Article

Dynamic Resistance and Flux Pumping

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Abstract: The dynamic resistance is an important parameter in flux pumping. It is simple to calculate in slab geometry but more complex in tapes. In this paper another simple geometry is analysed which is a wire loop. To do this it is essential to make clear what is meant by the term 'voltage', which is ambiguous when there are alternating magnetic fields present. The term can be used if it refers to the EMF along a specified path. However this means that results are dependent on how the voltage leads are attached. If the n value of the V-I characteristic is large simple analytic expressions can be obtained. This will be referred to as the 'Bean' model solution. A more exact approach uses Faraday's law to derive a differential equation which is solved and compared with the Bean model. The dynamic resistance of the loop can be used to drive a flux pump of the type proposed by Geng and Coombs [4] and again simple expressions can be obtained for the performance. If the load is of high inductance L_2 and the loop has an inductance L_1 then the load current increases exponentially with a characteristic number of cycles L_2/L_1. The final current can be close to the critical value.

Keywords: superconductors; flux pumps; electromagnetism.

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1. Introduction

The dynamic resistance in wires is reasonably well understood, but in tapes the current distribution within the tapes is complex and needs finite element analysis to explain the results in detail. Jiang et al [1,2] have measured the dynamic resistance of high T_c tapes in a range of fields and AC amplitudes and found agreement with critical state models for the tape geometry. Ainslie et. al. have compared finite element calculations of the current distribution and consequent dynamic resistance with experiment and also found agreement. [3]

The dynamic resistance has been used by Geng and Coombs to create a novel type of flux pump which has been used in a number of applications [4-7]. The principle is that a superconducting tape in parallel with a load magnet is placed in a perpendicular AC magnetic field. An AC bias current at a lower frequency is applied to the tape. When this current is in one direction the magnetic field is turned on and the dynamic resistance forces some current into the load magnet. When the bias current is in the opposite direction the AC magnetic field is turned off and the strip shorts the load magnet so that no current leaks out.

This process has been modelled in [7] using finite element analysis. However, in order to understand the process it is useful to find a simple system which contains the main parameters but which can be solved analytically. This can be done using a loop of thin wires instead of a tape or sheet. Since this is a circuit model it allows the load magnet to have a realistically large inductance. Finite element programs do not seem to be easily adapted to a boundary condition involving a large external inductance. Although a circuit model can be used, it is essential to be very careful about the use of the term 'voltage'. This is because the alternating fields produce inductive electric fields which are comparable to the resistive ones so that the reading on a voltmeter is dependent on the position of the voltage contacts and the leads to the meter. The unambiguous terms are the electric field and the EMF. The latter is the electric field integrated along a specified path. Therefore, the next section goes into the concept of a voltage in some detail.

2. Voltages

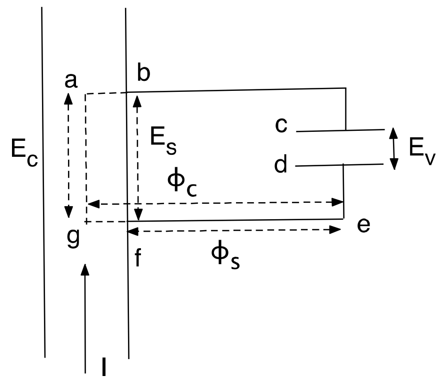

Figure 1. A current carrying wire with voltage leads and an electrostatic voltmeter.

Let us consider first the measurement of the 'voltage' along a wire carrying a current. In figure 1 a thick wire carries a current *I*, which may not be uniform, in a perpendicular changing magnetic field. High resistance wires are connected to an electrostatic voltmeter where the electric field E_v is calibrated into a voltage measurement, i.e. the measured voltage $V=E_v$. We assume the meter is well away from the magnetic field so that only the electrostatic field between the plates is indicated. The measured value is obviously independent of the nature of the voltmeter, but this picture allows us to apply Faraday's law directly. E_c is the electric field along the centre and E_s that along the surface. ϕ_s is the flux in the loop up to the surface and ϕ_c that to the centre.

Now apply Faraday's law $\int E.dl= \dot{\phi}$ to the circuit along the centre, i.e. along *abcdefga*. If the voltage contacts are unit distance apart then E_c-E_v= $\dot{\phi}_c$ so the measured voltage is E_c-$\dot{\phi}_c$. Similarly if we go round *bcdef* the measured voltage is E_s-$\dot{\phi}_s$. These different expressions give the same value since, again from Faraday's law, E_c-E_s= $\dot{\phi}_c$-$\dot{\phi}_s$.

In other words a voltmeter well away from the magnet will give a unique value for the voltage, but its relation to the various electric fields in the material requires a detailed analysis, including the flux changes in the leads. It means that the voltage measured from leads on the side of the tape will be quite different from that measured by leads on the top.

A common method of minimising the effect is to twist the leads near the sample. However, there is always a small gap. Note that it is not sufficient to subtract the voltage from the gap by measuring it with small currents in the Meissner state. Once the field begins to penetrate the superconductor there are higher Fourier components in the gap which are not taken into account by a low current sinusoidal balancing procedure. The way to eliminate this effect is use helical turns round the sample [9].

3. The Dynamic Resistance

3.1. Bean model Calculation

Although the dynamic resistance is normally measured in single conductors, it also appears in wire loops. Figure 2 shows a superconducting loop of inductance *L* and total circumference 2*a*. A bias current 2*I* is applied across it. A current *i* is induced by changing the external field. This can be done by applying a uniform field to the loop, by turning on a magnet with poles within the loop, or by moving a magnet with its poles outside the loop to inside the loop. In all cases the only parameter we need is the change of magnetic flux in the loop. (It was Faraday's genius to realise that this was the way to describe magnetic induction in all these very different procedures).

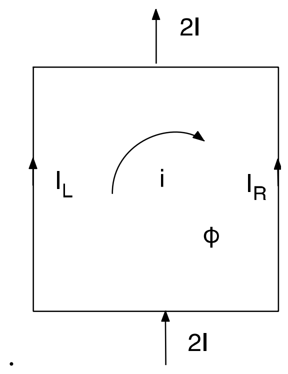

Figure 2. A Superconducting loop.

We can get a good idea of the behaviour using the Bean model. On changing the external field initially, a circulating current i is induced which keeps the total flux inside constant. If the applied flux is ϕ_0 then $Li = \phi_0$. There is no electric field in the wires. The currents in the left and right branches are:

$$I_L = I + i \tag{1}$$

And

$$I_R = I - i \tag{2}$$

respectively.

If either wire reaches the critical value I_c the currents remain constant and flux then enters the ring at the same rate is if the ring were not there. There is then an EMF $\int Edl = \dot{\varphi}$ round the ring provided by the total electric field in the wires. This EMF is determined by $\dot{\varphi}$ and is not the field at I_c as determined from the *V-I* characteristic, it will be significantly larger.

Now consider the effect of a linear cyclic variation of ϕ, figure 3.

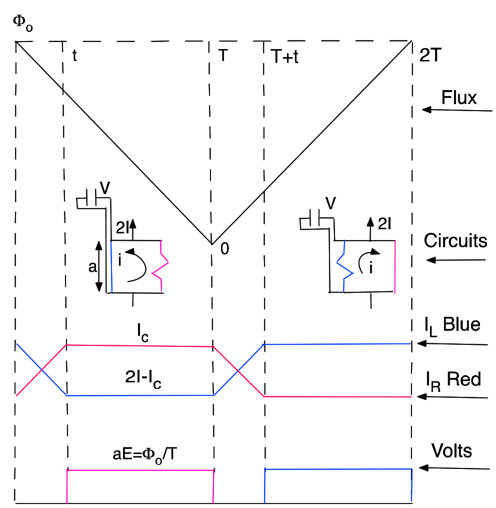

Figure 3. The flux variation, induced currents and voltages in figure 2.

In figure 3 the top lines show the flux starting at ϕ_0 and decreasing linearly to zero at a time T, then increasing back to ϕ_0 at $2T$. The induced circulating current, i, is anticlockwise during the decrease and clockwise during the increase. I_L decreases and I_R increases.

After a time t the current in the right branch reaches I_c and that in the left branch is then $2I$-I_c. These remain constant until a time T. When the flux now increases the left current increases to I_c and the left drops to $2I$-I_c. This is shown by the red (right conductor) and blue (left conductor) lines labelled I_c and $2I$- I_c.

The voltage V developed in a voltmeter connected to the left conductor, as shown in figure 3, is zero for the decreasing section, and on the increasing section it is ϕ_o/T once I_c is reached. This is after a time t from the peak. The average voltage is therefore :

$$V_a = \left(1 - t / T\right)\phi_o / T \tag{3}$$

(We assume the leads are close to the surface or twisted so no flux due to the magnet enters the free space between the leads). This is for a half cycle. The field over the other half cycle is zero since I is less than I_c. Therefore the mean voltage over time is half that of Equation (3). Had the voltmeter been connected to the opposite branch the voltage would have been generated only during the upward part of the cycle, but the average would be the same. If the leads were attached across the centre line and twisted, or brought up out of plane, the voltage measured would have been $\phi_o/2T$ on both parts of the cycle giving the same average.

While currents are below I_c then $L\Delta i=\Delta\phi$. The current in the right branch must increase from $2I$-I_c to I_c so $\Delta i=2(I_c$-$I)$ and:

$$t / T = 2L\left(I_c - I\right) / \phi_o \tag{4}$$

Hence the mean voltage over a cycle is:

$$V_m = \left[\phi_o - 2L\left(I_c - I\right)\right] / 2T \tag{5}$$

The dynamic resistance is:

$$R = \left[\phi_o - 2L\left(I_c - I\right))\right] / 4IT \tag{6}$$

Equation (5) applies provided $I<I_c$. If I is very slightly below I_c then the lower current in figure 3 is just below I_c and t is very small. This gives the maximum mean voltage $\phi_o/2T$. If I is greater than I_c there will be considerable dissipation which would be unwelcome.

There is a threshold amplitude of field below which there is no mean voltage. This is if $t>=T$ so that the current never reaches I_c. This is when:

$$\phi_o <= 2L\left(I_c - I\right) \tag{7}$$

To make the meaning clearer we define $V_o=\phi_o/T$. This is the EMF induced in the ring by the external field if the ring is resistive. Since a voltage only appears for half the cycle the maximum mean voltage is $V_o/2$. We can also define a dimensionless parameter K where:

$$K = LI_c / \phi_o \tag{8}$$

Then Equation (5) becomes:

$$V_m = 0.5V_o\left[1 - 2K\left(1 - I / I_c\right)\right] \tag{9}$$

K is the ratio of the flux in the ring at the critical current to that due to the applied flux. For large ϕ_o the mean voltage is $V_o/2$. If K=1 the applied flux can just induce I_c in the ring if the bias current is zero.

As the bias current increases from zero the threshold decreases with (I_c-I) until at $I=I_c$ it is zero. The mean voltage increases from zero to $V_o/2$.

We see this has all the correct characteristics of a dynamic resistance. Beyond the threshold the voltage rises linearly with V_o, the amplitude of the AC field and its frequency. It also rises linearly with the bias current until this reaches $2I_c$ (I_c in each branch). Beyond this the Bean model fails and the V-I characteristic is needed.

Equation (9) is essentially the same as that derived for a strip by Jiang et. al. [1] except that in the strip geometry, and also slab geometry, the voltage is proportional to the bias current while in the loop it is independent of the bias. This is because the area of flux movement per unit length in strips and slabs increases from zero to the width as the bias increases to I_c, while in the loop it is the width of the loop.

If the bias current is zero there is no dynamic resistance, but this does not follow from the equations above. The reason is that as *I* tends to zero the lower current in figure 3 tends to $-I_c$, but only when this is actually reached do we get oscillating plus and minus fields in each branch, and zero net voltage. Up to this point the equations apply. Again a detailed analysis requires the *V-I* characteristic.

A point worth making again is that in this calculation it makes no difference whether we apply the field by moving a magnet into the loop, taking it right across the loop, or turning a stationary magnet on and off. Another point is that it does not take a large field to induce I_c in a loop of the order of centimetres in size. Thus in practice most practical systems will be in the large ϕ_0 limit, i.e. small *K*, and the mean voltage is $V_o/2$ with a dynamic resistance of $V_o/4I$.

3.2. V-I calculation.

A more accurate result can be obtained using the *V-I* characteristic. I have used an exponential relation Equation (10) rather than a power law, but the results are very similar.

$$V(I) = \frac{aE_o sign(I)\left[\exp(n|I|/I_c)-1\right]}{\exp(n)-1} \tag{10}$$

The exponential has the advantage of going smoothly through zero to negative values and also gives an analytic solution, (although at the expense of some heavy algebra). The *n* is essentially the *n* value of the power law.

Applying Faraday's law to the loop gives:

$$\frac{LdI}{dt} = \dot{\phi}_o - V(I+i) + V(I-i) \tag{11}$$

From this we can immediately get the saturated current when dI/dt=0. this is:

$$I_s = (I_c/n)\operatorname{asinh}\left[\frac{\dot{\phi}_o(e^n-1)\exp(-nI/I_c)}{2aE_o}\right] \tag{12}$$

Surprisingly equation (11) can be integrated with Maple to give the time $t(I)$ to reach a current *I*:

$$t(I)/T = \frac{K}{n\sqrt{1+s^2}}\ln\left(\frac{s\exp(-nI/I_c)-1+\sqrt{1+s^2}}{s\exp(-nI/I_c)-1-\sqrt{1+s^2}}\right) \tag{13}$$

where:

$$s = \frac{2aE_o\exp(nI/I_c)}{\dot{\phi}_o(e^n-1)} \tag{14}$$

This equation is awkward to use directly since the time tends to infinity as the current approaches I_s. However we can extract the current as a function of time which is:

$$I(t) = \left(I_c / n\right) \ln\left(\frac{\sqrt{1+s^2} - 1 + f_o \exp\left(\alpha t\right)\left(\sqrt{1+s^2} + 1\right)}{s\left(f_o \exp\left(\alpha t\right) - 1\right)} \right) \tag{15}$$

Where

$$\alpha = n\sqrt{1+s^2} / \left(Kt\right) \tag{16}$$

and

$$f_o = \frac{s \exp\left(nI_m / Ic\right) - 1 + \sqrt{1+s^2}}{s \exp\left(nI_m / Ic\right) - 1 - \sqrt{1+s^2}} \tag{17}$$

3.3. Results

We choose the following values which are practically plausible. E_o=1μV/cm, B=5mT, a=10cm, I_c=100A, bias I=0.9I_c, T=0.5s, n value n=20.

The flux ϕ_o=Ba^2=5x10^{-5} Wb and the rate of change $\dot{\phi}$=Ba^2/T=10^{-4}V. The inductance L=3.7x10^{-7} H so the threshold flux =7.5x10^{-6} Wb which is less than ϕ_o and we are well over the threshold. In this limit the dynamic resistance/length/Hz is $V_o/8aTI$=2.8x10^{-6} ohm/m/Hz.

We can compare this with the value in [2] with an AC field of 100mT. This is 1x10^{-5} ohm/m/Hz, and if we divide by 20 to give the value at 5mT the result is 5x10^{-7} ohm/m/Hz. This is a factor five less than the theory for a loop which is plausible given the difference in width between the loop and the strip.

Figure 4 shows the induced circulating current on the first half cycle. It can be seen that as in many hysteretic systems there is an initial linear slope which changes quite sharply to a flat saturation. This behaviour lends itself to a good empirical approximation as two straight lines if required.

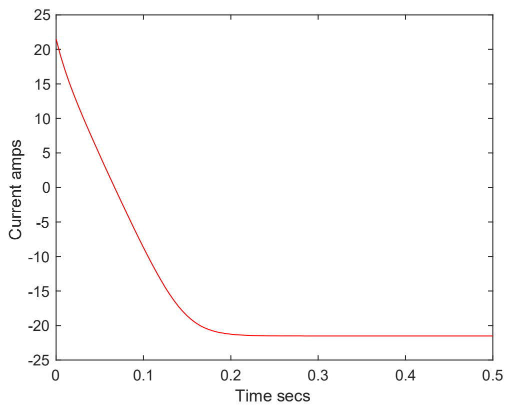

Figure 4. The induced circulating current.

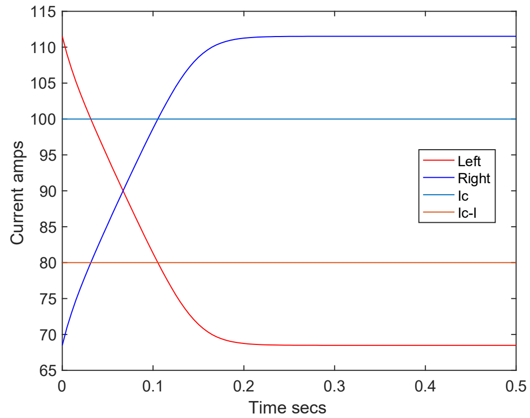

Figure 5. The currents in the two branches.

Figure 5 shows the currents in the left and right conductors. The currents are considerably larger, by about 10%, than those predicted by the Bean model. Ten percent above I_c implies quite large dissipation and much more would lead to thermal instability.

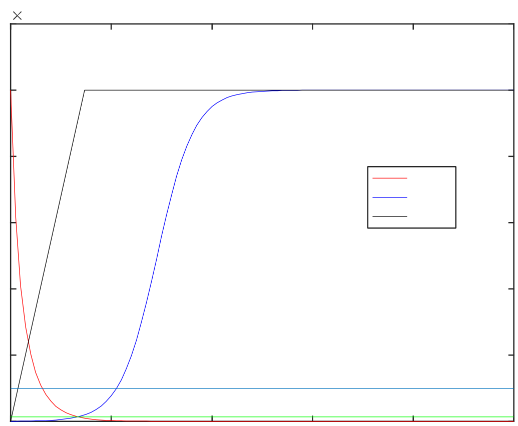

Figure 6. The voltages across the conductors.

Figure 6 shows the voltages across the conductors. These are much larger than both the voltage with no AC field and the 1μV/cm voltage at I_c, which is aE_o. However, the voltage in the conductor admitting flux corresponds closely to that expected from a normal wire, ϕ_o/T, which is what the Bean model predicts, because the current in the loop remains constant.

The inclined line labelled 'Bean' is found by considering the change in the current when the material is superconducting, which depends only on the loop inductance and the rate of change of applied flux. $I=t\phi_o/TL$ until I_c is reached, when the voltage stabilises at ϕ_o/T.

We see that there are two easy regimes. One is below the threshold when t>T, the loop is fully superconducting, and there is no average electric field. The other is well above the threshold when the mean voltage is just that generated by the change of flux in the loop. $V=\phi_o/T$, divided by two since it only appears in half of the cycle. The intermediate region is relatively small and requires a careful selection of geometric and material parameters to illustrate it. This became apparent in [10] where only a small load inductance could be used and it proved difficult to find the regime between which there was no flux pumping and the regime where it saturated in the first cycle.

Figure 7 shows the currents and voltages on the V-I curve for these values of the parameters. The critical current is 100A and the bias current, $2I$ =180A, (90A for each branch). If the induced voltage $\varphi^{\cdot}$ is 100μV then the high current branch, which provides this voltage, carries 112A so the circulating current is 22A and the low current branch, with negligible electric field, carries 68A.

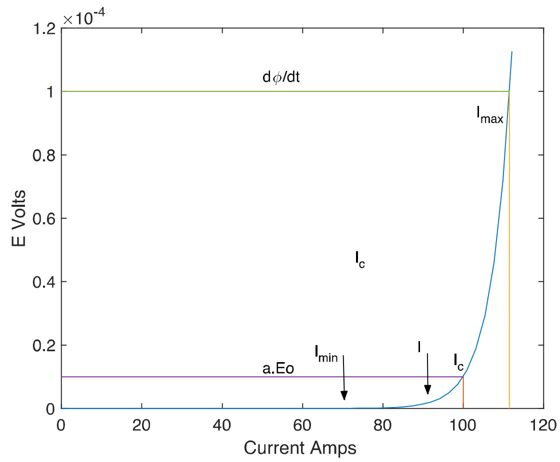

Figure 7. The voltages and currents on the V-I diagram.

4. Applications to flux pumping.

4.1. Bean model calculation.

In most flux pumps a magnet is moved across a thin conductor large compared with the size of the magnet into a space connected to the load, and then turned off. This induces currents in alternating directions which do not quite cancel. It is the imbalance due to the nonlinearity of the system which puts a small proportion of the induced currents into the magnetic load. The dynamic resistance must play a part in this process but the connection is not obvious. However, a similar circuit to the circuit in this paper has been used by Geng and Coombs [4] to build a successful flux pump in which the induced voltage is transferred directly to the load. It only requires the voltmeter in figure 2 to be replaced by a magnet with inductance L_2, as in figure 8. Here the electric field in the left conductor drives the current into the magnet. It behaves as a flux pump, although no flux from the driving coil goes directly into the magnet circuit.

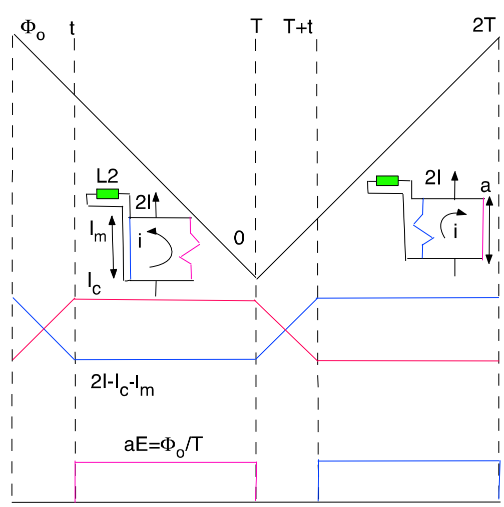

Figure 8. The voltage pumps into a magnet of inductance L_2.

At first sight this may not appear to be very useful since in figure 8 a bias current comparable to the critical current of the magnet must be supplied to the flux pump. However, this problem was solved by supplying an AC bias current inductively with an AC field at a lower frequency than the field applied to the pumping circuit. The high frequency magnetic field was turned on when the bias current was in the right direction and turned off when it was not, so that the pump could still be supplied inductively from outside the cryostat.

If in figure 8 the loop inductance is now L_1 and a current I_m goes into a load magnet with inductance L_2. If $L_2 \gg L_1$, as will be the case by a large factor, we can assume that this current does not change significantly within a cycle so that we can use the average voltage from the dynamic resistance to find the rate at which current is pumped into the magnet.

When the left current reaches I_c the current in the right branch is now $2I-I_c-I_m$. The change in the circuit current to change from I_c in one to I_c in the other is:

$$\Delta I = 2\left(I_c - I\right) + I_m \tag{18}$$

and the mean voltage is:

$$V_m = \left[\phi_o - 2L_1\left(I_c - I + I_m / 2\right)\right] / 2T \tag{19}$$

We see that the initial voltage is the same as before, but as the magnet current increases, the lower current in figure 5 gets lower so that it takes longer to reach I_c and the average voltage decreases. If there is a DC resistance in the magnet then $IR + L_2\, dI_m/dt = V_m$, so:

$$L_2 dI_m / dt = -I_m R + \left[\phi_o - 2L_1\left(I_c - I + I_m / 2\right)\right] / 2T \tag{20}$$

The solution for the magnet current is:

$$I_m = \frac{\phi_o / L_1 - 2\left(I_c - I\right)}{1 + 2RT / L_1} \exp\left[\left(\frac{tL_1}{2L_2 T}\right) - \left(\frac{tR}{L_2}\right)\right] \tag{21}$$

In what follows the resistance R will be assumed zero for simplicity, although for topping up a magnet with a measurable DC resistance it will be an important parameter.

If we neglect R, the number of complete cycles is $t/2T$ so we can write the exponential as $\exp(-n/n_o)$ where $n_o = L_2/L_1$ and n is the number of cycles. Now the inductance of a 10cm. square loop is about 10^{-8}H and that of a big magnet could be 0.1H so the number of cycles to charge it will be 10^7. Topping up requires fewer cycles, but clearly some scaling up of the size of the flux pump will be needed.

The other result is the saturation magnet current. This is when $dI_m/dt=0$. Again assuming R is small,

$$I_{sat} = \phi_o / L_1 - 2\left(I_c - I\right) \tag{22}$$

For rapid charging we make I close to I_c which also gives the maximum saturation current which is $I_{sat}/I_c = \phi_o / I_c L_1 = 1/K$. (Going above I_c will lead to a large dissipation). Now to get significant charging we need $\phi_o > I_c L_1$ so the saturation current will be close to I_c, assuming all conductors have the same I_c.

From these results it appears that although a large number of cycles is needed even to top up a magnet, the system can eventually supply a current comparable to the critical value.

In practice this may not be the criterion needed. If the field is decaying due to resistances in the circuit then what is needed is that the mean voltage should exceed the voltage generated by the magnet critical current in this resistance.

4.1.A more exact treatment.

In this section a nearly exact general solution is discussed. We apply Faraday's law to the loop including the magnet current and the V-I characteristic. From figure 8 the current in the left branch, which supplies the magnet, is I_L and in the right it is $2I-I_L-I_m$ where $2I$ is the bias current and I_m the magnet current.

Applying Faraday's law to the loop of wire:

$$V\left(I_L\right)-V\left(2I-I_L-I_m\right)=\left(\phi_o/T\right)-L_1dI_L/dt \tag{23}$$

and applying it to the magnet circuit:

$$V\left(I_L\right)=L_2dI_m/dt \tag{24}$$

The flux in the loop due to the rate of change of I_m has been neglected but could be included. These equations were solved numerically in Matlab with the starting condition $I_L=I$ and $I_m=0$.

4.2. Results

The first results are with $L_2=200L_1$ and $n=200$. Here we expect close agreement with the Bean model. Figure 9 shows the left and right currents for the first half cycle. Even with these values the left current is about 1% above I_c and the reduction in the left conductor due to the current going into the magnet is visible, but the agreement is pretty good.

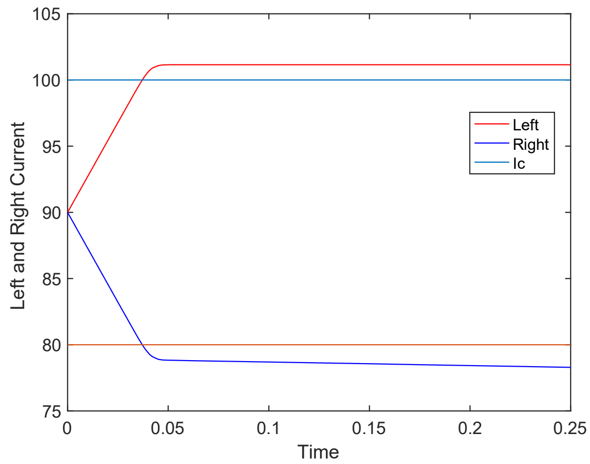

Figure 9. Left and right currents. Lines are at I_c and $2I-I_c$.

Figure 10 shows the voltages. The left voltage is ϕ_o/T in agreement with the theory and the right voltage virtually zero. The first is much higher than the *V-I* voltage at I_c and may cause significant loss. For these figures it is 0.1mV at 100A, i.e. 10 mW.

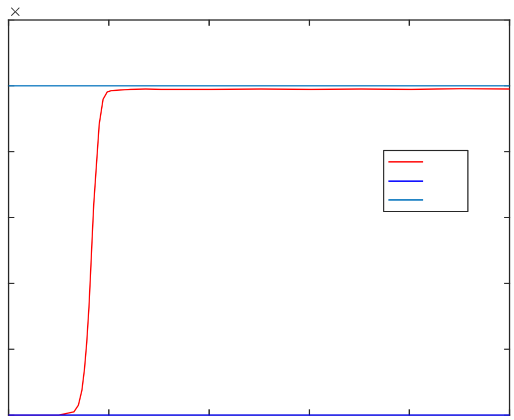

Figure 10. Left and right voltages and the Bean voltage ϕ_o/T.

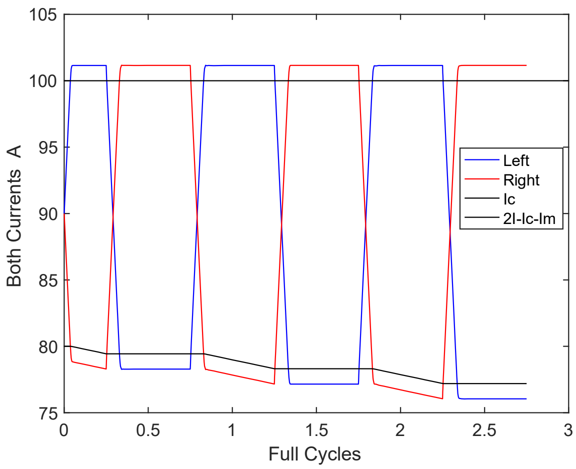

Figure 11. The currents after three cycles. The top line is I_c, the bottom $2I$-I_c-I_m.

Figure 11 shows the loop currents after three cycles. It shows the expected oscillations between I_c and $2I$-I_c-I_m as in figure 3. Figure 12 shows the magnet current after three cycles along with the start of the exponential solution, equation (21).

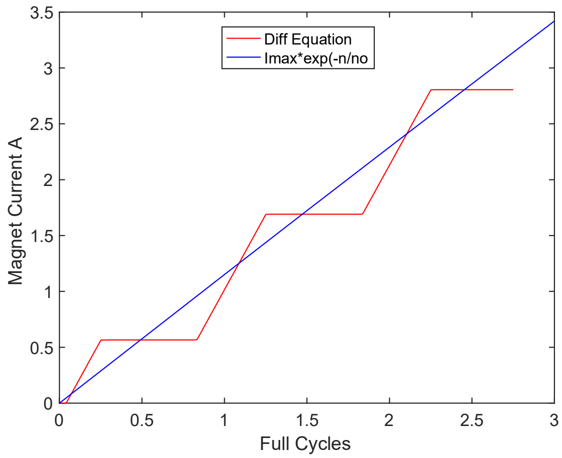

Figure 12. The magnet current for the first three cycles and the theoretical exponential.

Figure 13 shows the magnet current after 350 cycles with the DC resistance *R*=0. The agreement with equation (21) is excellent.

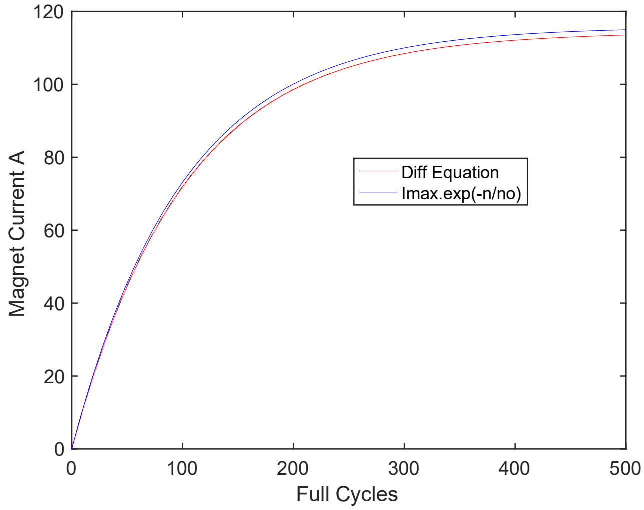

Figure 13. The magnet current and theory for 500 cycles. L_2/L_1 is 200.

4.2. High Tc and low inductance Results

High T_c materials have much lower n values. The following graphs show the results with n=20 and L_2=20 L_1. Figure 14 shows the loop currents for the first three cycles. The currents differ by 10% from the Bean value and the steady reduction in the lower current as current is transferred to the load is now clear.

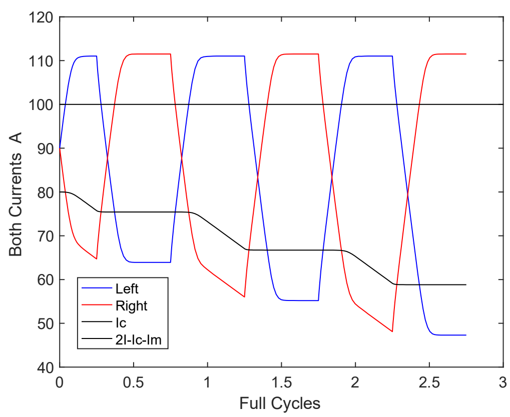

Figure 14. The first three cycles.

Figure 15 shows thirty-five cycles. It makes clear the mechanism of saturation. As the lower current decreases it requires a larger part of the cycle to get back to I_c until eventually it never reaches I_c and the dynamic voltage disappears.

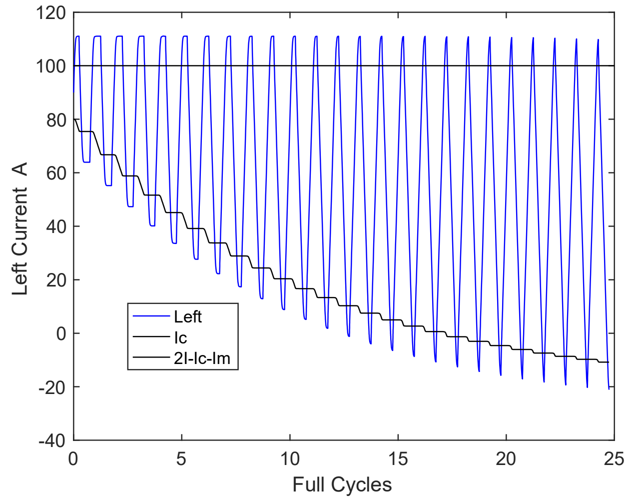

Figure 15. The left current after 35 cycles. Here L_2=20L_1.

This shows why even without a DC resistance in the magnet the pumping saturates. As the magnet current increases it takes current from the lower current in the left conductor in figures. 9 and 11 when the right conductor reaches I_c. This means it takes longer for the current in the left conductor to reach I_c on the increasing half cycle until eventually it never reaches it, we are below the threshold, and there is no further pumping.

Figure 16 shows the magnet current. It is still not a bad fit to the theoretical exponential.

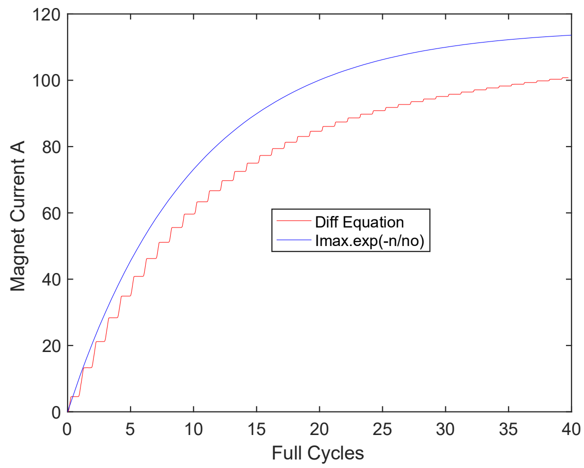

Figure 16. The magnet current.

5. Conclusions

A loop of superconductor in an AC magnetic field can be used to provide a dynamic resistance and flux pump. The characteristics can be derived simply from the Bean model, extended to give the voltage in the critical state, and are similar to those of the dynamic resistance in other geometries. The dynamic resistance is $R=(\phi_0-2L(Ic-I))/4IT$ where ϕ_0 is the change in flux, L the loop inductance, I_c the critical current, $2I$ the bias current and $2T$ the period. The results were confirmed by solving the differential equation of Faraday's law, which allows the effects of a low n value and low load inductance to be assessed.

The loop can be used to drive a flux pump and again simple formulae can be derived from the Bean model. The magnet current rises exponentially with a cycle constant equal to the ratio of the load magnet inductance to the loop inductance. The pumped current saturates because the current in the branch supplying the voltage is diverted to the magnet so a higher proportion of the cycle is needed before the current becomes critical, but nevertheless it can be close to the critical current if all wires have the same value. The theory can be easily extendable to include the DC resistance of the magnet and the effect of the variation of J_c with B. The qualitative similarities to flux pumps using tapes or sheets help in the understanding of these more complex systems.

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