 Budget constraints

$(G_8)$ Vendor 1	25,000	27,500
$(G_9)$ Vendor 2	100,000	110,000
$(G_{10})$ Vendor 3	35,000	38,500

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 Table 4 VSPLCC criteria and abbreviations (adopted and modified from Kumar et al., 2009)
 

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Criteria Number	Criteria	Abbreviation used
1	Cost of product	CP
2	Quality of product (based on rejection rate)	QP
3	Lead time (late deliveries)	LT
4	PLC cost	PL
5	Quality certification of the vendor	QC
6	Goodwill of the vendor	GV
7	Reliability of the vendor	RV
8	Price of product	RP
9	Transportation ease and cost	TC
10	Buffer stock of inventory required	BS

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Table 5 The geometric mean matrix for the criteria of the VSPLCC problems

Criteria	CP	QP	LT	PL	QC	GV	RV	RP	TC	BS
CP	1	6	4	9	3	4	9	9	8	2
QP	0.167	1	0.500	3	0.333	0.333	2	4	5	0.250
LT	0.250	2	1	4	0.500	0.500	3	5	6	0.333
PL	0.111	0.333	0.250	1	0.200	0.500	2	3	3	0.167
QC	0.333	3	2	5	1	1	4	6	7	0.500
GV	0.250	3	2	5	1	1	4	6	7	0.500
RV	0.111	0.500	0.333	2	0.250	0.250	1	3	4	0.200
RP	0.111	0.250	0.200	0.500	0.167	0.167	0.333	1	2	0.125
TC	0.125	0.200	0.167	0.333	0.143	0.143	0.250	0.500	1	0.111
BS	0.500	4	3	6	2	2	5	8	9	1
Total	2.9583	20.2833	13.45	35.833	8.5929	9.8929	30.5833	45.5	52	5.186

Table 6 AHP method weight and supertransitive approximation with geometric mean

Criteria number	Criteria	AHP method weight	Supertransitive proximation
		$\lambda_{\max} = 10.77$	$\lambda_{\max} = 9.94$
1	CP	0.2958	0.3020
2	QP	0.0579	0.0611
3	LT	0.0863	0.0810
4	PL	0.0365	0.0272
5	QC	0.1291	0.1226
6	GV	0.1254	0.1294
7	RV	0.0392	0.0376
8	RP	0.0199	0.0193
9	TC	0.0151	0.0142
10	BS	0.1949	0.2057

Table 7 PLCC model during the first stage (Introduction)

	$Z_1$	$Z_2$	$Z_3$	$Z_4$
Model 1	57,000	521	656	33,162
Model 2	57,000	515	655	33,125
Model 3	72,980	560	920	45,486
Model 4	72,980	560	920	45,486

Table 8 PLCC during the second stage (Growth)

	$Z_1$	$Z_2$	$Z_3$	$Z_4$
Model 1	57,000	521	656	29,438
Model 2	57,000	515	655	29,450
Model 3	71,980	440	880	39,187
Model 4	57,000	515	655	29,450

Table 9 PLCC during the third stage (Maturity)

	$Z_1$	$Z_2$	$Z_3$	$Z_4$
Model 1	57,000	521	656	26,465
Model 2	57,000	515	655	26,508
Model 3	71,980	440	880	34,709
Model 4	57,000	515	655	26,507

Table 10 PLCC during the fourth stage (Decline)

	$Z_1$	$Z_2$	$Z_3$	$Z_4$
Model 1	57,000	521	656	30,923
Model 2	57,000	515	655	30,880
Model 3	71,980	440	880	40,467
Model 4	57,000	515	655	30,880

Table 11 PLCC during the first period (Introduction)

	Order quantity $x_1$	Order quantity $x_2$	Order quantity $x_3$
Model 1	240	5,570	4,190
Model 2	0	5,570	4,250
Model 3	5,000	8,005	6,995
Model 4	0	15,750	4,250

Table 12 PLCC during the second period (Growth)

	Order quantity $x_1$	Order quantity $x_2$	Order quantity $x_3$
Model 1	240	15,570	4,190
Model 2	0	12,005	7,995
Model 3	0	12,005	7,995
Model 4	0	15,750	4,250

Table 13 All of the models for order quantity of vendor  $x_1$  in the fourth PLC stages

Stages of PLC	Model 1	Model 2	Model 3	Model 4
Introduction	240	0	5,000	0
Growth	240	0	0	0
Maturity	240	0	0	0
Decline	240	0	0	0

Table 14 All of the models for order quantity of vendor  $x_2$  in the four PLC stages

Stages of PLC	Model 1	Model 2	Model 3	Model 4
Introduction	15,570	15,570	8,005	15,570
Growth	15,570	12,005	12,005	15,750
Maturity	15,570	15,750	12,005	15,750
Decline	15,570	15,750	12,005	15,750

Table 15 All of the models for order quantity of vendor  $x_3$  in the four PLC stages

Stages of PLC	Model 1	Model 2	Model 3	Model 4
Introduction	4,190	4,250	6,995	4,250
Growth	4,190	7,995	7,995	4,250
Maturity	4,190	4,250	7,995	4,250
Decline	4,190	4,250	7,995	4,250