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Article

Quantum conditional strategies for prisoners' dilemmata under the EWL scheme

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Abstract: Classic game theory is an important field with a long tradition of useful results. Recently, the quantum versions of classical games, such as the *Prisoner's Dilemma* (PD), have attracted a lot of attention. Similarly, state machines and specifically finite automata have also been under constant and thorough study for plenty of reasons. The quantum analogues of these abstract machines, like the quantum finite automata, have been studied extensively. In this work, we examine some well-known game conditional strategies that have been studied within the framework of the repeated PD game. Then, we try to associate these strategies to proper quantum finite automata that receive them as inputs and recognize them with probability 1, achieving some interesting results. We also study the quantum version of PD under the Eisert-Wilkens-Lewenstein scheme, proposing a novel conditional strategy for the repeated version of this game.

Keywords: quantum game theory, quantum automata, prisoner's dilemma, conditional strategies, quantum strategies

1. Introduction

Quantum game theory has gained a lot of research interest since the first pioneering works of the late '90s [1–5]. Quantum game theory combines the well-known field of game theory with the emerging branch of computation theory that involves quantum principles. The processing of information can be thought as a physical phenomenon, and, hence, information theory is indissociable from applied and fundamental physics. In many cases a quantum system description offers certain advantages, at least in theory, compared to the classic situation [6,7].

In global terms, game theory has been developed to explore the strategic interactions between strategic decision-makers (modelled as players) who try to maximize their payoffs (or minimize their losses) [8,9]. The theory of games involves the scientific study of situations where there is conflict of interest and its principles are applied to various frameworks, such as military scenarios, economics, social sciences, and even biology [10–13]. The theory of games can be obviously seen as a branch of decision theory involving more than one rational decision makers (or "actors").

Quantum game theory is an extension of classical game theory in the quantum domain [1,2,5]. It differs in three primary ways from the classical game theory: superposed initial states, quantum entanglement of states, and superposition of actions to be used upon the initial states. Meyer [1] and Eisert et al. [2] are credited for the first influential works on quantum games considered from the perspective of quantum algorithms.

Strategies in classical game theory are either pure (deterministic) or mixed (probabilistic). While not every two-person zero-sum finite game has an equilibrium in the set of pure strategies, von Neumann showed that there is always an equilibrium at which each player follows a mixed strategy.

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A mixed strategy deviating from the equilibrium strategy cannot increase a player's expected payoff. Meyer demonstrated by the penny-flip game that a player who implements a quantum strategy can increase his expected payoff, and explained this result by the efficiency of quantum strategies compared to the classical ones [1]. Eisert et al. were the first to propose a quantum extension of the prisoner's dilemma game in [2], which is now commonly refereed to as the Eisert-Wilkens-Lewenstein (EWL for short) scheme in the literature.

Quantum computing advocates the use of quantum-mechanical methods for computation purposes. Under this perspective, quantum automata have also been thoroughly investigated [14–18] and have been associated with game theory in [19], where it was shown that they can be related to quantum strategies of repeated games.

In this work, the classical and the quantum variant of the prisoner's dilemma game are studied under their repeated versions. Known conditional strategies, like the Tit for Tat and Pavlov, are analyzed within the framework of quantum computing. Most importantly, a new "disruptive" quantum conditional strategy is proposed and compared against other ones. This proposed conditional strategy takes place upon the Eisert-Wilkens-Lewenstein [2] version of quantum Prisoner's Dilemma, where an irrational player tries to over-exploit the fact that a Nash equilibrium set of strategies coincide with the Pareto optimal set. Another critical aspect of the achieved contribution regards the fact that each game's outcome is then associated to a periodic quantum automaton inspired by the work in [17]. This result reveals a connection between these two concepts, a fact that goes along with other related works. The automata we use for this association are quite compact in terms of number of states, which can be useful since it makes the computation process simpler and easier to calculate. Numerous scenarios are then shown and the paper's contribution is finalized by discussing the outcomes of the proposed disruptive conditional strategy for the player who chooses to follow it. The quantum game in which the proposed work is based on, is the quantum prisoner's dilemma under the Eisert-Wilkens-Lewenstein scheme, where the Nash equilibrium point coincides with the with Pareto optimal point, a necessary condition in order to handle player's actions and introduce the "disruptive" conditional strategy for an irrational player.

The paper is structured as follows: Section 1 introduces the paper's scope and motivations, whereas in Section 2 we discuss related works from the recent literature. The required definitions and notation are presented in Sections 3 and 4. In Section 5 the classical PD strategies are associated to quantum automata and in Section 6 the quantum versions of conditional strategies are included. Section 7 contains the proposal and performance of the quantum "disruptive" conditional strategy and finally, Section 8 concludes the paper.

2. Related work

The *prisoner's dilemma*, called *PD* in short, is a well-known and extensively studied game that shows why two rational individuals might not cooperate, even if cooperation leads to the best combined outcome. In the general case of the PD setup, two criminals are arrested and then are separately interrogated without communicating with each other [9,20]. Each criminal (or player, since it has a game-theoretic aspect) is given the opportunity to either defect or cooperate, which results in 4 different outcomes (years in prison for each of them). Due to the fact that defecting offers a greater benefit for the player, it is rational for each prisoner to betray the other, leading to the worst outcome for them when they both choose to defect.

When this game-like setting is repeatedly played, then we have the *repeated prisoner's dilemma* game, or simply *RPD*. In this extension, the classic game is played repeatedly by the same players who still have the same sets of choices, i.e., to defect or cooperate with each another [21]. It is also customary to refer to this variation of PD as the *iterated prisoner's dilemma* [20,22,23] and thus both terms, repeated and iterated, eventually describe the same concept in the literature, unless otherwise stated. In this work, we choose to use the term *repeated*. In Section 3 we provide the formal definition of PD and RPD.

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Many researchers are keen on solving or simply modeling various problems via a game theoretic approach. For example, the control of access to resources is a fundamental task in computer systems. Evolutionary game theory, which has found application in biology [24], examines the evolution of strategies within games that take place over time. These games become even more interesting when their repeated versions are investigated [20].

The Repeated Prisoner's Dilemma has become the paradigm for the evolution of egoistic cooperation. The "win-stay, lose-shift" (or "Pavlov") strategy is a winning one when players act simultaneously. Experiments with humans demonstrate that collaboration does exist and that Pavlovian players seemed better at the same time than the generous Tit-for-Tat players in the alternative game [25].

Rubinstein in [26] proposed a novel association between automata and the repeated PD. In particular, he studied a variation of the RPD in which a Moore machine (a type of finite state transducer) is associated to each player's strategies, accompanying his research with numerical results. Later, Rubinstein and Abreu examined iterative games, i.e., games that are infinitely played, using Nash equilibrium as a solution concept, where players try to maximize their gain and in the same time minimize the complexity of the strategies they follow [27].

Landsburg defined a new two-player game $\mathbf{G}^{\mathbb{Q}}$ for any 2×2 game [28]. In the $\mathbf{G}^{\mathbb{Q}}$ game, each player's (mixed) strategy set consists of the set of all probability distributions on the 3-sphere $\mathbb{S}^3$. Nash equilibria in the game can be difficult to compute. In the Iterated Prisoner's Dilemma (IPD) game, it has been shown that the Pavlov conditional strategy can lead to cooperation, but it can also be manipulated by other players. To mitigate exploitation, Dyer et al. modify this strategy by proposing a *Rational Pavlov* as the resulting strategy [29].

New successful strategies, particularly outperforming the well known Tit for Tat strategy [22,30], are regularly proposed in the iterated prisoner dilemma game [31]. New ways of thinking were also introduced recently to analyze the game. Mathieu et al. deal with various conditional strategies, including the Tit for Tat, providing several experiments. They identify new experiments which suggest several ways to devise strategies that go one step further and highlight efficient strategies in prison iteration dilemmas and multi-agent systems in particular with software design technology [21].

Golbeck et al. examine the use of genetic algorithms as a tool to develop optimal strategies for the prisoner's dilemma. The Pavlov and Tit for Tat conditional strategies are two successful and well studied approaches that are capable of encompassing this characteristic. Their results show that in all evolved populations hypothesized characteristics are present [32].

Automata in general have already been shown to be related to games and strategies in various works [26,27,33,34]. Andronikos et al. established a sophisticated connection between finite automata and the PQ Penny game, constructing automata for various interesting variations of the game [34]. The proposed semiautomaton is capable of capturing the game's finite variations, introducing the notion of the winning and the complete automaton for either player.

Sousa et al. use a simplified multiplayer quantum game as an access controller to recognize the resource-sharing problem as a competition. The proposed quantum game can be used in quantum computer architecture [35]. An eminent work proposed by Eisert et al. examines how non-zero sum games are quantified, proposing a quantum PD variant [2] that is used in the present paper.

For the particular variant of the prisoner dilemma they propose, which is known in the literature as the Eisert-Wilkens-Lewenstein (EWL for short) protocol, they show that if quantum strategies are allowed, this game ceases to pose a dilemma. They also build a certain quantum strategy that always outperforms any classic strategy [2], leading to a Nash equilibrium set of strategies that are, also, Pareto optimal. This work serves as a basis for the underlying quantum variant of the prisoner's dilemma problem and is usually refereed to as the EWL scheme.

The EWL scheme has received criticism by Benjamin and Hayden in [3], due to the restrictions imposed to players' actions (in which Eisert et al. replied in [36]). In particular, they noted that the players' set of actions was, although technically correct, unnecessarily restricted, and, consequently, they proposed a more general form of the quantum PD game. In their version the operator that expresses a player's action is taken from a 3-parameter family of unitary operators instead of two. In this case, the Nash equilibrium was not altered compared to the the classical version. This more general setting would not allow a player to choose a dominant strategy and would effectively lead to the always defect case of the classical setup. Thus, in the present paper we solely rely on the EWL protocol.

The presence of thermal decoherence takes into account a two-player quantum game. It is shown by Dajka et al. [37] that the rigorous Davies approach to modeling thermal environment affects the players' returns. Conditions are identified for the beneficial or harmful effect of decoherence. Their quantitative version of the Prisoner's Dilemma illustrates the general considerations.

Quantum Game Theory is a relatively new field of intensive research [5,38]. In [39] Du et al. systematically study quantum games using the famous example of Prisoner's Dilemma. They present the remarkable properties of quantum play under different conditions, i.e., varied number of players, different players strategic spaces and levels, etc. For the quantum PD, each prisoner is given a single qubit which a referee can entangle. If a third qubit is introduced by the referee, then the underlying Hilbert space is enlarged. Siopsis et al. discuss an improved interrogation technique based on tripartite entanglement in order to analyze the Nash equilibrium. They calculate the Nash equilibrium for tripartite entanglement and show that it coincides with the Pareto-optimum choice of cooperation between the players [40].

Evolutionary games are becoming increasingly important on multilayer networks. While the role of quantum games in these infrastructures is still virtual among previous studies, it could become a fascinating problem in a myriad fields of research. A new framework of classical and quantum prisoners dilemma games on networks is introduced in [41] to compare two different interactive environment and mechanisms.

Li et al. found that quantum interlocking guarantees new kind of cooperation, super-cooperation, of the dilemma games of the quantum prisoners, and this interlocking is the guarantee for the emergence of co-operation in network dilemma games of the evolutionary prisoners [42]. Yong et al. study small-world networks with different values of entanglement to a generalized prisoner's quantum dilemma [43]. Li et al. proposed three generalized prisoners dilemmas (GPD, in short) versions, namely the GPDW, the GPDF and the GPDN, which are based on weak prisoner's dilemma games [44]. Those games are made up of 2 players, all of whom are equipped with 3 actions: Cooperate (C), defect (D) and Super Cooperate (identification by Q) and a μ parameter to measure the linkage between both players.

Cheon et al. examine classic quantum game contents. A quantum strategy with effective density dependent matrices consisting of transposed matrix elements can also be interpreted as a classic strategy [45]. The work in [46] studies the dynamics of a spatial quantum formulation of the dilemma of the iterated prisoner's game. In terms of the discrete quantum walk on the line, iterated bipartite quantum games are implemented by Abal et al., enabling conditional strategies, since two rational operators are choosing a limited set of unitary two-qubit operations [47]. A quantum version of Prisoner's Dilemma is presented as a specific example, in which both players use mixed strategies [47].

An interesting point made by the eminent work of Eisert et al. in [2] was that games of survival are met on a molecular level, where rules of quantum mechanics have already been observed. In this work, it is, also, argued that there is a connection between game theory and the theory of quantum communication. Another challenging work on two-player quantum games for multiple stages was discussed by Bolonek-Lasoń in [48], where an overview of related works is presented in a solid way. The author discusses aspects of quantum games related to the existence of Nash equilibria in general games.

A previous work in [19] discussed the connection between a quantum game and quantum automata. In particular, the game was the PQ Penny-flip game from [1] where one player outperforms the other by employing quantum actions. The iterative version of the game was studied and a dominant

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strategy for the quantum player was associated to quantum periodic ω-automata from [17]. These automata were restricted in the sense that they strictly recognized languages with probability 1. This restriction, albeit its computation drawback, did not affect the association. Instead, it was a perfect match for the computability needs of the association. Although the game's strategies used in [19] were simple, a slight (but required) modification of the definition used there can be apparently used in other games, like the one described here.

Frąckiewicz presented a different approach to repeated games that involve quantum rules, focusing on two-player games with two actions for each player [49,50]. In particular, this work followed the Marinatto–Weber approach for static quantum games [51], where the authors study quantum strategies rather than simple quantum actions, presenting a quantum version of the Battle of Sexes. Game theory is well known to have close ties with the field of economics. This fact is also stretched in the quantum versions of games [2,52,53]. In particular, the quantum prisoner's dilemma game has been studied as a means to enhance cooperation among different parties that try to achieve negotiation goals in [53].

Quantum phenomena, and especially quantum entanglement, are the key to achieve specific outcomes when it comes for quantum games. This was initially shown in the work of Eisert et al. [2] and since then numerous works have added further useful bits of information regarding this quantum supremacy over classical games even in fields other than strictly game-theoretic ones, like multi-agent communication protocols and resource allocation [54,55].

Regarding the related literature on quantum automata, several works have already been proposed. Since their initial proposal back in the late 1990s, when two different models of quantum finite automata, were described in [15] and [16]. In the model of [15], Moore and Crutchfield introduced the measure-once quantum automata in which a single measurement operator is applied in the end of the computation process. Kondacs and Watrous, on the other hand, described the measure-many quantum automata, where multiple measurements are applied, one after each read symbol. A comprehensive overview of these models, along with useful examples is provided By Ambainis and Yakaryilmaz in [14].

3. Definitions and background

This section provides the necessary notation and definitions used in later parts of the paper. Due to the fact that we use ideas from different fields, it is important to clearly state the notation we follow throughout the main sections of the work. This applies to both game-theoretic notions and the required automata definitions.

Game theory involves players which interact with each other, the standard assumption being that this is done in a rational way. The famous *minimax* theorem of von Neumann is widely considered as the starting point of game theory. von Neuman defined a game as something "involving two players who play against each other and their gains add up to zero" (which is now known as zero-sum game). Since then, game theory has been developed extensively in many fields and branches of scientific research. Evolutionary game theory studies the evolution of strategies over time and has been applied to biology [24]. These particular games are of interest when it comes to repeated versions of specific games [20].

Quantum game theory is an extension of classical game theory, based on the physics of information viewed as a physical process. In the simplest form of a classical game between two players with two actions for each of them, both of them can use a bit (a '0' or a '1') to express their choice of strategy. In the quantum version of the classical game, the bit is replaced by the qubit, which, generally, is in a superposition of the two basis "kets" $|0\rangle$ and $|1\rangle$, which are the quantum analogues of the classical 0 and 1, respectively. When two or more qubits are present, they may be entangled, which means that the quantum state of each qubit cannot be described independently of the state of the others, something that further complicates the expected pay-offs of the game.

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Games can be either non-cooperative or cooperative. Cooperative game theory describes the structure, strategies and payoffs of a set of players (coalitions) [20]. In a non-cooperative game cooperation may be incorporated only through the choice of strategies the players make. The strategies chosen by each player are of particular interest to us. We are also interested in the negotiation and formation of the coalition in the cooperative case. Full or incomplete information may also exist for games. When information is not perfect, players have to keep track of what players know, as a learning process.

One of them most famous games in Game Theory is the *Prisoner's Dilemma* or *PD*. The traditional story tells of two criminals that have been arrested and are interrogated separately. The police offer each of them the following deal: "If you confess to the crime (*defect*) and your partner fails to confess (he *cooperates*), you will be set free. If you fail to confess but your partner confesses, you will be sentenced to the maximum penalty. But if you both confess to the crime, you will each get an intermediate sentence. If neither of you confesses, you will be both sent in jail". The PD shows why two rational individuals might not cooperate even if cooperation is in their best interest, thus resulting in a sub-optimal outcome [20].

We define the *strategic form* of a game following the definition of [20].

Definition 1. *A game in strategic form is actually a triple* $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ *in which*

- $N = \{1, 2, \ldots, n\}$ *denotes a finite set of players.*
- S_i *denotes the set of strategies of player* i*, for each player* $i \in N$*.*
- $u_i \colon S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ *is a function that associates each vector of strategies* $s = (s_i)_{i \in N}$ *with the payoff* $u_i(s)$ *to player* i*, for every player* $i \in N$*.*

3.1. Classic PD

Having defined the strategic form a game in Section 3, we proceed to define the Prisoner's Dilemma (or PD) game. PD consists of:

- **A set of players** N, where $N = \{1, 2\}$. In a two player game 1 stands for "Player 1" and 2 stands for "Player 2."
- For each player N_i, there is a set of actions $S_i = \{C, D\}$, where C stands for "Cooperate" and D stands for "Defect."
- The notion of **strategy profiles** captures the combinations of players' strategies such as (DC), (CC), which represent all the combinations of actions that can arise based on the rules of the game. In turn, the actions of the players lead to the payoff utilities assigned to each one of them. Players seek to maximize their payoff.

The ordering of the strategy profiles, from best to worst, where the first action in parentheses represents player 1's action and the second action represents player 2's action, is:

1. (D, C), meaning that player 1 defects and player 2 cooperates (resulting in (s, t) payoff);
2. (C, C), where both 1 and 2 cooperate (resulting in (r, r) payoff);
3. (D, D), where both 1 and 2 defect (resulting in (p, p) payoff), and;
4. (C, D), where 1 cooperates and 2 defects (resulting in (t, s) payoff).

Payoffs in the prism of game theory are numbers (symbolic values can be also used) that reflect the player's motivations and expectations. Payoff values often represent the expected profit, loss, or a utility of the players. They can also stand for continuous measures or they may express a rank of preference of possible outcomes. Payoff is most often represented using a matrix form (like Table 1), but other types of depictions also exist (like the tree form as shown in Figure 1) [20,56].

The players' actions in prisoner's dilemma are represented according to the payoff matrix in Table 1 [2,20]. The symbols used are not chosen at random: r stands for *reward*, s for *sucker's payoff*,

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t represents the *temptation payoff*, and p is for *punishment*. The numerical values used later are in accordance to those by Eisert et al. in [2].

Table 1. Symbolic payoff matrix for the Prisoner's dilemma game.

		Player 1	
		Cooperate (C)	Defect (D)
Player 2	Cooperate (C)	(r, r)	(s, t)
	Defect (D)	(t, s)	(p, p)

If both players cooperate, they both receive the reward r. If both players defect, they both receive the punishment p. If player 1 defects while player 2 cooperates, then player 1 gets the temptation payoff t, while player 2 gets the "sucker's" payoff s. Symmetrically, if 1 cooperates while 2 defects, then 1 receives the sucker's payoff s, while 2 receives the temptation payoff t. Considering that the players are rational and care only about their individual payoff, "Defect" is the *dominant* strategy of the game [20].

The extensive form of the game is described using a game tree, depicted in Figure 1.

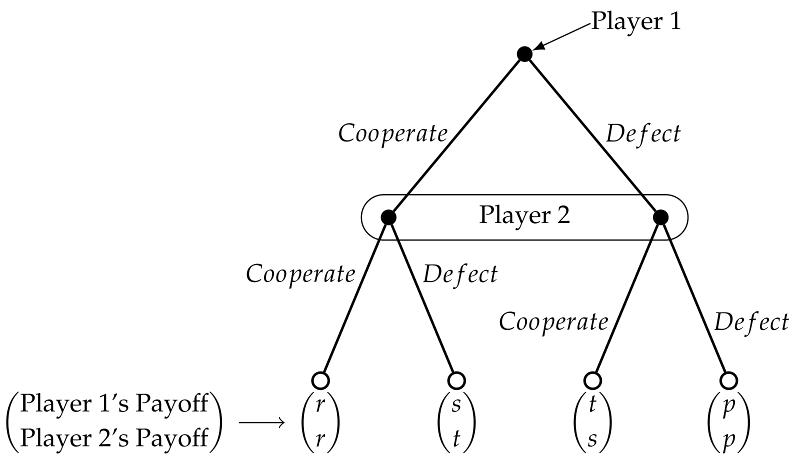

Figure 1. An extensive form of the classical Prisoner's Dilemma game. Two stages of the game are shown, but one can easily see how it evolves.

3.2. *Quantum computing background*

A detailed description of quantum computation can be found in various textbooks, like the one in [7] by Nielsen and Chuang.

In this paper the sets of real and complex numbers are denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively, whereas $\mathbb{C}^{n\times n}$ denotes the set of all $n \times n$ complex matrices. An n-dimensional vector space $\mathbb{C}_n$ endowed with an inner product $\langle x|y\rangle$ is called a Hilbert space $\mathcal{H}_n$ [57]. Typically, the state of a quantum system is represented by a *ket* $|\psi\rangle$ over a complex Hilbert space. Ket is the standard terminology, introduced by Dirac, for a column vector. Each ket $|\psi\rangle$ is a superposition of the basis kets: $|\psi\rangle = \sum_{i=1}^{n} c_i |i\rangle$, where $|i\rangle$ denotes the ith basis ket, $c_i \in \mathbb{C}$ the corresponding probability amplitude, and n is the dimension of the Hilbert space. We assume that kets are normalized, i.e., $|c_1|^2 + |c_2|^2 + \cdots + |c_n|^2 = 1$.

In the finite dimensional case, an operator T is just a matrix of $\mathbb{C}^{n\times n}$. For every operator T, $T^\dagger$ denotes its *adjoint* (conjugate transpose). Every observable is associated with a *Hermitian* operator

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$\mathcal{O} \in \mathbb{C}^{n\times n}$, where Hermitian means that $O^\dagger = O$. Evolution among quantum states is achieved through the application of *unitary* operators U, that is operators for which $U^{-1} = U^\dagger$ holds.

3.3. PD under quantum rules

The quantum version of Prisoner's Dilemma has been extensively investigated since the innovative work of Eisert et al. in [2]. It has been argued that their extension of the PD game is not general enough. Nevertheless, a more general form than the EWL scheme (as suggested in [3]) would not allow a player to act in a dominant way. Such a general setup would invariably end up being equivalent to the always defect case of the classical setup. For this reason, in this work we rely on the EWL protocol.

In the EWL scheme, a physical model of the Prisoners' Dilemma in the context of quantum mechanics gives players the chance to escape the classic dilemma to cooperate (C), or defect (D). The players' quantum actions are operators in a two-dimensional Hilbert space with basis kets $|C\rangle, |D\rangle$. According to the work of Eisert et al. in [2], i.e., the EWL scheme, the strategic space can be represented by the following 2-parameter family of operators:

$$U(\theta,\varphi) := \begin{pmatrix} e^{i\varphi}\cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\varphi}\cos(\frac{\theta}{2}) \end{pmatrix}, \tag{1}$$

where $\theta \in [0,\pi]$ and $\varphi \in [0,\frac{\pi}{2}]$.

Each player is given by a referee a qubit and may only act on it locally via operators chosen from the above family by picking specific values for the parameters θ and φ.

In theory the players' states $|\psi\rangle_i$, $(i = \{1,2\})$ can be entangled. A quantum strategy of a player i is represented as a general unit vector [2]. The operator C denotes the action "Cooperate", while D describes the action "Defect".

$$C \equiv U(0,0) \tag{2}$$

$$C \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \text{Cooperate} \tag{3}$$

$$D \equiv U(\pi,0) \tag{4}$$

$$D \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \rightarrow \text{Defect} \tag{5}$$

Again, we note that the operators described in Equations (1)-(5) are based on the work of Eisert et al. in [2] (that is the EWL scheme), whose quantum version of PD is the basis for the work described in this paper.

3.3.1. How it is "quantumly" played

The game is initialized and each player (i.e., player 1 and 2) is given a qubit. The gate $J(\gamma)$ represents an operator which produces entanglement between the two qubits, and, for this reason, it is usually called an *entangler*, where γ is the degree of entanglement. For no entanglement, $\gamma = 0$, whereas for maximal entanglement it should be $\gamma = \pi/2$ (in general, $\gamma \in [0,\pi/2]$). The actions are executed by operators U_1 and U_2 for each player respectively. Gate $J^\dagger(\gamma)$ is a reversible two-bit gate, that "disentangles" the states. The quantum circuit that corresponds to the quantum PD is depicted in Figure 2 (initially proposed by Eisert et al. [2]).

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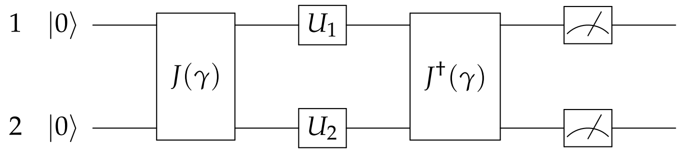

Figure 2. A depiction of the the circuit model of quantum Prisoner's Dilemma as proposed by Eisert et al. in [2].

The main difference between classical and quantum game theory is that in Hilbert space $\mathcal{H}$ the two quantum states $|\psi\rangle_A \in \mathcal{H}_A$ and $|\psi\rangle_B \in \mathcal{H}_B$ are entangled. The overall state of the system is described as a 2-player quantum state $|\Psi\rangle \in \mathcal{H}$. The four basis vectors of the Hilbert space $\mathcal{H}$, defined as the classical game outcomes ($|s_1 s_2\rangle := (1,0,0,0)$, $|s_1 s_2\rangle := (0,-1,0,0)$, $|s_1 s_2\rangle := (0,0,-1,0)$ and $|s_1 s_2\rangle := (0,0,0,1)$) [58].

These vectors are derived from the fact that Equations (3) and (5) correspond to the classical actions of Cooperate and Defect, respectively [2,58]. Assuming that the initial state of both players is $|s_1 s_2\rangle$, the unitary operator J, known to both players, entangles the 2-player system (see [2,59]), leading to the state $J\,|s_1 s_2\rangle$. As shown in these works, it is possible to have a Nash equilibrium point that coincides with the Pareto optimal.

In the quantum version of PD, the states the players are after the actions cooperation or defection are represented by the two basis kets $|C\rangle$ and $|D\rangle$, respectively, in the Hilbert space of a two-state system. The classical action U_1 is selected by setting $\theta = 0$ and $\varphi = 0$, whereas the strategy U_2 is selected by setting $\theta = \pi$ and $\varphi = 0$. The quantum action Q_i is given by $Q_i := U_i(0, \pi/2)$. The two players choose their individual quantum actions ($U_1 := U(\theta_1, \varphi_1)$ and $U_2 := U(\theta_2, \varphi_2)$) and the disentangling operator $J^\dagger$ acts before the measurement of the players' state, which determines their pay-off. The state prior to measurement (i.e., after each qubit is entangled and each player has applied each chosen action) is:

$$\left|\Psi_f\right\rangle = J^\dagger\,(U_1 \otimes U_2)\,J\,|00\rangle \tag{6}$$

The expected payoff depends on the payoff matrix and the joint probability to observe the four observable outcomes $P_{s_1 s_1}, P_{s_1 s_2}, P_{s_2 s_1}$ and $P_{s_2 s_2}$ of the game [52].

$$\begin{aligned} \$_1 &= \$_{11}\,P_{s_1 s_1} + \$_{12}\,P_{s_1 s_2} + \$_{21}\,P_{s_2 s_1} + \$_{22}\,P_{s_2 s_2} \\ \$_2 &= \$_{11}\,P_{s_1 s_1} + \$_{21}\,P_{s_2 s_1} + \$_{12}\,P_{s_1 s_2} + \$_{22}\,P_{s_2 s_2} \end{aligned}$$

with $P_{\sigma\sigma'} = \left|\langle\sigma\sigma'|\sigma\sigma'\rangle\,\Psi_f\right|^2,\ \sigma, \sigma' \in \{s_1, s_2\}$.

3.4. Repeated PD and conditional strategies

A classic play of the PD game (i.e., one-stage PD) has a limited range of possible strategies, since there is only one round of the game. The fact that there are only 2 possible actions for each player yields 4 possible outcomes. Thus, each player's strategy is trivial and cannot be widely sophisticated. Therefore, the iterated or repeated variant of this game is of greater interest compared to the simple one-step setting [20]. The outcome of the first play is observed before the second stage begins. The payoff for the entire game is simply the sum of the payoffs from the two stages. This can easily be extended for n stages of the game (for $n > 2$).

In a single round of Prisoner's Dilemma game, two rational individuals might not cooperate, even though it is in their mutual best interest, while according to the payoff matrix they will be driven to defect. Nevertheless, cooperation may be rewarded in the repeated PD. The game is repeatedly played by the same participants [60]. In a repeated PD participants can learn about their counterpart and base their strategy on past moves with no regular logical convention. It is easy to show by induction that defect is the unique Nash equilibrium for the repeated version of the PD game (similar to the one stage version).

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For the scope of this paper, iterative and repeated mean the same thing, although it is customary for iterative to mean a game with infinite stages, whereas in the repeated version the number of stages is finite. Conditional strategies are a set of rules that each player follows when deciding what action to choose having as a condition the other player's behavior in previous stages. For example, the altruistic "always Cooperate" strategy can be exploited by the "always Defect" strategy. Note that the repeated version requires that $2a > b + c$ (according to the payoff as shown in Table 1), in order to prevent alternating of cooperation and defection that would lead to a greater reward than mutual cooperation.

3.4.1. Tit for Tat conditional strategy

"Tit for tat" is a common repeated prisoner's dilemma conditional strategy (found in other games, too), in which a player cooperates in the first round and then chooses the action that the opposing player has chosen in the previous round. In a repeated game, Tit for Tat highlights that cooperation between participants may have a more favorable outcome than a non-cooperative strategy [11,22,30,61]. The reverse Tit for Tat conditional strategy, proposed in [62], is a variant of the standard Tit for Tat, where a player defects on the first move and in consequent stages of the game he plays the reverse action of the opponent's last move.

3.4.2. Pavlov conditional strategy

A completely different strategy, known as Pavlov strategy, is the "win-stay, lose-shift." A Pavlov strategy is based on the principle that if the most recent payoff was high, the same choice would be repeated, otherwise the choice would be changed. In its simplest form, the Pavlov sequence tends to cooperation, unless the previous move had unfavorable outcome (i.e., player cooperated but the other defected). Both strategies (Tit for Tat and Pavlov) reinforce mutual cooperation [61,63].

3.5. Repeated quantum PD

In a repeated quantum game based on the PD, three rules are applied:

- Each player has a choice at each round (or stage), either to cooperate $|0\rangle \rightarrow C$, or to defect $|1\rangle \rightarrow D$.
- A unitary operation (or an action), denoted by U_1 or U_2, respectively, is applied by each player to his/her qubits. Players are unable to communicate with each other.
- Introducing "gates" to entangle the qubits. The actions of the players in the quantum game will result in a final state which will be a superposition of the basis kets. When the final state is measured, the payoff is determined from the table.

The evolutionary aspect in this repeated form of the game allows for cooperative solutions, even in the absence of communication. As discussed in [2,59], the quantum players could escape the dominant strategy to "Defect." As commented in [64] this quantum advantage can become a disadvantage when the game's external qubit source is corrupted by a noisy "demon."

4. Quantum automata

First we point out that two models of quantum finite automata, i.e., the quantum extension of the classical finite state automata, were initially proposed in [15] and [16]. The former model is known as measure-once quantum automata (meaning that a single measurement operator is applied in the end of the computation with respect to the orthonormal kets representing the automaton's accepting states), whereas the latter is known as measure-many quantum automata (since a measurement is applied after each read symbol). A comprehensive overview of these models, along with further insights and examples is provided in [14]. In this work, we follow the measure-once principle, since the periodic quantum automata of [17] include one and only one measurement after reading the last symbol.

We now present the definitions for the periodic quantum ω-automata as proposed in [17]. Based on this formalization, we provide the definition for the case of finite inputs, which is derived in a

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straightforward way. For a deeper introduction to classical automata and computation theory, the reader is referred to [65].

At first, we have the definition of the periodic quantum ω-automata as proposed in [17]. Albeit its simplicity and its inherited limitations, this model is capable of recognizing particular languages with probability 1 (this restriction could be lifted or modified, but it would not contribute anything for the purposes of this work).

Definition 2. *A simple μ-periodic, 1-way quantum ω-automaton with periodic measurements is a tuple $(Q, \Sigma, U_{\alpha\in\Sigma}, q_0, \mu, F, P, Acc)$, where Q is the finite set of states, Σ is the input alphabet, $U_{\alpha\in\Sigma}$ is the $n \times n$ unitary matrix that describes the transitions among the states for each symbol $\alpha \in \Sigma$, $q_0 \in Q$ is the initial state, μ is a positive integer that defines the measurement period, $F \subseteq Q$ is the set of final states, P is the set $\{P_0, P_1, \ldots, P_n\}$ of the projectors to the automaton states, and Acc is the almost-sure periodic quantum acceptance condition.*

Periodic quantum ω-automata recognize with probability 1 languages like the $(a^m b)^\omega$. That is, they recognize a language with a probability cut-point $\lambda = 1$. This class of languages was described in [17] and [19]. For the purposes of this work, a cut-point other than $\lambda = 1$ (e.g., $\lambda = c$ with $0 < c < 1$ or $\lambda > 0$ that are usually discussed) would not be necessary, since the game strategies can be associated to the aforementioned automata model. Measurement is performed infinitely often after the reading of μ input symbols (hence, the term periodic) with respect to the orthonormal basis of the automaton's states. Thus, the set P of projection matrices is the same for every automaton used later in this paper.

If we lift the infinite character of the inputs in the previous definition, then we have the μ-periodic quantum automata, where a single measurement is applied after reading μ letters. We keep the notion of periodicity in order to avoid any confusion with the standard measure-once quantum automata of [15]. Periodicity is also somehow "embedded" in the functionality of the periodic quantum automata due to the periodic-like evolution of the operators.

Definition 3. *A simple μ-periodic 1-way quantum automaton is a tuple $(Q, \Sigma, U_{\alpha\in\Sigma}, q_0, \mu, F, P, Acc)$, where Q is the finite set of states, Σ is the input alphabet, $U_{\alpha\in\Sigma}$ is the $n \times n$ unitary matrix that describes the transitions among the states for each symbol $\alpha \in \Sigma$, $q_0 \in Q$ is the initial state, μ is a positive integer that defines the measurement period, $F \subseteq Q$ is the set of final states, and P is the set $\{P_0, P_1, \ldots, P_n\}$ of the projectors to the automaton states.*

Periodic quantum automata accept words of the form $a^m b$, ba^m, etc., with a probability cut-point $\lambda = 1$ (like in the infinite case). Again, measurement is performed after the reading of μ symbols (hence, the term periodic) with respect to the orthonormal basis of the states (but this time only once, unlike the infinite case). Similarly to the definition of the infinite variant, the set P of projection matrices is the same for every automaton used later in this paper, thus it is omitted.

Note that the value of the period μ has to be a positive integer in order for the computation to have a physical meaning (we cannot perform a measurement after reading 2.1 symbols!). For the finite case, it is obvious that the period μ is actually the length of the read sequence of letters, but in order to be consistent with the original definition and due to the connection with the angle factor of each read symbol, we keep the notation "periodic."

5. Quantum automata for the classical repeated PD

The classical PD game has 2 different actions for every player which means that there are 2^2 possible outcomes. We note again that for the purposes of the repeated version of the PD game the outcome of the first play is observed before the second stage begins for all n stages of the game (for $n > 2$). This holds for the quantum version, too. We remind that the game's payoff matrix for each player is shown in Table 2.

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Table 2. Symbolic payoff matrix for the Prisoner's dilemma game.

		Player 1	
		Cooperate	Defect
Player 2	Cooperate	(r, r)	(s, t)
	Defect	(t, s)	(p, p)

It is obvious that the symbolic values can be replaced with numerical values. For the purposes of this work, we use the numeric payoff matrix shown in Table 3.

Table 3. Numerical payoff matrix for the Prisoner's dilemma game.

		Player 1	
		Cooperate	Defect
Player 2	Cooperate	$(3, 3)$	$(0, 5)$
	Defect	$(5, 0)$	$(1, 1)$

This corresponds to the one-stage version of the game. We represent the choice of "Cooperation" with "C" and with "D" the "Defection." Consequently, the game's four possible outcomes are $\{CC, CD, DC, DD\}$. Then, it is easy to map each one of them to a letter of the alphabet $\Sigma = \{a, b, c, d\}$ (as depicted in Table 5).

Table 4. Representation of actions using letters.

Action	representation
Cooperation	C
Defect	D

Table 5. Associating a letter to each possible outcome of the PD game.

Pair of actions	Letter
CC	a
CD	b
DC	c
DD	d

Below, we present a graphical depiction of a periodic automaton like those that will be associated to repeated stages of the PD game. This is shown in Figure 3.

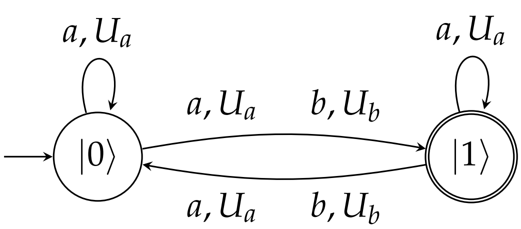

Figure 3. A simple quantum periodic automaton inspired from the work in [17]. The measurement period is $\mu = 5$ (the period is equal to the sum 4 + 1 of the exponents) and it accepts with probability 1 the language $(a^4b)^\mu$ (or in case the input is infinite, it accepts the $(a^4b)^\omega$).

5.1. Classic Pavlov in PD and automata

In this part we discuss the outputs of the the repeated version of the classical game for μ stages, where μ is an arbitrary positive integer that defines the number of plays. Note that this number is *unknown* to the players in order not to have a predefined series of actions [22] which would lead to a dominant strategy for both of them. We follow the actions to letter association as shown in Table 5. The associated automata in every case recognize the respective language with probability cut-point $\lambda = 1$.

We clarify that the projective measurement, with respect to the orthonormal basis of the automaton's states, is performed after the reading of the μ input symbols. Furthermore, in all scenarios, the measurement period μ is equal to the number of stages or, equivalently, to the number of input symbols (and consequently, in the case of repeated versions of the game, $\mu > 1$). For example, if the accepted language is $a^m b^n c^z$, this means that there are $m + n + z$ input symbols in total, that is $m + n + z$ stages of the game are played, which, in turn, implies that $\mu = m + n + z$.

5.1.1. Player 1 plays Pavlov

Initially, we assume that Player 1 chooses to follow the Pavlov strategy. At first, Player 2 always defects, which means that regardless of what Player 1 chooses, Player 2 defects. As seen from the series of plays, the Pavlov strategy can be exploited by the all-defect strategy.

$$\begin{aligned} Player1: &\quad CDDDDDDDDDD \\ Player2: &\quad DDDDDDDDDDD \\ LetterSequence: &\quad bdddddddddd \end{aligned}$$

The regular expression for this sequence can be described as (bd^{m-1}). This enables us to associate it with a variant of the quantum periodic automata that are able to recognize such languages, even if they are infinitely repeated. The automaton defined below can achieve this task.

The language bd^{m-1} is recognized with probability 1 by the μ-periodic quantum automaton M_1 that is defined as the tuple $(Q = \{q_0, q_1\}, \Sigma = \{b, d\}, U_{\alpha \in \Sigma} = \{U_b, U_d\}, q_0, F = \{q_0\}, \mu = m)$, where: $U_b = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$ and $U_d = \begin{pmatrix} \cos(\pi/(m-1)) & \sin(\pi/(m-1)) \\ -\sin(\pi/(m-1)) & \cos(\pi/(m-1)) \end{pmatrix}$.

For the next scenario, we assume that Player 2 always cooperates.

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

The regular expression for this sequence can be described as a^m. This leads us to associate it with a variant of the quantum periodic automata that are able to recognize such languages, even if they are infinitely repeated. The following automaton does the work.

The language a^m is recognized with probability 1 by the μ-periodic quantum automaton M_2 that is defined as the tuple $(Q = \{q_0, q_1\}, \Sigma = \{a\}, \{U_a\}, q_0, F = \{q_0\}, \mu = m)$, where $U_a = \begin{pmatrix} \cos(\pi/m) & \sin(\pi/m) \\ -\sin(\pi/m) & \cos(\pi/m) \end{pmatrix}$.

Next, suppose that Player 2 chooses to follow the Pavlov strategy, similarly to Player 1.

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

For this case, the associated automaton is the same with the above scenario (i.e., M_2), since the underlying language is the same, i.e., a^m.

Next, Player 2 applies the Tit for Tat strategy.

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

Again, the associated automaton is the same as the previous, since the accepted language is a^m.

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Finally, we consider the case where Player 2 follows the Reversed Tit for Tat strategy.

$$\begin{aligned} Player1: &\quad CDDDDDDDDDD \\ Player2: &\quad DDCCCCCCCCC \\ LetterSequence: &\quad bdccccccccc \end{aligned}$$

The regular expression for this sequence is bdc^{m-2}, which is recognized with probability 1 by the μ-periodic quantum automaton M_3 that is defined as the tuple $(Q = \{q_0, q_1\}, \Sigma = \{b, c, d\}, U_{\alpha\in\Sigma} = \{U_b, U_c, U_d\}, q_0, F = \{q_0\}, \mu = m)$, where $U_b = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$, $U_c = \begin{pmatrix} \cos(\pi/(m-2)) & \sin(\pi/(m-2)) \\ -\sin(\pi/(m-2)) & \cos(\pi/(m-2)) \end{pmatrix}$, and $U_d = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$.

5.1.2. Player 1 plays Tit for Tat

In this series of PD games we assume that Player 1 chooses to follow the Tit for Tat strategy, whereas Player 2 always defects.

$$\begin{aligned} Player1: &\quad CDDDDDDDDDD \\ Player2: &\quad DDDDDDDDDDD \\ LetterSequence: &\quad bdddddddddd \end{aligned}$$

The regular expression for this sequence is the bd^{m-1} and the corresponding μ-periodic quantum automaton is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{b, d\}, U_{\alpha\in\Sigma} = \{U_b, U_d\}, q_0, F = \{q_0\}, \mu = m)$, where $U_b = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$ and $U_d = \begin{pmatrix} \cos(\pi/(m-1)) & \sin(\pi/(m-1)) \\ -\sin(\pi/(m-1)) & \cos(\pi/(m-1)) \end{pmatrix}$.

Clearly, this is the same as the previous M_1 automaton.

When Player 2 always cooperates, we have:

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

Its associated automaton is again the same as M_2.

When Player 2 follows the Pavlov logic, we have:

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

Its associated automaton is again the same as M_2.

Then, if Player 2 applies the Tit for Tat strategy, we have:

$$\begin{aligned} Player1: &\quad CCCCCCCCCCC \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad aaaaaaaaaaa \end{aligned}$$

The corresponding automaton is the same as M_2.

For the scenario where Player 2 follows the Reversed Tit for Tat strategy, we have:

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$$\begin{aligned} Player1: &\quad CDDCCDDCCDD \\ Player2: &\quad DDCCDDCCDDC \\ LetterSequence: &\quad bdcabdcabdc \end{aligned}$$

In this case, the resulting automaton is a bit more complicated than those previously encountered, since this series yields the regular language $(bdca)^m$. With $|bdca|$ we denote the length of string $bdca$ (in this case it is equal to 4, but we prefer to use $|bdca|$ in order to emphasize its derivation). The associated μ-periodic quantum automaton M_3 is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{a, b, c, d\}, U_{\alpha\in\Sigma} = \{U_a, U_b, U_c, U_d\}, q_0, F = \{q_0\}, \mu)$, where

$U_a = \begin{pmatrix} \cos(\pi/(m/|bdca|)) & \sin(\pi/(m/|bdca|)) \\ -\sin(\pi/(m/|bdca|)) & \cos(\pi/(m/|bdca|)) \end{pmatrix}$,

$U_b = \begin{pmatrix} \cos(\pi/(m/|bdca|)) & \sin(\pi/(m/|bdca|)) \\ -\sin(\pi/(m/|bdca|)) & \cos(\pi/(m/|bdca|)) \end{pmatrix}$,

$U_c = \begin{pmatrix} \cos(\pi/(m/|bdca|)) & \sin(\pi/(m/|bdca|)) \\ -\sin(\pi/(m/|bdca|)) & \cos(\pi/(m/|bdca|)) \end{pmatrix}$, and

$U_d = \begin{pmatrix} \cos(\pi/(m/|bdca|)) & \sin(\pi/(m/|bdca|)) \\ -\sin(\pi/(m/|bdca|)) & \cos(\pi/(m/|bdca|)) \end{pmatrix}$, and

μ is a positive multiple of $|bdca|$, i.e., $\mu = 4k$, for some positive integer k.

For this particular scenario that seems a bit more complicated, it is important to emphasize that the measurement period μ has to be a multiple of the length of the substring $bdca$ in order to guarantee that the measurement will return an accepting outcome with probability 1 (as stated in the definition of μ-periodic quantum automata in Section 4), otherwise the acceptance cut-point Λ is $0 \leq \lambda \leq 1$. This means that the read symbols (or the number of PD stages) have to be multiples of this length (in this case $|bdca| = 4$).

5.1.3. Player 1 plays Reversed Tit for Tat

Assuming Player 1 chooses to follow the Reversed Tit for Tat strategy, whereas Player 2 always defects, we have:

$$\begin{aligned} Player1: &\quad DCCCCCCCCCC \\ Player2: &\quad DDDDDDDDDDD \\ LetterSequence: &\quad dbbbbbbbbbb \end{aligned}$$

The regular expression for this sequence is db^{m-1} and the associated μ-periodic quantum automaton M_4 is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{b, d\}, U_{\alpha\in\Sigma} = \{U_b, U_d\}, q_0, F = \{q_0\}, \mu = m)$, where

$U_d = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$ and $U_b = \begin{pmatrix} \cos(\pi/(m-1)) & \sin(\pi/(m-1)) \\ -\sin(\pi/(m-1)) & \cos(\pi/(m-1)) \end{pmatrix}$.

This is the same as M_1 by swapping the letter b with the d.

When Player 2 always cooperates, we have:

$$\begin{aligned} Player1: &\quad DDDDDDDDDDD \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad ccccccccccc \end{aligned}$$

The associated automaton for this scenario is the same as M_2, where the unary language a^m is accepted with probability 1. For this scenario all we have to do is to replace the letter a with c.

When the Pavlov strategy is followed by Player 2, we have:

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$$\begin{aligned} Player1: &\quad DDCCCCCCCCC \\ Player2: &\quad CDDDDDDDDDD \\ LetterSequence: &\quad cdbbbbbbbbb \end{aligned}$$

The regular expression for this sequence is the cdb^{m-2}, which is recognized with probability 1 by the μ-periodic quantum automaton M_5 that is defined as the tuple $(Q = \{q_0, q_1\}, \Sigma = \{b, c, d\}, U_{\alpha\in\Sigma} = \{U_b, U_c, U_d\}, q_0, F = \{q_0\}, \mu = m)$, where

$U_c = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$, $U_d = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$, and

$U_b = \begin{pmatrix} \cos(\pi/(m-2)) & \sin(\pi/(m-2)) \\ -\sin(\pi/(m-2)) & \cos(\pi/(m-2)) \end{pmatrix}$.

When Player 2 follows the Tit for Tat, we have the below series of plays:

$$\begin{aligned} Player1: &\quad DDCCDDCCDDC \\ Player2: &\quad CDDCCDDCCDD \\ LetterSequence: &\quad cdbacdbacdb \end{aligned}$$

The associated automaton for this scenario is the same as M_3, where instead of the substring $bdca$ we now have the $cdba$. The language for this scenario is the regular language $(cdba)^m$. Regarding the period μ, the same restrictions with those of M_3 must also hold.

Finally, if Player 2 follows the Reversed Tit for Tat behaviour, it holds:

$$\begin{aligned} Player1: &\quad DCDCDCDCDCD \\ Player2: &\quad DCDCDCDCDCD \\ LetterSequence: &\quad dadadadadad \end{aligned}$$

This series yields the language $(da)^m$. The associated μ-periodic quantum automaton M_6 is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{a, d\}, U_{\alpha\in\Sigma} = \{U_a, U_d\}, q_0, F = \{q_0\}, \mu = 2m)$, where

$U_a = \begin{pmatrix} \cos(\pi/(m/|da|)) & \sin(\pi/(m/|da|)) \\ -\sin(\pi/(m/|da|)) & \cos(\pi/(m/|da|)) \end{pmatrix}$ and $U_d = \begin{pmatrix} \cos(\pi/(m/|da|)) & \sin(\pi/(m/|da|)) \\ -\sin(\pi/(m/|da|)) & \cos(\pi/(m/|da|)) \end{pmatrix}$.

For this scenario the measurement period μ has to be a multiple of the length of the substring da in order to guarantee with probability 1 the desired outcome. This is equivalent with saying that the input symbols (or the number of PD stages) have to be multiples of this length (in this case $|da| = 2$).

6. Quantum version of conditional strategies

In this section we study the quantum version of the repeated prisoner's dilemma game. Now, each player chooses an action that is expressed as a unitary operator that acts on his own qubit. Each player has its own qubit that is entangled using the known J operator. In this case, Alice (or Player 1) has her qubit in state $|0\rangle$ and Bob (or Player 2) has his in state $|1\rangle$.

The actions of each player are expressed using unitary operators that can take infinitely many values according to the scheme described by Eisert et al. in [2]. This scheme was criticized in [3] (in which a subsequent reply from Eisert et al. followed [36]), due to the restrictions imposed to players' actions. Albeit the fact that this limitation leads to a slightly different version of the quantum PD game, it is technically correct and has served as a basis model for several other works [5,28,50,66,67].

We follow the standard quantum setting of the PD game as described in the previous section of this paper (based on the EWL scheme). It is assumed here that each player gets informed about the other player's choice regarding his/her chosen operator. If we combine this assumption with the described strategies of the classical version (like Pavlov, Tit for Tat, etc.) it seems like we pose restrictions to the range of players' actions. This is due to the fact that, although the players are free to choose any action they wish in the first step, their next moves are affected by the previous moves of the other player. This leads to their response being either to continue with their previous action, or

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copy the other player's move, or "flip" the latter's action. As a result, each player's range of actions seemingly converges to a subset of all the possible actions.

Each player's action is expressed with a unitary operator from the two-parameter family of operators of the form:

$$U(\theta,\varphi) := \begin{pmatrix} e^{i\varphi}\cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\varphi}\cos(\frac{\theta}{2}) \end{pmatrix} \tag{7}$$

Player 1 applies the U_1 operator and Players 2 the U_2 operator. Like the classical version of the PD game (and generally of every two-player non cooperative game), conditional strategies like Tit for Tat or Pavlov are defined in a straightforward way. In this part, we consider proper alternations of the conditional strategies from the previous sections and we enrich them with new ones. Next, we proceed to the association with quantum finite automata, as we did for the classical PD.

6.1. Strategies for the quantum PD

Here we establish the strategies for the quantum repeated PD game and then we associate them to inputs of quantum automata. First, the symbolic payoff matrix for the quantum PD game is shown in Table 6, whereas in Table 7 the numerical values for each pair of actions are depicted.

Table 6. The payoff matrix for the quantum PD game, where Players can choose to apply four different actions. C stands for Cooperate, D for Defect, M for the Miracle move, and Q for the Q move.

		Player 1			
		C	D	M	Q
Player 2	C	r,r	s,t	(s,t) or (p,p)	p,p
	D	t,s	p,p	(s,t) or (p,p)	s,t
	M	(t,s) or (p,p)	(t,s) or (p,p)	All	$(r,r) or (s,t)$
	Q	p,p	t,s	(r,r) or (t,s)	r,r

Table 7. The numerical values for the quantum PD game.

		Player 1			
		C	D	M	Q
Player 2	C	$3,3$	$0,5$	$(0,5)$ or $(1,1)$	$1,1$
	D	$5,0$	$1,1$	$(0,5)$ or $(1,1)$	$0,5$
	M	$(5,0)$ or $(1,1)$	$(5,0)$ or $(1,1)$	All	$(3,3) or (0,5)$
	Q	$1,1$	$5,0$	$(3,3)$ or $(5,0)$	$3,3$

When multiple stages of a game are taking place and conditional strategies have to be designed, each player tries to avoid successive losses. To achieve this, Player 1 has to know the action taken by Player 2 in the previous round of the game (and vice versa). This cannot be directly observed by himself since the measurement of the state dissolves any information regarding the choices of the actions of both players. Thus, in our setting, every player announces the unitary operator they used in the previous part and it is assumed that they are not lying about it.

As already noted, the pure classical strategies Cooperate and Defect, are expressed as $U(0,0)$ and $U(\pi,0)$, where U is the operator of Equation (7).

We observe that the cooperate operator:

$$U(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{8}$$

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is the identity operator and the defect operator

$$U(\pi, 0) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{9}$$

resembles the bit-flip operator.

Aside from the two classical actions, we have to introduce two purely quantum actions. The first one is the *Miracle* action [2] that allows the player to always win against the other player's classical strategy. The other action is the Q move and enables the existence of a new Nash equilibrium point that is also a Pareto optimal.

We associate the player's strategies with the following unitary matrices:

$$\begin{aligned} C &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ M &= \frac{1}{\sqrt{(2)}} \begin{pmatrix} i & 1 \\ -1 & -i \end{pmatrix}, \quad Q = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \end{aligned} \tag{10}$$

7. Defining the quantum "disruptive" conditional strategy

In this part of the paper we define the "disruptive" conditional strategy, where one players tries to disrupt the other player's rational choices by choosing the quantum action that deviates from the Pareto optimal. This novel quantum strategy is the first time to be defined and analyzed through comparisons with other conditional strategies. The player who chooses to follow this conditional strategy is no longer rational, instead he tries to sabotage the whole procedure. Bob assumes the role of Player 1 and Alice the role of Player 2.

Definition 4. *A disruptive (or sabotage) quantum strategy for the quantum version of the Prisoner's Dilemma is defined as a series of actions by Bob $S_{1,B}, S_{2,B}, ..., S_{n,B}$, in which he starts with the unitary operator $Q = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ and every time Alice chooses to use Q, he switches to $M = \frac{1}{\sqrt{(2)}} \begin{pmatrix} i & 1 \\ -1 & -i \end{pmatrix}$ for his next turn until Alice either defects, cooperates or uses the miracle move. Then, he continues with Q until Alice chooses again Q.*

A strategy is actually a rule that dictates a player's reaction to another one's actions. This particular strategy tries to puzzle the other player in order to disrupt the pairs of actions as much as possible, taking into account the fact that for most sequences of strategies, the QQ point will be met since it is the Pareto optimal (and Nash equilibrium in the same time). Thus, Player 1 acts in a disruptive way trying to take advantage of the fact that a rational Player 2 will strive for the Pareto optimal condition. From this observation, it is obvious that the disruptive Player is actually an irrational one, aiming at disorientation of the other Player, leading less advantageous outcomes, even for himself.

Such a strategy does not exist in the classical version of the PD game, since in that case the Pareto point does not coincide with the Nash equilibrium. Repeated games differ from one-stage games because the players' actions can ensure retaliation or provide them with rewards according to their preferred strategies.

7.1. Quantum disruptive strategy against others

Similarly to the classical case, we now consider variations of the quantum game played using the disruptive strategy against straightforward quantum variants of the well-known strategies, as was done in the previous section of this paper. A quantum finite automaton is shown to accept the output for each scenario. We note that the associated automata recognize the respective language

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with a probability cut-point $\lambda = 1$ in each case. Again, measurement is performed with respect to the orthonormal basis of the automaton's states, after μ input symbols are consumed by the automaton.

7.1.1. Player 1 plays quantum "disruptive"

To begin with, we have to revisit the letter association to each action. Now each player applies an operator U_1 for Player 1 and U_2 for Player 2. These operators have two parameters for each of them, θ and ϕ. For appropriate values of them, we have the Defect and Cooperate analogs of the standard versions, as well as the Miracle and Q moves. Hence, the possible actions for each player as shown in Table 8 arise.

Table 8. Representation of quantum actions using letters.

Action	representation
Cooperation	C
Defect	D
Miracle Move	M
Q move	Q

This in turn leads to Table 9 of possible outcomes of the game (according to the payoff matrix of Tables 6 and 8), where each possible pair of outcomes is associated to a letter:

Table 9. Associating a letter to each possible outcome of the quantum PD game.

Pair of actions	Letter
CC or QQ	a
CD or DQ	b
CM or DM	c
DC or QD	d
DD or CQ or QC	e
QM	f
MQ	g
MC or MD	h
MM	i

We note that for some particular pairs of actions there are two or more possible outcomes due to the probabilistic nature of the quantum game. Therefore, conditional strategies are affected by the measurement outcome. For the examples below we choose an arbitrary value of every possible outcome.

Assuming Player 1 chooses to follow the quantum "disruptive" strategy, whereas Player 2 always defects, we have:

$$\begin{aligned} Player1: &\quad QQQQQQQQQQQ \\ Player2: &\quad DDDDDDDDDDD \\ LetterSequence: &\quad ddddddddddd \end{aligned}$$

The automaton for this scenario is the same as M_2, where the unary language a^m is accepted with probability 1. For this scenario all we have to do is to replace the letter a with d (that is d^m).

For the below scenario, Player 2 always cooperates.

$$\begin{aligned} Player1: &\quad QQQQQQQQQQQ \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad eeeeeeeeeee \end{aligned}$$

Again, we have a similar association with a previous automaton, in particular the M_2. This time the language is the e^m. This is not that strange considering the quite simplistic strategy chosen by Player 2.

Next, Player 2 always aplies the Miracle move.

$$\begin{aligned} Player1 &: \; QQQQQQQQQQQ \\ Player2 &: \; MMMMMMMMMMM \\ LetterSequence &: \; fffffffffff \end{aligned}$$

Again, we have a similar association with a previous automaton, in particular the M_2. This time the language is f^m.

Next, Player 2 always chooses the Q move.

$$\begin{aligned} Player1 &: \; QMMMMMMMMMM \\ Player2 &: \; QQQQQQQQQQQ \\ LetterSequence &: \; aggggggggggg \end{aligned}$$

This series yields the language ag^{m-1}. The associated μ-periodic quantum automaton M_8 (which is similar to M_4) is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{a, g\}, U_{\alpha\in\Sigma} = \{U_a, U_g\}, q_0, F = \{q_0\}, \mu = m)$, where $U_a = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$ and $U_g = \begin{pmatrix} \cos(\pi/(m-1)) & \sin(\pi/(m-1)) \\ -\sin(\pi/(m-1)) & \cos(\pi/(m-1)) \end{pmatrix}$.

If Player 2 follows the classical Pavlov strategy (meaning that he begins with cooperate), we have:

$$\begin{aligned} Player1 &: \; QQMMMMMMMMM \\ Player2 &: \; CQQQQQQQQQQ \\ LetterSequence &: \; eaggggggggg \end{aligned}$$

This series yields the language eag^{m-2}. The associated μ-periodic quantum automaton M_9 is the tuple $(Q = \{q_0, q_1\}, \Sigma = \{e, a, g\}, U_{\alpha\in\Sigma} = \{U_e, U_a, U_g\}, q_0, F = \{q_0\}, \mu = m)$, where
$U_e = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$, $U_a = \begin{pmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{pmatrix}$, and
$U_g = \begin{pmatrix} \cos(\pi/(m-2)) & \sin(\pi/(m-2)) \\ -\sin(\pi/(m-2)) & \cos(\pi/(m-2)) \end{pmatrix}$.

Next, Player 2 uses again the Pavlov strategy but this time he starts with Q instead of C. Then, we have:

$$\begin{aligned} Player1 &: \; QMMMMMMMMMM \\ Player2 &: \; QQQQQQQQQQQ \\ LetterSequence &: \; aggggggggggg \end{aligned}$$

This series yields the language, which is the same one with the μ-periodic quantum automaton M_8.

Next, Player 2 follows the classic Tit for Tat, that is he starts with cooperate.

$$\begin{aligned} Player1 &: \; QQMMMMMMMMM \\ Player2 &: \; CQQQQQQQQQQ \\ LetterSequence &: \; eaggggggggg \end{aligned}$$

This series yields the language eag^{m-2}, which is identical to the one accepted by the μ-periodic quantum automaton M_9.

Next, Player 2 follows the Tit for Tat starting with the Q move.

$$\begin{aligned} Player1 &: \; QMMMMMMMMMM \\ Player2 &: \; QQQQQQQQQQQ \\ LetterSequence &: \; aggggggggggg \end{aligned}$$

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This series of plays yields the same language with the one of M_8 (i.e., the ag^{m-1}).

Finally, for the below scenario Player 2 uses the Reversed Tit for Tat strategy where he responds with cooperation to defection (and vice versa) and the miracle move to the Q (and vice versa).

$$\begin{aligned} Player1: &\quad QQQQQQQQQQQ \\ Player2: &\quad DMMMMMMMMMM \\ LetterSequence: &\quad dffffffffff \end{aligned}$$

This series yields the language (df^{m-1}), which is similar to (ai^{m-1}) of the M_4 automaton, where instead of the letters a and i we now have d and f, respectively.

7.2. Changing his action to C when the other played M

In this part we propose a "more disruptive" strategy, where Player 1 not only responds with an M move to the other player's Q, he also responds with a C move to the other player's M. Similarly to the previous scenarios, we begin with the setting where Player 2 always defects.

$$\begin{aligned} Player1: &\quad QQQQQQQQQQQ \\ Player2: &\quad DDDDDDDDDDD \\ LetterSequence: &\quad ddddddddddd \end{aligned}$$

Again, we have a similar association with a previous automaton, in particular the M_2. This time the language is the (d^m).

Next, Player 2 always cooperates:

$$\begin{aligned} Player1: &\quad QQQQQQQQQQQ \\ Player2: &\quad CCCCCCCCCCC \\ LetterSequence: &\quad eeeeeeeeeee \end{aligned}$$

Again, we have a similar association with the M_2, since the language is the (e^m).

In the same manner, below there is the setting where Player 2 always applies the miracle move.

$$\begin{aligned} Player1: &\quad QCCCCCCCCCC \\ Player2: &\quad MMMMMMMMMMM \\ LetterSequence: &\quad fcccccccccc \end{aligned}$$

Again, we meet a mirror of the M_1 automaton, for the language (fc^{m-1}).

When Player 2 always applies the Q move, we have:

$$\begin{aligned} Player1: &\quad QMMMMMMMMMM \\ Player2: &\quad QQQQQQQQQQQ \\ LetterSequence: &\quad aggggggggggg \end{aligned}$$

Which yields the association with the M_1 automaton again, this time for (ag^{m-1}).

Things are getting more complicated when Player 2 is forced to follow the Pavlov strategy (with initial cooperation). In this case, we have:

$$\begin{aligned} Player1: &\quad QMMMMMMMMMM \\ Player2: &\quad CQQQQQQQQQQ \\ LetterSequence: &\quad eggggggggggg \end{aligned}$$

This leads to the association with the M_1 automaton again, this time for (eg^{m-1}).

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When Player 2 follows the Pavlov strategy (with initial Q move), we have:

$$\begin{aligned} Player1: &\quad QMMMMMMMMMM \\ Player2: &\quad QQQQQQQQQQQ \\ LetterSequence: &\quad aggggggggggg \end{aligned}$$

This again yields the association with the M_1 automaton, this time for (ag^{m-1}) (as in other scenarios).

Then, Player 2 follows the Tit for Tat strategy choosing to cooperate as a first move.

$$\begin{aligned} Player1: &\quad QQMMCCQQMMC \\ Player2: &\quad CQQMMCCQQMM \\ LetterSequence: &\quad eagicaeagic \end{aligned}$$

This series yields the language $(eagica)^m$), where $|eagica|$ is the length of the substring *eagica*. The associated μ-periodic quantum automaton M_{10} is the tuple (Q={q_0, q_1}, Σ={e,a,g,i,c}, U_δ={U_e, U_a, U_i, U_h}, q_0, μ), where

$$U_a = \begin{pmatrix} \cos(\pi/((m/|eagica|)/2)) & \sin(\pi/((m/|eagica|)/2)) \\ -\sin(\pi/((m/|eagica|)/2)) & \cos(\pi/((m/|eagica|)/2)) \end{pmatrix},$$

$$U_e = \begin{pmatrix} \cos(\pi/((m/|eagica|))) & \sin(\pi/((m/|eagica|))) \\ -\sin(\pi/((m/|eagica|))) & \cos(\pi/((m/|eagica|))) \end{pmatrix},$$

$$U_g = \begin{pmatrix} \cos(\pi/((m/|eagica|))) & \sin(\pi/((m/|eagica|))) \\ -\sin(\pi/((m/|eagica|))) & \cos(\pi/((m/|eagica|))) \end{pmatrix},$$

$$U_i = \begin{pmatrix} \cos(\pi/((m/|eagica|))) & \sin(\pi/((m/|eagica|))) \\ -\sin(\pi/((m/|eagica|))) & \cos(\pi/((m/|eagica|))) \end{pmatrix},$$

and

$$U_c = \begin{pmatrix} \cos(\pi/((m/|eagica|))) & \sin(\pi/((m/|eagica|))) \\ -\sin(\pi/((m/|eagica|))) & \cos(\pi/((m/|eagica|))) \end{pmatrix}.$$

For this particular scenario the measurement period μ has to be a multiple of the length of the substring *eagica* for the measurement to return an accepting outcome with probability 1. This means that the read symbols (or the number of PD stages) have to be multiples of this length (in this case $|eagica| = 6$).

Then, Player 2 chooses the Tit for Tat with the Q as an initial move.

$$\begin{aligned} Player1: &\quad QMMCCQQMMCC \\ Player2: &\quad QQMMCCQQMMC \\ LetterSequence: &\quad agicaeagica \end{aligned}$$

In this scenario, the language is similar to the previous one, thus, it is associated to the same automaton M_{10}, this time the language is $((agicae)^m)$.

Finally, for the next scenario Player 2 chooses the reversed Tit for Tat (swapping actions for C/D and M/Q, starting action is irrelevant).

$$\begin{aligned} Player1: &\quad QQCCQQCCQQC \\ Player2: &\quad DMMDDMMDDMM \\ LetterSequence: &\quad dfcbdfcbdfc \end{aligned}$$

For this case the language is $(dfcb^m)$, which is quite similar to the one of the M_3 automaton (which was for the $(bdca)^m$).

7.3. Notes from applying the "disruptive" conditional strategy

In the previous section we analyzed the evolution of the quantum PD game when it is repeatedly played, assuming that each player follows a deterministic conditional strategy, different in each scenario. In particular, we tested the proposed "disruptive" quantum strategy in contrast to other pro-cooperation quantum strategies, like the quantum Tit for Tat, etc. These strategies (always cooperate, always defect, quantum Tit for Tat, quantum Pavlov, and quantum reversed Tit for Tat) were simply the quantum analogues of the standard ones, whereas new ones were also introduced (the always miracle action and always Q action).

At first, we observe that the proposed strategy leads to low level of disruption for the Player who chooses to follow it against the always defect strategy, whereas it does not seem to have any impact when always cooperate is followed by the other player. This highlights the fact that the disruptive strategy can have no effect when the other player has purely cooperative purposes and does not care for rational actions. This also holds in case Player 2 follows the always miracle action. For these two scenarios the disruptive strategy is weak, but we note that since the quantum PD game as presented in [2] has a Nash pair of actions (Q, Q), the disruptive player assumes that the other player would rationally choose the Q move. That is the reason for choosing the EWL model of quantum PD, since the more general one as proposed in [3] could not serve the purposes of the disruptive conditional strategy (there would not be any rational strategy to "disrupt").

On the other hand, the quantum disruptive conditional strategy fails to complete its mission for both the quantum versions of Pavlov (which is not surprising given the Pavlov's character). Against the quantum Tit for Tat versions, there is a balance regarding the gains for each player, which also holds for the quantum reversed Tit for Tat.

When the disruptive player becomes more disruptive in subsection 7.2, we now have a better result for Player 1 when he competes against the always miracle and always Q strategies (i.e., disruption is achieved). Player 1 has also better results against the quantum Pavlov case, whereas for the Tit for Tat case, the results are roughly the same as in the simple disruptive policy.

8. Conclusion

Game theory is an umbrella term that covers many scientific fields and branches. When infused with traces of quantum computation (or more generally quantum mechanics), then we may speak of quantum game theory. The well-known prisoner's dilemma problem is one of the most studied games and its quantum analogue has been already proposed [1,2,5,19,38]. Since then, it has attracted a lot of attention due to some interesting and counter-intuitive results, like the coincidence of a Pareto optimal point with a Nash equilibrium point for particular instances of this game.

The repeated versions of these games, especially in the classical setting, have also been under the spotlight for various reasons. Trying to shed more light into their quantum analogues, in this paper we have introduced and analyzed the conditional quantum strategies. The most dominant of them are the Tit for Tat and the Pavlov. Repeated games differ from simple games of one stage, since in the former case, the players' actions can lead to retaliation or reward through appropriate strategic choices (some of them favor cooperation and others not).

In this work we propose a new conditional strategy that is applied in the quantum setup of the prisoner's dilemma game. This "disruptive" quantum strategy is defined and, subsequently, its behavior is compared against other strategies that are properly adjusted to fit in the quantum frame of the game. It is shown that this strategy is able to diverse the other player, under the assumption that Player 2 follows some known conditional strategy. Finally, the outcomes of these scenarios, along with scenarios from the classical setting of the PD game, are associated to a variant of quantum automata, namely the periodic quantum automata, which emphasizes the connection among formal methods of state machines with aspects of game theory.

We expect this association to prove quite handy in analyzing distinct plays by players using compact computation schemes, since the periodic quantum automata, albeit weak regarding the recognizing spectrum, are small in size. The results presented in this paper should be useful to anyone studying the repeated version of the quantum PD, providing some noteworthy material and new insights for particular cases. In general, having quantum automata that are able to process and decide inputs related to the repeated form of PD could have an impact on evaluating efficiently and in a sophisticated way such conditional strategies.

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Abbreviations

The following abbreviations are used in this manuscript:

EWL scheme	Eisert-Wilkens-Lewenstein scheme (from [2])
PD	Prisoner's Dilemma
RPD	Repeated Prisoner's Dilemma
C	Cooperate
D	Defect

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